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Estimating Turbulent Wall Shear and Boundary Layer Thickness for Hydro-dynamically Rough Surfaces by Perturbing Known Smooth Results

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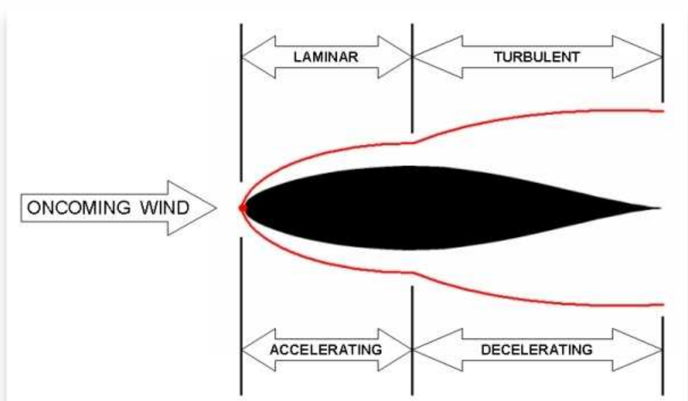

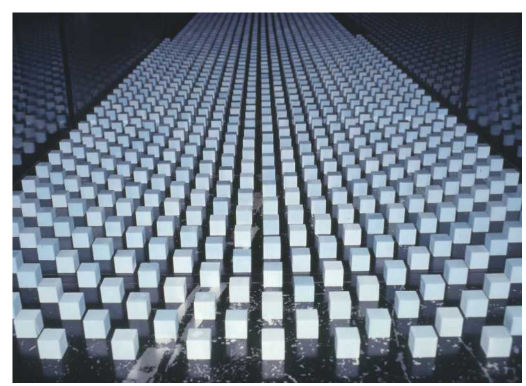

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Estimating Rough Wall Effects from Smooth Flow Information

"When utilizing hydrodynamically smooth results sometimes one would like to estimate effect of wall roughness without performing new experiments/computations"

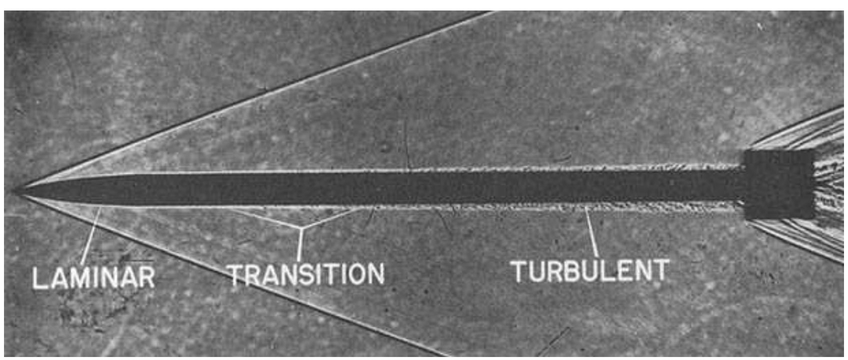
- **We consider a simple analytical model based on inner law methods which extends smooth wall skin friction and boundary layer thickness to be valid for rough wall flows:**
  - **Formal perturbation-based approaches**
  - **Approximate solution of inner-law expressions**
  - **Numerical inversion of the inner-law model**
  - **Compare to experiments/Empirical correlations (Fang et. al. 2003)**

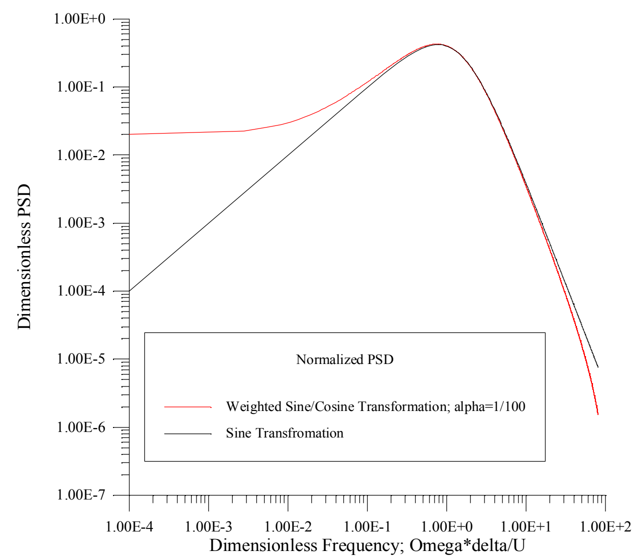
Estimating Rough Wall Effects from Smooth Flow Information
Motivation

