

Using LAMMPS to Derive Crystal Structure Rule

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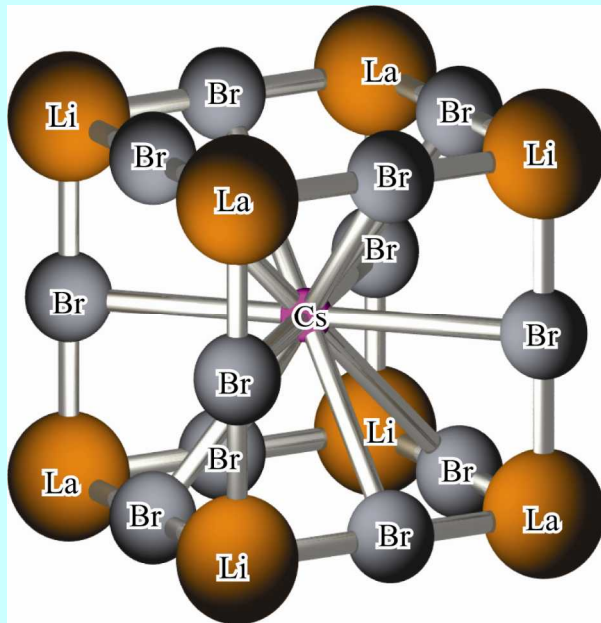
Example: Alkali Halides

To predict alkali halide crystals, our embedded-ion method interatomic potential database

- 1. contains all nine alkali halide elements (Li, Na, K, Rb, Cs F, Cl, Br, I),**
- 2. incorporates atomic size, electronegativity, and bond energy effects,**
- 3. uses elemental properties directly as model parameters without parameterization, and**
- 4. gives good trend of charge, energy, and bond length predictions.**

Role of Electronegativity

Double Perovskite $\text{Cs}_2\text{LiLaBr}_6$



Observations:

1. Lattice constant of $\text{Cs}_2\text{LiLaBr}_6$ is $a = 2r_{\text{LiLa}} = 11.289 \text{ \AA}^{[1]}$, i.e., $r_{\text{LaLa}} = 7.983 \text{ \AA}$.
2. fcc La has a lattice constant $a = 5.307 \text{ \AA}$ and a cohesive energy $E_c = -4.446 \text{ eV/atom}^{[2]}$. In fcc La, $r_{\text{LaLa}} = 3.753 \text{ \AA}$.
3. It is difficult to for a potential to be transferrable to both $\text{Cs}_2\text{LiLaBr}_6$ and La.

Solution:

The electronegativity difference-induced ionization can increase the bond length and reduce the bond strength.

[1]. P. Yang, M. A. Rodriguez, F. P. Doty, X. Zhou, M. R. Sanchez, and K. S. Shah, submitted.

[2]. X. W. Zhou, and F. P. Doty, *Phys. Rev. B*, **78**, 224307 (2008).

Embedded Ion Method (EIM)

$$E = \frac{1}{2} \sum_{i=1}^N \sum_{j=i_1}^{i_N} \phi_{ij}(r_{ij}) + \sum_{i=1}^N E_i(q_i, \sigma_i)$$

system total energy

$$E_i(q_i, \sigma_i) = \frac{1}{2} q_i \cdot \sigma_i^*$$

embedding energy at i

$$q_i = \sum_{j=i_1}^{i_N} \eta_{ji}(r_{ij})$$

charge on atom i

$$\sigma_i = \sum_{j=i_1}^{i_N} q_j \cdot \phi_{ij}(r_{ij})$$

electrical potential (in voltage) at i

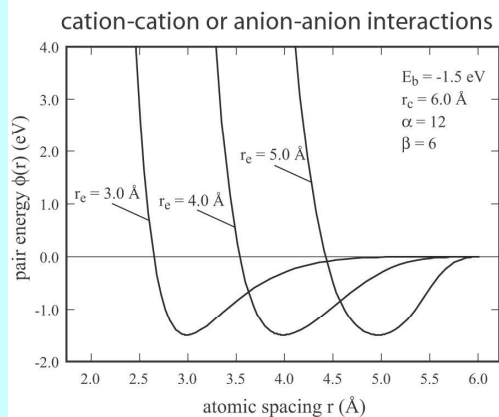
$$E = \frac{1}{2} \sum_{i=1}^N \sum_{j=i_1}^{i_N} \phi_{ij}(r_{ij}) + \sum_{i=1}^N \left\{ \left(\sum_{j=i_1}^{i_N} \eta_{ji}(r_{ij}) \right) \cdot \sum_{j=i_1}^{i_N} \left[\left(\sum_{k=j_1}^{j_N} \eta_{kj}(r_{jk}) \right) \cdot \phi(r_{ij}) \right] \right\}$$

*when $\phi_{ij}(r) \sim r^{-1}$, the embedding energy reduces to Coulomb interactions between i and its neighbors:

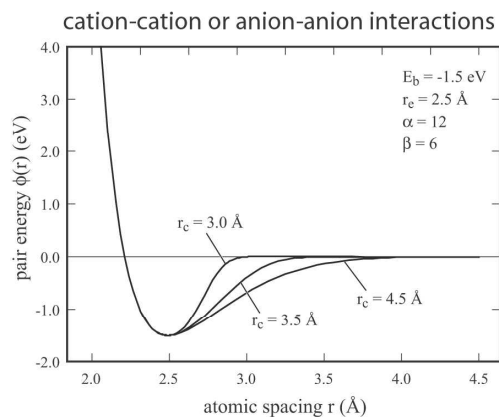
$$E_i(q_i, \sigma_i) = \frac{1}{2} \sum_{j=i_1}^{i_N} \frac{q_i \cdot q_j}{r_{ij}}$$

