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Engineering optical forces in waveguides and cavities based on optical response

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ABSTRACT

We present a new treatment of optical forces, revealing that the forces in virtually all optomechanically variable systems can be computed exactly and simply from only the optical phase and amplitude response of the system. This treatment, termed the response theory of optical forces (or RTOF), provides conceptual clarity to the essential physics of optomechanical systems, which computationally intensive Maxwell stress-tensor analyses leave obscured, enabling the construction simple models with which optical forces and trapping potentials can be synthesized based on the optical response of optomechanical systems. A theory of optical forces, based on the optical response of systems, is advantageous since the phase and amplitude response of virtually any optomechanical system (involving waveguides, ring resonators or photonic crystals) can be derived, with relative ease, through well-established analytical theories. In contrast, conventional Maxwell stress tensor methods require the computation of complex 3-dimensional electromagnetic field distributions; making a theory for the synthesis of optical forces exceedingly difficult. Through numerous examples, we illustrate that the optical forces generated in complex waveguide and microcavity systems can be computed exactly through use of analytical scattering-matrix methods and compared to Maxwell stress-tensor methods.

Keywords: Optomechanics, optical forces, gradient forces, microcavity, waveguide

1. INTRODUCTION

Radiation pressure has been thoroughly studied for optical trapping and manipulation of microscopic objects[1–3] and parametric processes in interferometers.[4–7] With recent advances in nanophotonics, the mass and the dimensions of optical devices have been miniaturized to the degree that device tuning through optical actuation is possible at micro- to milli-watt power levels.[8–12] In many of these cases, optically induced forces can scale to large values when optical fields are enhanced through high-Q resonances.[13–15] Such observations have sparked significant scientific interest in light-driven mechanically variable systems that can perform trapping,[13,16] actuation,[10,12,14,15,17] transduction[8,9,11,17,18] and manipulation[13,19] of nanoscale objects. Since the mechanical state of such systems is intimately linked to its optical state, these mechanical functions can lead to variable directional couplers,[10,16] parametric optical processes[8,9,12,14,15] in cavities, ultra-widely tunable microcavities,[13] and microcavity athermalization through self-adaptive optomechanical behaviors.[13,20] A general analytical formalism capable of handling such complex optical systems is therefore essential in tailoring optical forces at nanoscales.

It is typically believed that the knowledge of the full electromagnetic field distribution in such mechanically variable optical systems is a prerequisite for the computation of optical forces. In these electromagnetic field-based calculations, the Maxwell stress-tensor (MST) is numerically integrated over a closed surface surrounding the movable components in the system to compute the optical forces acting on them.[21] While the MST method is reliable, it offers little intuition for the design of a system with a desired optical force profile as system is tuned through different optomechanical states. In addition, no simplification can be made through MST methods to unify systems with similar optical response but different field distributions. With the trend of increasing complexity in nanooptical systems, a full numerical approach is computationally intensive, and therefore limits the scale of the system that can be studied (since the electromagnetic field distributions at every mechanical configuration must be solved to evaluate the force).

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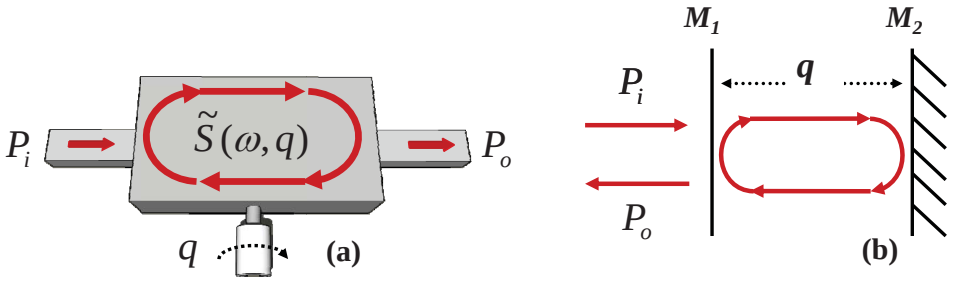

Figure 1. (a) Schematic showing a generic open photonic system that can be mechanically varied through displacement of q. Optical power flows into and out of system. The response of the device, $\tilde{S}(\omega, q)$, is also a function of frequency ω. (b) An example of a mechanically-variable open system in the form of an ideal lossless Gires-Tournois interferometer with a fixed partial mirror *M1* and a movable mirror *M2*. Here, q is taken to be the separation of mirror *M2* from *M1*.

In this paper, we develop a simple analytical approach to calculating optically induced forces and potentials in open, lossless, mechanically variable optical systems (consisting of only linear media) which possess a single mechanical degree of freedom.[22] By "open" systems, we refer to systems with optical input and output ports, which allow electromagnetic energy to enter and exit the system. In such systems, we show that the optical force acting on the mechanical degree of freedom is determined by the optical response of the system, and is completely independent of structural implementation. In contrast to the computation of complex electromagnetic field distributions, it is generally straightforward to derive an analytical expression for the optical response of nontrivial optomechanically variable systems (involving ring resonators, photonic crystals, waveguides, etc.) using temporal coupled mode theory (CMT) and/or scattering matrix (S-matrix) methods.[23–27] Given the relative ease with which the response of optical systems can be treated by these and other methods, our formalism, which we term the Response Theory of Optical Forces (or RTOF), provides a very unique and powerful analytical alternative to MST methods. Furthermore, the RTOF method can leverage analytical theories through which optical system response can be *synthesized* (independent of the underlying electromagnetic complexity),[28] yielding a means of engineering optical forces profiles as well. Moreover, since the optical response captures the salient physics of optomechanical systems, universal designs which are independent of structural implementation are possible using the RTOF method.

In what follows, we first discuss the definition of "open" mechanically variable optical systems and describe a formalism for energy and photon-number conservation in the context of such systems. The optical forces are first derived for single-input single-output systems under monochromatic excitation and are reduced to a simple analytical expression of the optical phase and amplitude response of the system. As an extension of this result, a scalar optical potential can be defined from the force profile, allowing formulation of a generalized optical trapping potential in these systems. We then expand the theory to include open systems under polychromatic excitation and open systems with multiple inputs and multiple outputs. Examples involving waveguides (possessing a continuum of modes) and resonant systems (supporting a discrete set of modes) demonstrate a perfect agreement between our analytical theory and MST methods.

