

# **Solution of Multivariate Inverse Radiation Transport Problems**

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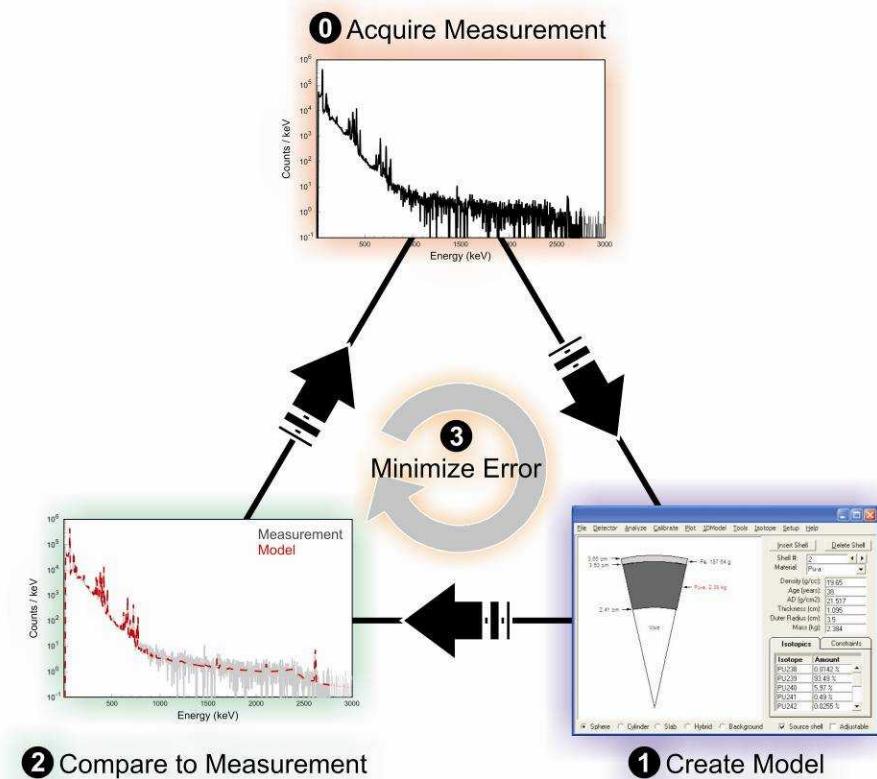
# Inverse Radiation Transport Problems

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- Objective: infer the configuration of an unknown radiation source from its measured radiation signatures
- Source features
  - Isotopic composition
  - Fissile mass & multiplication
  - Geometric arrangement of radiating and shielding materials
- Radiation signatures
  - Gamma spectrometry
  - Neutron time-correlation and multiplicity counting
- Applications
  - Non-intrusive, non-destructive inspection & interrogation
  - Nonproliferation & counterterrorism

# Solution Method

- Start with an initial model of the source; treat some model parameters as variable
- Estimate the radiation field incident on the detector(s) using radiation transport calculations
- Fold the radiation field with a model of the detector response function(s) to calculate the radiation signature(s)
- Iteratively minimize the error between the calculated and measured signature(s) using nonlinear regression



# Components of the Inverse Transport Framework

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## Radiation Transport

- Discrete ordinates transport solvers are used to compute the neutron, electron, and photon radiation fields
- Neutron transport
  - Neutron flux & leakage
  - Induced fission gammas
  - Capture & inelastic scatter gammas
- Electron transport
  - Bremsstrahlung
- Photon transport
  - Photon flux & leakage
  - Decay gammas
  - Spontaneous fission gammas
  - $(\alpha, n)$  gammas

## Detector Response

- Point models are used to estimate the response to photons & neutrons
  - Detector material & dimensions
  - Energy calibration & resolution
  - Near- & far-field scatter
  - Shielding & collimation

## Nonlinear Regression

- The Levenberg-Marquardt regression solver is used to find transport model parameters that minimize the error in the calculated detector response

# Solution Requirements

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- Forward computations (radiation field and detector response) must be accurate
- Minimize bias in the solution due to systematic errors in the model
- Forward computations must be fast
- Minimize the time per iteration required to find the solution
- The model must have a finite number of numeric parameters
- Minimize the degrees of freedom/dimensionality of solution
- Accuracy requires high-fidelity spectral synthesis: coupled neutron/electron/photon transport calculations
- Speed requires explicit solution of transport problem: deterministic transport
- Tractable problems do not have arbitrary geometry

# Radiation Observables

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- Most externally observable radiation signatures result from gamma and neutron emissions
- Observables are usually differential over one or more independent variables (e.g., energy, position, time)
- Gamma spectrometry measures the distribution of photons versus energy
- Neutron multiplicity counting measures the distribution of neutrons versus number and counting time