"When utilizing hydrodynamically smooth results sometimes one would like to estimate effect of wall roughness without performing new experiments/computations"

- **Skin friction not only important for drag but also for local pressure fluctuation/direct normal pressure vibratory loading magnitude**

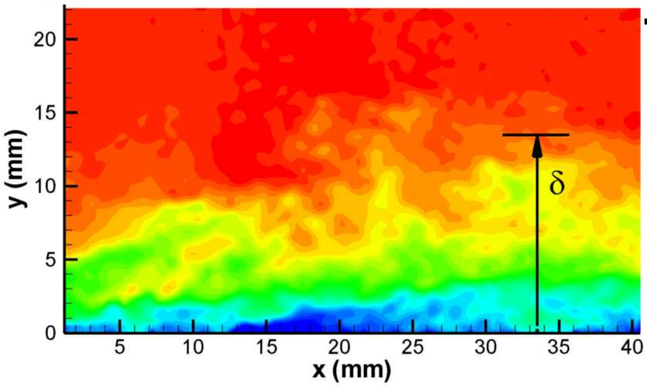
$$\frac{p'_{rms}}{\frac{1}{2}\rho U^2} = \alpha C_f \quad ; \quad \alpha = O(3)$$

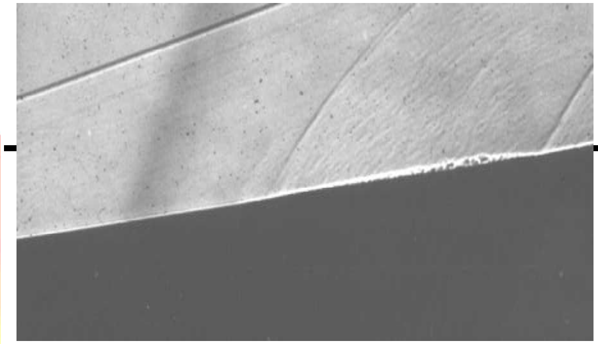




Estimating Rough Wall Effects from Smooth Flow Information
Sources of Unsteadiness

