SAND2010-1209C

# Unstructured Hexahedra Mesh Generation and Modification via Direct Sheet Manipulation (i.e. no templates)

**Hex Meshing and CFD blocking seminar**
**Cambridge, England**
**March 9-10, 2010**

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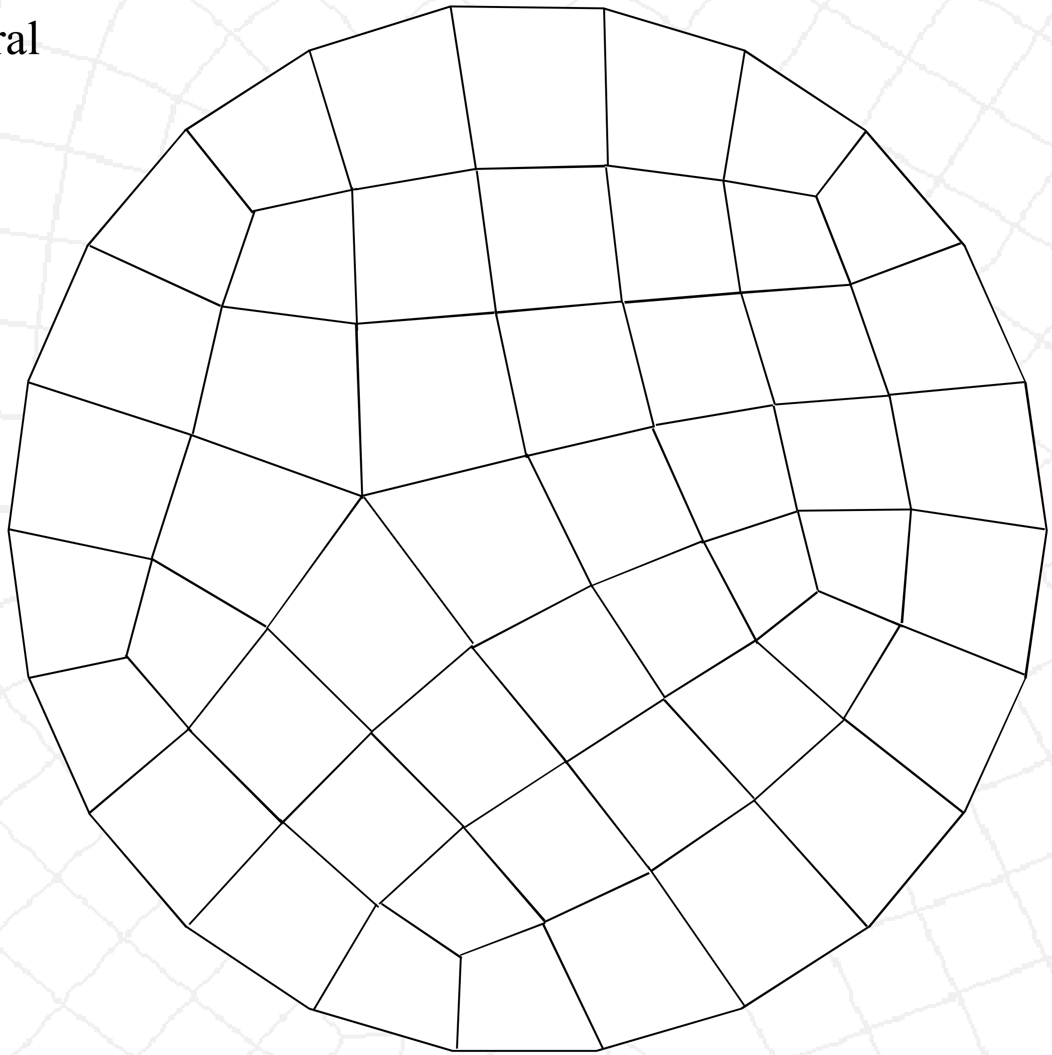
National Nuclear Security Administration

Sandia National Laboratories

# The Dual: An Alternate Representation of a mesh

Quadrilateral/hexahedral meshes have a dual representation, similar to the voroni skeleton of a triangular delaunay mesh.



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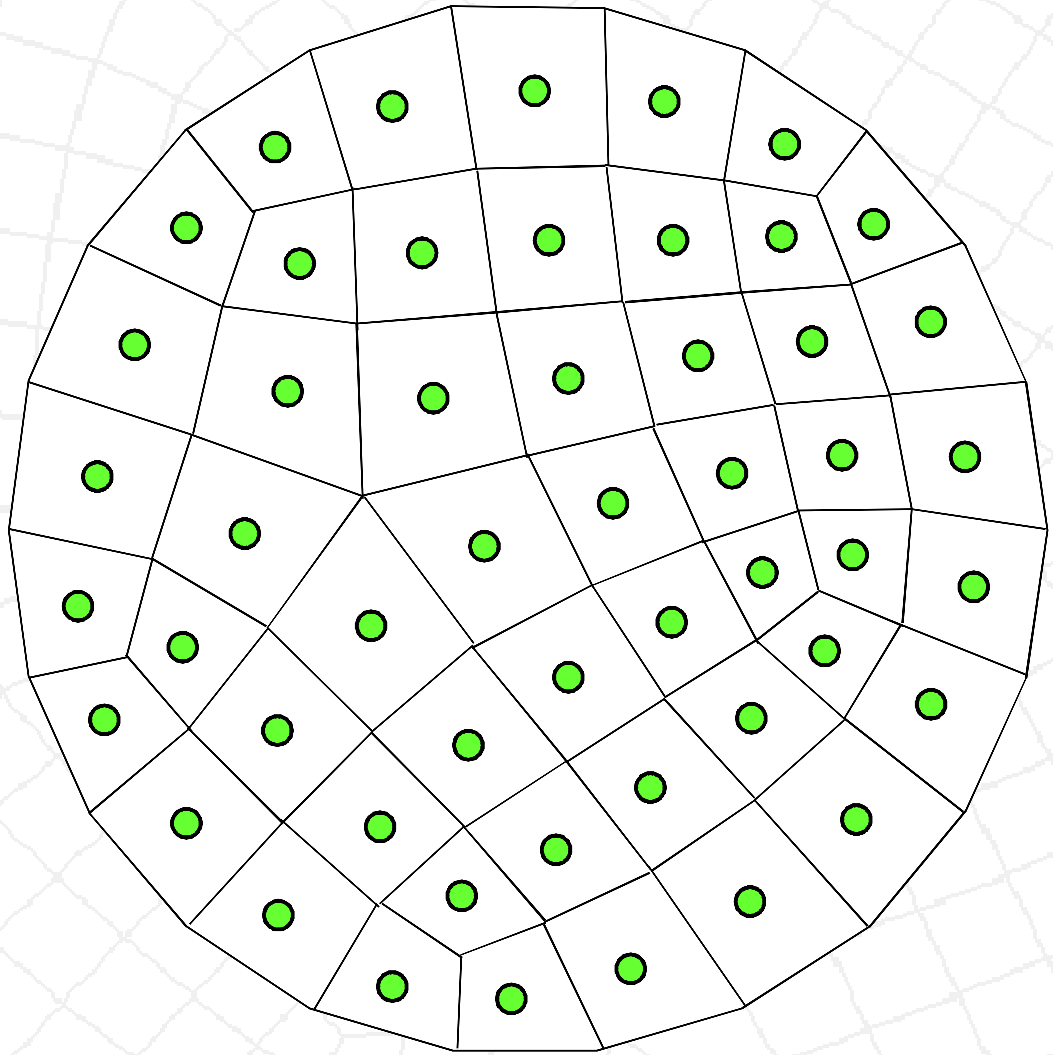
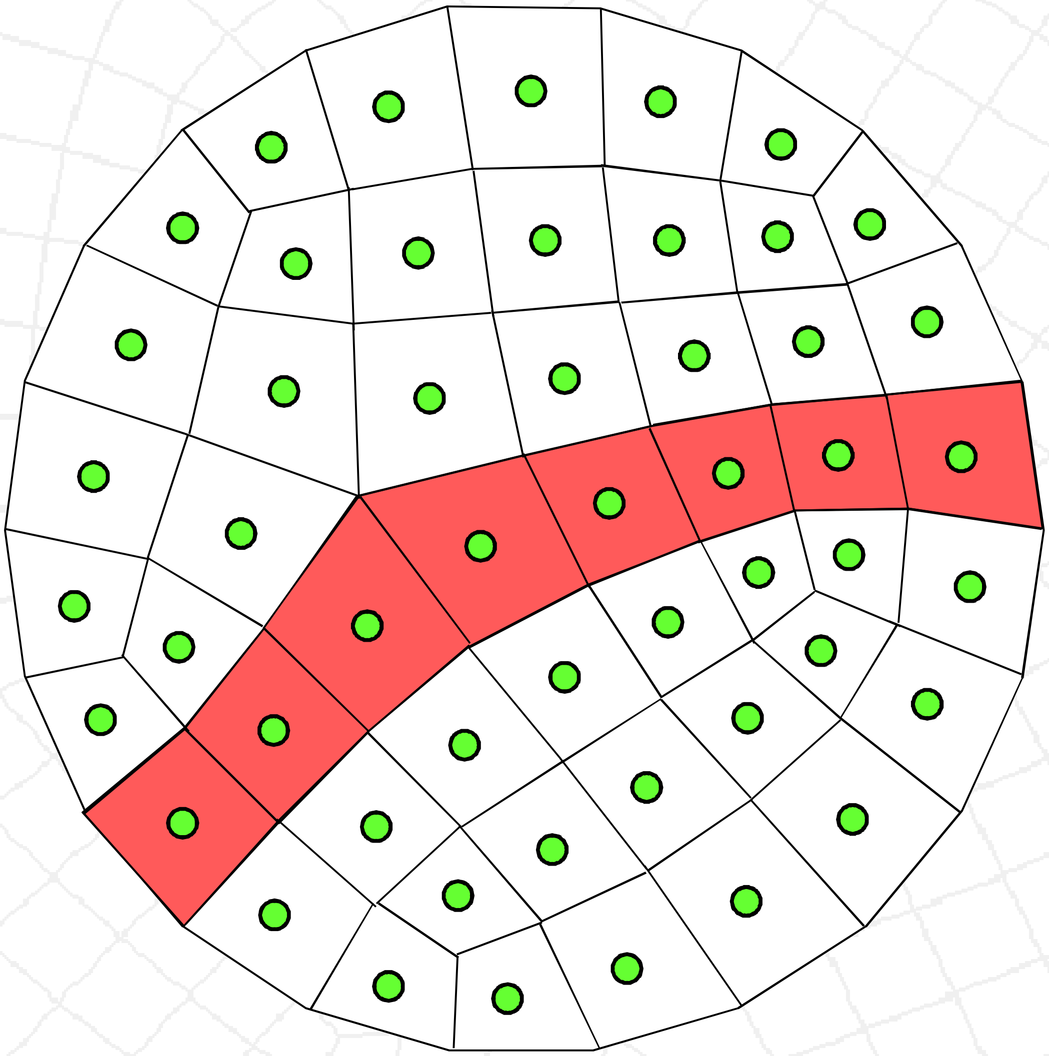
# The Dual: An Alternate Representation of a mesh

A dual vertex, $v_i$, is defined at the centroid of each quadrilateral element



# The Dual: An Alternate Representation of a mesh

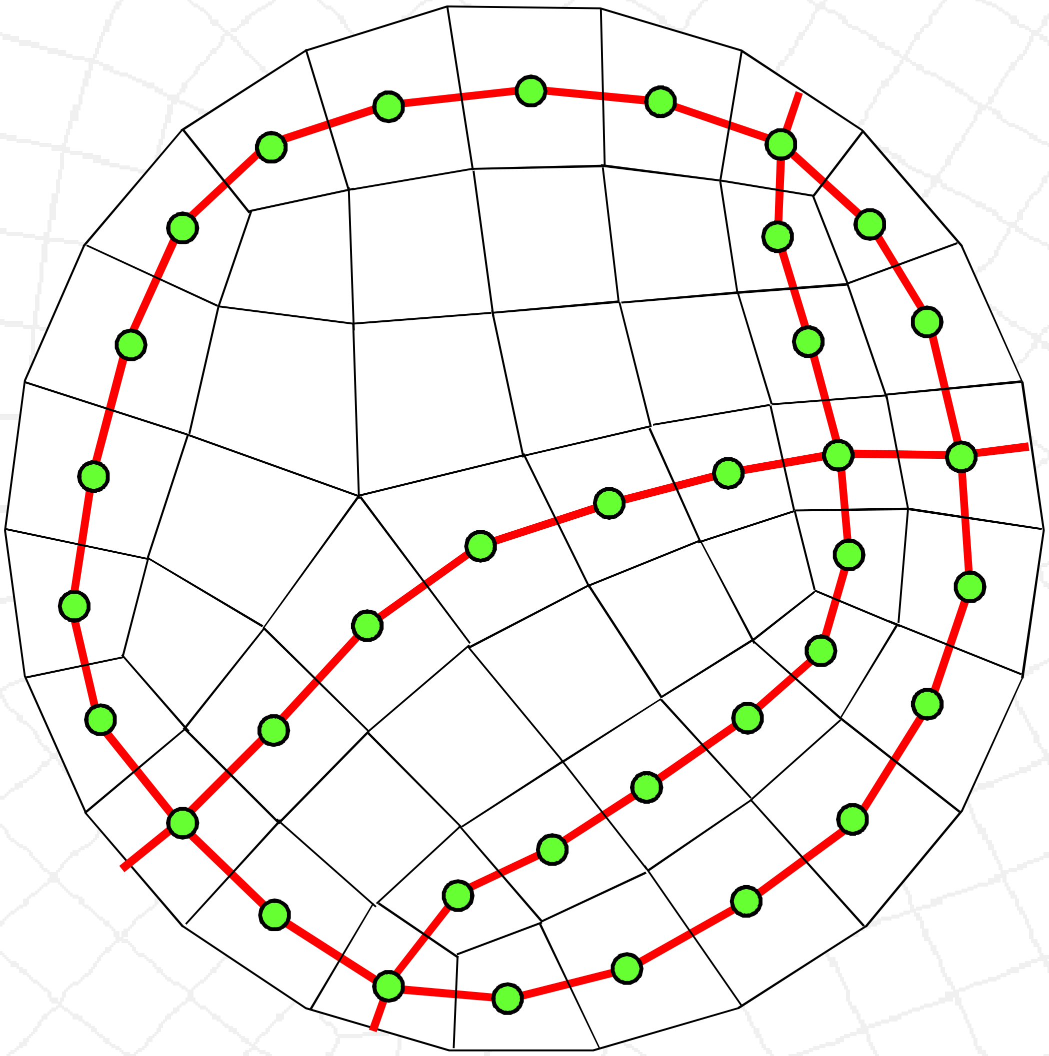
Quadrilateral meshes have an inherent row structure. For example, the red quadrilaterals form a single row of quads.