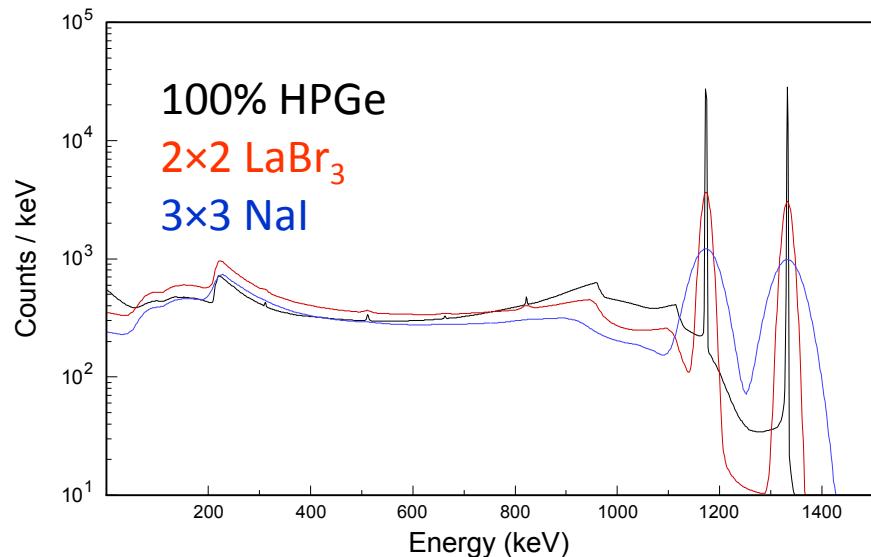
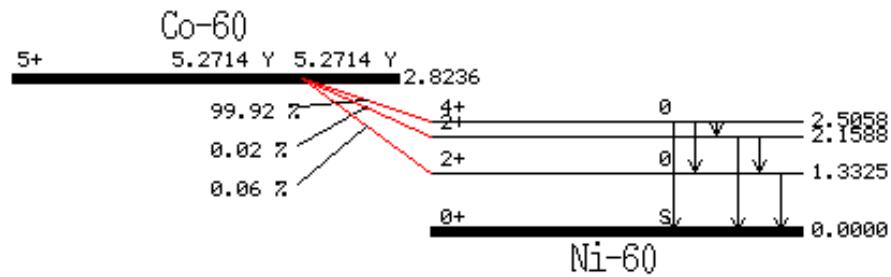
Gamma Spectrometers



Neutron Multiplicity Counters

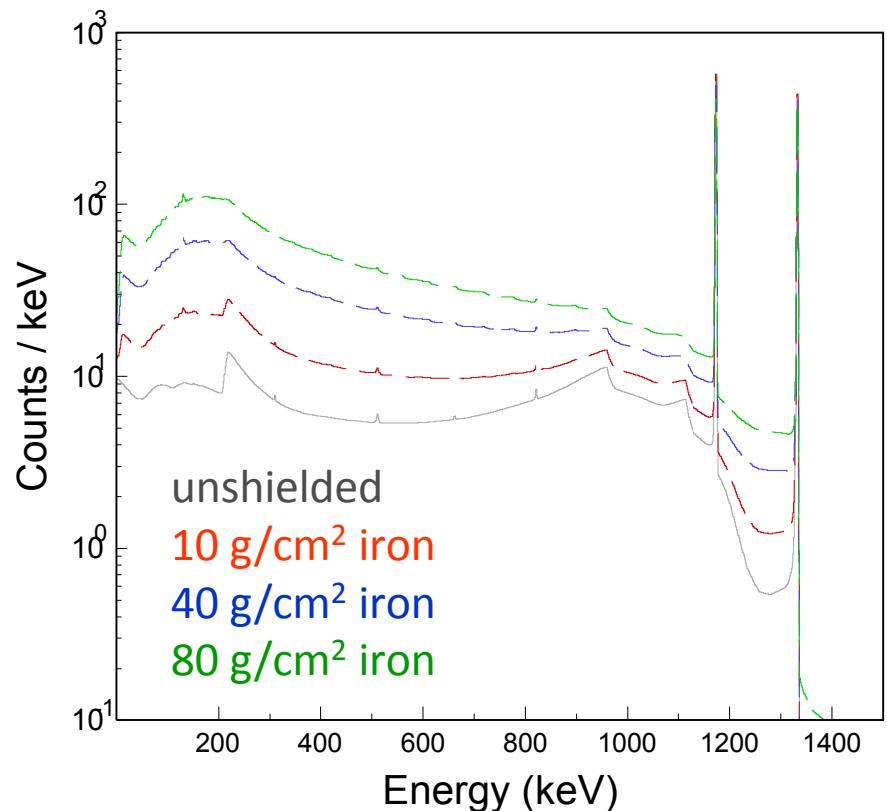
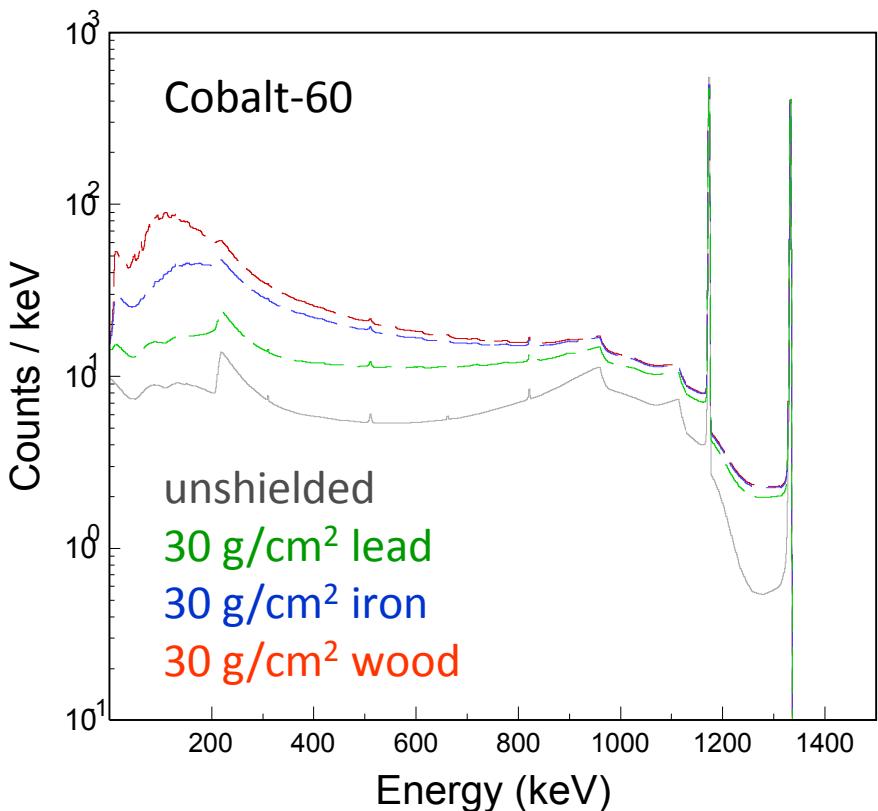
# Gamma Spectrometry