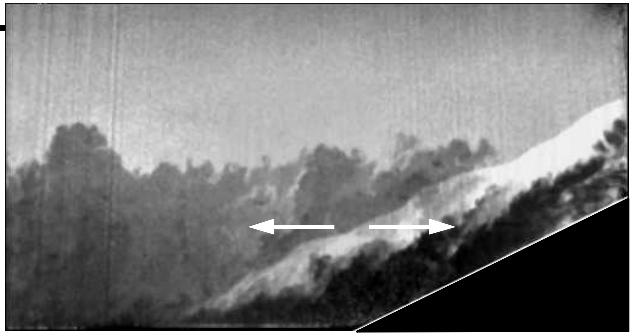
Boundary Layer Turbulence

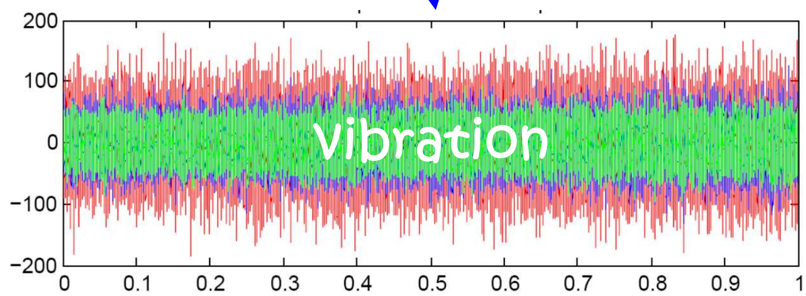


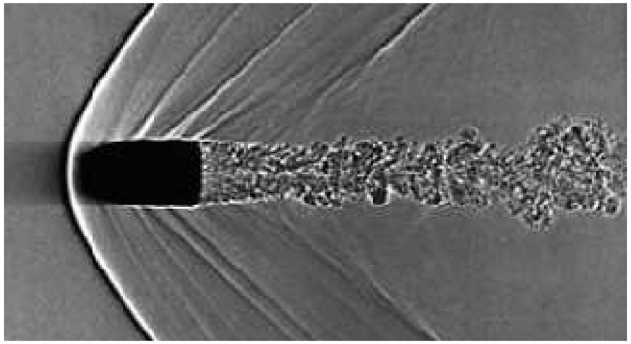


Transition

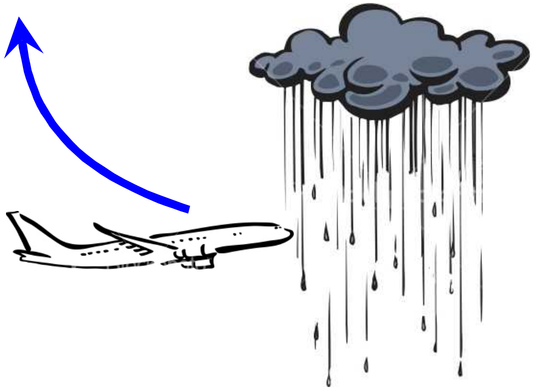
Unsteady Shocks







Vortex Shedding



Weather Encounters

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Estimating Rough Wall Effects from Smooth Flow Information
Governing Equations

Streamwise-momentum equation(inner law variable assumption, integrate "y" and "x"):

$$\mathrm{Re}_x = 1.73(1+0.3k^+)e^z(z^2 - 4z + 6 - \frac{0.3k^+}{1+0.3k^+})(z-1)$$

$$z \equiv \kappa\lambda \quad ; \quad \lambda \equiv \left(\frac{2}{c_f}\right)^{1/2} \quad ; \quad k^+ = \mathrm{Re}_x\left(\frac{k}{x}\right)\lambda^{-1}$$

Compressibility transformation (adiabatic), Van Driest:

$$c_f = \frac{1}{F_c} c_{f,inc}(F_{\mathrm{Re}\,x}\,\mathrm{Re}_x) \equiv \frac{1}{F_c} c_{f,inc}(\widetilde{\mathrm{Re}}_x)$$

$$F_c = \frac{\frac{\gamma-1}{2}M_\infty^2}{\arcsin(a)} \quad ; \quad F_{rex} = \frac{\mu_\infty}{\mu_w} F_c^{-1}$$

Estimating Rough Wall Effects from Smooth Flow Information
Formal Perturbation

Classical Perturbation Expansion from smooth results

$$c_f = c_{f0} + k^+ c_{f1} + \ldots + O(k^{+2})$$

$$\delta = \delta_0 + k^+ \delta_1 + \ldots + O(k^{+2})$$

Utilizing (momentum and B.L. Thickness)

$$c_{f1} = -\frac{\sqrt{2}}{\kappa} c_{f0}^{3/2} z_1 = 0.3 \frac{\sqrt{2}}{\kappa} c_{f0}^{3/2} \frac{\left(z_0^3 - 5z_0^2 + 9z_0 - 5\right)}{\left(z_0^3 - 2z_0^2 + 4\right)} \quad ; \quad z_0 \equiv \kappa \left(\frac{2}{c_{f0}}\right)^{1/2}$$

$$\delta_1 = 16 c_{f1} \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) x$$

Though interesting; Perturbation solutions are impractical since k⁺ is NOT always small

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Approximate Solutions for k+ >>1