# The Dual: An Alternate Representation of a mesh

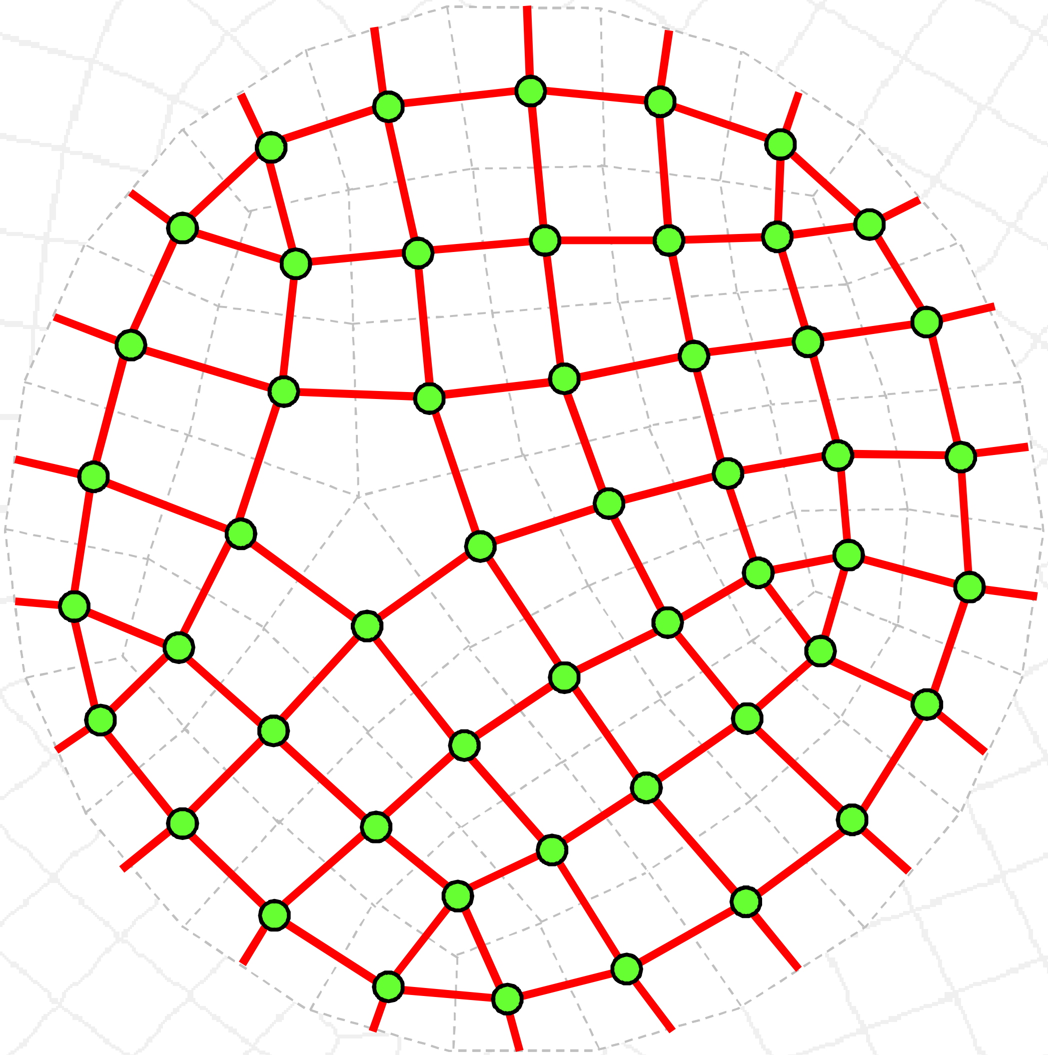
Connecting the dual vertices of adjacent quads in a row forms what we call a dual chord, $c_i$.

We can form a dual chord, $c_i$, for each row in the quad mesh.

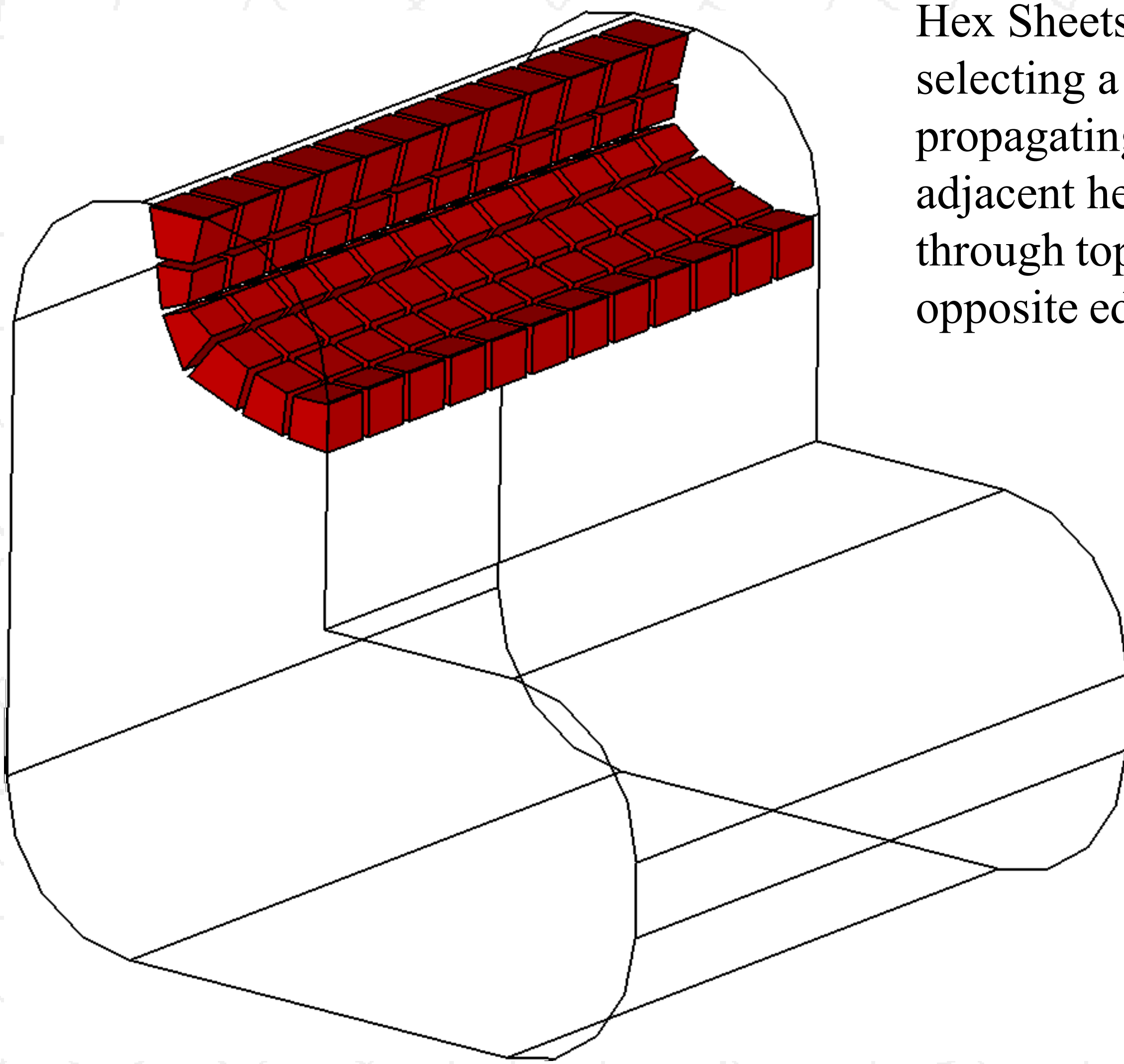


# The Dual: An Alternate Representation of a mesh

The collection of all dual chords, $c_i$, forms the dual.

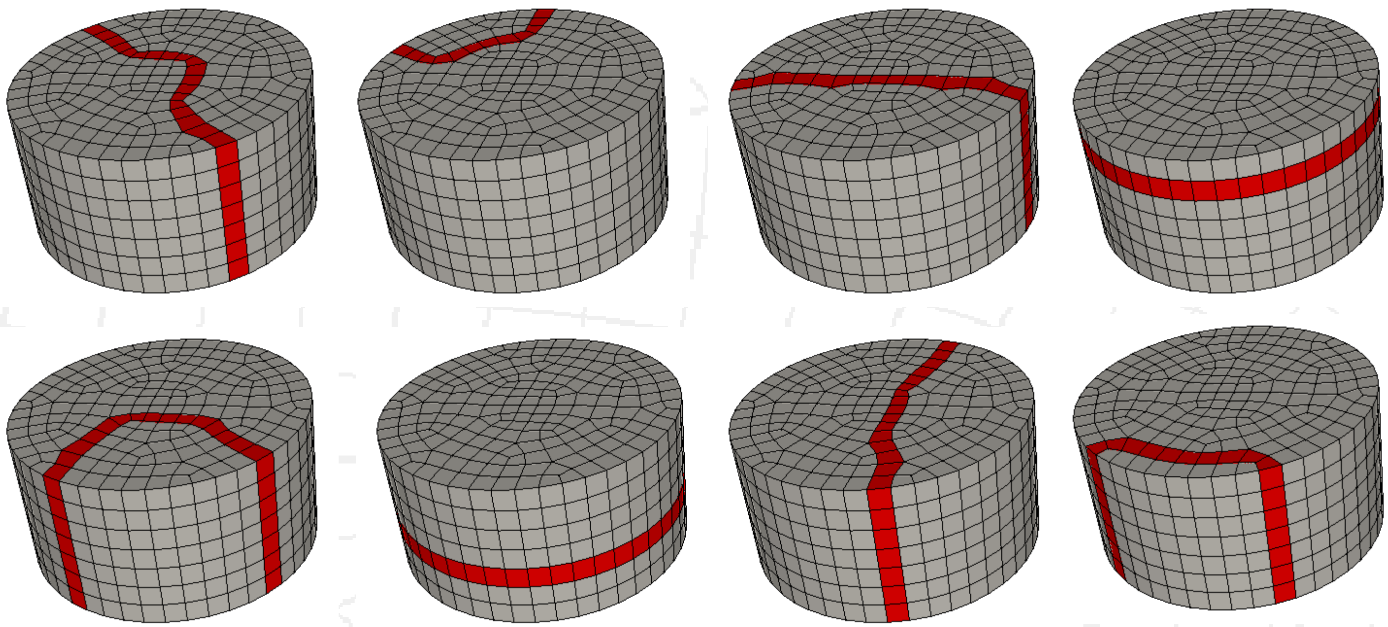


# Basic Definitions
# Hex Sheets



Hex Sheets are defined by selecting a single edge, and propagating through adjacent hex elements and through topologically opposite edges.

# Basic Definitions
# Hex Sheets



**Hex Sheets**

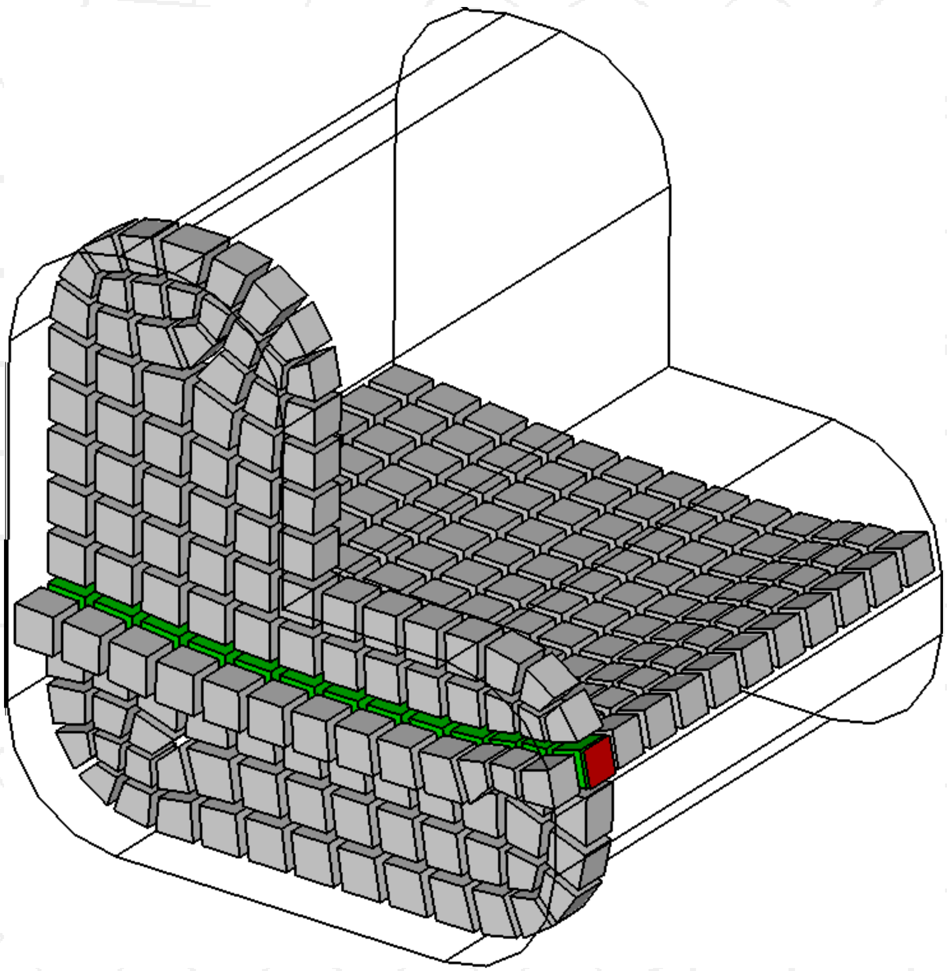
***Primal Mesh***

The set of all quads or hexes in a mesh

***Mesh Dual***

The set of all chords or sheets in a mesh

# Basic Definitions
# Hex Columns



Hex columns are defined by selecting a single face. We iteratively propagate through adjacent hexahedra and opposite faces until we return back to the starting face, or terminate on the boundary.

Hex Columns define the intersection of two hex sheets.