- A radionuclide decays from its unstable state through a series of discrete energy levels
- Decay between levels of a single daughter nucleus is achieved via emission of discrete energy gammas
- The gammas are characteristic of the daughter level scheme
- Gamma spectrometers measure the distribution of photon energies
- The gamma spectrum can be used to identify radionuclides and shielding



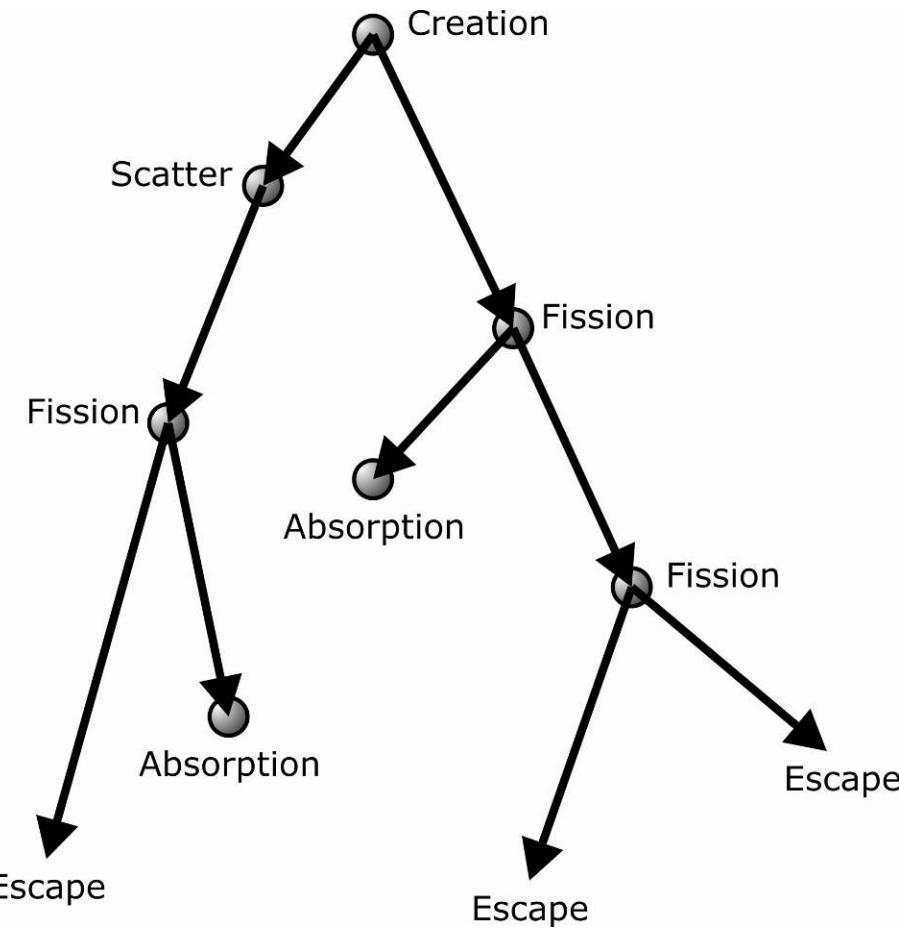
# Photopeaks and Compton Continua

- Photopeak positions identify source
- Differential attenuation of photopeaks and Compton continua identify shielding
- Fitting the full spectrum enables the source and shielding to be characterized



# Fission Chain-Reactions

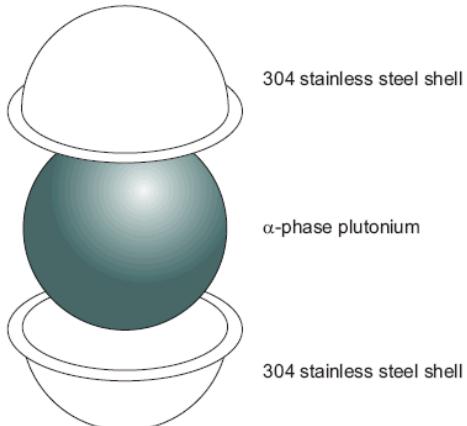
- Fission chain reactions multiply the number of neutrons in fissile transport medium
- Chain reaction characteristics:
  - Number of neutrons made during the chain reaction: neutron multiplication
  - Speed of chain reaction evolution: neutron generation time
- Neutron multiplicity measurements are sensitive to both characteristics
- Neutron multiplicity counting can be used to estimate kinetics properties of the source
  - Source strength
  - Multiplication
  - Neutron lifetime
  - Leakage probability