Fitting-Free Parameters

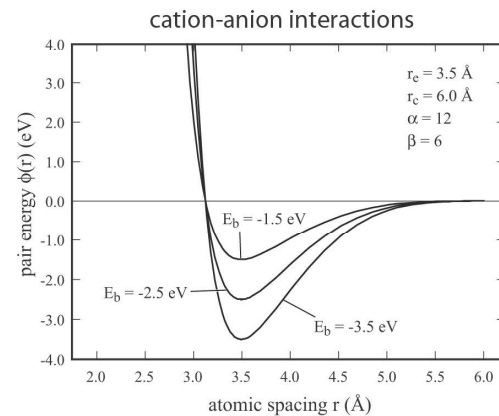
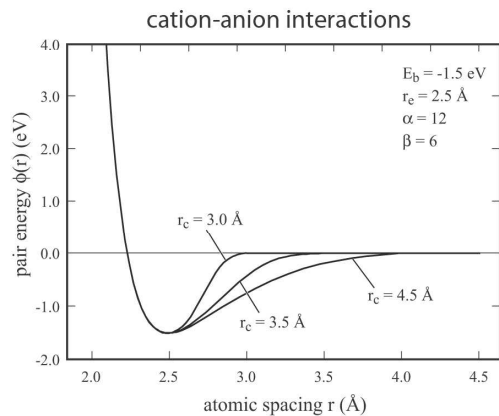
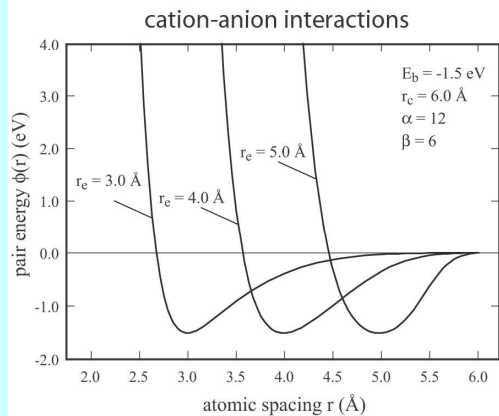
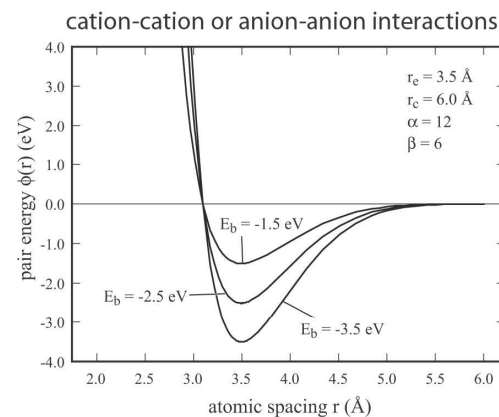
(a) effect of r_e