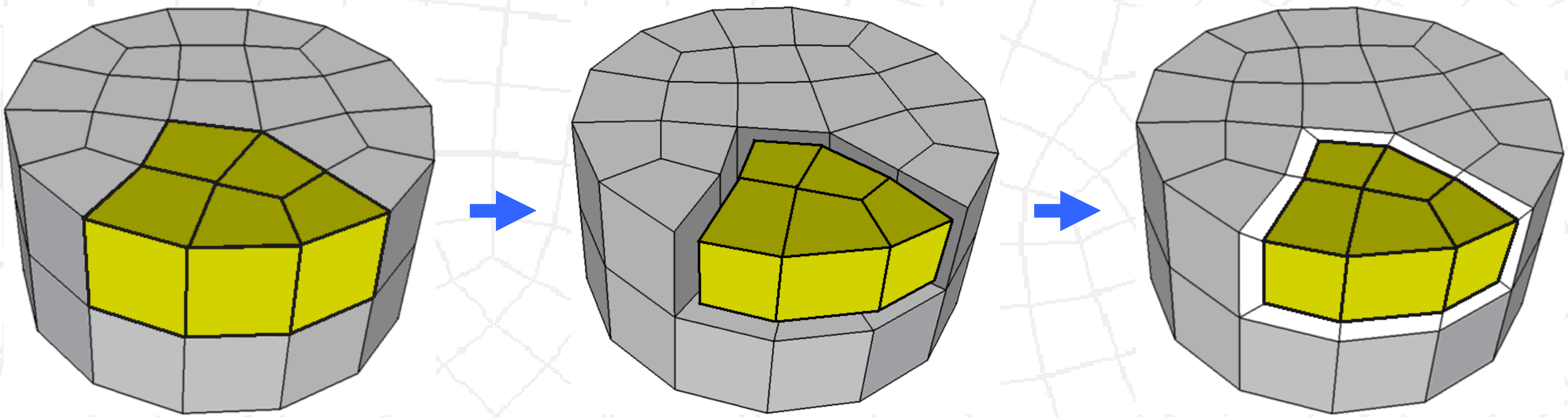
# Mesh Modification Toolbox

- Toolbox
  - Sheet Insertion (Pillowing
  - Sheet Extraction
  - Column  Collapse

- No Templates

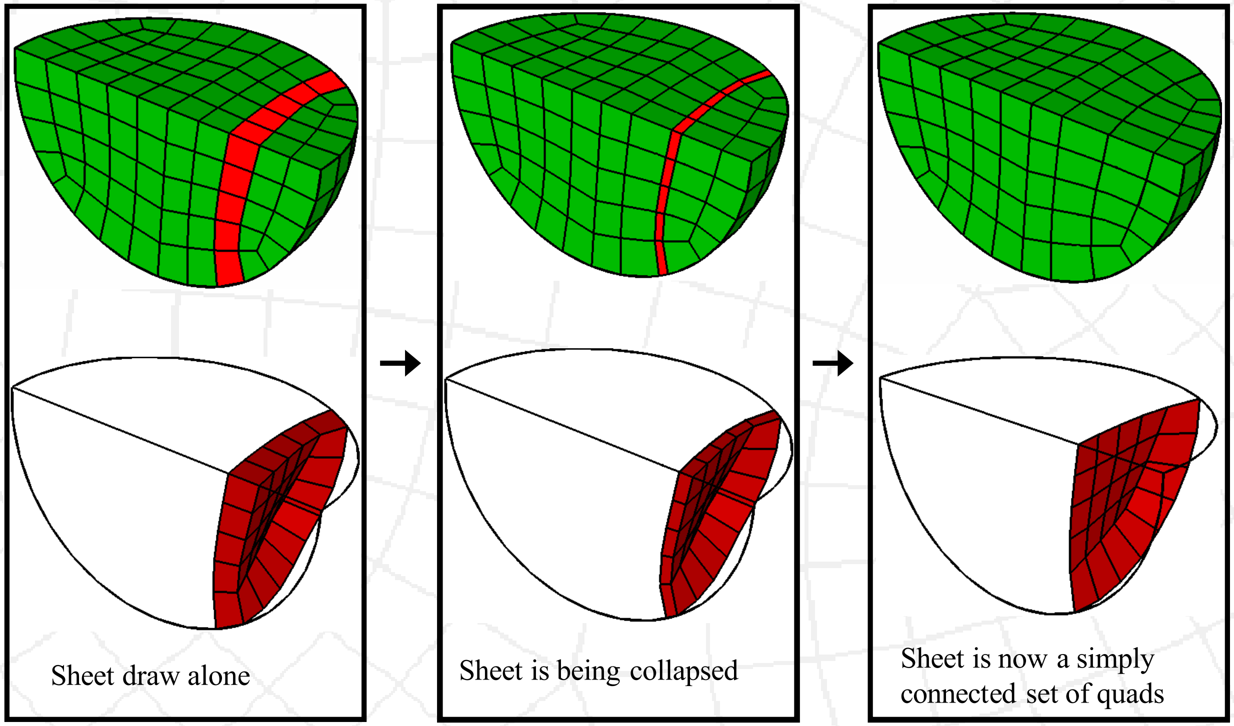
# Toolbox: Sheet Insertion (Pillowing)

- Pillowing – Mitchell et. al., 4th IMR 1995
  - Inserts arbitrary "Regular" sheets only, by defining a shrink set, creating a gap, and filling with new sheet.



# Toolbox: Sheet Extraction

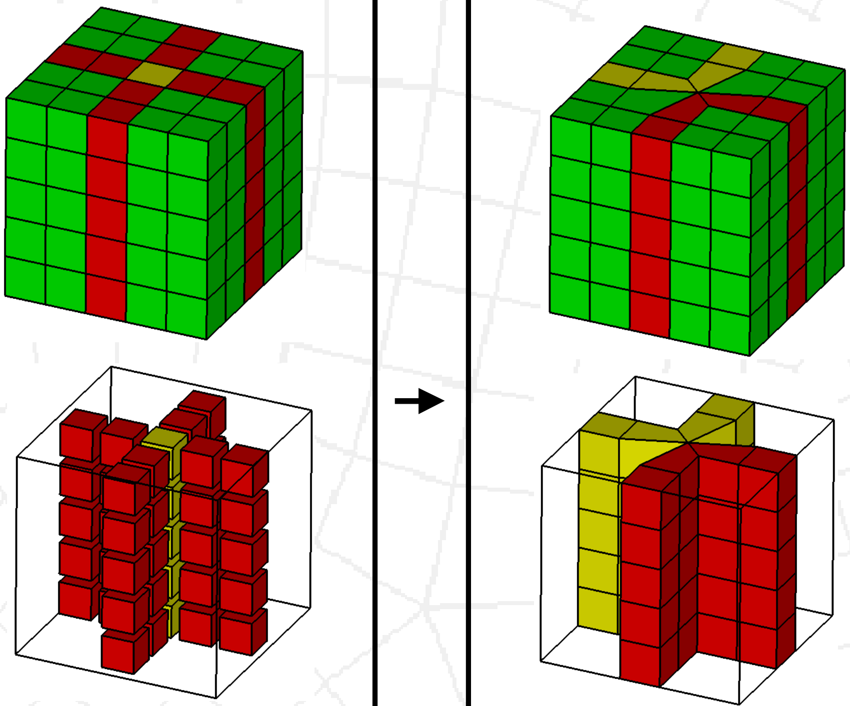
- Sheet Extract – Borden et. al. 11th IMR, 2002




- Any sheet can be extracted, including self-intersecting, and self-touching
- Extract can lead to geometric node- associativity problems.

# Toolbox: Column Collapse

**A column can be collapsed … which removes the intersection of the sheets. But collapse propagates along entire length of hex column.**



The yellow column at the intersection of the 2 red sheets is selected for collapse

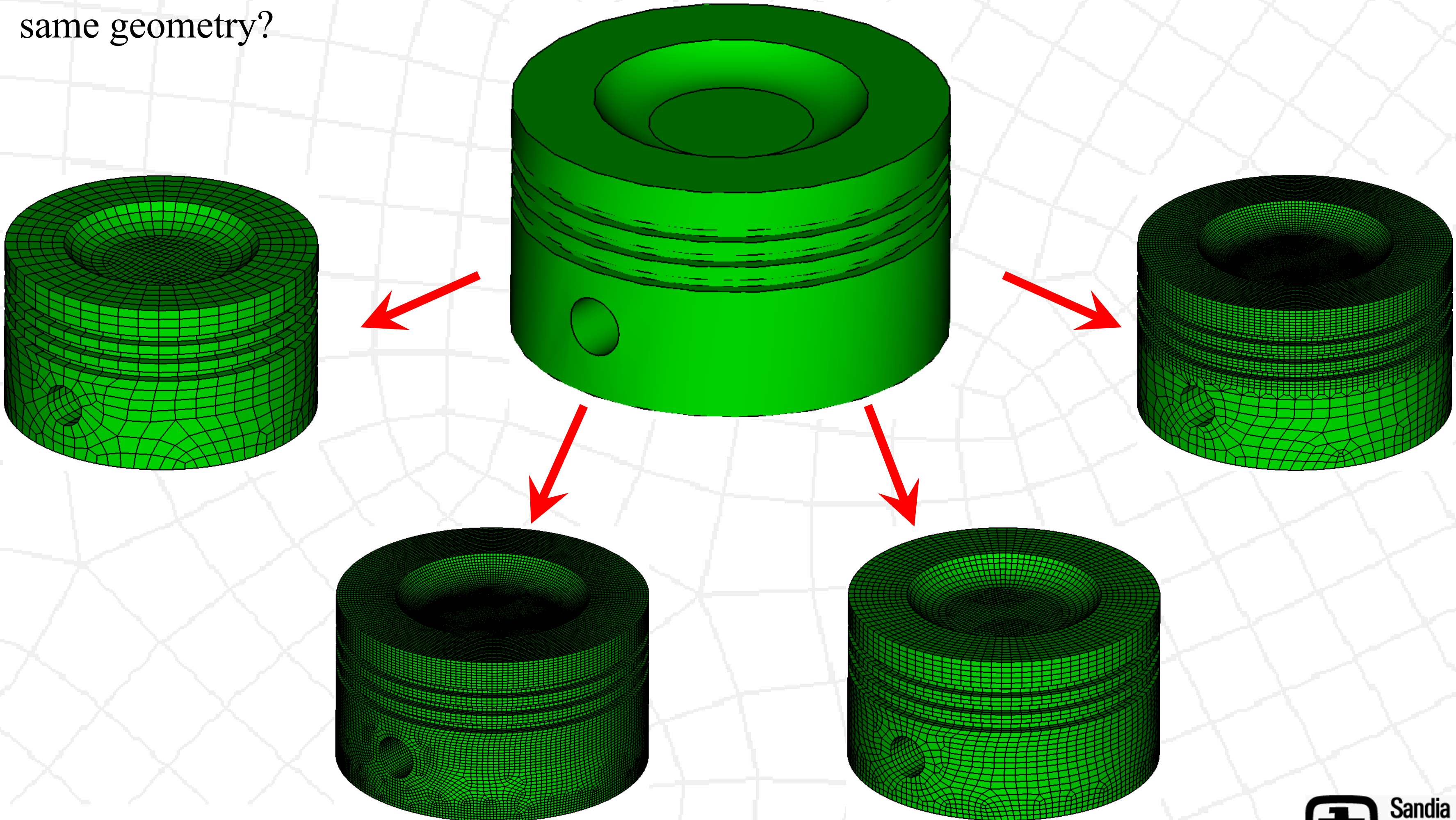
The column is collapsed and the intersecting sheets no longer intersect

# Toolbox Application?

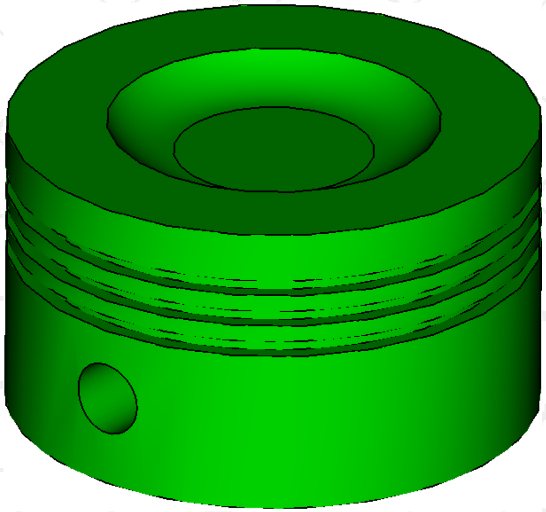
- Conversion of any mesh, $M_1$, on a geometry $G$, into any other mesh $M_2$, on the same geometry.
- Mesh Matching, creation of conforming component interfaces
- Localized all-hexahedral coarsening
- Fun Sheet Matching, automatic generation of hexahedral meshes.

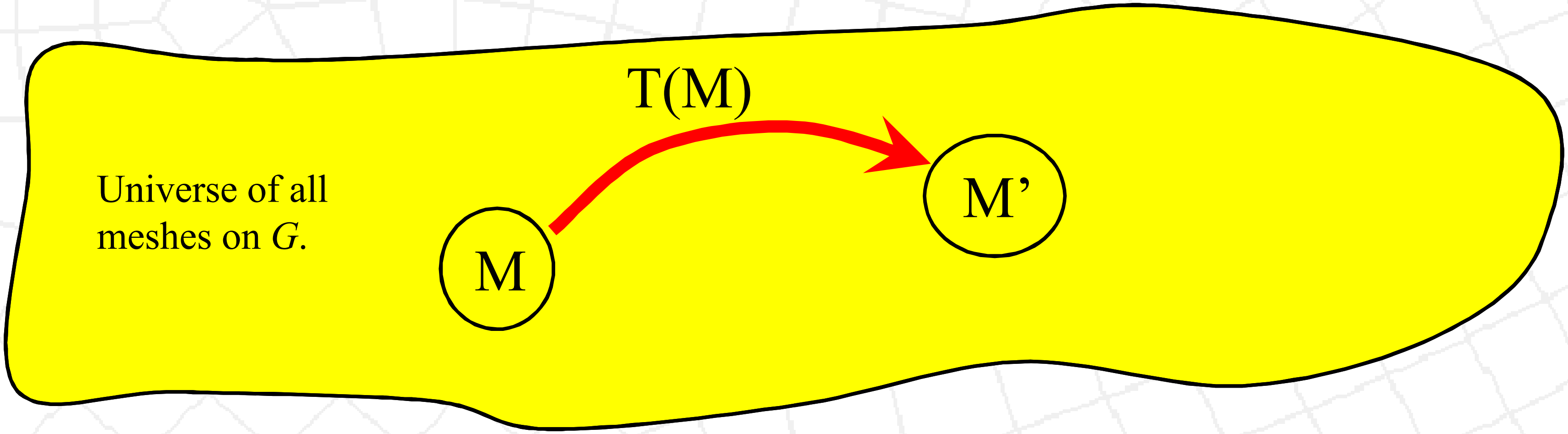
# Mesh Transformation

There are an infinite number of possible meshes on a given geometry, *G*. Given one of these meshes, $M_1$, is there a transformation which will convert it into any other mesh $M_2$, on the same geometry?



# Mesh Transformation





# Mesh Transformation

**Theorem:** *Let $M_1$ and $M_2$ be two hexahedral meshes of the same geometric model. There exists a series of transformations based on sheet insertions and extractions allowing the conversion of $M_1$ into $M_2$.*

2 proofs available for this:

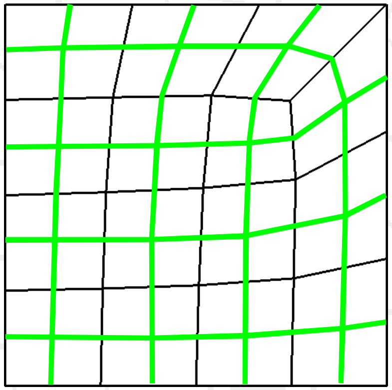
- Ledoux et al. [EngWithComp, 2010]
- Staten et al. [IJNME, 2010]

Proofs are based on sheet insertions and sheet extractions.