2. DEFINITION OF AN *OPEN* MECHANICALLY VARIABLE OPTICAL SYSTEM

We define an open photonic (or electromagnetic) system as one which exchanges electromagnetic energy with the environment through input and output ports. The power in-flux is determined by optical sources, typically lasers, at a fixed frequency and power level. At steady state, the output power can be related to the input power by the optical response of the system, for example, as expressed by a scattering matrix.[23] We focus on systems with few input and output ports in the forms of single-mode waveguides or collimated Gaussian beams in free space, as in typical experimental settings.

The optical response of these systems can be varied by mechanical movement of a sub-component. Here we consider simple cases where the movement can be characterized by a change in the scalar coordinate, q. A one-port example is illustrated in Fig. 1(a), in which the mechanical degree of freedom q, represented iconically as a "knob", affects the optical response $\tilde{S}(\omega, q)$. Here, the optical response, $\tilde{S}(\omega, q)$, is a 1×1 scattering-matrix,[23]

which relates the complex-valued amplitudes of the transmitted wave (b) to that of the incident waves (a) at a steady state: $\tilde{b} = \tilde{S}(\omega, q)\tilde{a}$. (Note, while the specific examples used in this paper exhibit linear displacements through motion of q, we may consider q to be a generalized coordinate.[29] Within a Lagrangian dynamics framework, the generalized coordinate might represent a rotation, translation, or motion along an arbitrary path along which the degree of freedom is constrained to move.[29,30]) In a dynamic regime (i.e., where q is time-varying) the electromagnetic energy in the system can be converted into/from mechanical energy within the system. In general, such a conversion is accompanied by a change in the frequency of transiently stored photons, a change in the frequency of the photons exiting the system through its output ports, and a change in the output power level. We show that the energy conversion, and the related optical forces, can be analytically calculated from the changes in the optical response of the system alone, provided that the changes occur adiabatically. (For definition of an adiabatic process, see.[31])

It is worth emphasizing that the analytical treatment that we will develop here (or RTOF method) extends the existing analytical models, such as scattering matrix and coupled mode theory to yield analytical solutions of optical forces. The optical response, $\tilde{S}(\omega, q)$, of open systems can be treated very precisely and completely through use of S-matrix and/or temporal coupled mode theory (CMT) methods in common nano-photonic systems, such as ring resonators, photonic crystals and coupled waveguides.[23–27] Simple analytical models of these nontrivial open systems can be constructed in terms of these formalisms, capturing all aspects of energy storage in that system (which can be related to the group delay of an optical system – a quantity which is naturally computed through these methods). Furthermore, both S-matrix and CMT methods are capable of treating the external degrees of freedom associated with coupling to the outside world.

As a concrete example of an open mechanically variable optical system, we consider a lossless Gires-Tournois interferometer (seen in Fig. 1(b)) excited by a monochromatic source. When the movable mirror (*M2*) of the Gires-Tournois interferometer is displaced, a cavity resonance, of frequency $\omega_o(q)$, can be tuned through the source frequency, allowing the cavity to store a large number of photons at a range of positions coinciding with the resonant excitation of the cavity. Through this process, photons and energy are exchanged with the outside world. Resonant systems of this type, possessing high Q (quality factor), are also interesting for their ability to generate large optical force. In this system, the optical forces could perform mechanical work in this system (i.e., convert energy for electromagnetic to mechanical domains), by displacing the movable mirror, *M2*.

Through coupling to the outside world, the optical nature of the forces allows one to drive the system in a number of unique ways. For instance one could excite the system with a laser that is off resonance with the cavity, with several lasers at various frequencies and various power levels, or with a source consisting of a continuum of frequencies (such as a white light source). These inherent flexibilities, as we will see later, are compatible with the assumptions in our open-system analysis, and do not require any approximation or equivalent systems necessary in a closed system analysis.

3. ENERGY CONSERVATION AND PHOTON-NUMBER CONSERVATION

Conservation of energy and photon number form the basis for the response theory of optical forces (RTOF). The simplest statement which captures the essential physics of the response theory of optical forces is: "If work performed against an optically induced force, the energy of the electromagnetic wave (responsible for the optical force) must change by an amount equivalent to the work done". Therefore, from knowledge of the change in electromagnetic energy of the system (which we will show, can be related to the mechanically variable optical response of the system), and the change in a mechanical degree of freedom through which the work is performed, one can compute the optical forces generated in the system. In addition, through development of RTOF, we assume that the system evolves slowly enough that photon conservation applies. Through conservation of energy and conservation of photon number, we will show that the energy and force in optomechanically variable systems can be greatly simplified.

Through development of RTOF, we will begin with an analysis of power conservation in slowly time-varying mechanically variable optical systems. Development of conservation of energy and photon number will be critical in establishing the time-dependent form of the energy conversion (between electromagnetic and mechanical domains) and energy exchange (i.e., electromagnetic energy exchange with the outside world) derived in this

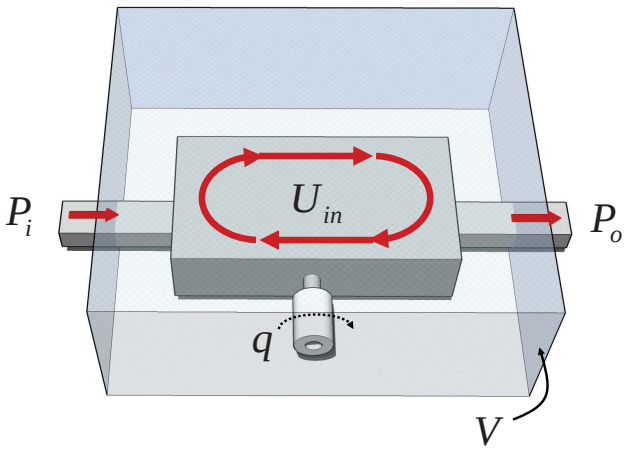

Figure 2. Schematic showing generic optomechanically-variable open-system within a closed surface forming the boundary to the volume, V. This system can be seen as a reflectionless one-port system. Optical power flows into and out of system (no power is reflected). Here q represents the mechanical degree of freedom of the system, which impacts the optical response of the system in some manner. Through the motion of q against internally generated optically-induced forces, work can be done on the electromagnetic field.

paper. We will show that, photon and energy conservation dictate that the optical forces in a system are uniquely determined by the optical response of the system. We first consider the conservation of energy, known as Poynting's theorem in classical electromagnetics.[21] For *linear* systems containing movable components,[32] Poynting's theorem can be written as

$$\oint_{\partial V} \vec{S}\cdot\hat{n}da + \frac{\partial U_{in}}{\partial t} = -\int_V \vec{J}\cdot\vec{E}dv. \tag{1}$$

Here $\vec{S}$ is the Poynting vector, ∂V is the bounding surface to the volume (V) under consideration, U_{in} is defined as the total electromagnetic energy contained within the volume, V, and $\vec{J}\cdot\vec{E}$ represents the mechanical work done per unit time per unit volume by the field. Here $\vec{J}$ represents the polarization currents generated by dielectric moving parts, and the surface charge currents generated by moving parts consisting of perfect electric conductors.[32] We note that, although there are several different formulations of the Maxwell's equations and constitutive relations for moving media,[32] one can show that Eq. 1 is applicable to any *lossless* system absent of magnetic materials.