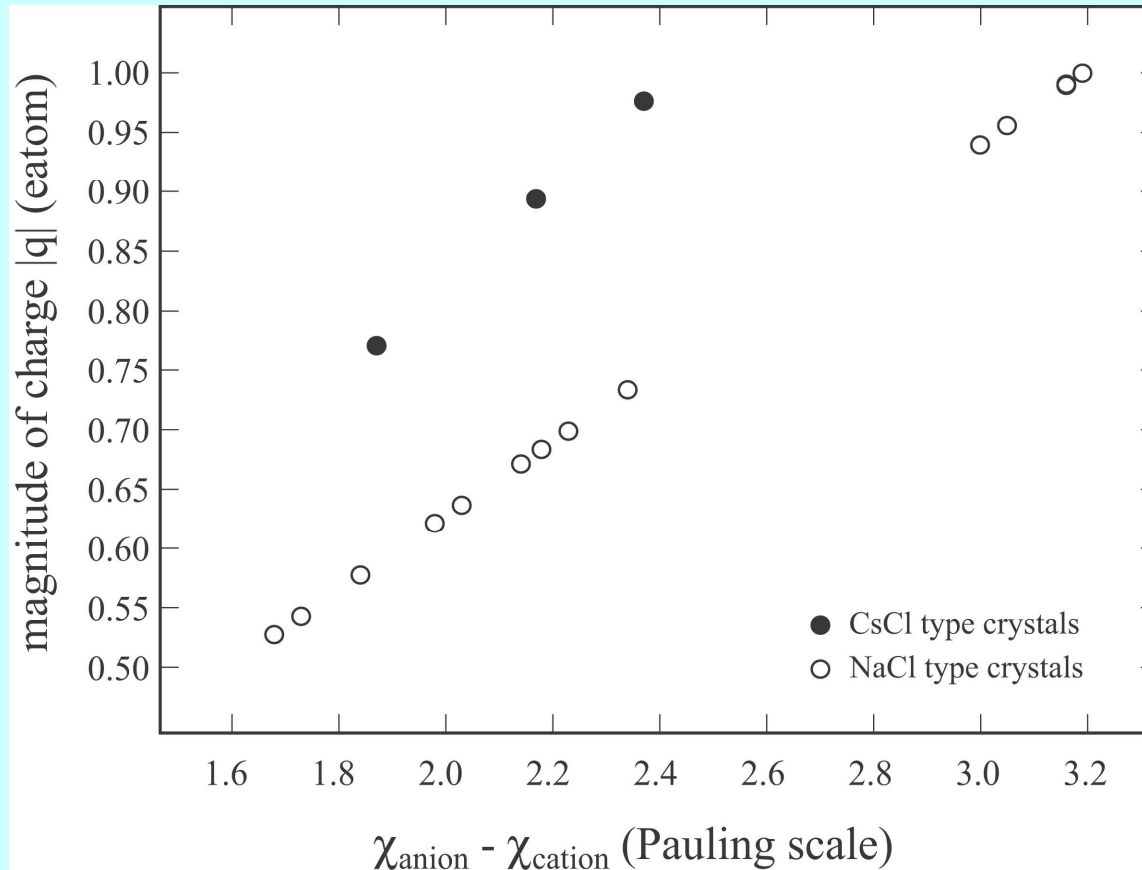
(b) effect of r_c



(c) effect of E_b

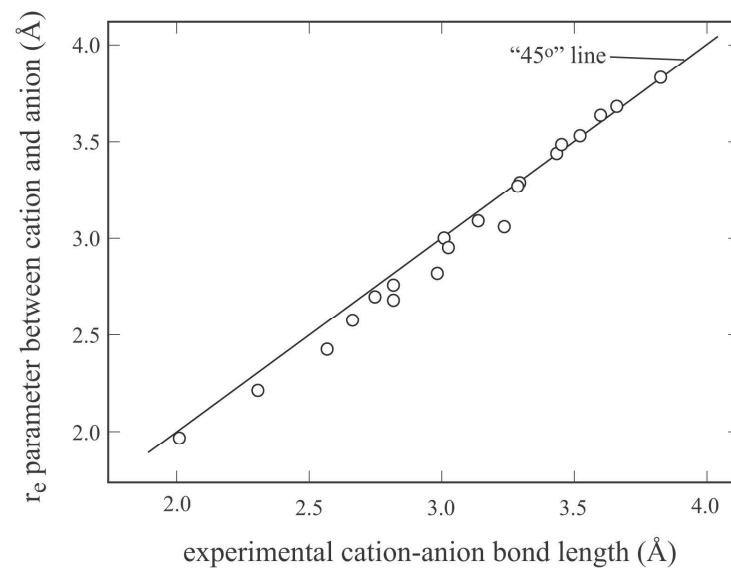
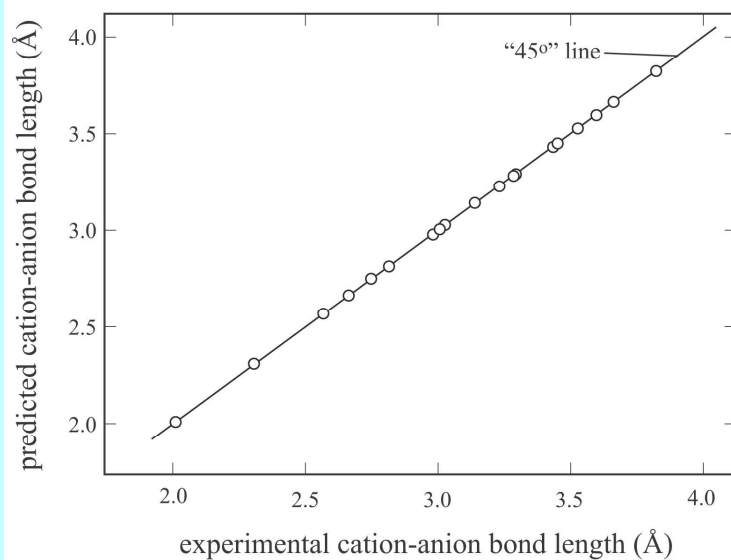