# Example:
# We seek a transformation from M to M’

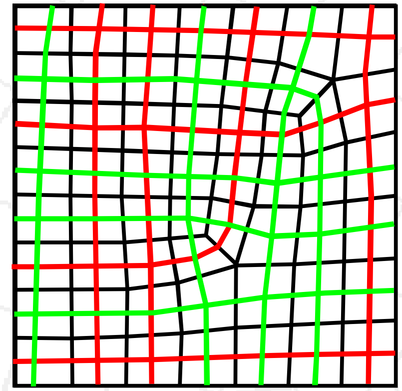
M=

D=



M∪M’=M”=

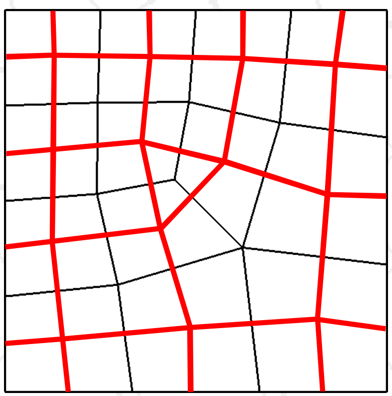
D∪D’=D”=



Not Unique

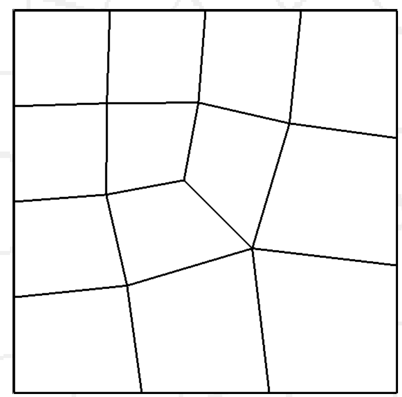
M’=

D’=



M”-M=M’=

D”-D=D’=

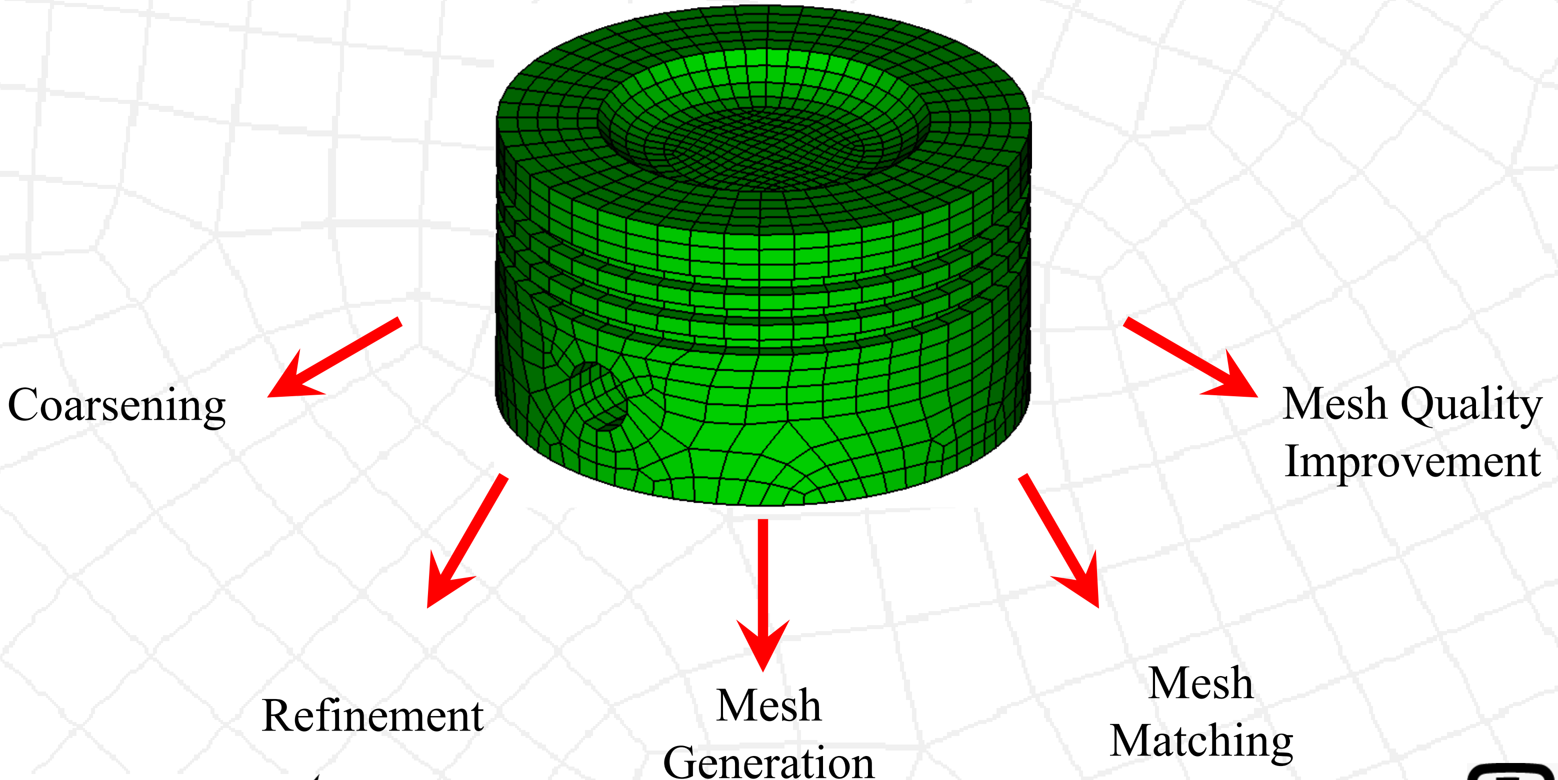


Unique!

# Mesh Transformation

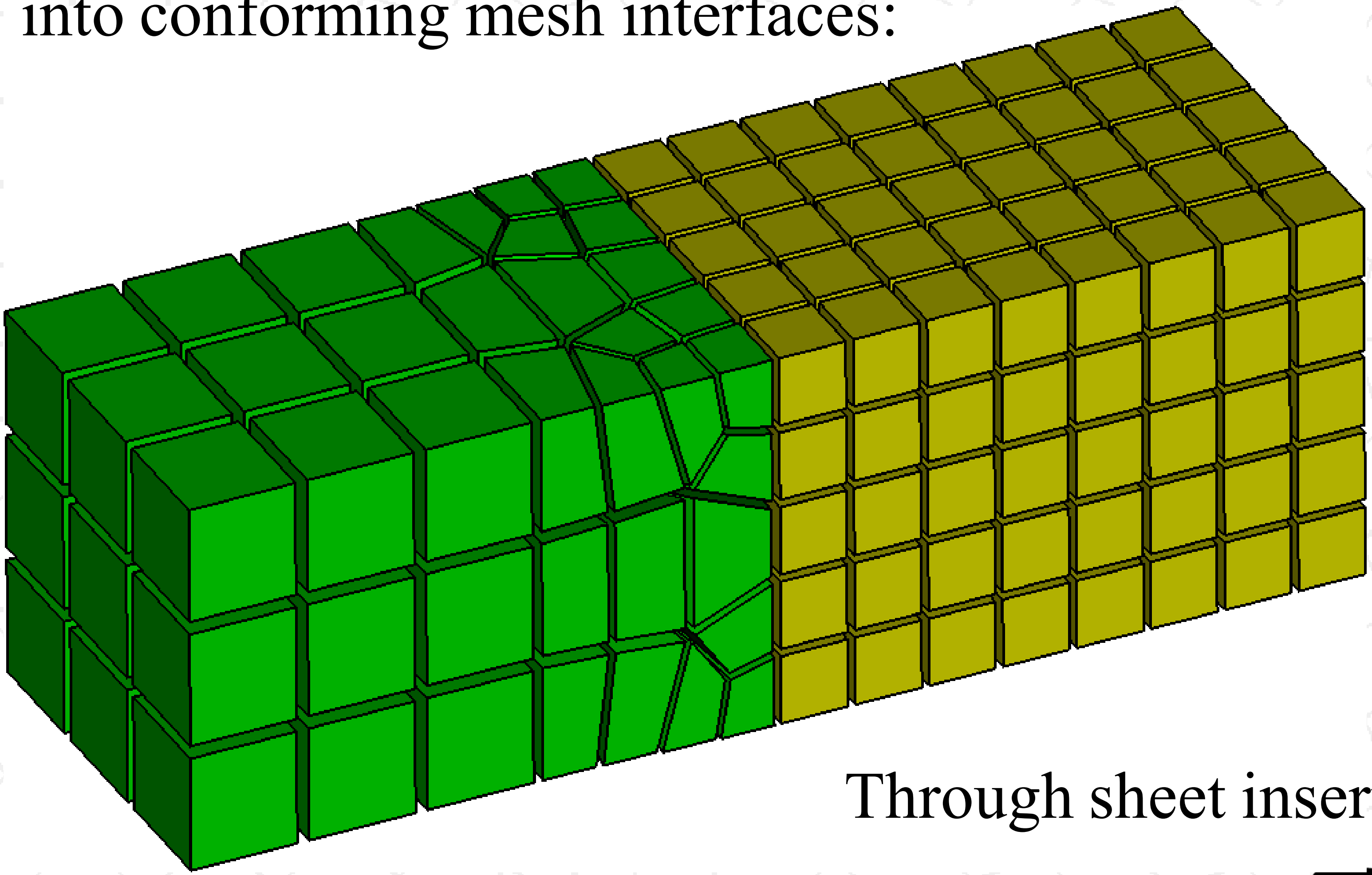
Extraction of all sheets in $M_1$ requires that the boundary of the input mesh change. Some previous hex topology optimizations were constrained by a fixed boundary.

We are free to define what our goal mesh, $M_2$, is, for whatever objective we have.



# Mesh Matching

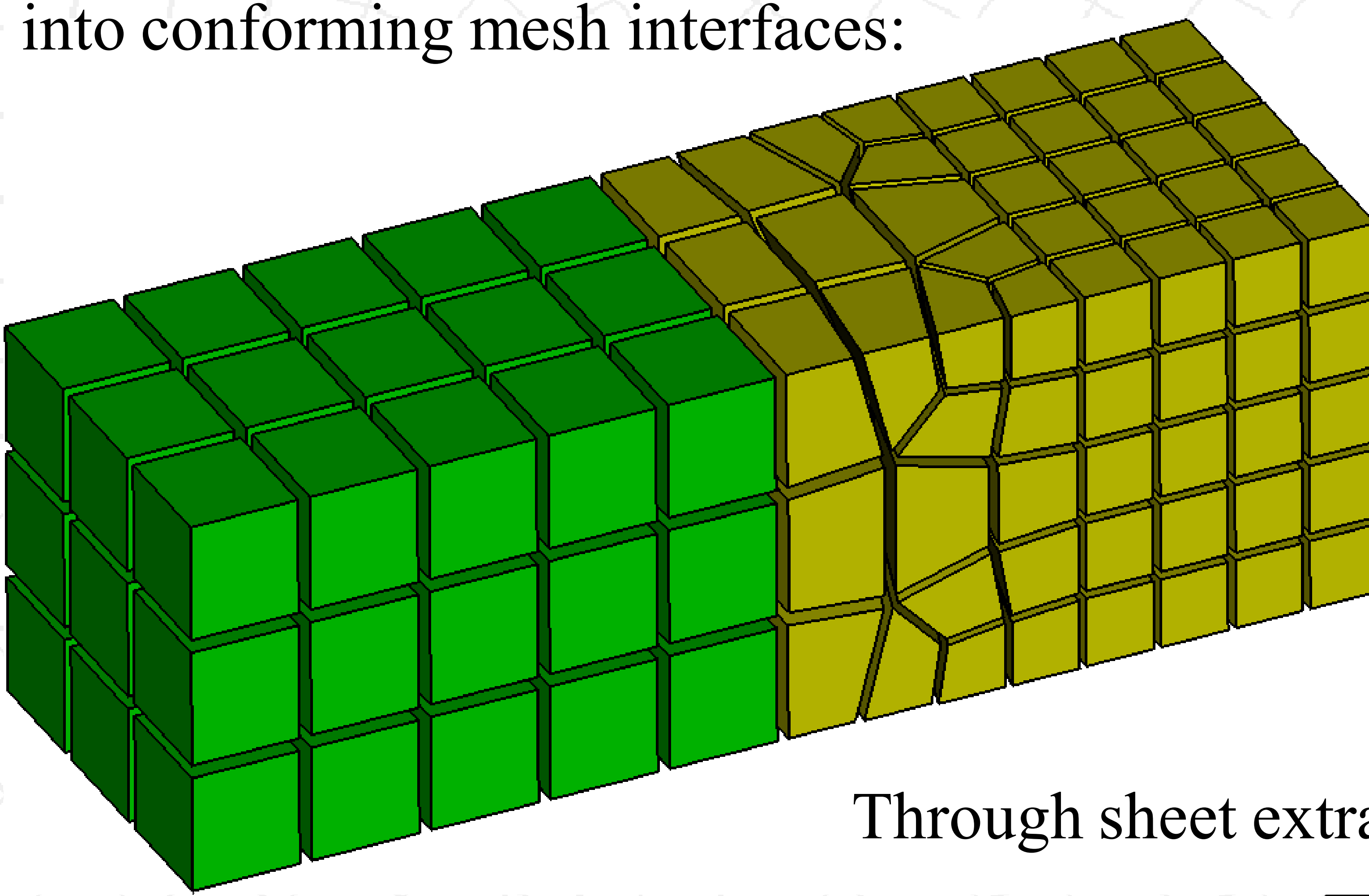
Converting non-conforming mesh interfaces into conforming mesh interfaces:



Through sheet insertion:

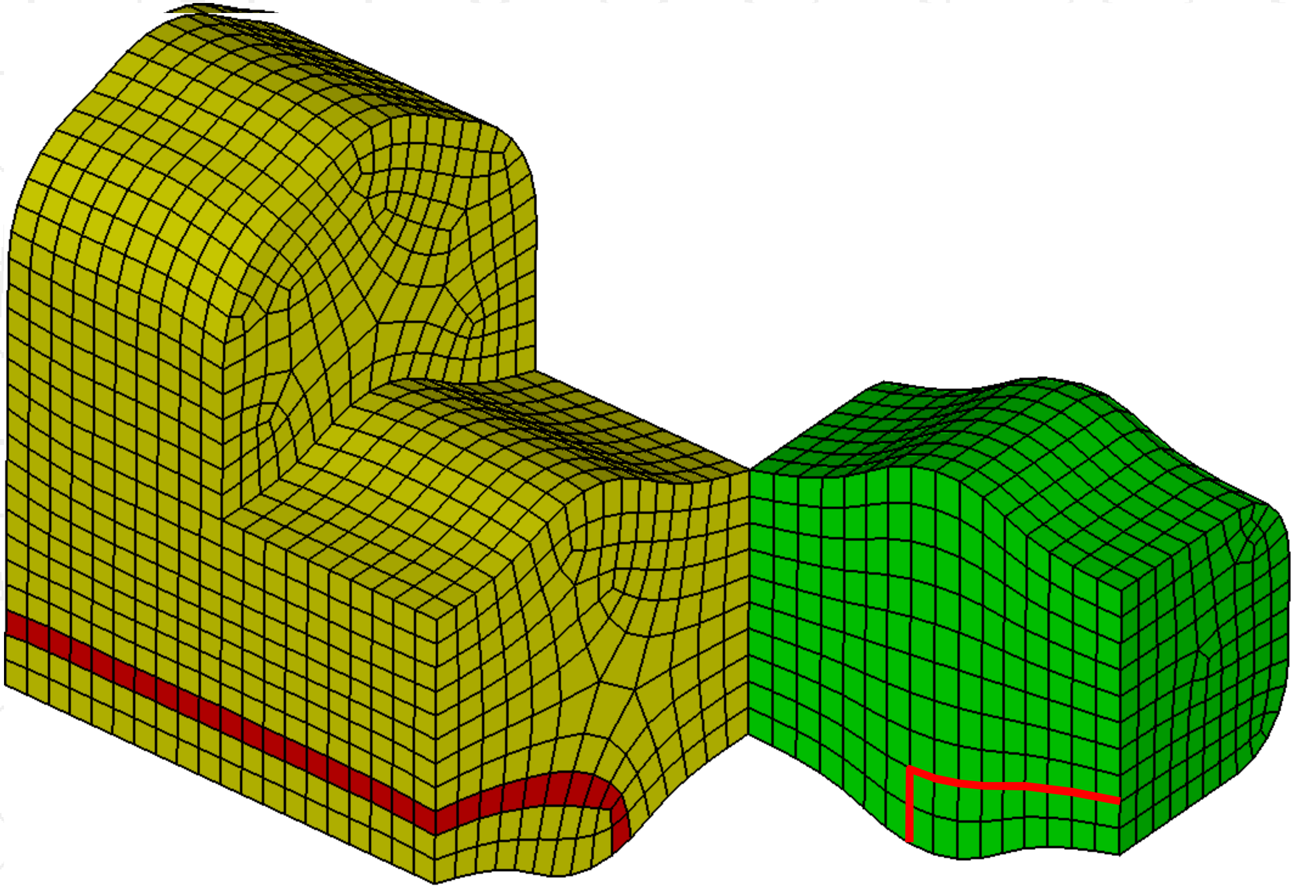
# Mesh Matching

Converting non-conforming mesh interfaces into conforming mesh interfaces:

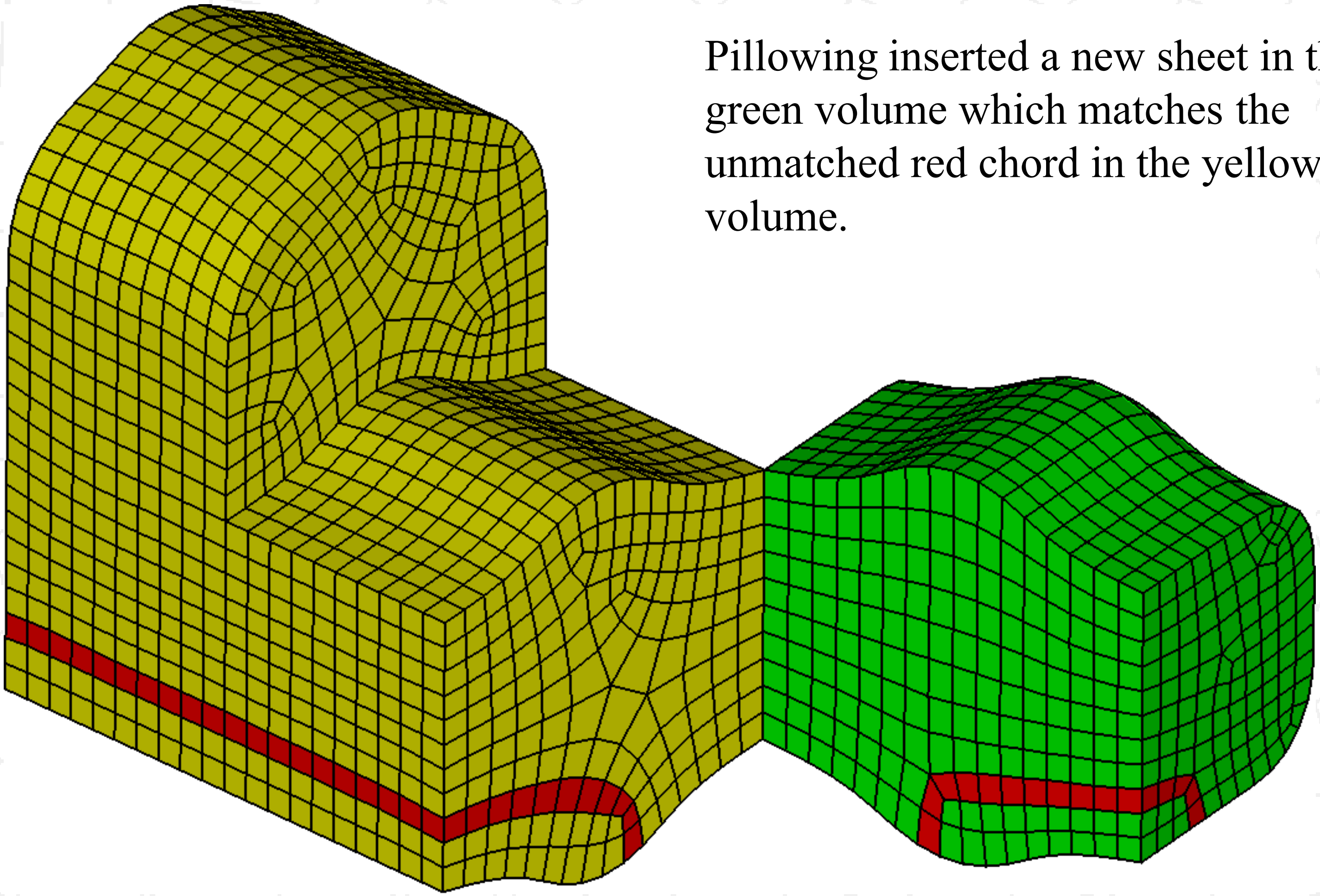


Through sheet extraction:

# Sheet Insertion in Mesh Matching



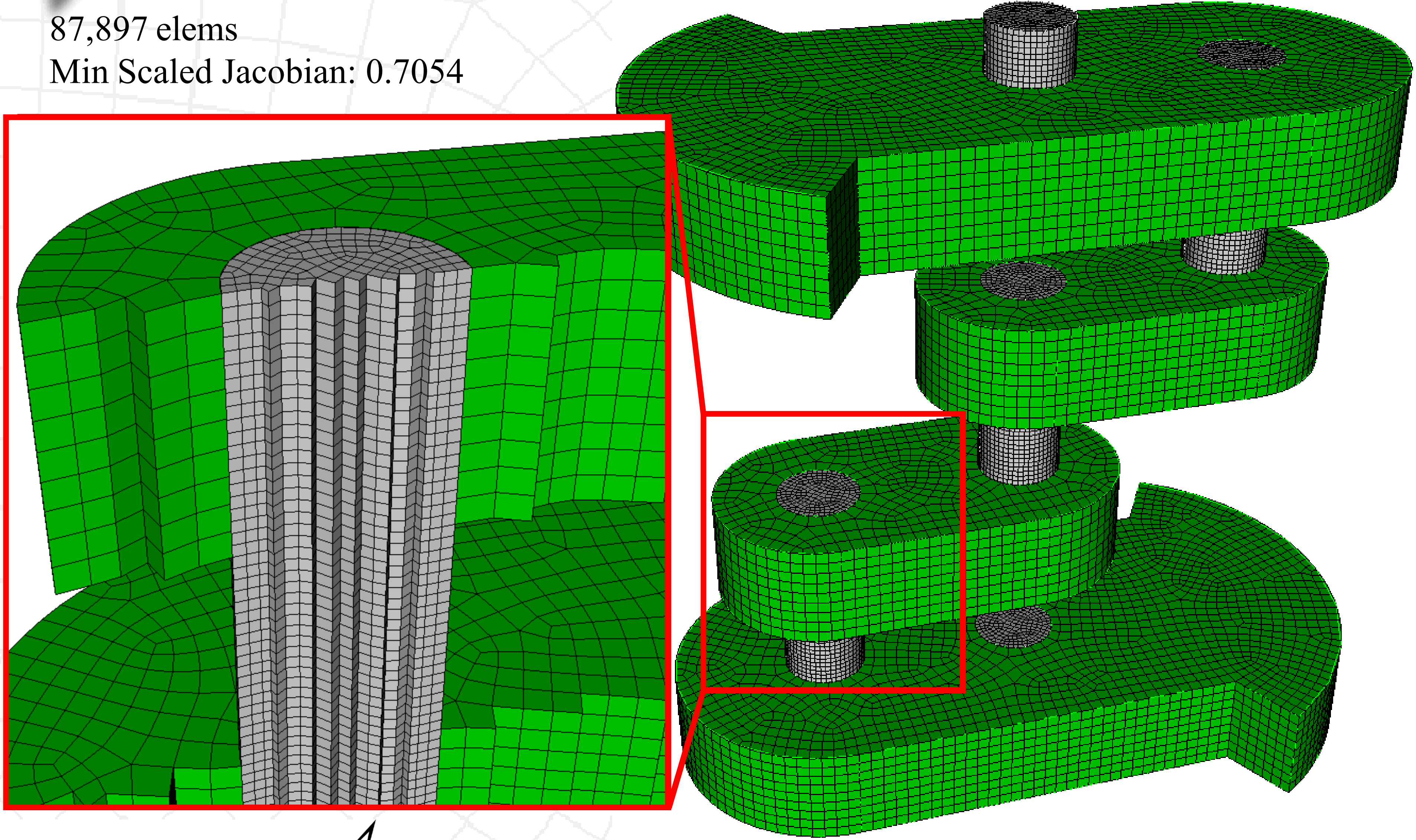
# Sheet Insertion in Mesh Matching



Pillowing inserted a new sheet in the green volume which matches the unmatched red chord in the yellow volume.

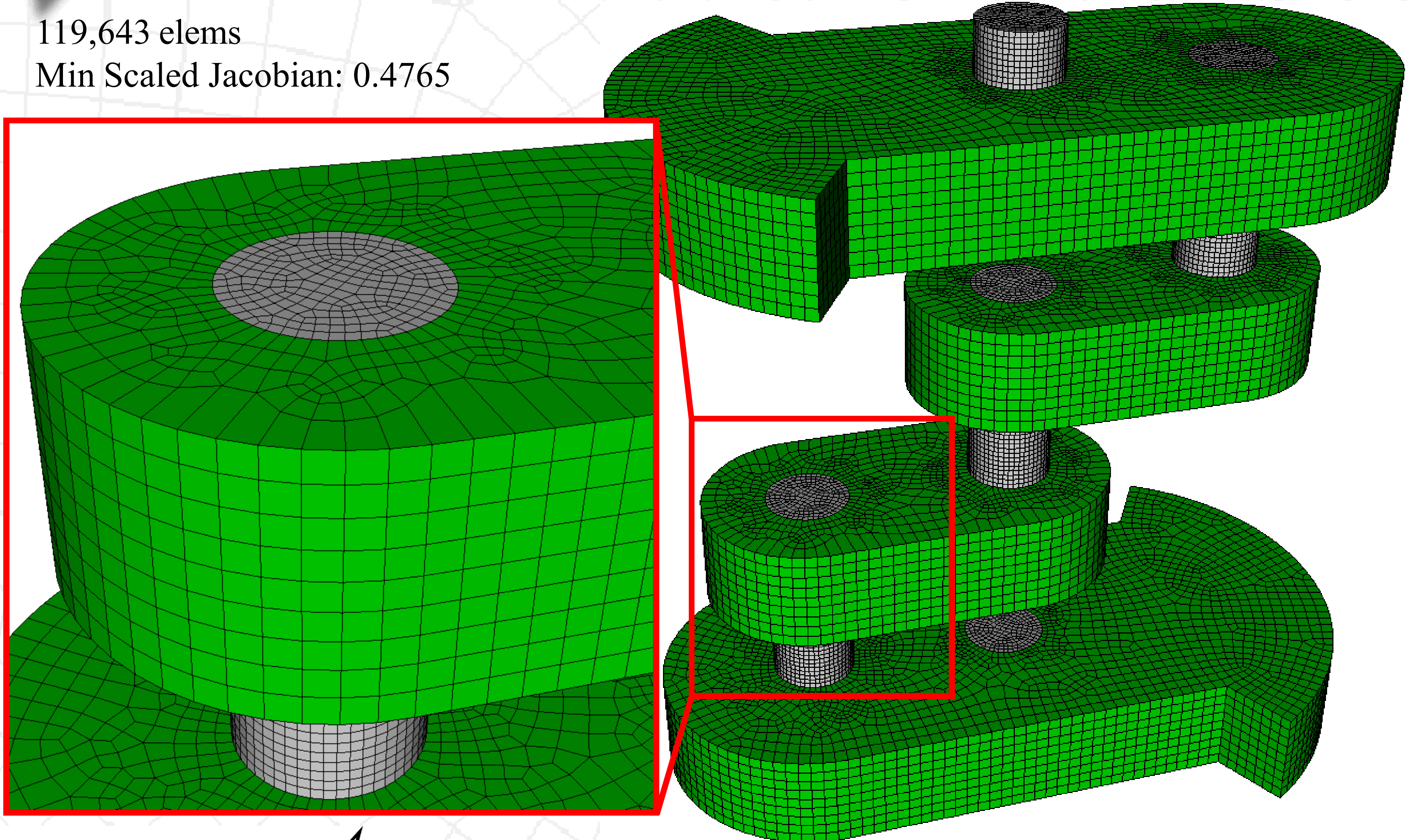
# Example #2 – Crankshaft - Before

87,897 elems
Min Scaled Jacobian: 0.7054



# Example #2 – Crankshaft - After

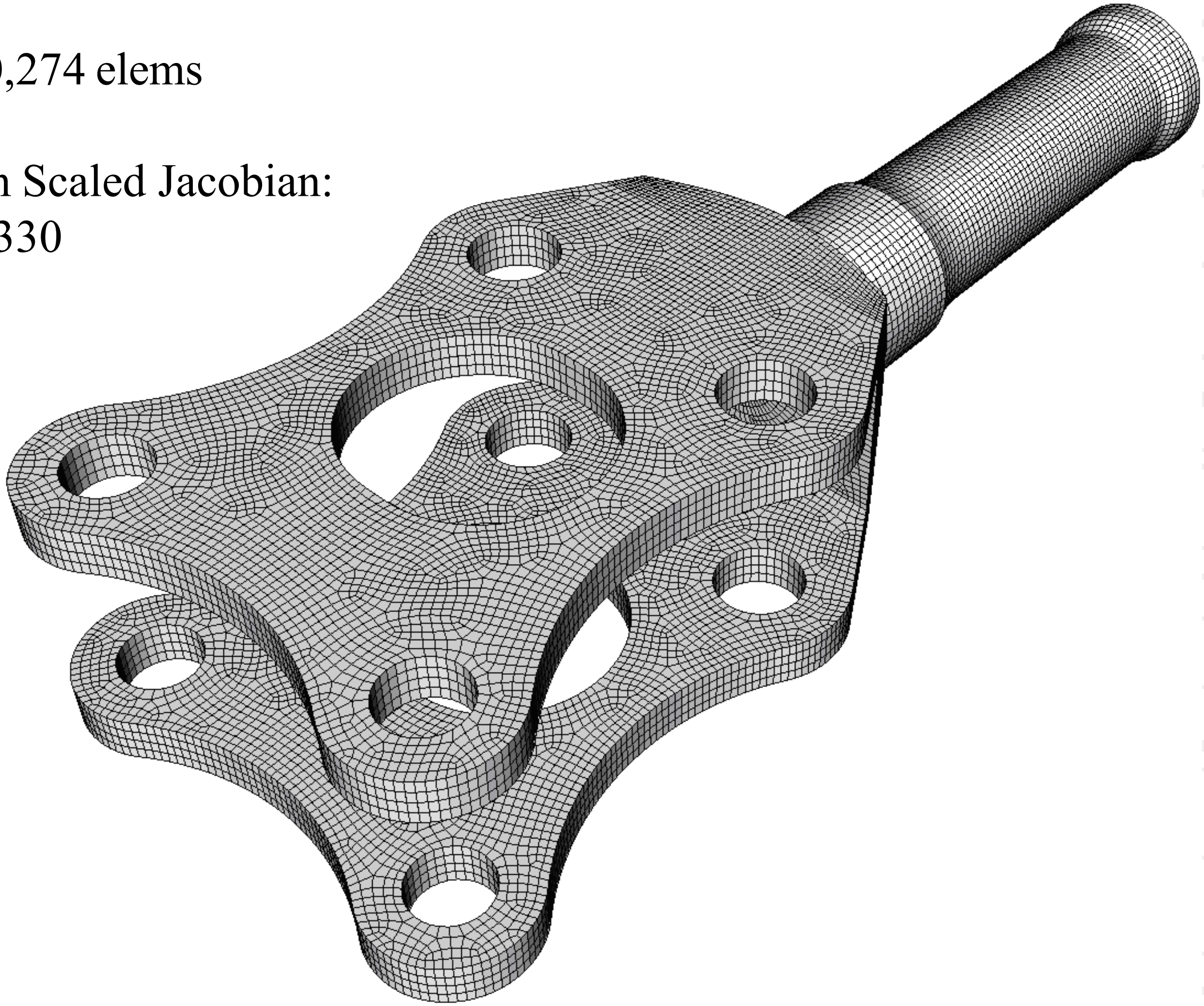
119,643 elems
Min Scaled Jacobian: 0.4765