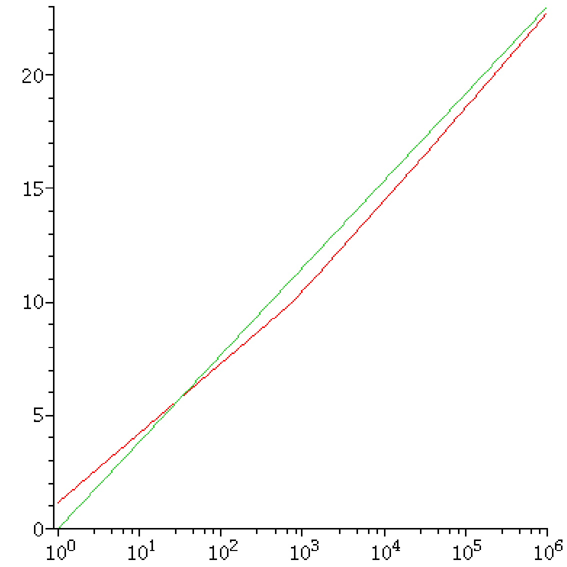
Asymptotic Solutions for k+ >>1

Approximate $\mathrm{Re}_x = 1.73(1+0.3k^+)e^z(z^2 - 4z + 6 - \frac{0.3k^+}{1+0.3k^+}(z-1))$ **as:**

$$\mathrm{Re}_x = 1.73(1+0.3k^+)e^z z^2 + \ldots$$

Solvable (transcendental) via LambertW(x), which can be approximated via Ln(x)

$$x = 2LambertW\left[\left(\frac{\lambda}{4a}\right)^{1/2}\right] \approx \ln\left[\left(\frac{\lambda}{4a}\right)^{5/6}\right] \quad ; \quad \frac{\lambda}{4a} >> 1$$



Estimating Rough Wall Effects from Smooth Flow Information

Approximate Solutions for $k^+ \gg 1$

Solutions are straight forward:

- **Smooth (limit, $k^+ = 0$)**

$$c_f \approx \frac{2\kappa^2}{\ln^2\left[\left(\frac{\mathrm{Re}_x}{8(1.73)}\right)^{5/6}\right]} \approx \frac{0.3362}{\ln^2\left[0.12\,\mathrm{Re}_x^{5/6}\right]}$$