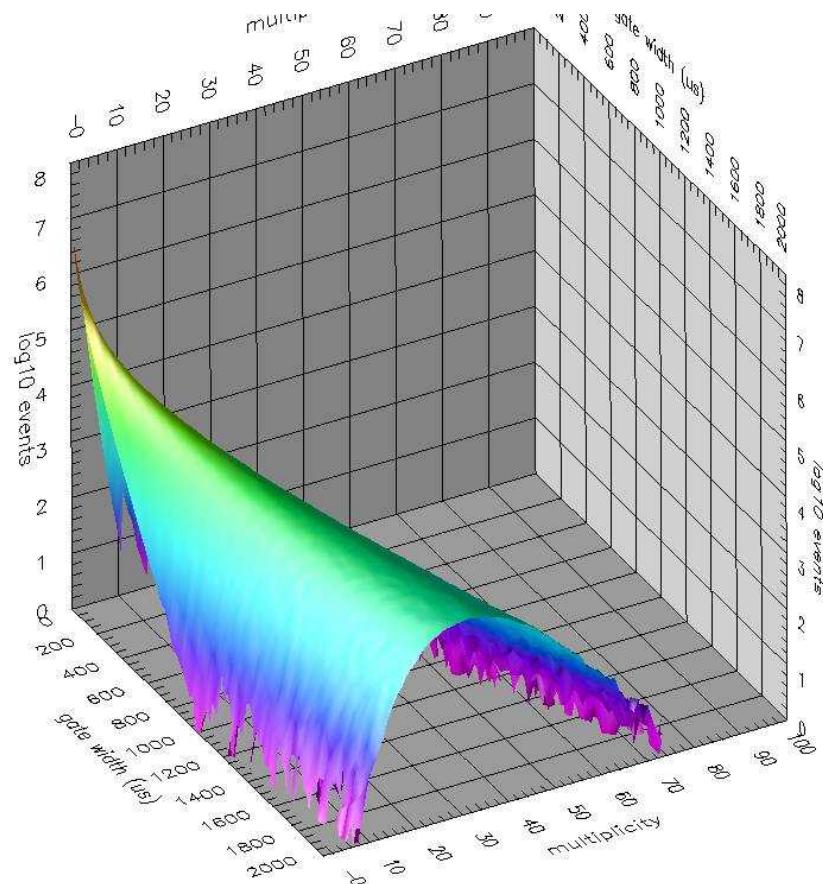
# Neutron Multiplicity Counting

- Neutron multiplicity counting measures the frequency of neutron detection versus:
  - Counting time, a.k.a. coincidence gate width, usually on order of microseconds
  - Number of coincident counts, a.k.a. multiplicity, usually between 10's and 100's of coincident neutrons

LANL BeRP Ball

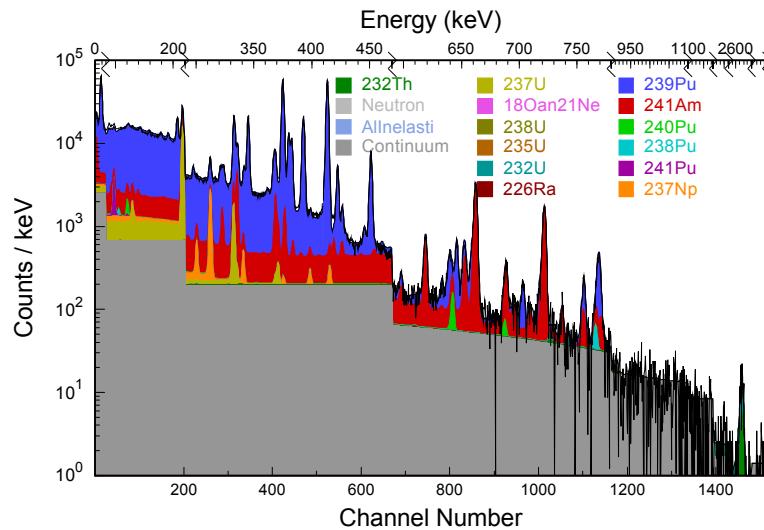
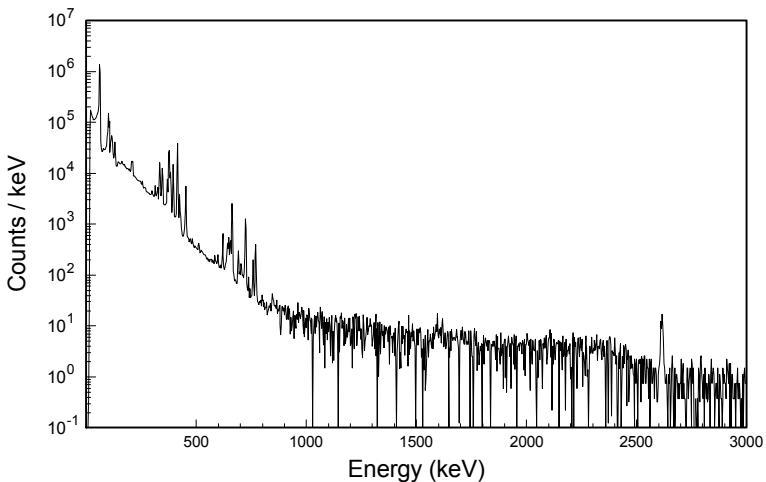


LANL BeRP Ball / 3.8 cm Poly Reflector



# Example Problem

- The gamma spectrum below exhibits features consistent with plutonium
- The spectrum can be fit (top-right) via nonlinear regression using variable isotopics, volume, shielding, and age
- The regression analysis (bottom-right) provides approximate model of source