First, we consider a reflectionless one-input, one-output system (Fig. 2) enclosed within a volume V, outside of which the electromagnetic fields associated with the system are negligible. A net power of P_i (P_o) enters (exits) the system through its input (output) port. The energy converted from the electromagnetic to mechanical domains (corresponding to $\vec{J}\cdot\vec{E}$) must be equivalent to the work done through displacement of, q, by the optically induced force (F_{opt}). Thus, the volume integral over $\vec{J}\cdot\vec{E}$ can be equated to $F_{opt}\cdot\dot{q}$, allowing us to express Poynting's theorem as

$$\oint_{\partial V} \vec{S}\cdot\hat{n}da + \frac{\partial U_{in}}{\partial t} = -F_{opt}\cdot\dot{q}. \tag{2}$$

Defining W as the work done, through displacement of q, on the electromagnetic field, we have $\partial W/\partial t = -F_{opt}\cdot\dot{q}$. Here, F_{opt} is taken to be the instantaneous optical-force component in the direction of displacement of the generalized coordinate (subject to the constraints on the generalized coordinate). Thus, Eq. 2 can be expressed as

$$(P_o(t) - P_i) + \frac{\partial U_{in}}{\partial t} = \frac{\partial W}{\partial t}. \tag{3}$$

Above, we have replaced the surface integral over $\vec{S}$ with $(P_o(t) - P_i)$, where P_i and $P_o(t)$ are defined as the electromagnetic powers entering and exiting the system respectively. Note, P_i is assumed to be a constant (even

as the mechanical state of the system changes). In contrast, P_o can be time dependent, because the motion of q changes the frequency of the photons transiently stored in the system and the energy storage capacity (or group delay) of the system.

It is important to note that, in general, the instantaneous optical force, F_{opt}, expressed on the coordinate, q, will be velocity dependent (i.e., $F_{opt} = F_{opt}(q, \dot{q})$) when q evolves rapidly. This can be seen from the fact that the optically induced force generated by an optomechanically-variable system depends on the instantaneous electromagnetic-field distribution produced within the system (e.g., F_{opt} acting on the movable component can be related to the fields through MST methods). Thus, if q evolves on a time-scale that is rapid with respect to the photon lifetime of the system, the internal field distribution may significantly differ from the steady-state field distribution, implying that F_{opt} is velocity dependent. However, in the gradual (or adiabatic) limit of motion, the instantaneous field distribution approaches the steady-state field distribution, meaning that the optically induced forces become state-dependent (or strictly a function of q), approaching those produced within the static system. In this paper, we seek only to develop an analytical method of computing the forces in the *static-limit*. Thus, while the velocity dependence of the forces should be noted, treatment of them beyond the scope of this paper. As we will show later, in the adiabatic limit of motion, the optical forces (F_{opt}) are conservative, and can be expressed in terms of an effective (opto-)mechanical potential ($U_{opt}(q)$) as $F_{opt} = -\partial U_{opt}/\partial q$.

In section 4 we will derive an explicit relationship for the optical forces (F_{opt}) in the steady state using Eq. 3. Key to these derivations are the further simplifications that can be made to Eq. 3 by expressing energy and power in terms of the photon construct. In the classical limit (i.e., in the limit of large photon flux), one can generally express the powers entering and exiting the device as $P_i = \Phi_i \cdot \hbar\omega$ and $P_o = \Phi_o(t) \cdot \hbar\omega'(t)$ respectively, through the photon-picture. Here $\hbar$ is the Planck constant, while, ω (ω') and Φ_i (Φ_o) are the mean frequency and the flux of incident (transmitted) photons respectively. The electromagnetic energy transiently stored in volume V can be expressed as $U_{in} = N\hbar\omega_{in}$, where N is the number of photons transiently stored at a mean frequency ω_{in}. Note that ω_{in} and ω' must be interpreted as a *mean* photon frequency, since energy conservation is considered for the entire system (which we treat as an ensemble). This subtlety is important because dynamic variation in q can shift the frequencies of the transiently stored and outflowing photons (For further discussion, see Section 4).

Through semiclassical treatment of this time-varying system (i.e., as q evolves), we assume that photons experience no inelastic scattering,[33–35] meaning that photon number is conserved. (Note, photon conservation is known to be a valid assumption in many time-varying optomechanical systems provided that the motion of the mechanical degree of freedom, q, is sufficiently gradual.[33–35]) Thus, photon conservation requires that

$$\Phi_o(t) = \Phi_i - dN/dt. \tag{4}$$

In other words, the difference between the incident and transmitted photon fluxes must be accounted for by the rate of change of transiently stored photons (dN/dt), if photon number is conserved. For simplicity, we focus on the case when the system is driven by a monochromatic source of constant frequency and power. Mathematically, this corresponds to a constant flux of photons (Φ_i) of fixed frequency (ω) entering the system.

4. A LOSSLESS ONE-PORT SYSTEM DRIVEN AT A SINGLE FREQUENCY

Having established the mathematical forms of energy conservation and photon number conservation in a mechanically tunable optical system, we next analyze the conversion of energy (between electromagnetic and mechanical domains) as q varies. By relating the energy and the power flow to the optical response of the system, an analytical form will be derived, allowing us to express optically induced forces and potentials as a function of the optical response and the mechanical coordinate, q.

For the most general form of a lossless, reflectionless system with one input and one output (seen in Fig. 3), at steady-state (i.e., assuming q is constant and U_{in} is constant), the incident wave simply experiences a coordinate-dependent phase-shift, $\phi(q, \omega)$, in traversing the system:

$$exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \phi(q, \omega))]. \tag{5}$$

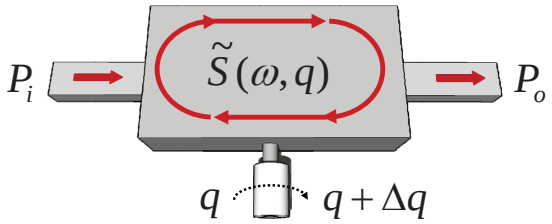

Figure 3. Schematic showing lossless optomechanically variable open system consisting of linear media. Optical power flows into and out of system. Here q represents a generalized coordinate which changes the response of the device, $\tilde{S}(\omega, q)$. Here it is assumed that this is a reflectionless system.