Compare to White's incompressible flat plate result

$$c_f = \frac{0.455}{\ln^2\left[0.06\,\mathrm{Re}_x\right]}$$

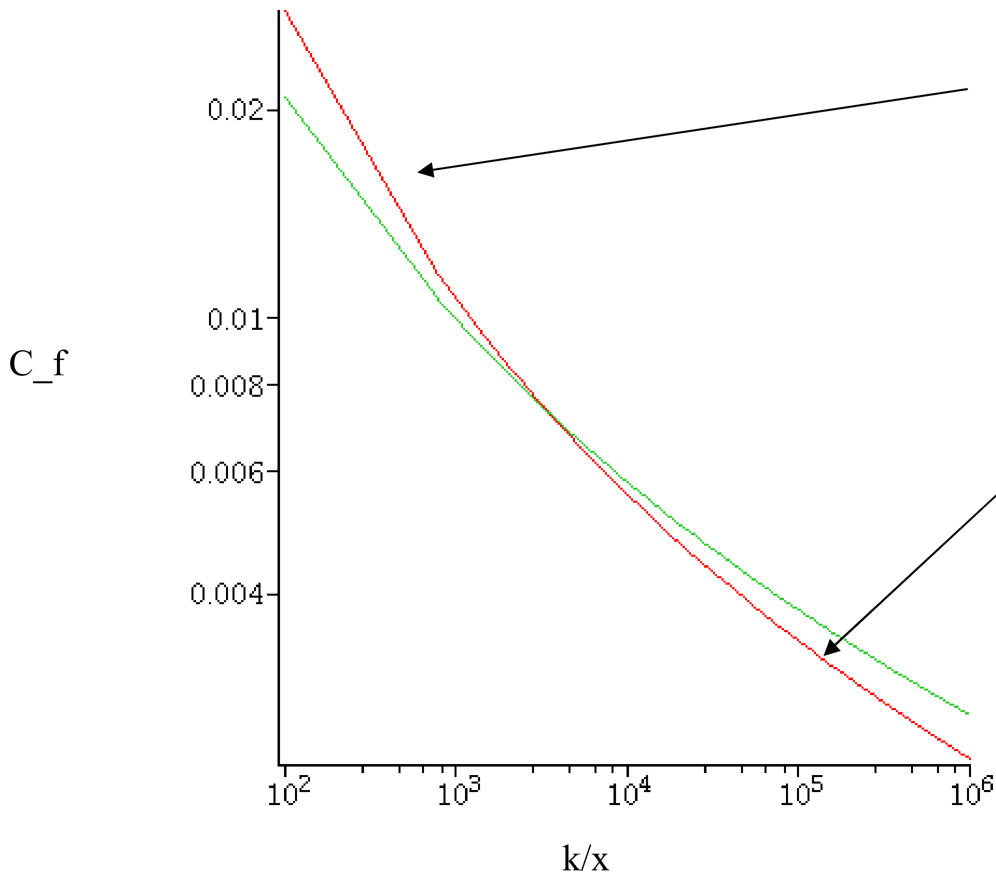
- **Fully Rough**

$$c_{f\_rough} = \frac{2\kappa^2}{\ln^2\left[\left(\frac{\mathrm{Re}_x}{8(1.73(1+0.3k^+))}\right)^{5/6}\right]}$$

Estimating Rough Wall Effects from Smooth Flow Information

Approximate Solutions for $k^+ >> 1$

Fully Rough (Re_x independent) Solution Versus Correlation



$$c_{f\_rough} = \frac{2\kappa^2}{\ln^2\left[\left(\frac{\mathrm{Re}_x}{8(1.73(1+0.3k^+))}\right)^{5/6}\right]}$$

$$c_{f\_full\_rough} \approx \left(1.4 + 3.7\log_{10}\left(\frac{x}{k}\right)\right)^{-2}$$

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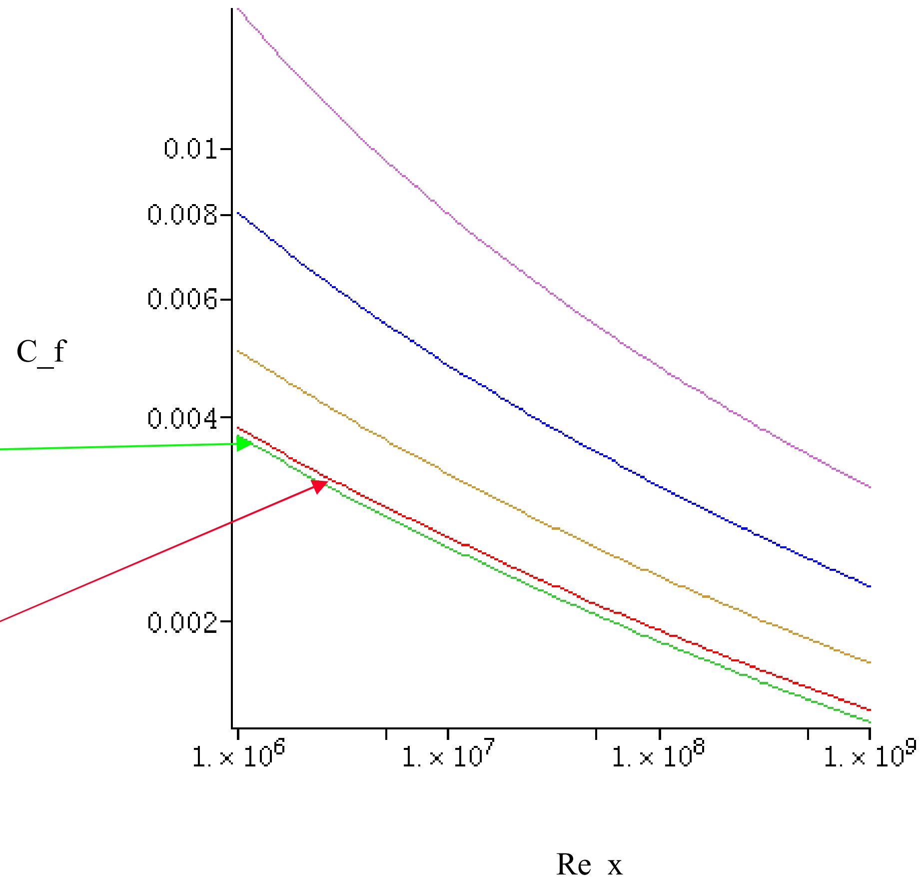
Estimating Rough Wall Effects from Smooth Flow Information
Approximate solution Results