# Example #3 – Spindle - Refinement

160,274 elems

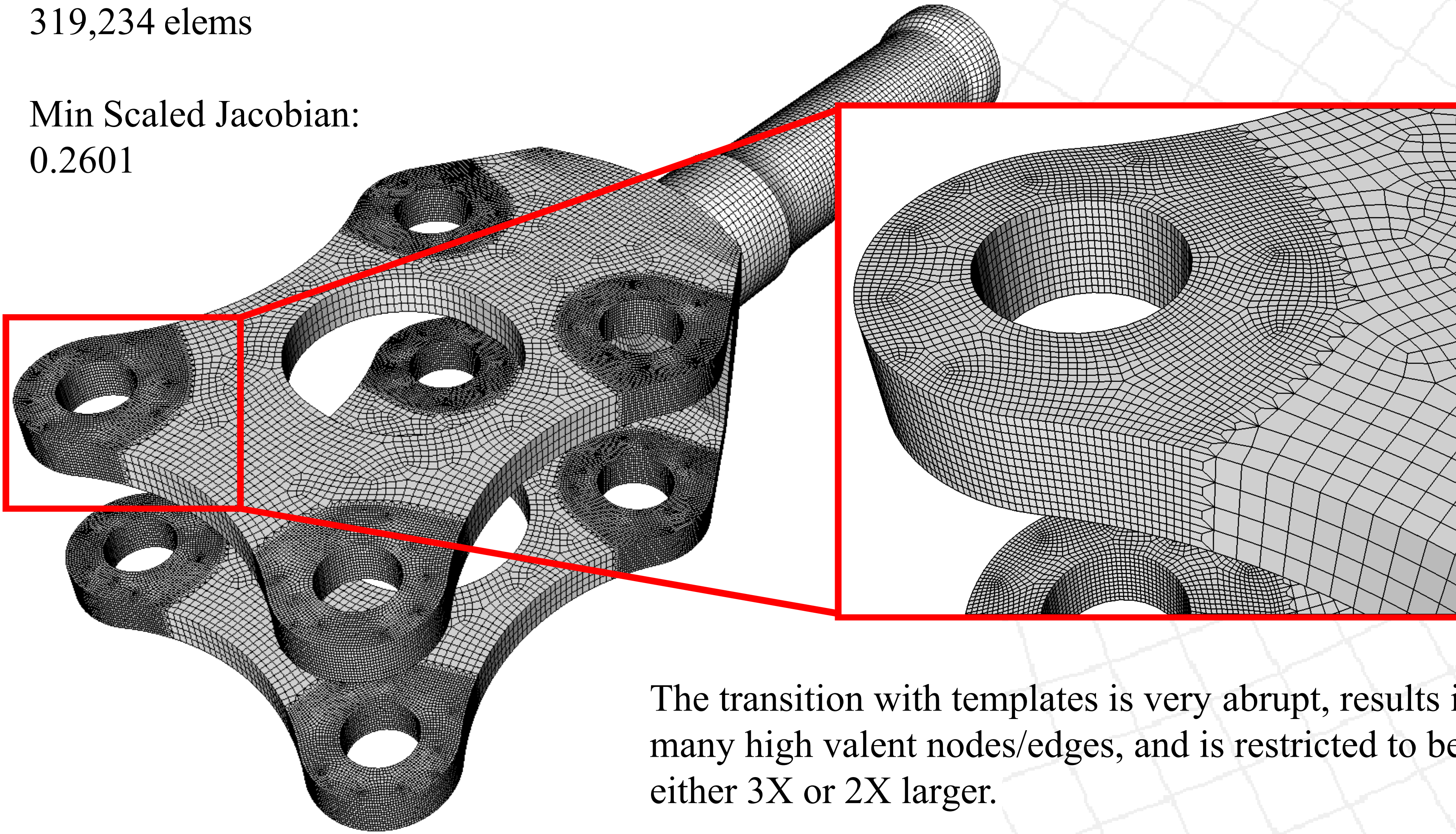
Min Scaled Jacobian:
0.4330



# Example #3 – Spindle – With Templates

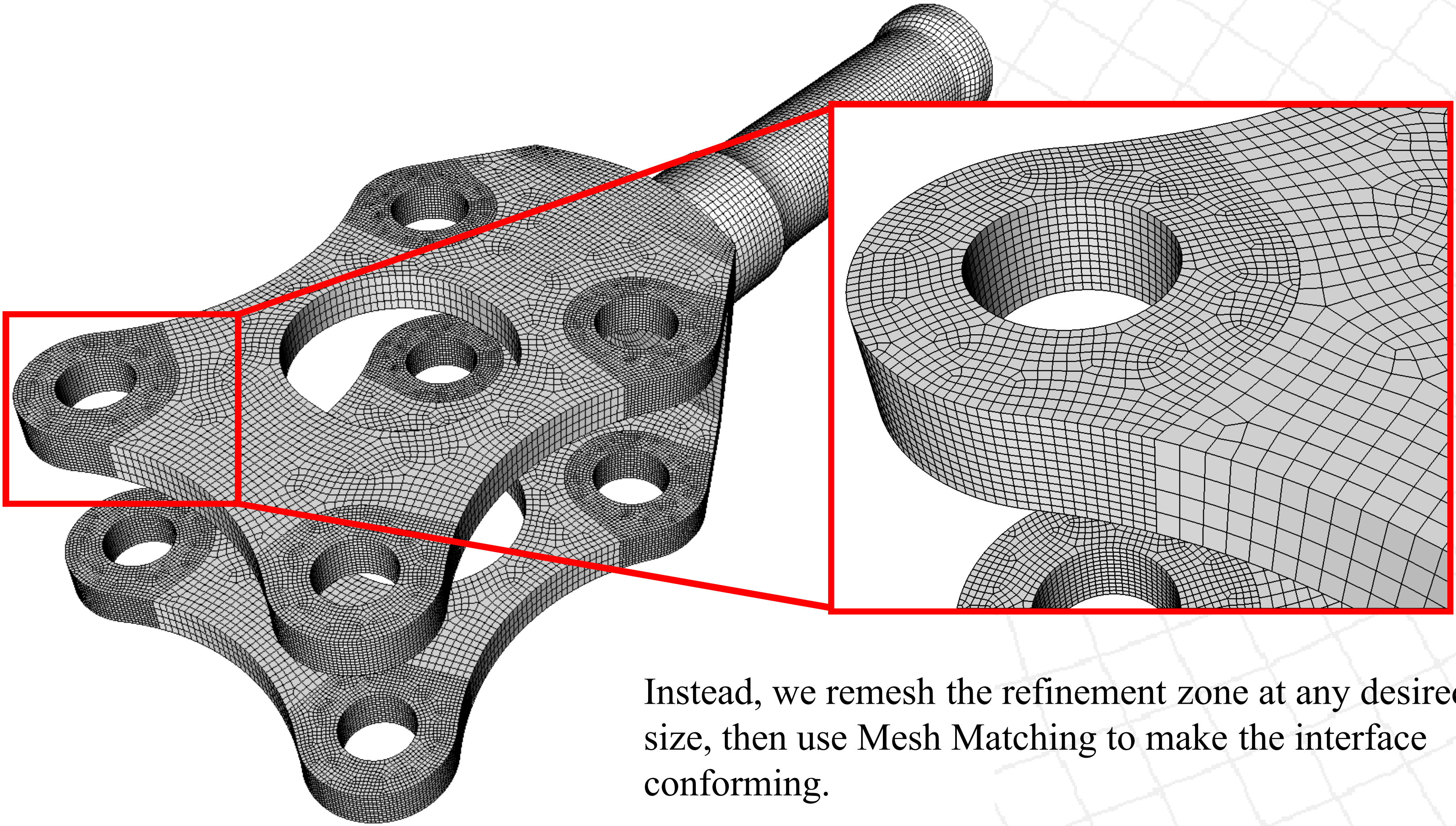
319,234 elems

Min Scaled Jacobian:
0.2601



The transition with templates is very abrupt, results in many high valent nodes/edges, and is restricted to be either 3X or 2X larger.

# Example #3 – Spindle – With Mesh Matching

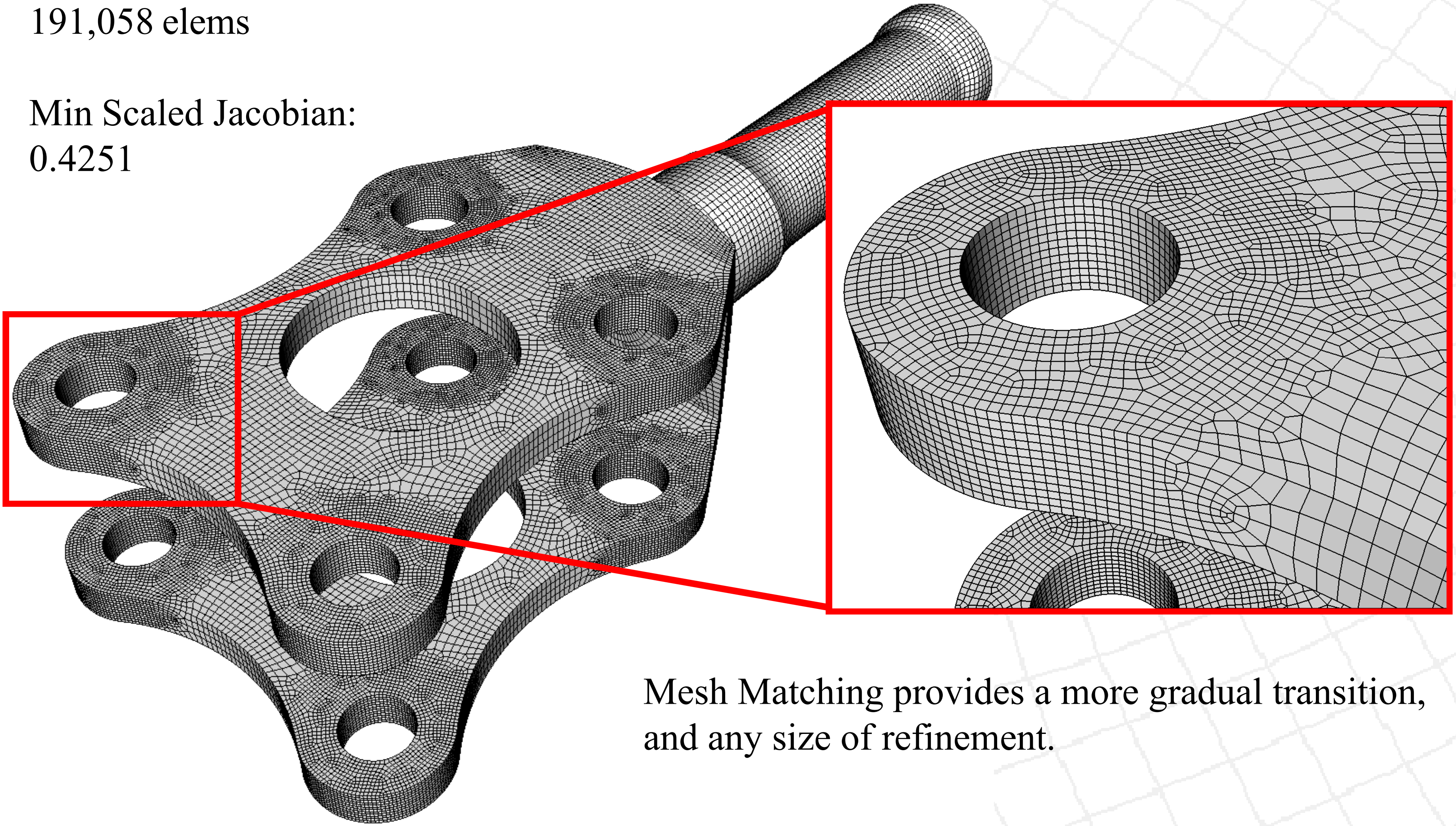


Instead, we remesh the refinement zone at any desired size, then use Mesh Matching to make the interface conforming.

# Example #3 – Spindle – With Mesh Matching

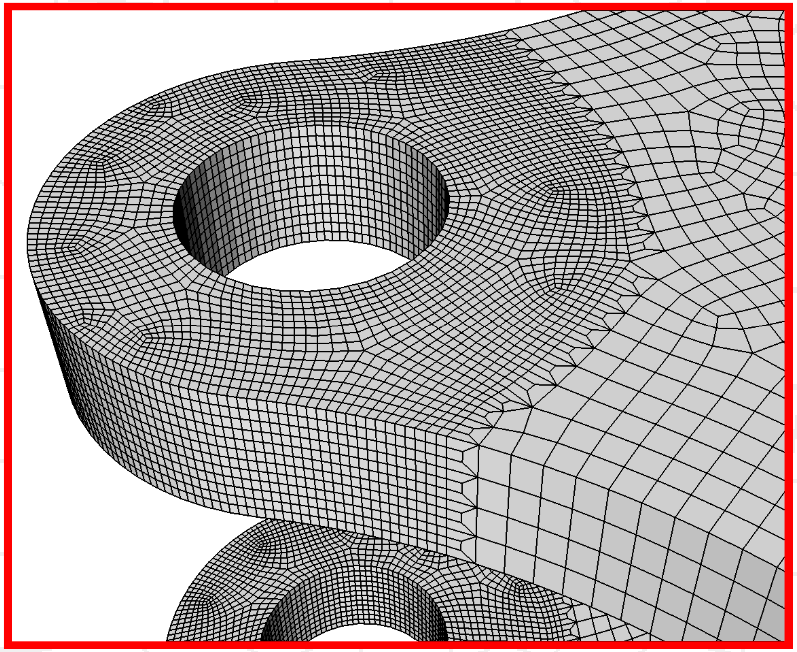
191,058 elems

Min Scaled Jacobian:
0.4251

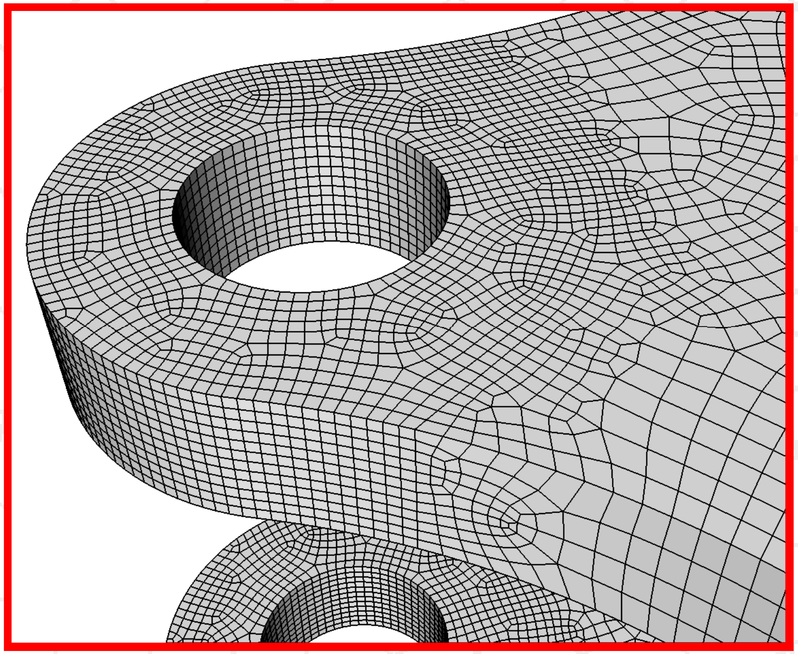


Mesh Matching provides a more gradual transition, and any size of refinement.