Since power and photon number are conserved at steady state in this lossless system, the amplitude of the transmitted wave and the transmitted photon flux are constant when q is static. We assume that the incident photon flux (Φ_i) is a constant given by Φ. Thus, $\omega = \omega'$ and $\Phi_i = \Phi_o \equiv \Phi$ in the static case. (In other words, the steady-state response can be expressed as $\tilde{S}(\omega, q) = exp[i\phi(\omega, q)]$.)

In the case when q is time varying, however, the behavior of the system is somewhat different. In general, both the amplitude and phase of the wave exiting the system will take on a nontrivial time dependence. We define the phase imparted by the time-varying system as

$$exp[-i(\omega t)] \rightarrow exp[-i(\omega t - \psi(t, \omega))]. \tag{6}$$

Generally speaking, determination of the functional forms of the transmitted wave amplitude and phase ($\psi(t, \omega)$) requires solution of complex nonlinear differential equations, which is beyond the scope of this paper. Nevertheless, in an adiabatic condition of motion, we will show that the time-varying phase ($\psi(t, \omega)$) differs from the time dependent version of the steady-state response, $\phi(q(t), \omega)$, only by a small correction (which will be discussed in more detail). As a general consequence of the nontrivial time-varying phase, $\psi(t, \omega)$, light exiting the system is no longer monochromatic. To take this into account through our photon model, we simply interpret ω' as the *mean* photon frequency exiting the system. While photon number must be conserved in the framework of this semi-classical model (for gradual evolution of q), photon flux is not necessarily conserved. This is because the stored energy (or number of stored photons) may vary with time. As a consequence, the transmitted photon flux ($\Phi_o(t)$), and the number of transiently stored photons (N) are generally time dependent.

To examine the energetics of this system, we assume that q takes on explicit time dependence, and seek a general relation for the work done on the electromagnetic field dW in unit time dt. Energy conservation (Eq. 3) allows us to express the time-varying electromagnetic energy as

$$\frac{dW}{dt} = \hbar\Phi_i[\omega'(t) - \omega] + \frac{d}{dt}\cdot[N(t)\cdot\hbar\omega_{in}(t)] - \frac{dN}{dt}\cdot\hbar\omega'(t). \tag{7}$$

Above, we have recast Eq. 3 in terms of N,Φ_i, Φ_o, ω, ω_{in} and ω' through the semiclassical model developed here. It is critical to note that, to arrive at the above expression, we have used the relation $\Phi_o(t) = \Phi_i - dN/dt$, which results from photon conservation in the time varying-case.

Through integration of Eq. 7, we now seek an expression for the static optomechanical potential as the energy of the system changes through gradual (or adiabatic) motion of the spatial coordinate. In a manner similar to that employed to derive the quantum mechanical adiabaticity theorem,[31] we consider the work done on the electromagnetic field (ΔW) by a small change of our spatial coordinate, Δq, over a time, Δt, where $q(t)$ is defined as $q(t) = q_i + f(t)\cdot\Delta q$. Here, $f(t)$ is continuous function defining the rate of coordinate change along the interval $[0, \Delta t]$, with values $f(t) = 0$ for $t : (-\infty, 0]$, $f(t) = [0, 1]$ for $t : [0, \Delta t]$ and $f(t) = 1$ for $t : [\Delta t, \infty)$. Integrating Eq. 7 over a time interval $t : [0, \Delta t + T]$, one can show that the work done against optical forces through change in coordinate of Δq is[22]

$$\begin{aligned} \Delta W(q) &= \int_{t_i=0}^{t_f=\Delta t+T} \frac{dW}{dt}\cdot dt \\ &= \Phi\cdot\hbar[\phi(q_o) - \phi(q)] \end{aligned} \tag{8}$$

Here, q_o is an arbitrary point of origin. For a detailed derivation of this expression, see Ref. 22. Since the change in energy of the system corresponds to mechanical work performed through motion along the generalized coordinate, q, $\Delta W(q)$ can be interpreted as the (opto-)mechanical potential $U_{opt}(q)$ of the system. Dropping the superfluous constant term, $\Phi \cdot \hbar\phi(q_o)$, the time-averaged static potential is

$$U_{opt}(q,\omega) = -\Phi \cdot \hbar\phi(q,\omega), \tag{9}$$

for any fixed frequency of excitation, ω. Thus, in a lossless system consisting of linear media, we see that the exact optomechanical potential is given by the phase change imparted on the transmitted wave as the generalized coordinate varies. From the effective potential, the time-averaged optically induced force acting on the coordinate q is found to be

$$F_q = -\partial U_{opt}/\partial q = \Phi \cdot \hbar \cdot \frac{d\phi(q)}{dq} \tag{10}$$

Note also, these expressions for optical force and potential (or Eqs. 10 and 9) can be generalized for the treatment of systems with an arbitrary number of mechanical degrees of freedom. In the following sections, we will show that the forces computed through this formalism (which we term the Response Theory of Optical Forces, or the RTOF method) are found to be identical to those computed through exact closed-system and Maxwell stress tensor analyses of the equivalent physical systems, indicating that this formulation is consistent with all of the essential physics necessary to describe static forces and potentials.

Finally, although no assumption is made about the phase response of the lossless linear system examined here, a sharp resonance can greatly enhance the forces (both attractive and repulsive) at various positions in space, enabling the creation of nontrivial, and tailorable potential wells.[13] Such potentials can be adiabatically transformed by varying the conditions of excitation, enabling ultra-precise manipulation of nanomechanical systems.[13] While Maxwell stress tensor methods could be employed to numerically examine nontrivial behaviors of this form, analysis of them becomes exceedingly complicated to examine and highly computationally intensive. In contrast, the formalism derived here provides tremendous simplicity and insight as it offers an analytical means of describing the optically induced force and potential in terms the optical response of the system.