Typical Roughness curves

Smooth (limit)

$$c_f = \frac{0.455}{\ln^2\left[0.06\,\mathrm{Re}_x\right]}$$

$$c_f \approx \frac{2\kappa^2}{\ln^2\left[\left(\frac{\mathrm{Re}_x}{8(1.73)}\right)^{5/6}\right]} \approx \frac{0.3362}{\ln^2\left[0.12\,\mathrm{Re}_x^{5/6}\right]}$$



Estimating Rough Wall Effects from Smooth Flow Information
Approximate solution Results

Example computation: known: C_{f_smooth}=0.0036, k=4 mils, Re_x=5E6

1. **Estimate "z" and k^+ (using smooth and fully rough approximation)**

$$k^{+} = z^{-1}\,\mathrm{Re}_x\left(\frac{k}{x}\right)\sqrt{\frac{c_{f0}}{2}} = z^{-1}_{fully\_rough}\,\mathrm{Re}_x\left(\frac{k}{x}\right)\sqrt{\frac{c_{f0}}{2}}$$

$$z_{full\_rough} \approx \kappa\sqrt{2}\left(\frac{0.55}{\ln^2\left[1.7\left(\frac{k}{x}\right)\right]}\right)^{-1/2}$$

2. **Compute C_{f_rough}**

$$c_{f\_rough} = \frac{2\kappa^2}{\ln^2\left[\left(\frac{\mathrm{Re}_x}{8(1.73(1+0.3k^+))}\right)^{5/6}\right]}$$

$$c_f = \left(\frac{0.0045}{0.0036}\right)c_{f0} = 1.25c_{f0}$$

3. **Compare to empirical correlation (Fang et. al. 2003)**

$$\frac{c_{f\_rough}}{c_{f0}} = 1 + 0.889(\log_{10}(k^+) - 1)$$

$$\frac{c_{f\_rough}}{c_{f0}} = 1 + 0.889(\log_{10}(15.56) - 1) = 1.17$$

Estimating Rough Wall Effects from Smooth Flow Information
Numerical Inversion and Compressibility

The previous discussion is incompressible. Using Van-Driest's transformation

$$c_f = \frac{1}{F_c} c_{f,inc}(F_{\mathrm{Re}\,x} \mathrm{Re}_x) \equiv \frac{1}{F_c} c_{f,inc}(\widetilde{\mathrm{Re}}_x)$$

$$F_c = \frac{\frac{\gamma - 1}{2} M_\infty^2}{\arcsin(a)} \quad ; \quad F_{rex} = \frac{\mu_\infty}{\mu_w} F_c^{-1}$$

We readily include compressibility effects.

But, we are getting beyond a back of the envelope computation… so let's solve the nonlinear equation numerically (Newton's method)

$$\mathrm{Re}_x = 1.73(1 + 0.3k^+)e^z(z^2 - 4z + 6 - \frac{0.3k^+}{1 + 0.3k^+})(z - 1)$$