# Example #3 – Spindle – With Mesh Matching



With Template-based Refinement
319,234 elems
Min Scaled Jacobian: 0.2601

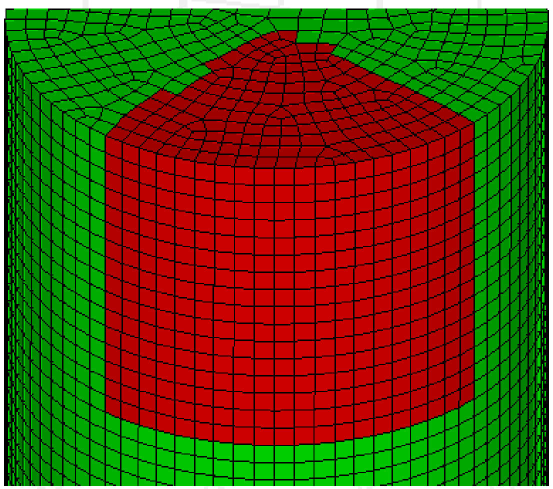


With Mesh Matching
191,058 elems
Min Scaled Jacobian: 0.4251

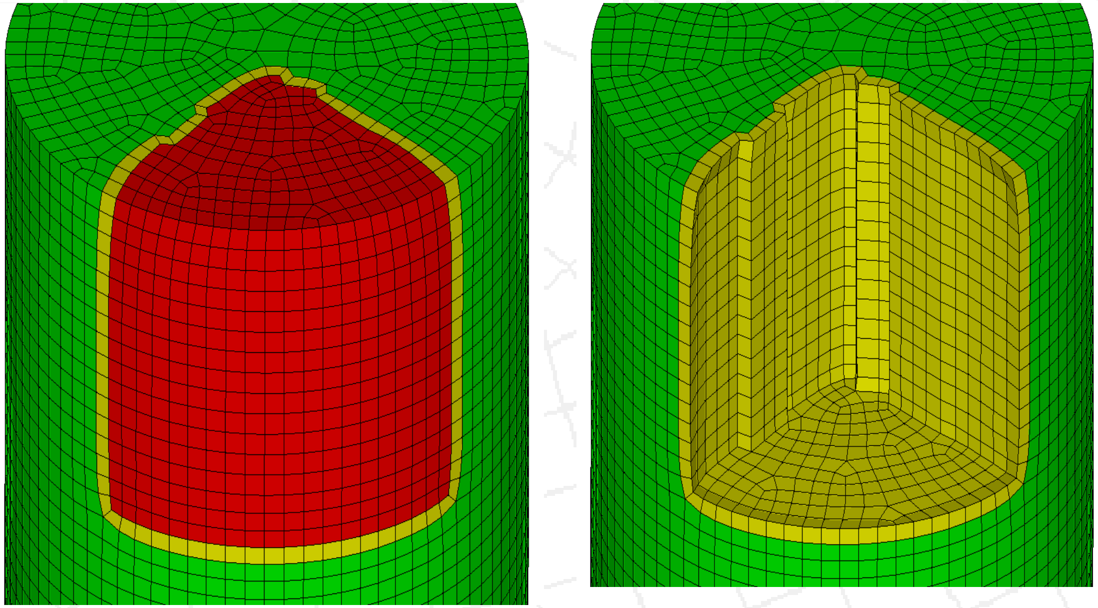
Mesh Matching provides a more gradual, higher quality, transition, and any size of refinement. However, templates provide a more consistent and predictable result, which is better for parallel.

# Hex Coarsening

- Pillow-Collapse-Extract Method



Coarsening Region

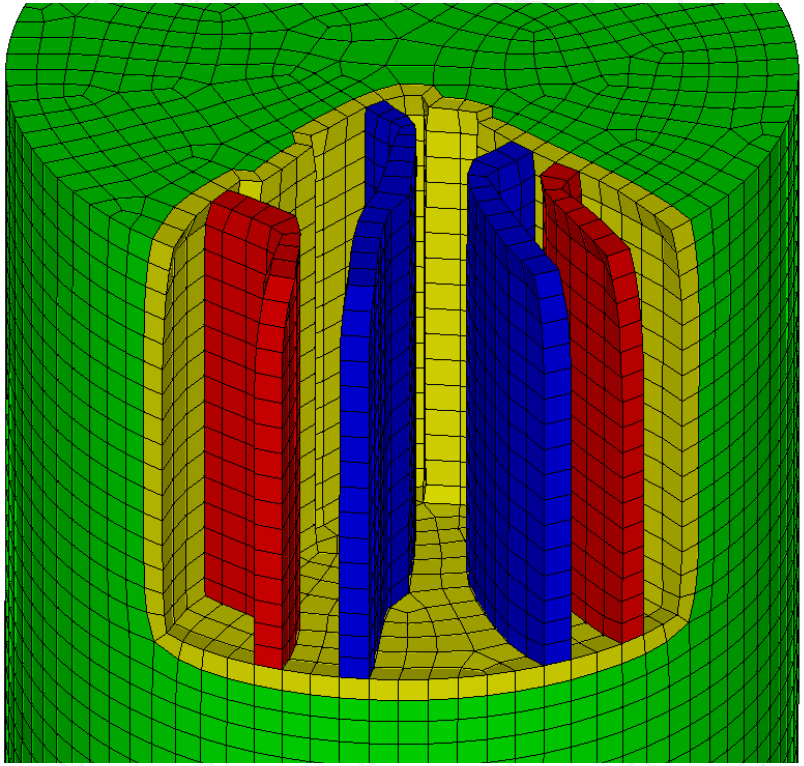


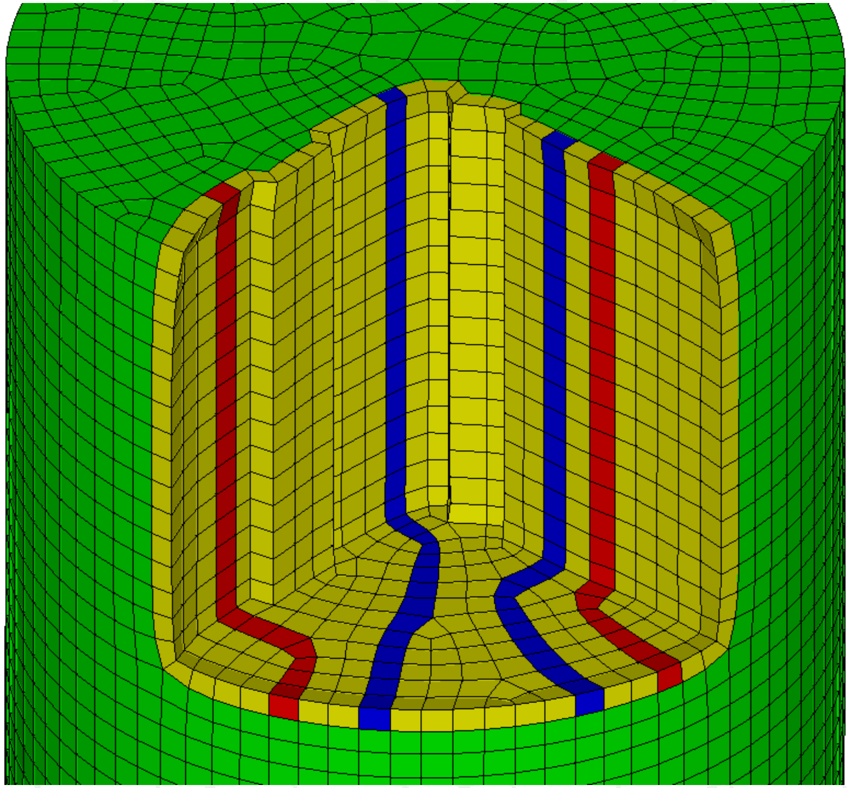
Pillow is added surrounding the coarsening region

# Hex Coarsening

Pillow-Collapse-Extract Method

This pillow gives us dozens of “Hex Columns” which intersect sheets in the coarsening region. All of these columns are local to coarsening region.



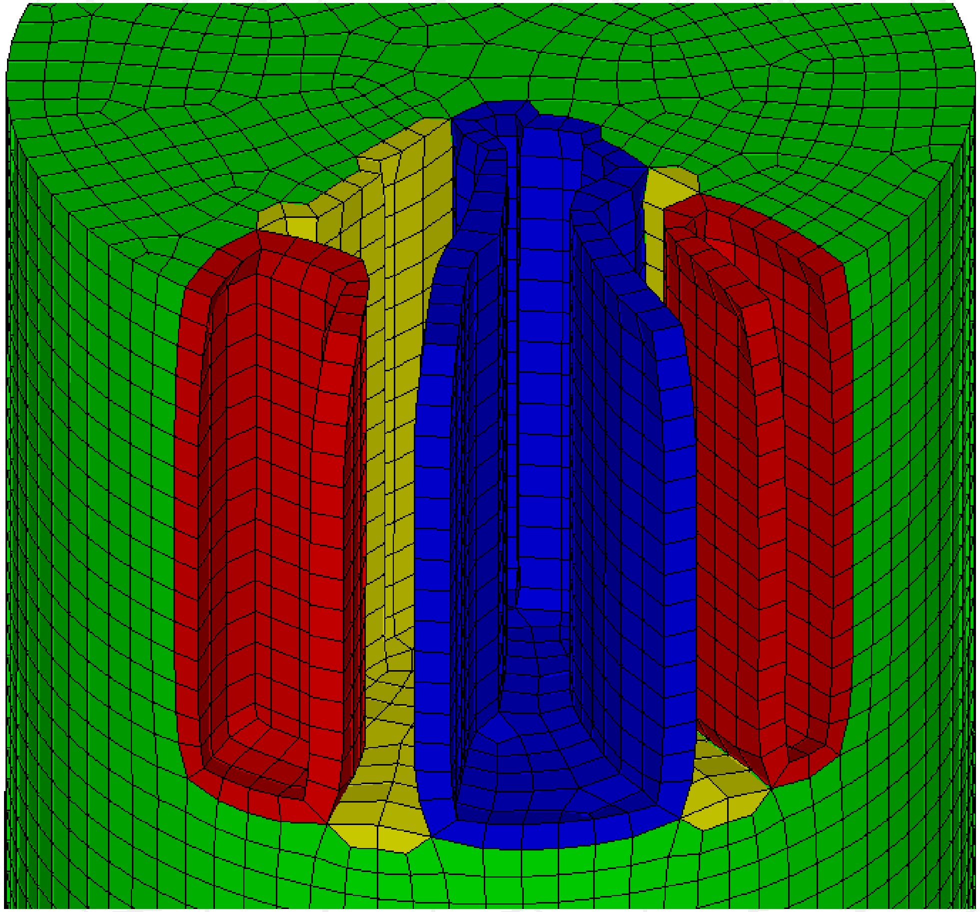


# Hex Coarsening

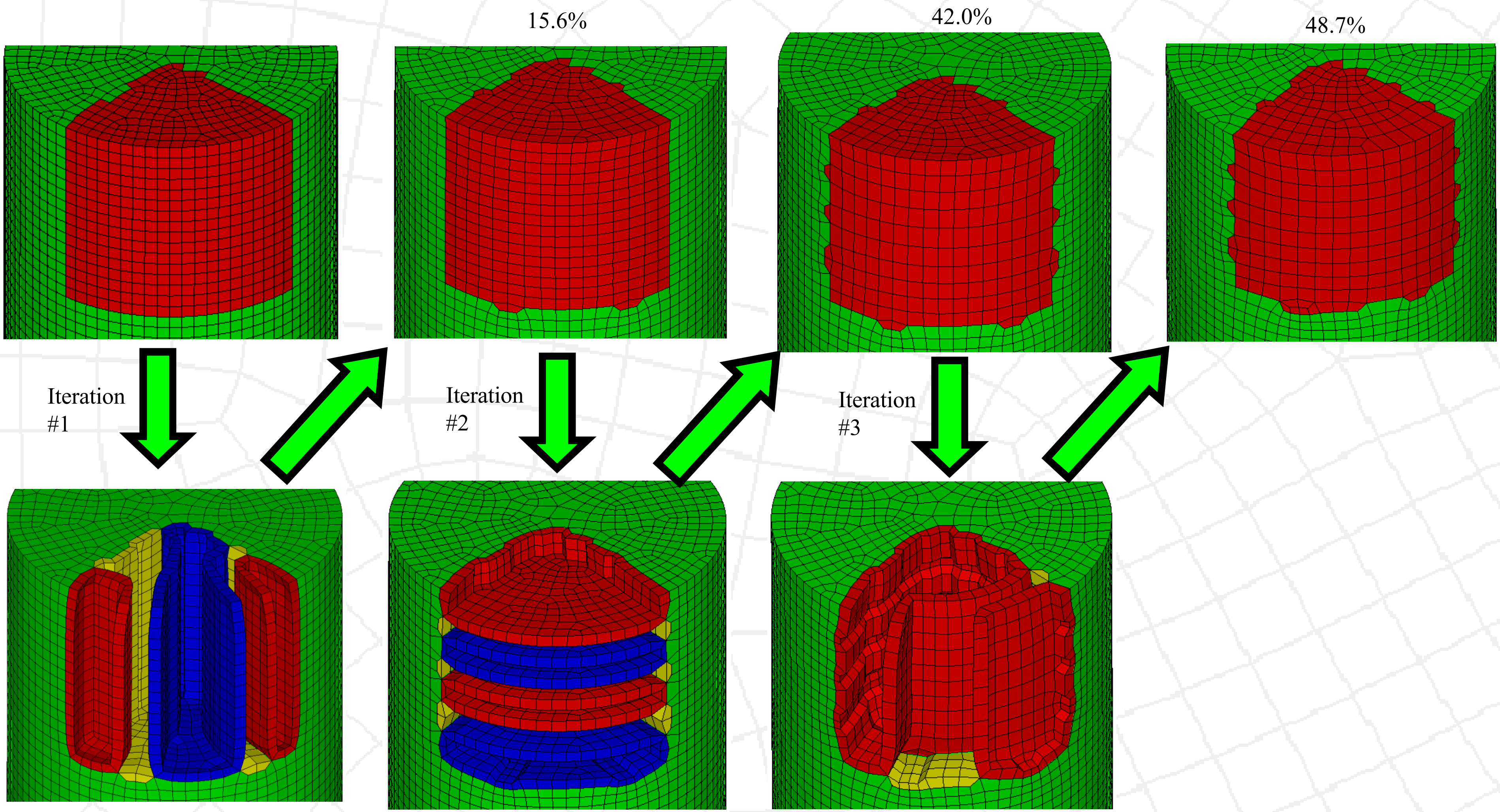
Pillow-Collapse-Extract Method

Collapsing these columns gives us sheets that are completely contained in the coarsening region and can be extracted from the mesh.