5. TAILORING POTENTIALS VIA POLYCHROMATIC EXCITATION AND USE OF OPTICAL RESONANCES

Through use of the RTOF method (and the relations derived above), it becomes apparent that an open system (of the form we have analyzed here) offers numerous unique degrees of freedom with which one can tailor force and potential profiles. For instance, through degrees of freedom offered by external coupling, one can think of simultaneously exciting the system with a superposition of wavelengths, or varying the wavelength of excitation of a monochromatic source to produce adiabatic transformation of the potential.[13] Since the forces and potentials are additive for distinct wavelengths, multiple excitation frequencies yield a time-averaged (in this case, averaging over the period of ploychromatic oscillation) potential of the form

$$U_{opt}^{tot}(q) = \sum_{l=1}^{N} U_{opt}(q,\omega_l) = -\hbar \sum_{l=1}^{N} \Phi_l \cdot \phi(q,\omega_l). \tag{11}$$

Above, we have assumed that N distinct sources of frequency ω_l, and flux Φ_l simultaneously drive the system. The above sum can be made into a continuous integral, enabling the treatment of broadband sources as well.

6. GENERALIZATION TO LOSSLESS MULTI-PORT SYSTEMS

Next, we generalize the RTOF method from the case of single-input single-output (or reflectionless one-port) systems to the more general case of multi-input- multi-output-port optomechanically variable systems. The multi-port system under consideration is schematically illustrated in Fig. 4(a), showing N-independent input and output ports in the device. (However, the analysis presented here would be identical for N bi-directional ports). Similar to the one-port system examined in Section 5, we assume that the steady-state response of the system is variable through motion of the generalized coordinate, q, shown as a rotational degree of freedom in

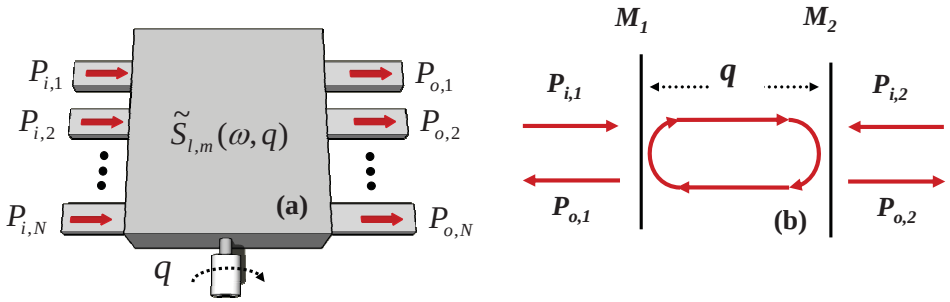

Figure 4. (a) Sketch of a generic lossless and linear optomechanically-variable open-system with N inputs and N outputs. Light of constant intensity, frequency and phase flows into system. Here q represents a generalized coordinate which changes the response of the device, which can be expressed in terms of a scattering matrix of the form $\tilde{S}_{l,m}(\omega, q)$. (b) A lossless Fabry-Perot interferometer, which serves as a specific example of a multi-port system of this form.

Fig. 4(a). Figure 4(b) shows a simple example of an optomechanically variable Fabry-Perot cavity, where the generalized coordinate, q, is taken to be the mirror separation. In contrast to Fig. 4(a) this system has two bi-directional optical ports. However, since the RTOF method is based on power conservation, the orientation of the incoming and outgoing waves is unimportant in the multi-port treatment shown here.

Through this analysis of force and potential, we assume that N input signals, of fixed frequency and amplitude, enter the multi-port system from the left with powers specified by $P_{i,k} = \Phi_{i,k} \cdot \hbar\omega$. This implies that the incident photon fluxes, $\Phi_{i,k}$, must be fixed; however, a change in q will, in general, effect the output photon fluxes, $\Phi_{o,k}(q)$, as power can be redistributed among the output ports at steady-state in a multi port system. Thus, the steady-state power exiting the k^{th} output port can be expressed as $P_{o,k}(q) = \Phi_{o,k}(q) \cdot \hbar\omega$. We assume that the system is lossless; thus, at steady state, photon flux is conserved, requiring that $\sum_k \Phi_{i,k} = \sum_k \Phi_{t,k}(q) = \Phi_{tot}$ is satisfied for all values of q.

To examine $F_q(q)$ and $U_{opt}(q)$, we again assume that q is time dependent, and integrate $\partial W/\partial t = (P_o(t) - P_i) + \partial U_{in}/\partial t$. However, in the multi-port case, the incident power (P_i) is given by $P_i = \hbar \cdot \sum_k \Phi_{i,k} \cdot \omega = \Phi_{tot} \cdot \hbar\omega$ and transmitted (P_o) powers is $P_o(t) = \hbar \cdot \sum_k \Phi_{o,k}(t) \cdot \omega'_k(t)$. Note that the instantaneous frequency ($\omega'_k(t)$) of the photons exiting the k^{th} output port will, in general, be distinct in a multi-port system since the time-dependent phase from each port can be different. Through a similar derivation to that used to analyze the single port system, the optically induced force is seen to be

$$-F_q = \frac{dU_{opt}}{dq} = -\hbar \cdot \sum_k \Phi_{o,k}(q) \cdot \frac{d\phi_{o,k}}{dq}. \tag{12}$$

Integration of Eq. 12, yields the multi-port potential

$$U_{opt}(q) = -\hbar \cdot \int \left[\sum_k \Phi_{o,k}(q) \cdot \frac{d\phi_{o,k}(q)}{dq} \right] \cdot dq. \tag{13}$$

Here, $\phi_{o,k}(q)$ is defined as the phase response of the k^{th} output port. The above is a general form of the optomechanical potential for a lossless optomechanically variable system with N inputs and N outputs, and having an arbitrary optical excitation of the inputs – all at a single fixed frequency, ω. Apparently, no explicit knowledge of the field distribution generated within is necessary to compute the force and potential created by light. For fixed-input conditions, one need only know the steady-state amplitude and phase response of the system as the generalized optomechanical coordinate, q, is varied.

Since the optical response of multi-port systems are most often expressed in terms of scattering matrices,[23,27] we illustrate how the optical forces and potentials (calculated using the RTOF method) can be also expressed in terms of scattering-matrix elements. Through the scattering-matrix formalism, the steady-state response of the system can be expressed as

$$\tilde{b}_l = \sum_m \tilde{S}_{l,m}(\omega, q) \tilde{a}_m, \tag{14}$$

where $\tilde{a}_k$ ($\tilde{b}_k$) is the complex wave amplitude entering (exiting) the k^{th} input port seen on the left (right) of the Fig. 4(a). The scattering amplitudes can be related to the photon fluxes entering the k^{th} input-port as $\Phi_{i,k} = |\tilde{a}_k|^2/\hbar\omega$ and exiting the k^{th} output-port as $\Phi_{o,k}(q) = |\tilde{b}_k|^2/\hbar\omega$. Furthermore, the steady-state phase of the exiting wave is given by

$$\phi_{o,k}(\omega, q) = tan^{-1}(Im(\tilde{b}_k)/Re(\tilde{b}_k)). \tag{15}$$

Through use of these simple relations, we see that, provided the scattering matrix ($\tilde{S}_{l,m}(\omega, q)$) of the lossless system of interest is known, the optically induced force and potential can be computed in a straightforward manner.