Predicted Charge vs. Electronegativity

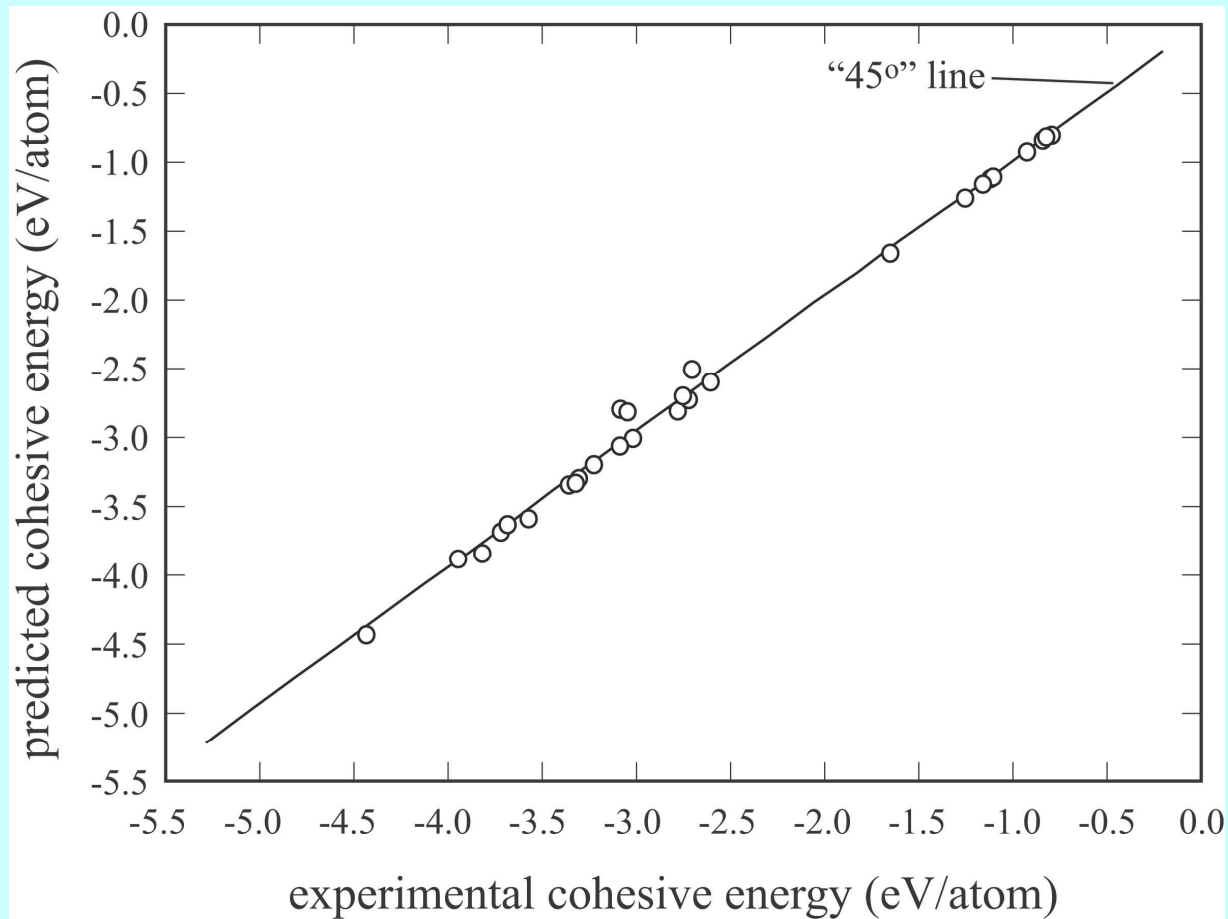


Cation-Anion Spacing

(a) predicted and experimental cation-anion bond length (b) r_e vs. experimental bond length for cation-anion pair