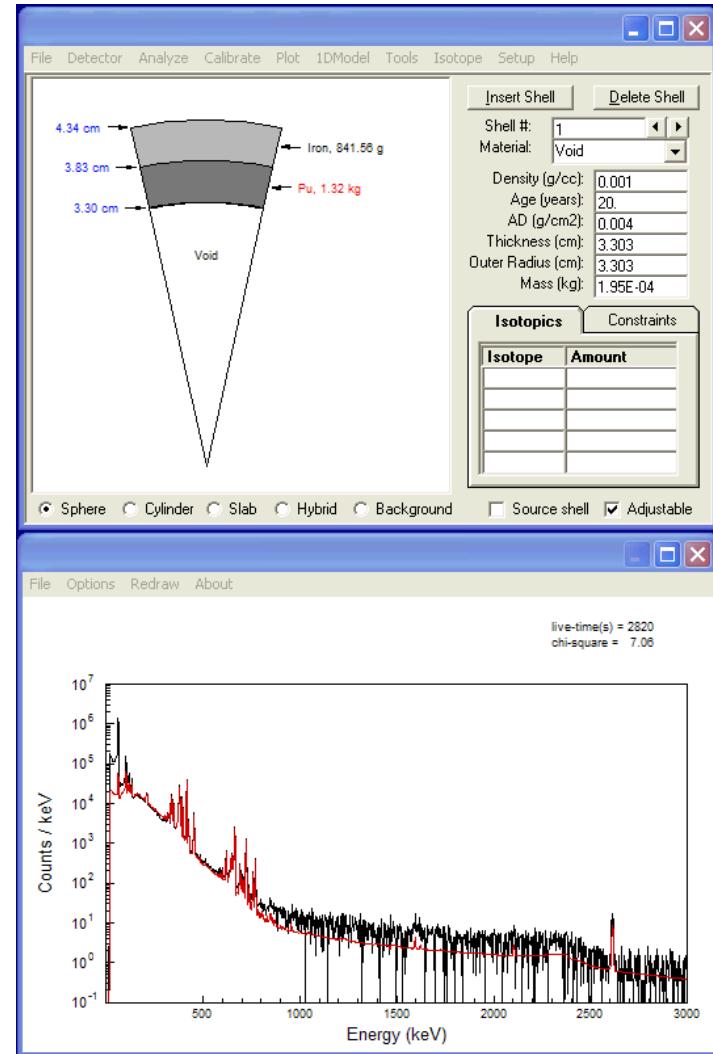
Plutonium wt.% (@ 33 +/- 1 years) _____ @ t=0 _____						
Pu-236:	2.69E-12	+/-	1.19E-12	8.98E-09	+/-	4.75E-09
Pu-238:	0.014	+/-	0.008	0.019	+/-	0.010
Pu-239:	94.205	+/-	0.603	94.295	+/-	0.604
Pu-240:	5.279	+/-	1.157	5.298	+/-	1.161
Pu-241:	0.095	+/-	0.006	0.513	+/-	0.038
Pu-242:	[ 0.010]			[ 0.010]		
Am-241:	0.396	+/-	0.008			
Np-237:	0.017	+/-	0.002			
U-237:	2.94E-09	+/-	1.92E-10			
U-232:	8.98E-09	+/-	3.97E-09			

Confidence for measured  $^{180}\text{An-21Ne}$  gammas: -1.6 sigma  
Measured (a.n) relative to expected for oxide: -3.0 sigma  
Aluminum-inelastic peak: 1011.7 keV @ 2.3 sigma  
Chemical form: METALLIC  
External shielding: AN = 30 +/- 1; AD = 3.9 +/- 0.1  
Outer radius if plutonium is spherical: 3.8 cm  
Estimated void radius: 3.3 cm  
Estimated mass if delta-phase plutonium: 1.3 kg

There is no evidence that uranium is present.

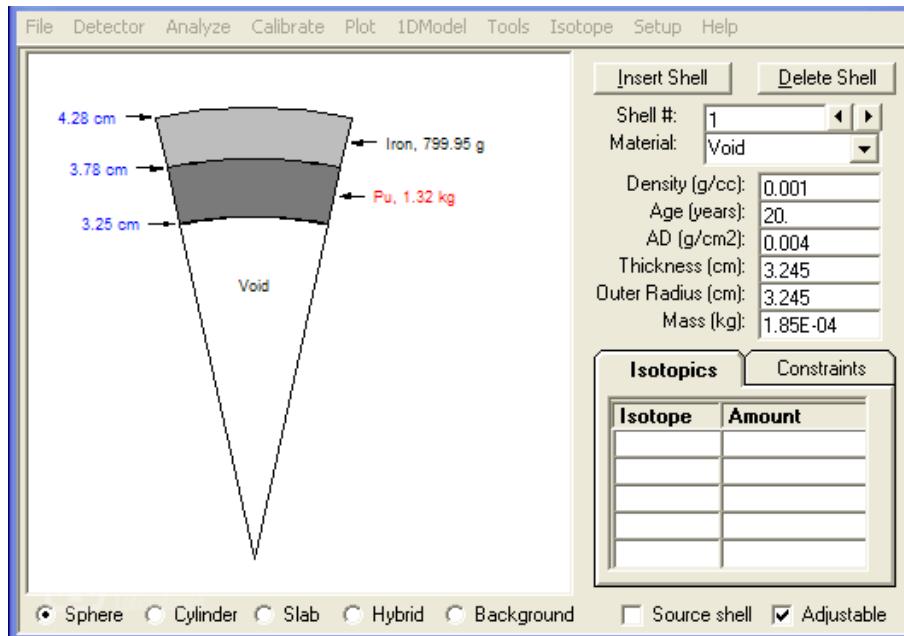
# Example Problem

- A one-dimensional transport model (top) can be generated from regression analysis
  - The model is displayed as a section of sphere with the center at the bottom and outer surface at the top
  - The dimensions of the model layers are treated as variables for nonlinear regression
- An initial estimate of the gamma spectrum (bottom) is generated from coupled neutron/electron/photon transport calculations
- Nonlinear optimization procedures are used to find the model dimensions that minimize the error in the calculated spectrum