7. DEMONSTRATION OF EQUIVALENCE WITH MAXWELL STRESS TENSOR METHODS

Next, we apply the RTOF-method to calculate optical forces in several systems with exact analytical solutions. Through these examples, we are able to demonstrate exact equivalence between the forces computed through Maxwell stress tensor analysis and the RTOF formalism derived here. We begin by examining the optical forces (or gradient forces) produced in mechanically variable waveguide systems whose guided modes are modified by a geometric change. We also explore, resonantly enhanced forces generated within an optical cavity through analysis of a lossless Gires-Tournois interferometer.

7.1 Optical forces produced in optomechanically variable waveguide geometries

An attractive or repulsive optical force can be generated by the light guided in two parallel evanescently coupled waveguides, depending on the symmetry of the compound mode which is excited (see Figs. 5(b)-(d)). The attractive and repulsive forces in such waveguide systems were first studied by Povinelli *et. al* (for more details on such systems, see ref[10]). Examples of two different optomechanically variable waveguide geometries of Fig. 5. Figures 5(b)-(d) show schematics of the waveguide cross-section and mode-profiles for the coupled dual-waveguide system examined here. These waveguide systems can be treated as reflectionless and lossless optomechanically variable one-port devices for treatment with the RTOF method, provided that light is coupled into a single waveguide eigenmode (coupling into more that one mode would require a multi-port analysis). In both cases, the generalized coordinate (q), which effects the response of the system, is taken to be the separation between the two bodies. As a first application of our open-system formulation of force and potential, we show that perfect agreement is found between MST methods and the analytical RTOF method derived here.

Open system treatment of optical forces from guided modes

In applying the RTOF method to optomechanically variable waveguide modes, we consider the forces generated between the waveguide segments (of length L) enclosed in the surface seen in Fig. 5. To evaluate the optically induced force through Eq. 10, we must determine the photon flux, Φ, entering (and exiting) the closed surface through the waveguide mode, and the coordinate-dependent phase-response, $\phi(\omega, q)$, that our optomechanically variable waveguide segment (within the surface) imparts on the transmitted wave. If we define P as the power entering our waveguide, the photon flux is $\Phi = P/(\hbar\omega)$. Thus, the phase response of the system is simply dictated by the optical path length of the system:

$$\phi(\omega, q) = \frac{\omega}{c} \cdot n_{eff}(\omega, q) \cdot L. \tag{16}$$

Here, $n_{eff}(\omega, q)$ is defined as the effective index of the waveguide mode of interest, L is the waveguide length, and c represents the speed of light in vacuum. Thus, applying Eq. 10 yields a relationship for the optically induced force of the form

$$F_q^o(\omega, q) = L \cdot \frac{P}{c} \frac{\partial n_{eff}}{\partial q}. \tag{17}$$

Above, $F_q^o(\omega, q)$ represents the optically induced force generated between the two bodies seen in either Fig. 5(b) or Fig. 5(e). Note that if the symmetric mode of Fig. 5(c) or the perturbed mode in Fig. 5(f) is excited for, $\partial n/\partial q < 0$, then the force tends to be attractive. Interestingly, as illustrated by Povinelli *et. al*, the anti-symmetric mode can yield $\partial n/\partial q > 0$ over some range of motion, generating repulsive forces between the waveguides.

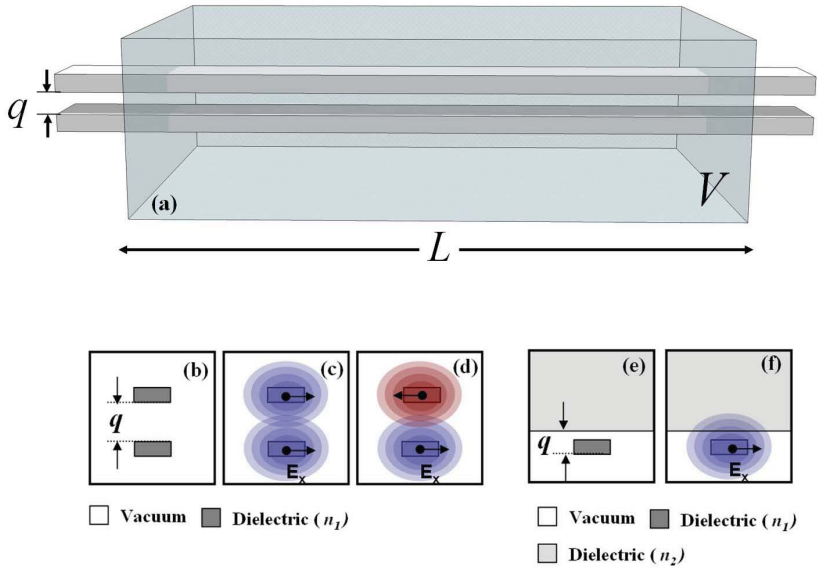

Figure 5. (a) Diagram of an optomechanically variable dual-waveguide system within a closed surface of length L. (b-d) and (e-f) show two different optomechanically variable waveguide geometries which might be examined in a nearly identical manner. (b-d) are schematics of the waveguide cross-section and mode-profiles for the coupled dual-waveguide system treated in ref,[10] and (d-e) is a schematic representation of a waveguide mode whose effective index is modified through evanescent perturbation by a uniform dielectric body (treated in ref[18]). In the latter system it is assumed that $n_1 > n_2$.

Comparison of RTOF and Maxwell stress-tensor methods

In the previous section, we derived a simple and general analytical relation (Eq. 17) for computing the optical forces produced within optomechanically variable waveguide systems using the RTOF method. Next, we compute the optical forces generated in a specific optomechanically variable waveguide system using Maxwell stress tensor method, and compare with those found using the Eq. 17.