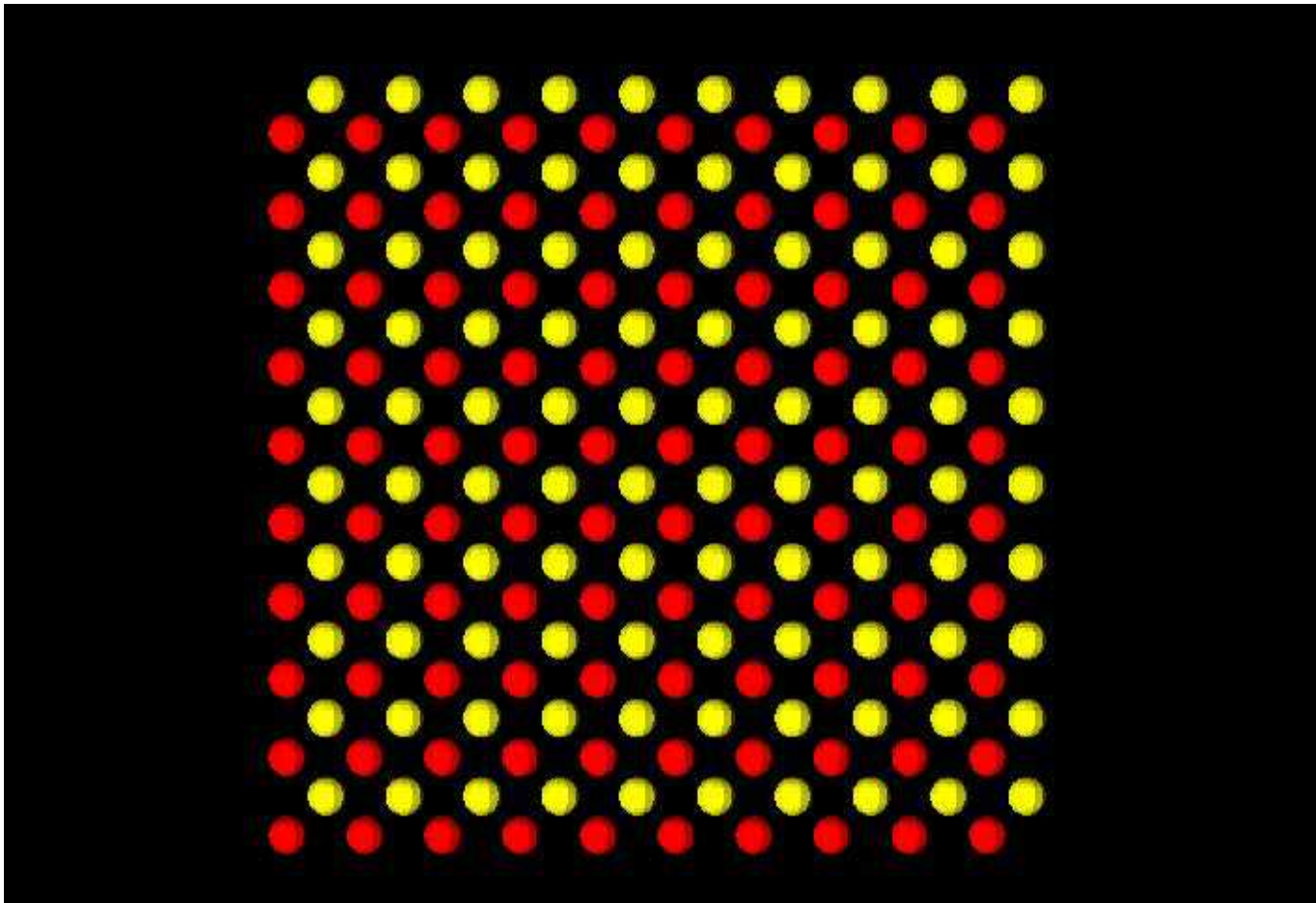


Cohesive Energy Prediction



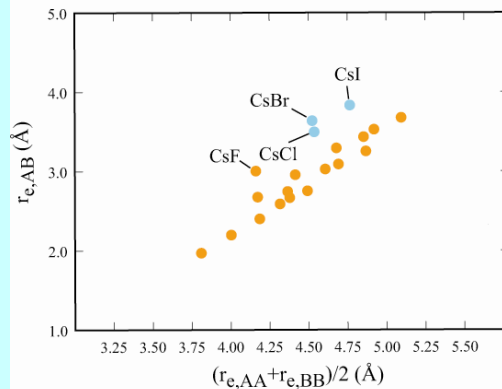
Simulated Annealing: NaCl at 300 K

Over a 25 ps period

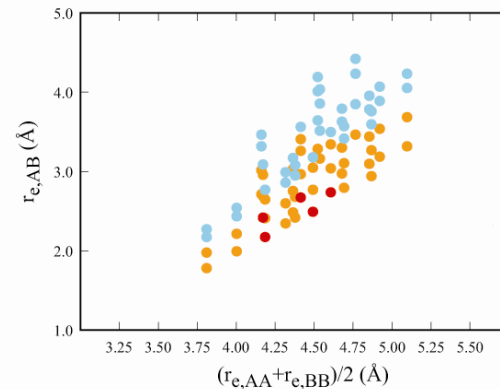


Crystal Phase Diagram

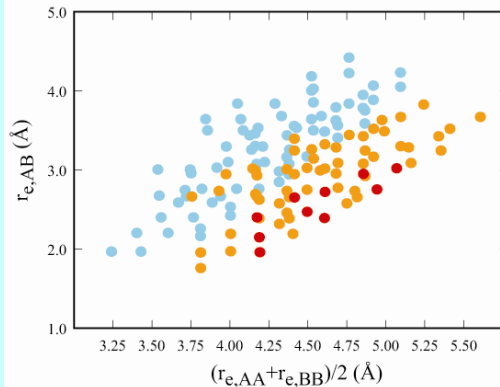
(a) 20 alkali halide structures



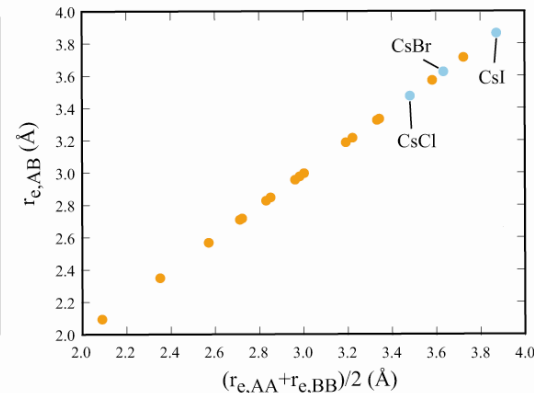
(b) varying $r_{e,AB}$



(c) varying $(r_{e,AA} + r_{e,BB})/2$

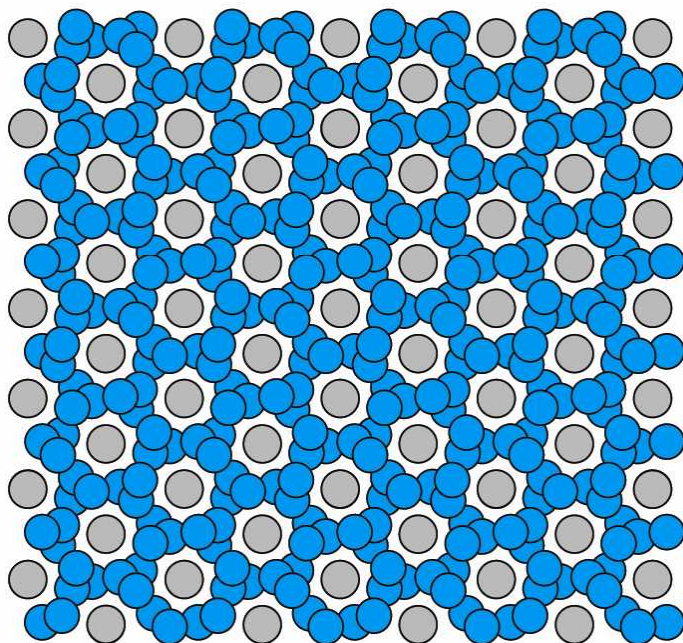


(d) the hard sphere model

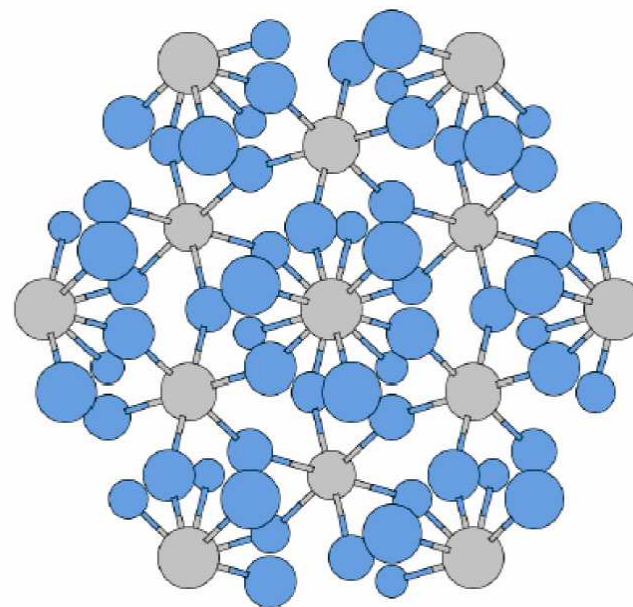


Working on Lanthanide

Predicted structure when r_{AB} becomes small



(0001) image of R32 structure
(compounds with small atoms, e.g., ScF_3)





Conclusions

- LAMMPS can now be used to explore crystal rule in the atomic size-electronegativity-bond energy space.
- Preliminary application of LAMMPS in the atomic size space already results in a new crystal rule beyond the hard sphere model.
- The new rule is mathematically very robust in that in AB binary compounds, larger A-B spacing with respect to A-A or B-B spacings favors the CsCl type of crystals and smaller A-B spacing with respect to A-A or B-B spacings favors the NaCl type of crystals.

Derivative Calculation

$$\begin{aligned}\sum_{i=1}^N \frac{\partial E_i(q_i, \sigma_i)}{\partial X} &= \frac{1}{2} \sum_{i=1}^N \sum_{j=i_1}^{i_N} \left[\frac{\partial q_i}{\partial X} \cdot q_j \cdot \phi(r_{ij}) + q_i \cdot \frac{\partial q_j}{\partial X} \cdot \phi(r_{ij}) + q_i \cdot q_j \cdot \frac{\partial \phi(r_{ij})}{\partial X} \right] \\ &= \frac{1}{2} \sum_{i=1}^N \left(\frac{\partial q_i}{\partial X} \cdot \sigma_i \right) + \frac{1}{2} \sum_{j=1}^N \left(\frac{\partial q_j}{\partial X} \cdot \sigma_j \right) + \frac{1}{2} \sum_{i=1}^N \sum_{j=i_1}^{i_N} \left[q_i \cdot q_j \cdot \frac{\partial \phi(r_{ij})}{\partial X} \right] \\ &= \sum_{i=1}^N \left(\frac{\partial q_i}{\partial X} \cdot \sigma_i \right) + \frac{1}{2} \sum_{i=1}^N \sum_{j=i_1}^{i_N} \left[q_i \cdot q_j \cdot \frac{\partial \phi(r_{ij})}{\partial X} \right]\end{aligned}$$

Pair Functions

$$f_c(r, r_p, r_c) = \left[0.510204 \cdot \operatorname{erfc} \left(\frac{1.64498(2r - r_p - r_c)}{r_c - r_p} \right) - 0.010204 \right]$$

cutoff function (r_c : cutoff distance)

$$\phi_{ij}(r) = \left[\frac{E_{b,ij} \cdot \beta_{ij}}{\beta_{ij} - \alpha_{ij}} \cdot \left(\frac{r_{e,ij}}{r} \right)^{\alpha_{ij}} - \frac{E_{b,ij} \cdot \alpha_{ij}}{\beta_{ij} - \alpha_{ij}} \cdot \left(\frac{r_{e,ij}}{r} \right)^{\beta_{ij}} \right] \cdot f_c(r, r_{e,ij}, r_{c\phi,ij})$$

pair energy function between cation and anion

$$\phi_{ij}(r) = \left[\frac{E_{b,ij} \cdot \beta_{ij}}{\beta_{ij} - \alpha_{ij}} \cdot \exp \left(-\alpha_{ij} \cdot \frac{r - r_{e,ij}}{r_{e,ij}} \right) - \frac{E_{b,ij} \cdot \alpha_{ij}}{\beta_{ij} - \alpha_{ij}} \cdot \exp \left(-\beta_{ij} \cdot \frac{r - r_{e,ij}}{r_{e,ij}} \right) \right] \cdot f_c(r, r_{e,ij}, r_{c\phi,ij})$$