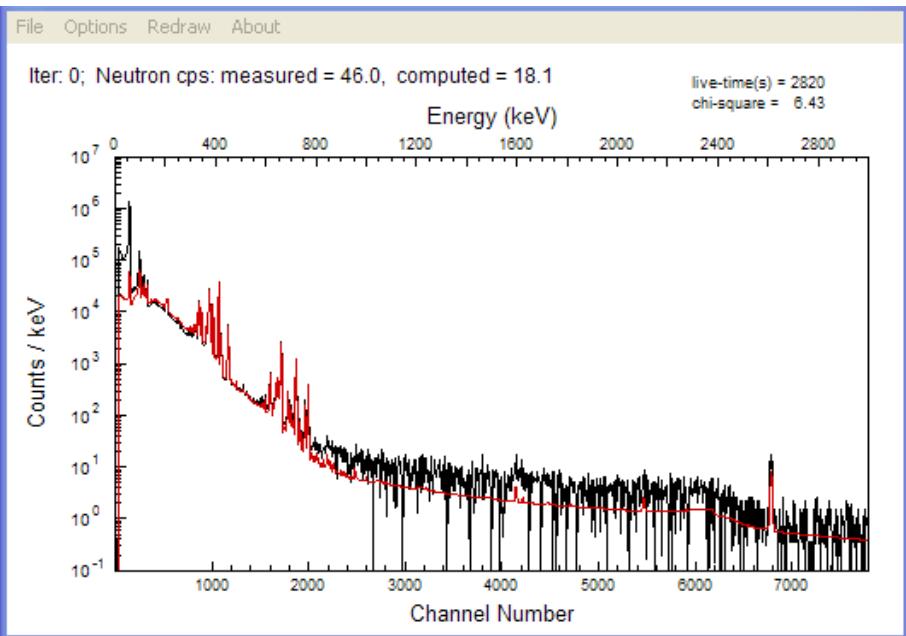


# Solution Optimization

## Transport Model

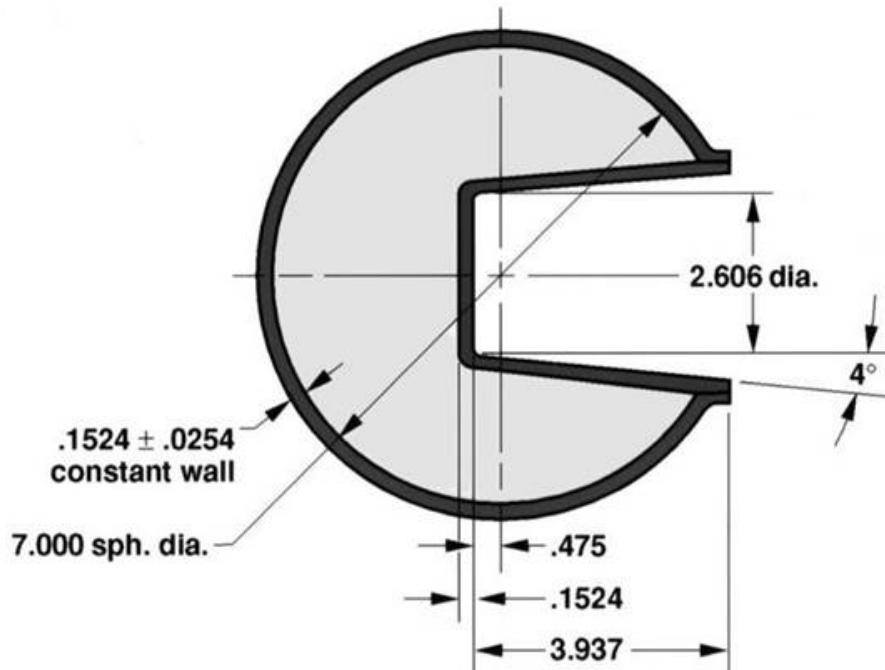


## Measurement vs. Calculation

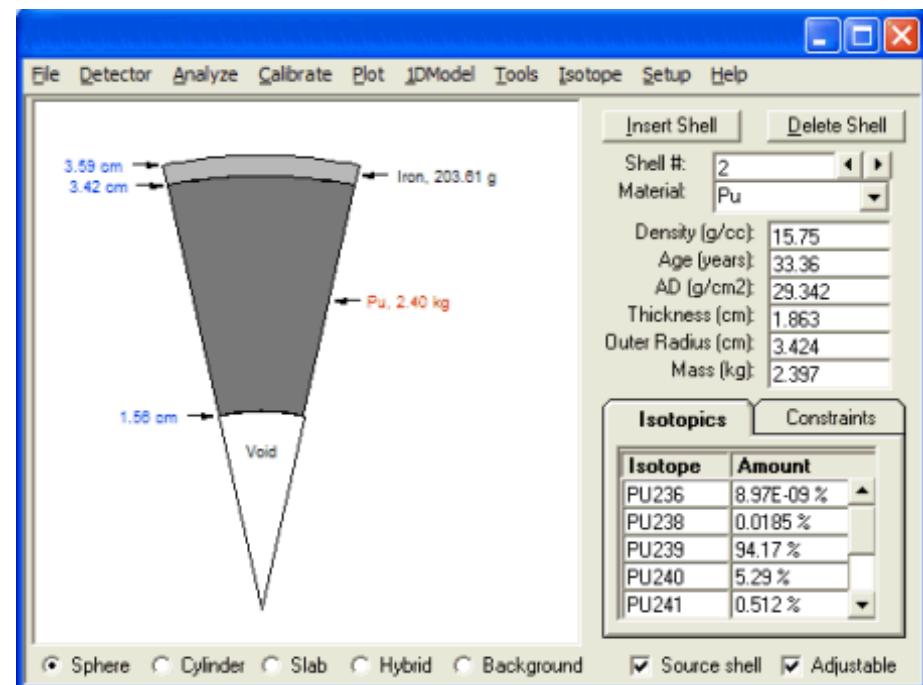


# Actual Source

Plutonium Sphere

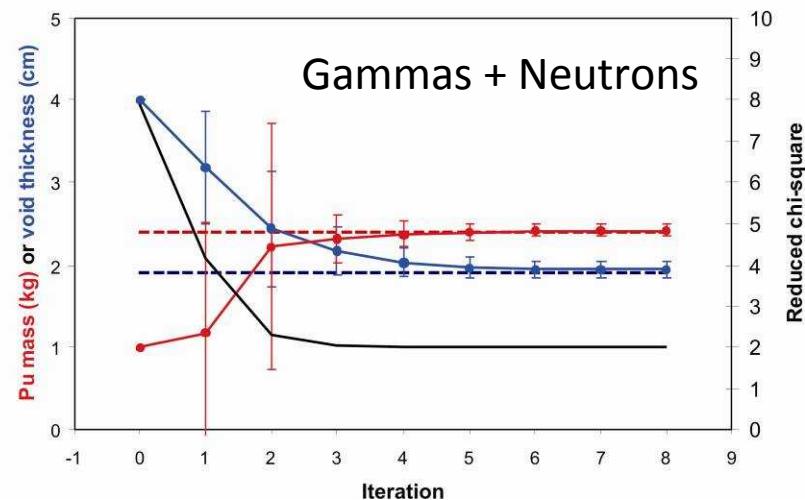
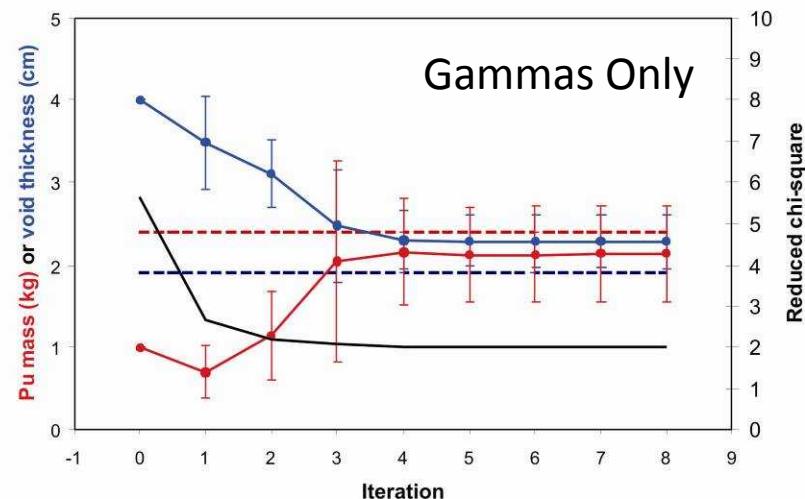


Optimized Transport Model



# Solution using Gammas and Neutrons

- The gamma spectrum is primarily sensitive to the outer surface of source
- The solution based on the gamma spectrum alone is weakly constrained
- Neutron measurements (e.g., count rate) are more sensitive to the entire volume of source
- A solution based on the simultaneous analysis of gamma and neutron signatures is better constrained
- Neutron multiplicity counting provides a fairly rich signature of the neutron field



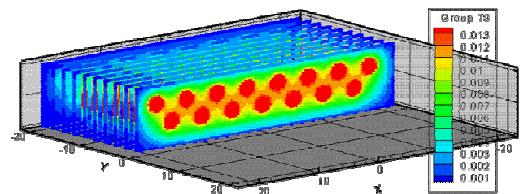
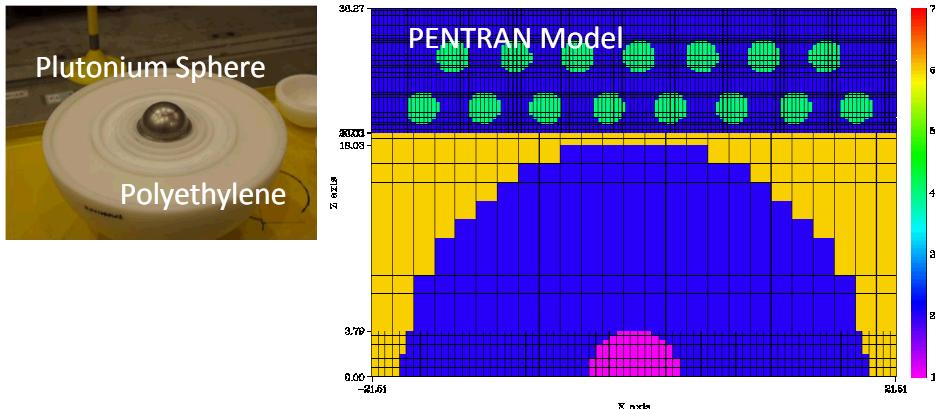
# Challenges to Solving Multivariate Inverse Radiation Transport Problems

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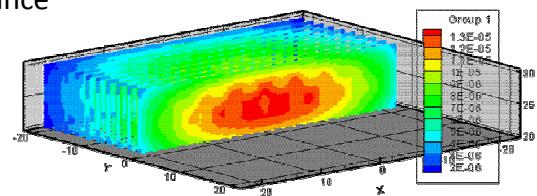
- Need to accelerate deterministic transport calculations
  - Typical computational time for neutron multiplicity statistics can be as much as 10 – 20 seconds
  - Need to investigate collapsed cross-section libraries and other approximations to speed the transport calculations
- Need to validate deterministic transport calculations
  - Approximations used to accelerate the calculations can introduce systematic errors
  - Need to validate approximations against measurements and higher fidelity (e.g., 3D Monte Carlo) simulations
- Need to develop a systematic approach to combining the analysis of complementary, correlated signatures
  - Errors in gamma spectrum and neutron multiplicity calculations are correlated
  - Need to correctly weight each signature's contribution to the error metric

# Accelerating Deterministic Transport Calculations

- University of Florida (UF) is developing a platform that will be used to test alternative methods of accelerating deterministic transport calculations
- The platform is based on UF codes PENTRAN ( $S_N$ ) and TITAN (hybrid  $S_N$ /ray-trace)
- UF is currently investigating contribution-weighted methods to collapse cross-sections
- So far, UF has been able to reduce the number of energy groups by a factor of 2 without introducing much error into neutron multiplicity calculations