Through this example, we examine the forces generated through excitation of the symmetric waveguide mode seen in the dual waveguide system shown in Fig. 5. The waveguide system considered here assumes optically coupled waveguides of width, $w = 450$nm, height $h = 200$nm, core (cladding) refractive index of 3.5 (1.0), and a free-space optical wavelength of $\lambda = 1.55\mu$m. As before, q is defined to be the waveguide separation. A computed mode profile (corresponding to the E_x component of symmetric mode) generated by this structure is shown in the inset of Fig. 6. The optically induced forces between the waveguides (shown as circles in Fig. 6) were found by integrating the Maxwell stress tensor over a closed surface containing one of the waveguides at a number of waveguide separations. For comparison, the forces were also computed using Eq. 17 (corresponding to the dashed line). Upon examination of Fig. 6, perfect agreement is seen between the two methods. Thus we see that the RTOF method provides a valid and greatly simplified means of computing optically induced forces in this dual-waveguide system, and numerous other complex waveguide topologies. In addition, we note that an analytical expression which is exactly equivalent to Eq. 17 can be derived through closed-system energetics, confirming the validity of this expression in an independent manner. (For further details, see Appendices A and B.)

7.2 Optical forces in optomechanically variable interferometers

Next we demonstrate the utility of the RTOF method by computing the optically induced forces generated on a mirror within a lossless Gires-Tournois interferometer (GTI). The GTI under consideration is shown in Fig. 7(a), and consists of partially reflecting mirror (M_1) and a perfectly reflecting mirror (M_2) separated by a distance q. We assume that a plane wave with complex amplitude $E_i \cdot exp[i(kz - \omega t)]$ is incident from the left. Since the system is lossless, the reflected wave can be written as $E_o \cdot exp[i(-kz - \omega t + \phi(\omega, q))]$, where $E_i = E_o = E$. We

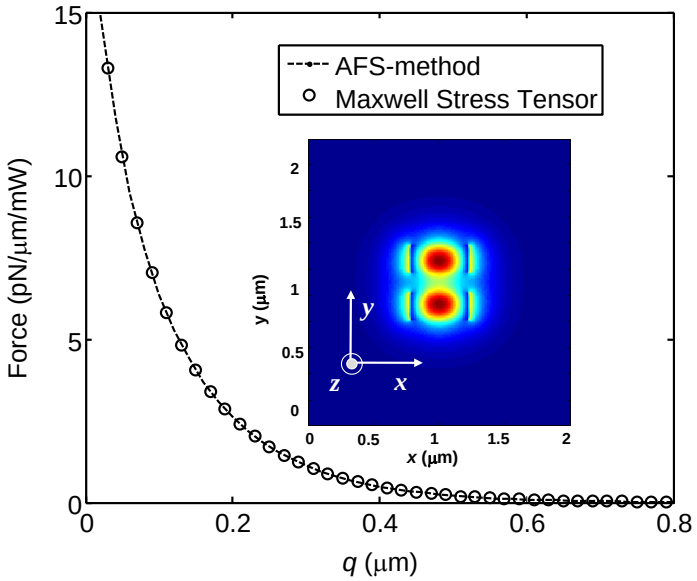

Figure 6. Shows the computed forces (pN/μm/mW) generated by a symmetric waveguide mode on the dual waveguide system. Forces computed by RTOF method (dashed line) and Maxwell stress-tensor method (circles) are over-layed, revealing perfect agreement. Inset shows and intensity-map of the E_x electric-field component computed with a full-vectorial mode-solver.

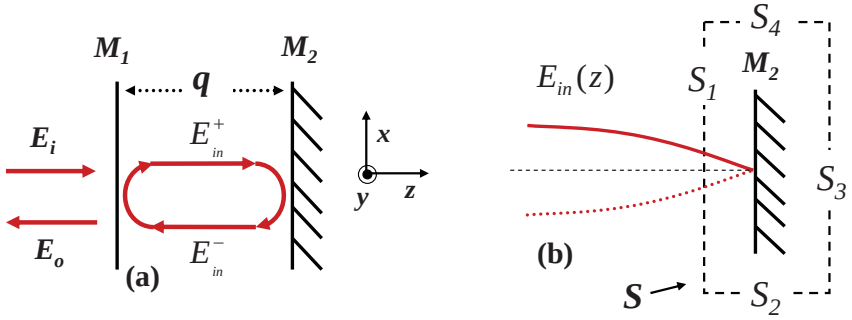

Figure 7. (a) and (b) are schematics of the same lossless Gires-Tournois interferometer. In both diagrams, the generalized coordinate q is taken to be the separation between mirrors M_1 and M_2. (a) Shows the incident (E_i) and exiting (E_o) field amplitudes, while (b) shows the internal fields impinging on (E_{in}^{+}) and receeding from (E_{in}^{-}) mirror M_2.

assume that M_1 has an amplitude reflectivity, r, and consider the optically induced forces acting on M_2. Note that the electromagnetic energy stored in the GTI, which is proportional to the number of photons transiently stored in the interferometer, may vary by orders of magnitude as the mirror separation tunes the cavity through resonance with the incident plane wave.

Optical forces in a Gires-Tournois interferometer via the RTOF method

Since the GTI is a lossless system with a single input and output, we can simply compute the phase response of the system, $\phi(\omega, q)$, and evaluate the optically induced force on M_2 using Eq. 10. Numerous methods can be employed to compute the phase response of a GTI interferometer.[23,36] For instance, the general scattering matrix for a two-mirror interferometer is derived in Ref.[23] From this result, one can show that the phase response of our GTI is given by

$$\phi(\omega, q) \quad = \quad tan^{-1}\left[\frac{(1-r^2)sin(\delta)}{2r-(r^2+1)cos(\delta)}\right]. \tag{18}$$

Here $\delta \equiv 2q(\omega/c)$. To evaluate the force using Eq. 10, we must also compute the incident photon flux. From the time averaged Poynting vector, one can show that the incident power per unit area is given by $P_i/A = |E_i|^2/(2\mu_o c)$. Here, μ_o is the magnetic permeability of vacuum, and A is the area under consideration. Thus, the incident photon flux is

$$\Phi_i = \frac{P_i}{\hbar\omega} = \frac{|E_i|^2}{2\mu_o c}\cdot\frac{A}{\hbar\omega}. \tag{19}$$

Substituting Eq. 19 and Eq. 18 into Eq. 10, we find that the force per unit area acting on M_2 is given by

$$\frac{F_q(\omega,q)}{A} = -\frac{E^2}{\mu_o c^2}\left[\frac{(1-r^2)}{2r\cdot cos[2q(\omega/c)]-(r^2+1)}\right]. \tag{20}$$

Notice that, as one might expect, on resonance (i.e. when $\delta = 2\pi\cdot m$), the forces on M_2 reach a maximum value.