pair energy function between cations or between anions

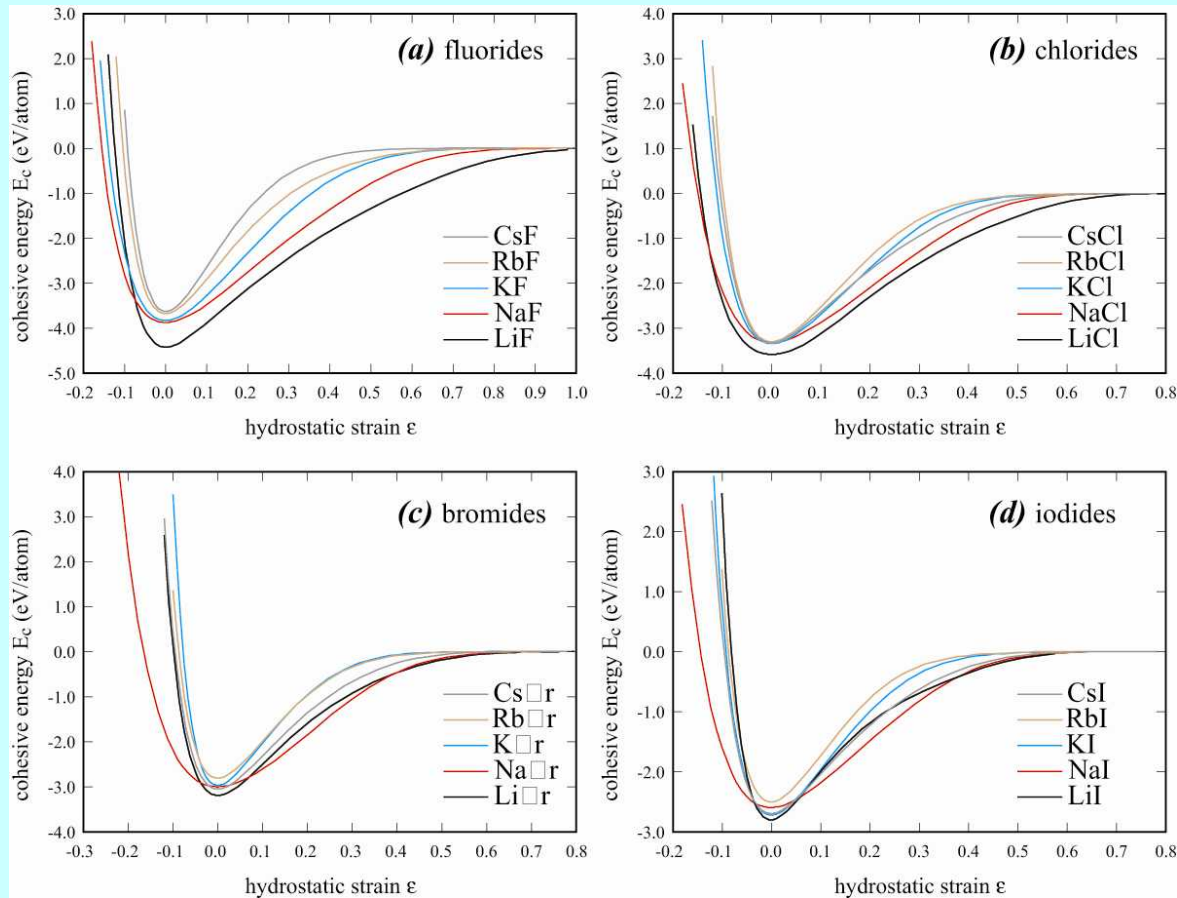
$$\eta_{ji}(r) = A_{\eta_{ij}} (\chi_j - \chi_i) \cdot f_c(r, r_{s\eta,ij}, r_{c\eta,ij})$$

charge transfer function (χ : electronegativity)

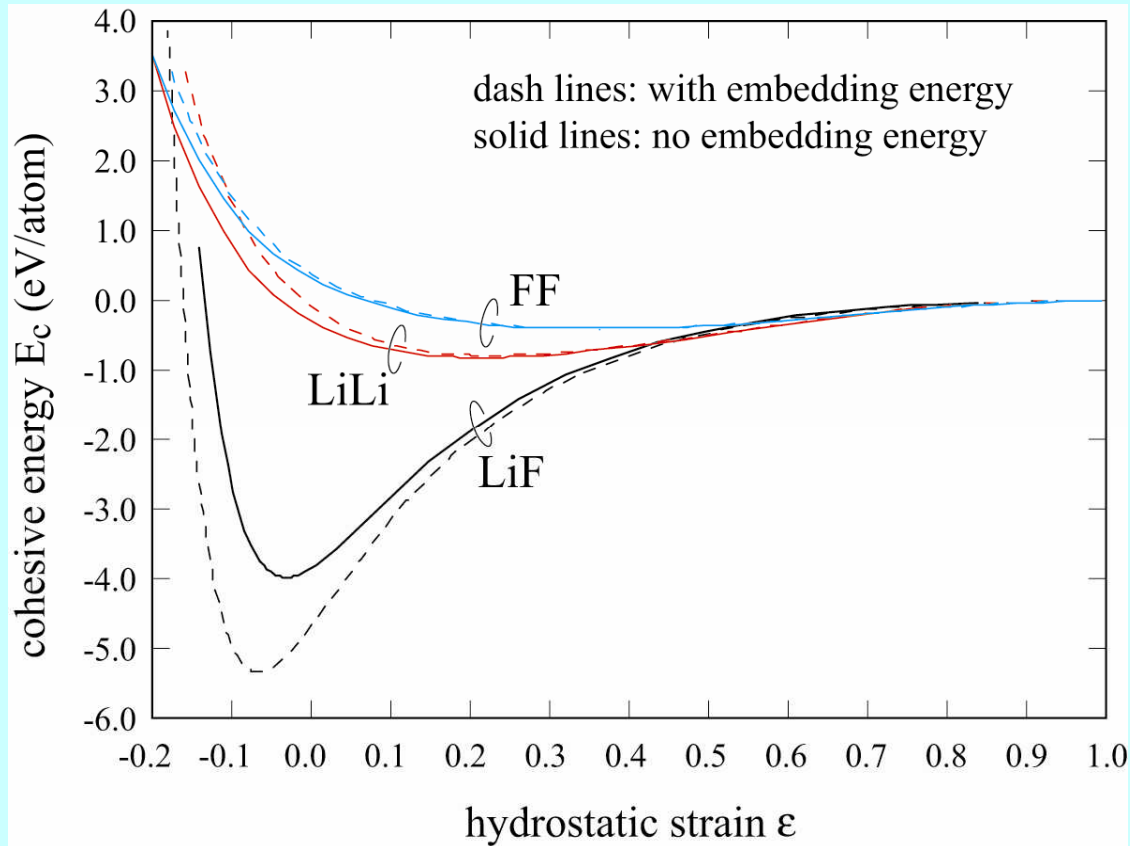
$$\varphi_{ij}(r) = A_{\varphi_{ij}} \cdot \exp(-\zeta_{ij} \cdot r) \cdot f_c(r, r_{s\varphi,ij}, r_{c\varphi,ij})$$