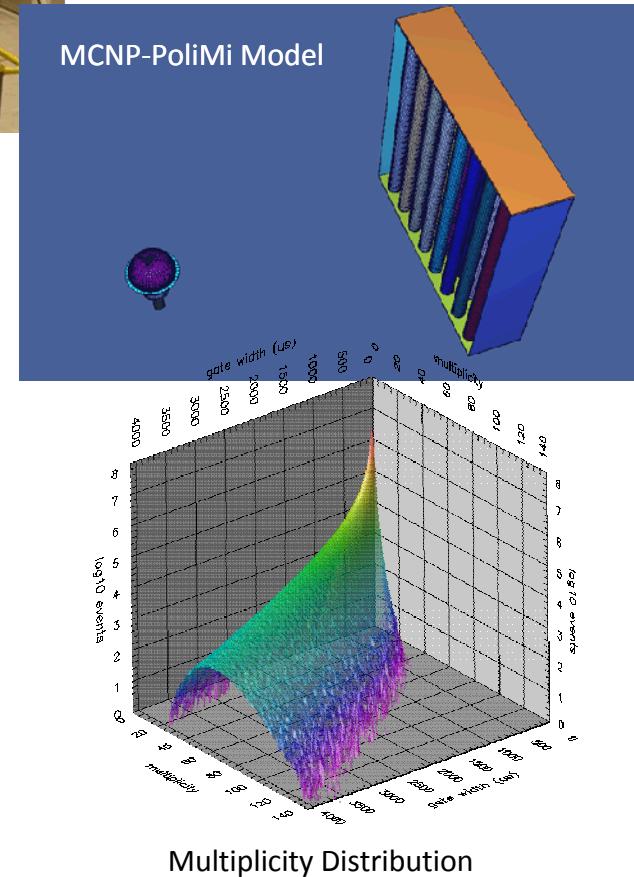
Thermal Neutron Importance



Fast Neutron Importance

# Using Monte Carlo to Validate Deterministic Calculations

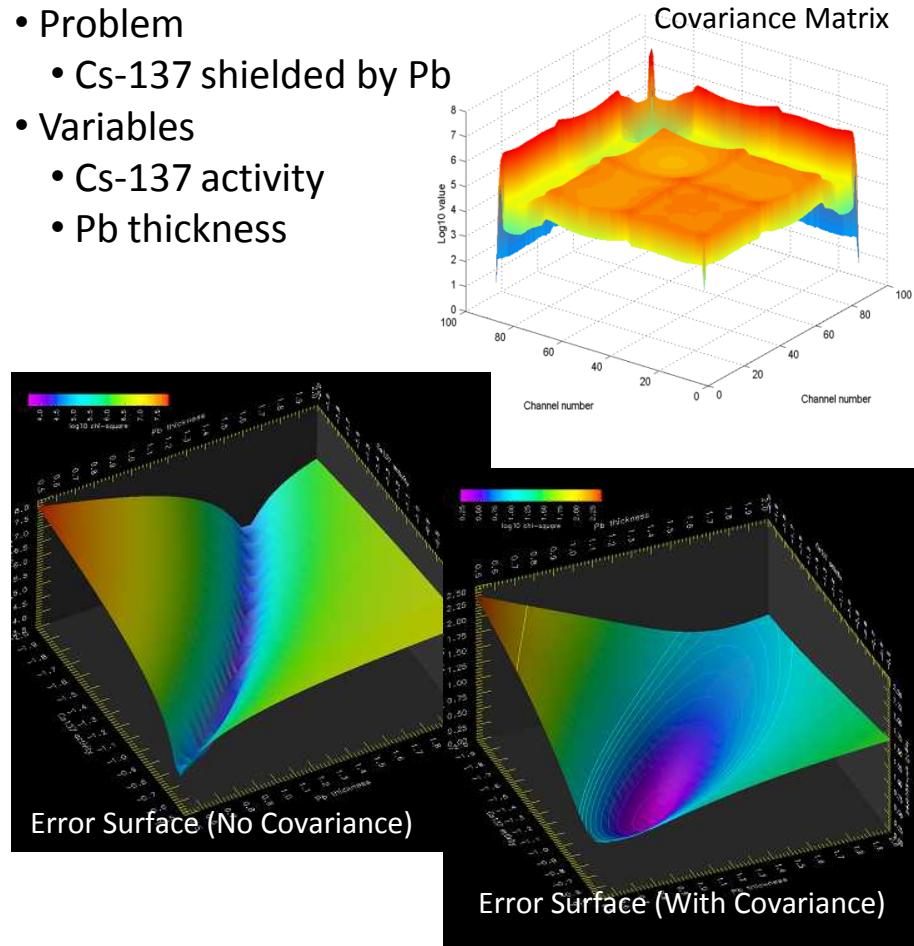
- University of Michigan (UM) is modifying MCNP-PoliMi and post-processing software to simulate neutron multiplicity counting experiments
- UM developed an MCNP-PoliMi post-processor that accumulates the neutron multiplicity distribution
- UM is currently validating their calculations against experiments conducted with reflected plutonium
- So far, UM has been able to match the experiments within about 15% (using the Feynman variance-to-mean statistic)



# Combining Gamma Spectral and Neutron Multiplicity Analyses

- Sandia is studying alternative regression error metrics to systematically combine the analysis of gamma spectrometry and neutron multiplicity measurements
- We developed a methodology that computes the covariance between model errors – the covariance matrix is used to weight the contribution of individual model errors to the total error
- Currently, we are investigating the effect of model error covariance on the topography of the solution space

- Problem
  - Cs-137 shielded by Pb
- Variables
  - Cs-137 activity
  - Pb thickness



$$\chi^2 = (x - x_0)^T \Sigma^{-1} (x - x_0)$$

# Summary

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- It is possible to infer the configuration of an unknown radiation source from its radiation signatures
- Sandia has developed and implemented techniques to solve this inverse transport problem based on gamma spectrometry
- Solutions based on multiple complementary signatures are generally better constrained
- Sandia is working with University of Florida and University of Michigan to develop methods to solve inverse transport problems using gamma spectrometry and neutron multiplicity signatures