Forces in a Gires-Tournois interferometer via Maxwell stress tensor methods

Next, we compute the forces on M_2 via Maxwell stress tensor (MST) methods. The Maxwell stress tensor (in vacuum) is defined as $T_{i,j} = \epsilon_o[E_iE_j - (1/2)\delta_{i,j}|E|^2] + [B_iB_j - (1/2)\delta_{i,j}|B|^2]/\mu_o$,[21] and can be related to the force on M_2 through the surface integral

$$\vec{F} = \oint_S \overleftrightarrow{T}\cdot d\vec{a}. \tag{21}$$

Here, S represents the closed surface, seen in Fig. 7(b), which consists of parts $S_1 - S_4$. If we take M_2 to be a perfect electric conductor of infinite extent, the only nonvanishing contribution to the integral will come from S_1. Within the interferometer, we represent the light impinging on M_2 as a plane wave of the form $E_{in}e^{ikz}$. Therefore, taking the front face of M_2 to be at position $z = 0$, we can express the electric and magnetic fields near M_2 as

$$\vec{E}_{in}(z) = \hat{x}\left[E_{in}e^{ikz} - E_{in}e^{-ikz}\right] \tag{22}$$

$$\vec{B}_{in}(z) = \hat{y}\left[E_{in}e^{ikz} + E_{in}e^{-ikz}\right]/c. \tag{23}$$

Using the above field amplitudes, and evaluating the Maxwell stress tensor over S_1, we see that the time-averaged force per unit area acting on M_2 is given by

$$\frac{F_z}{A} = \frac{1}{2}T_{z,z} = \frac{1}{2}\left(\frac{2\cdot E_{in}^2}{\mu_o c^2}\right) = \frac{2\cdot P_{in}}{c\cdot A}. \tag{24}$$

Here, P_{in} is defined as the net power impinging on M_2 from within the GTI. It is convenient to express the force in terms of the incident optical power since we can use the interferometer scattering matrix analysis presented in[23] to compute P_{in}. Using the results of Ref.[23] we find that

$$\frac{P_{in}(\omega,q)}{A} = -\frac{E^2}{2\mu_o c}\left[\frac{(1-r^2)}{2r\cdot cos[2q(\omega/c)]-(r^2+1)}\right]. \tag{25}$$

Using Eq. 24 to evaluate the time-averaged force per unit area on M_2, we have

$$\frac{F_q(\omega,q)}{A} = -\frac{E^2}{\mu_o c^2}\left[\frac{(1-r^2)}{2r\cdot cos[2q(\omega/c)]-(r^2+1)}\right]. \tag{26}$$

In comparing Eq. 26 and Eq. 20, we see that identical relations are found, illustrating exact equivalence between the RTOF method (derived here) and the Maxwell stress tensor analysis, through examination of this nontrivial resonant system. Furthermore, one can validate the multi-port formalism derived in Section 6 by assuming that M_2 is partially reflecting, and evaluating the force using the multi-port relation (Eq. 12) derived in the previous section. Through this exercise, equivalence is again found between the two formalisms.

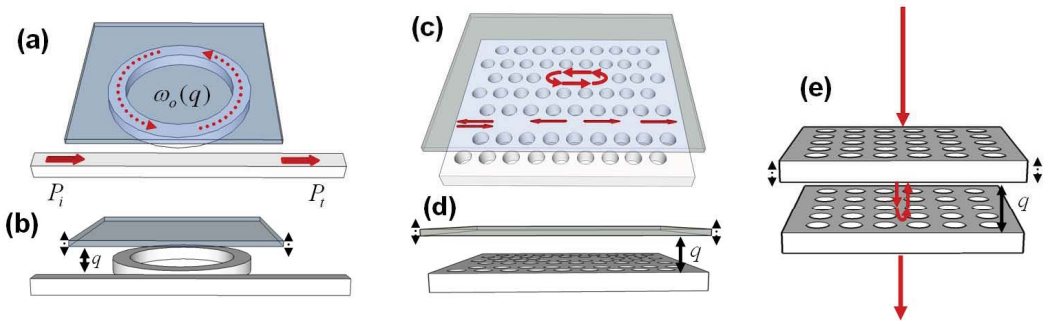

Figure 8. Examples of single- and multi-port systems which are analytically treatable with RTOF method. (a) and (b) show an all-pass ring resonator which is optomechanically tunable via dielectric perturbation, after Ref.[37] (c) and (d) show the analagous optomechanically variable geometry for a photonic cyrstal defect cavity. (e) shows a tunable resonant structure utilizing photonic crystal guided-slab resonances (see Ref.[26]

8. SUMMARY AND CONCLUSIONS

In this paper, we have derived a general formalism for the treatment of optically induced forces and potentials (within a Lagrangian mechanics framework) in the context of lossless optomechancially variable open systems with a single mechanical degree of freedom (consisting of linear media). The proposed and derived RTOF method allows the optically induced forces to be calculated solely from the optical response of the system, thereby offering tremendous simplicity and insight when compared to computationally intensive Maxwell stress tensor methods, which require explicit computation of complex internal field distributions. Through application of the RTOF method to examples having exact analytical solutions, we have shown that, although simple, this method yields exact correspondence with conventional Maxwell stress tensors methods.

A key insight of the RTOF method is that, provided the scattering matrix response of a lossless optomechanically variable system (i.e., an S-matrix of the form $S_{i,j}(\omega, q)$) is known, the force and potential energy expressed on the generalized coordinate can be computed analytically. An important corollary of the RTOF method is that, provided any two systems have an identical response (or s-matrix $S_{i,j}(\omega, q)$), identical forces are generated within. Thus, equivalent systems can be created in the context of photonic crystals, microrings and free-space optics, revealing that the general property which determines the optical force is the optical response, not the complex field distributions (an insight that MST methods leave obscured).

Finally, while the examples explored through use of the RTOF method were chosen for their simplicity, this method is applicable to any lossless open-system with a single mechanical degree of freedom which consists of linear media and conserves photon number. For example, Fig. 8(a)-(c) shows optomechanically variable ring resonator and photonic crystal structures that could be treated using the RTOF method. Provided these topologies (which are more complex that a Gires-Tournois) abide by the basic assumptions of the RTOF method, and response of these systems are known, one can compute the resulting force and potential profiles exactly. Lastly, because this formalism establishes a direct correspondence between the optical response of a system and the forces and potentials produced by light, well-established theories of optical filter systems[28] could therefore be extended to provide tools for synthesizing complex optical force and potential profiles.

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