Charge interaction function

Cohesive Energy Profiles

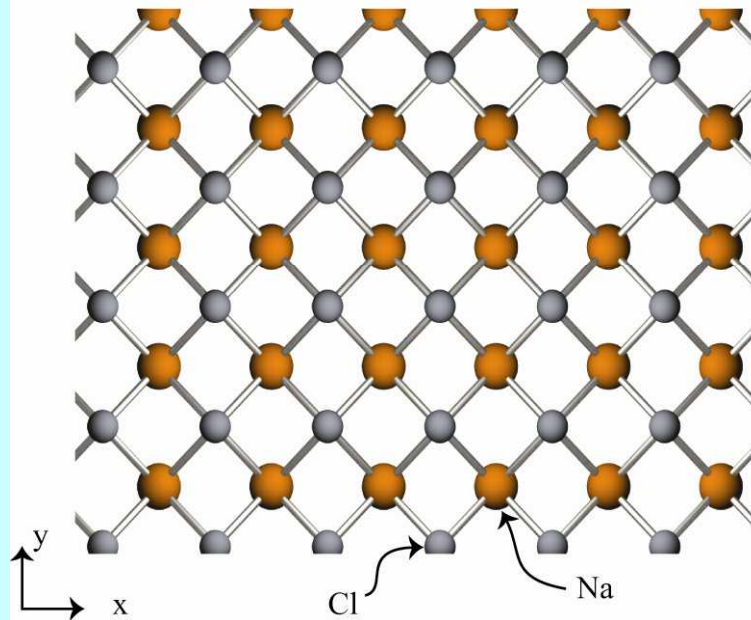


Reactivity of EIM

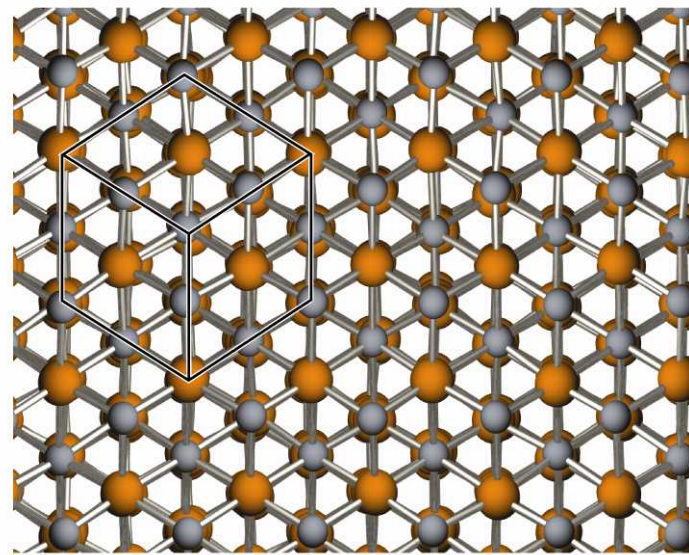


Simulated Annealing

(a) initial “CsCl” crystal structure of NaCl

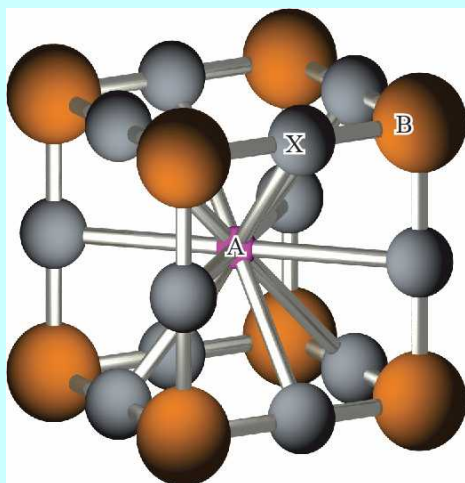


(b) after 10 ns 1400-to-300 K annealing



Goldschmidt Criteria^{1,2}

Perovskite



$$r_{AX} / \sqrt{2} \cdot r_{BX} = 1$$

Hard sphere mode: $r_{AX} = R_A + R_X$; $r_{BX} = R_B + R_X$.

Basic criterion: $0.77 < \frac{R_A + R_X}{\sqrt{2}(R_B + R_X)} < 1.00$

Problems:

1. Hard sphere model not true.
2. Environment-independent atomic size not true.
3. Electronegativity effects not incorporated.
4. Bond energy effects not incorporated.

Limitations: Goldschmidt criterion has only been successful for perovskite crystals. Even so, a large number of non-perovskite crystals were found to satisfy the criterion, and a large number of perovskite crystals were found to not satisfy the criterion.

- [1]. V. M. Goldschmidt, *Geochemische Verhältnisse der Elemente*. Norske Videnskap, Oslo, 1927;
[2]. L. Liang, L. Wencong, and C. Nianyi, *J. Phys. Chem. Sol.*, **65**, 855 (2004).