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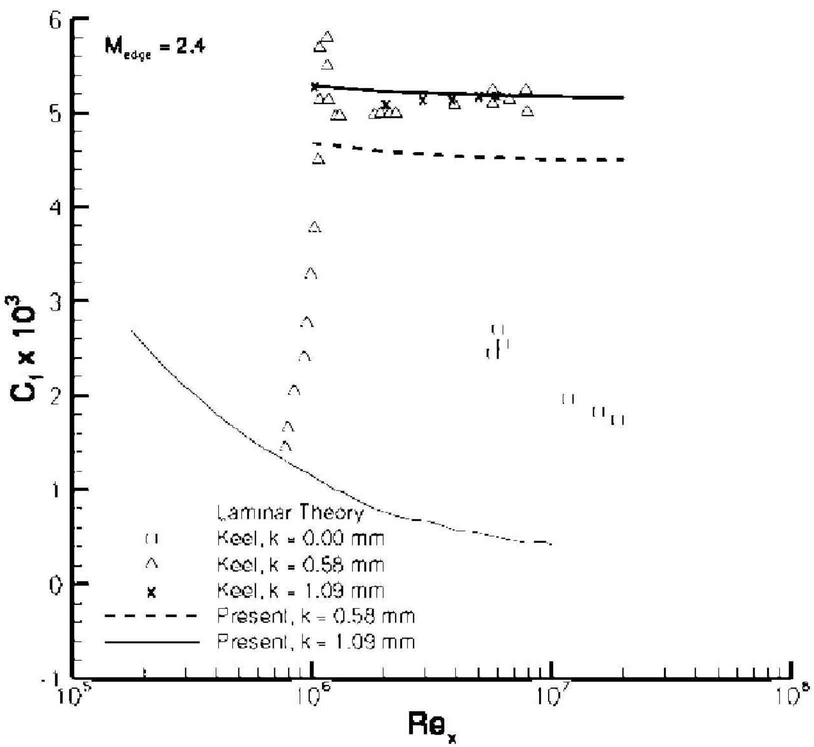
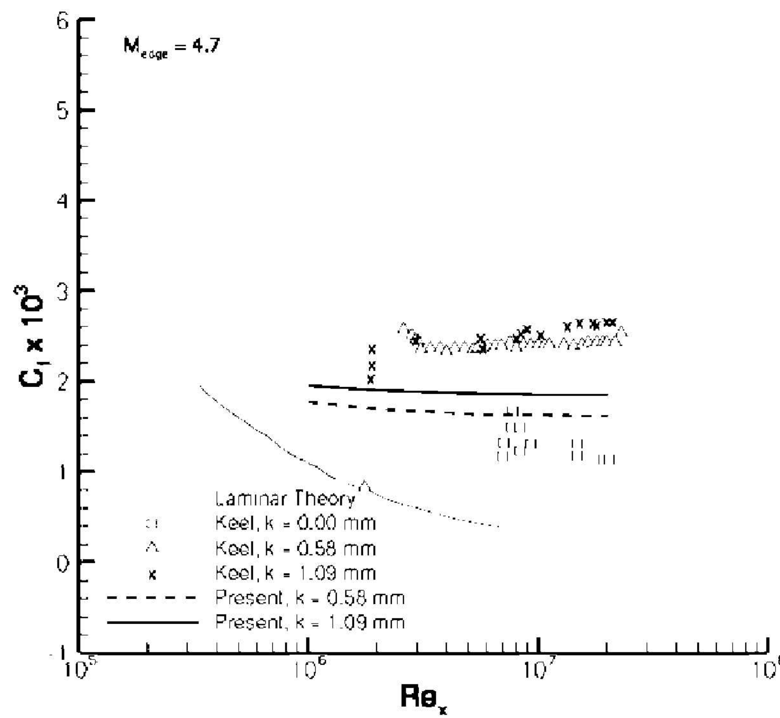
Estimating Rough Wall Effects from Smooth Flow Information
(Numerical Solution for Rough Flow with Compressibility)

Rough sharp cone 5 degree cone/adiabatic

(M=2.4)



(M=4.7)



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Estimating Rough Wall Effects from Smooth Flow Information

Conclusions

1. An inner law based model that maps smooth flow information, i.e. C_{f_smooth}=0.0036, k=4 mils, Re_x=5E6 to rough flow behavior have been derived and examined

2. Classical perturbation methods are not useful here

3. More complete solutions (dominant balance or fully numerical) are necessary

4. The solutions are in moderate-good agreement with data and empirical expressions (Fang 2003).

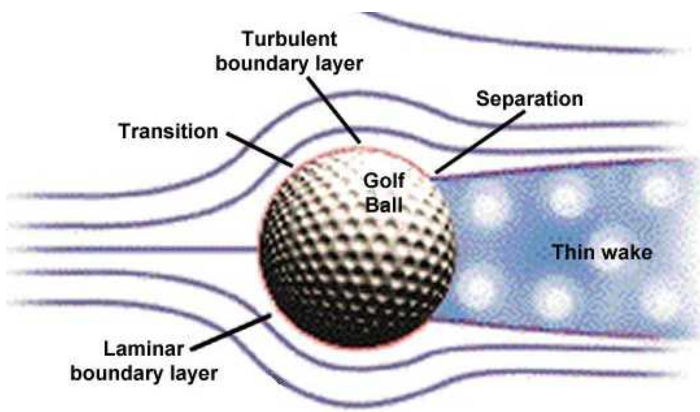
5. The empirical expressions, though not based on first principles, are accurate and convenient.

Estimating Rough Wall Effects from Smooth Flow Information
Further Work

The simple approach here can be extended (modifications for inner law theory)

1. **Blowing and suction**
2. **Curvature**
3. **Streamwise pressure gradient dp/dx**

Implementation does not need to be "stand alone" but might be better directly incorporated in a wall layer formulation (two-way coupling of wall behavior and flow)



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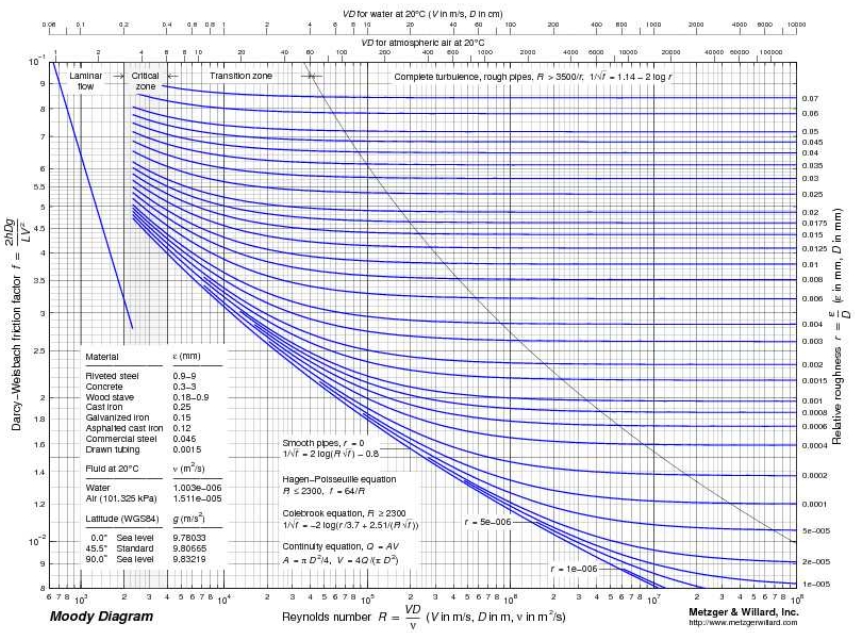
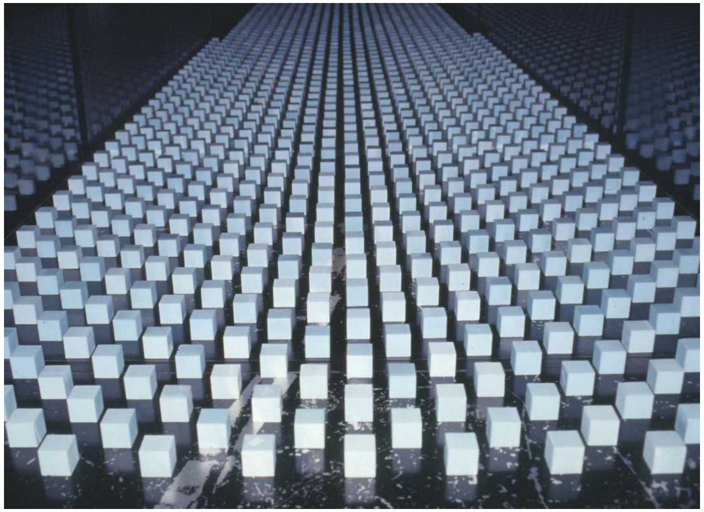
Estimating Rough Wall Effects from Smooth Flow Information

Further Work (roughness geometry and physics)

Roughness physics has always been empirical and capture by a shift in the inner law and parameterized by a roughness height

But can we tie more physics to the roughness geometry?





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