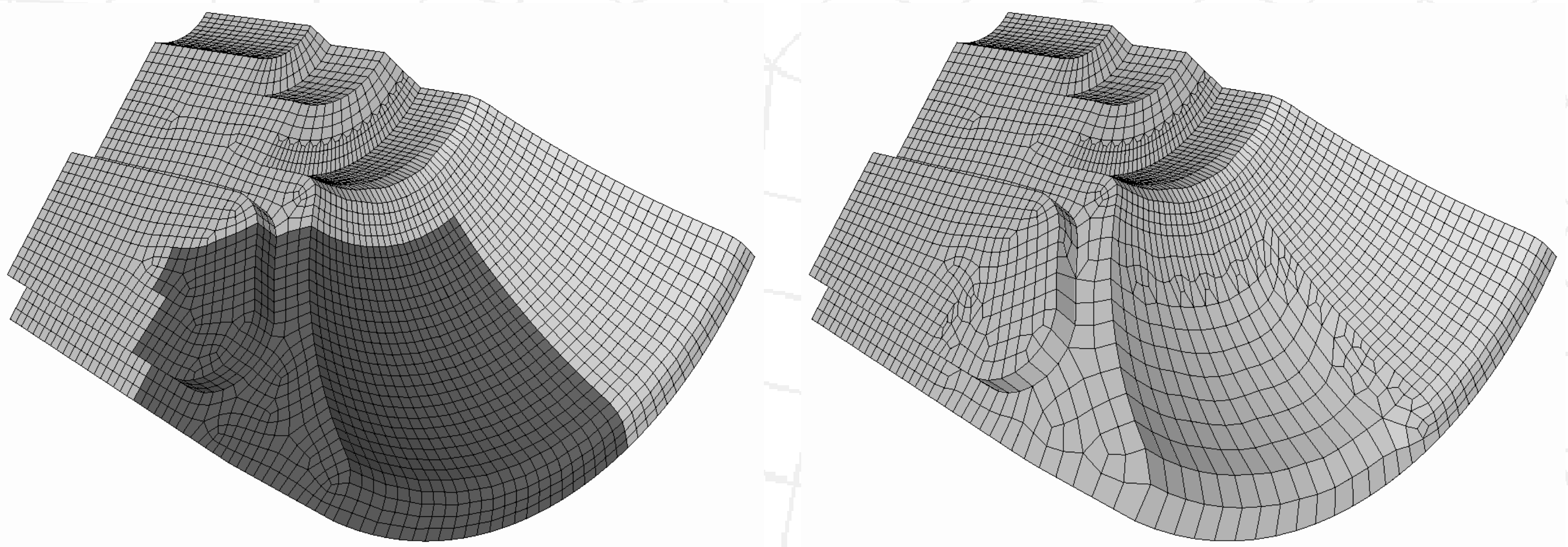
In this example, the blue and red elements are removed.  Yellow pillow elements remain.



# Pillow-Collapse-Extract Method Example



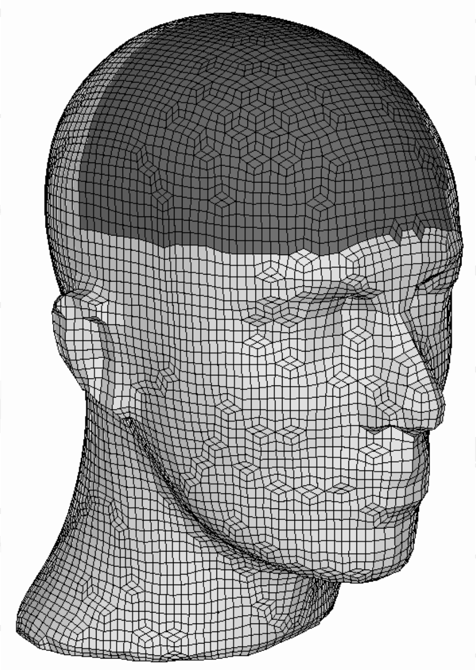
# Examples



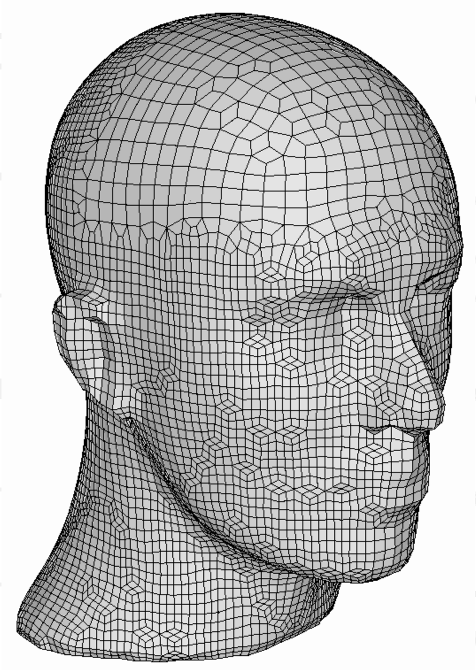
(a) (b)

| Model | Number of Elements | | % Removal | Minimum Scaled Jacobian | |
|---|---|---|---|---|---|
| | Before | After | | Before | After |
| Mechanical Part | 7641 | 2205 | 71.1 | 0.77 | 0.22 |
| Human Head | 10080 | 2615 | 74.1 | 0.48 | 0.23 |

# Examples



(a)



(b)

| Model | Number of Elements | | % Removal | Minimum Scaled Jacobian | |
|---|---|---|---|---|---|
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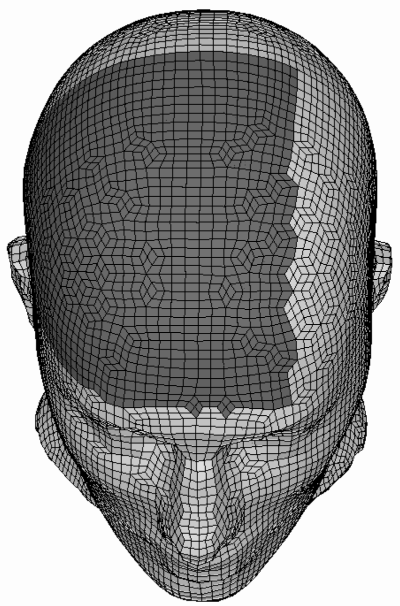
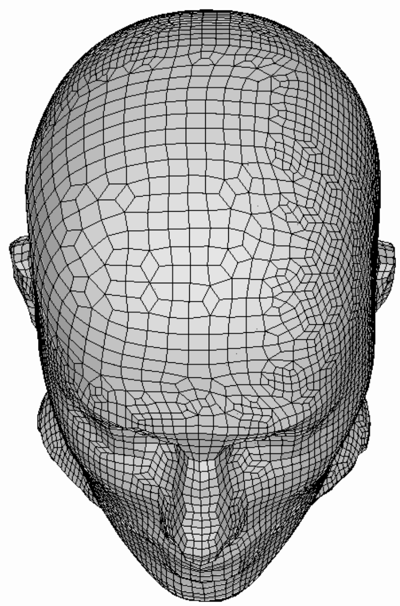
# Examples



(a)



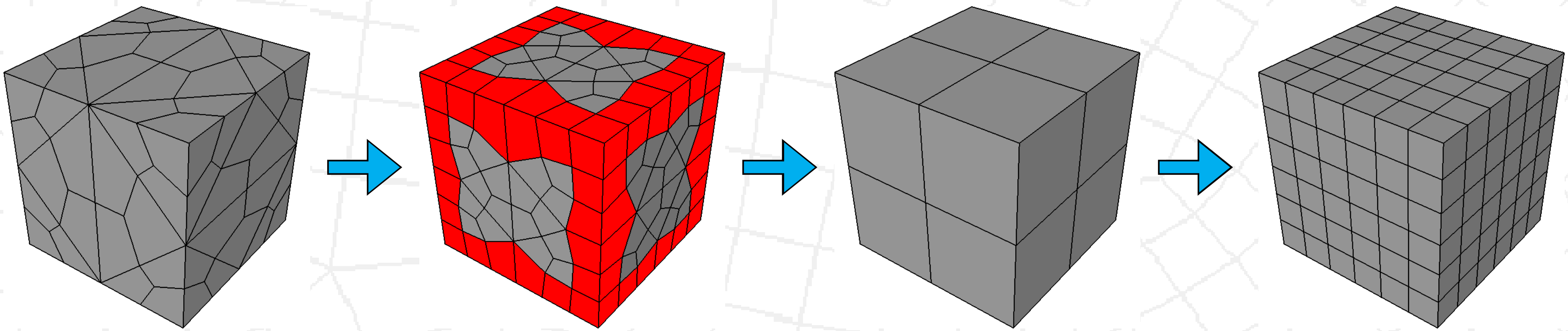
(b)

| Model | Number of Elements | | % Removal | Minimum Scaled Jacobian | |
|---|---|---|---|---|---|
| | Before | After | | Before | After |
| Mechanical Part | 7641 | 2205 | 71.1 | 0.77 | 0.22 |
| Human Head | 10080 | 2615 | 74.1 | 0.48 | 0.23 |

# Fun Sheet Matching

Collaboration with Franck Ledoux and Nicolas Kowalski, CEA, France

Goal: Automatic block decomposition of mechanical objects.



THex Mesh | Fundamental sheets inserted | All THex sheets extracted | Block decomposition is refined

With all convex, 3-valent vertices:

- Each curve has 1 adjacent column
- Each vertex has 1 adjacent hex

Principle Challenges:

1. Non-3-valent vertices
2. Model concavities

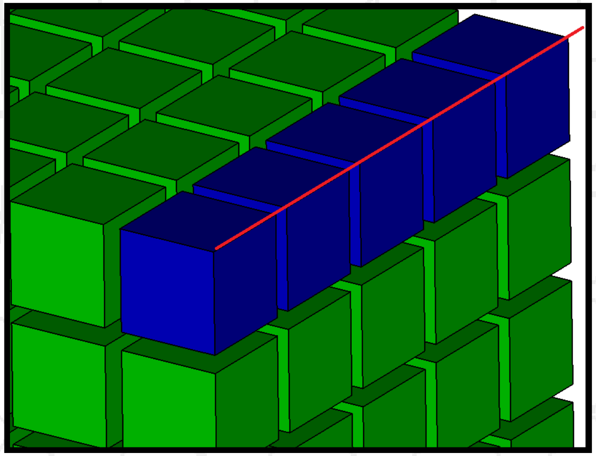
# Curve Types

We define …

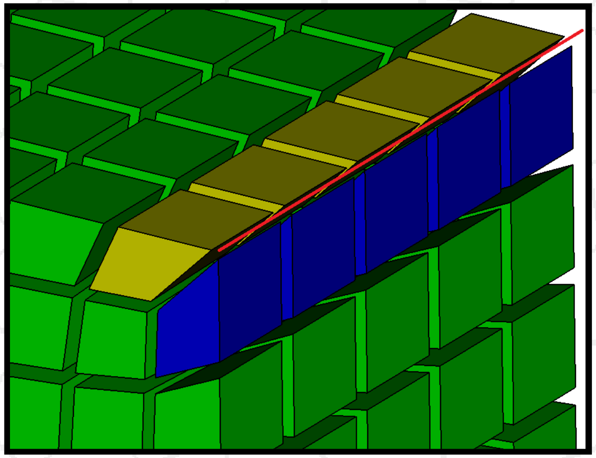
Curve Types:

- Type 1 – Curve has 1 adjacent hexahedral column
- Type 2 – Curve has 2 adjacent hexahedral columns
- Type 3 – Curve has 3 adjacent hexahedral columns
- …
- Type N – Curve has N adjacent hexahedral columns

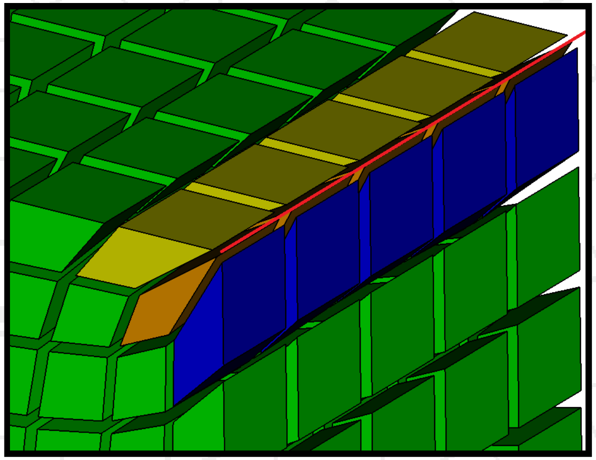
Type 1



Type 2



Type 3

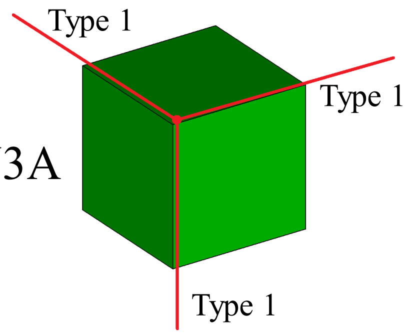


# Vertex Types

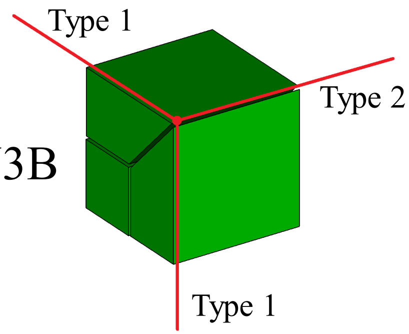
## Define Valid Hex Configurations at Vertices

3-Valent Vertices:

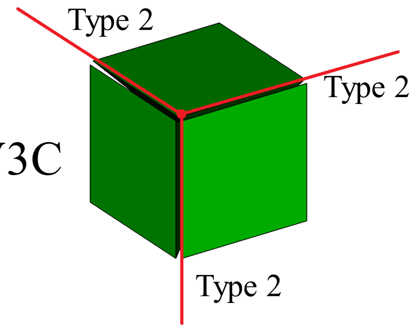
V3A




V3B



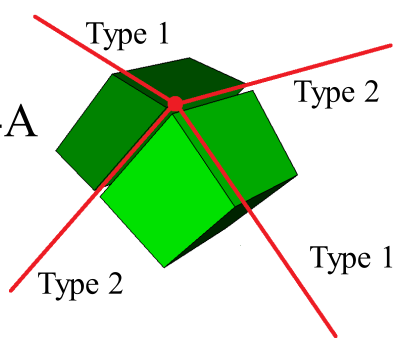

V3C



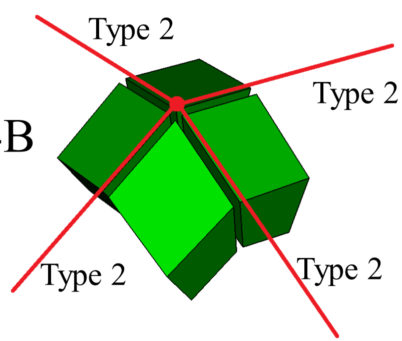

etc.

4-Valent Vertices:

V4A




V4B

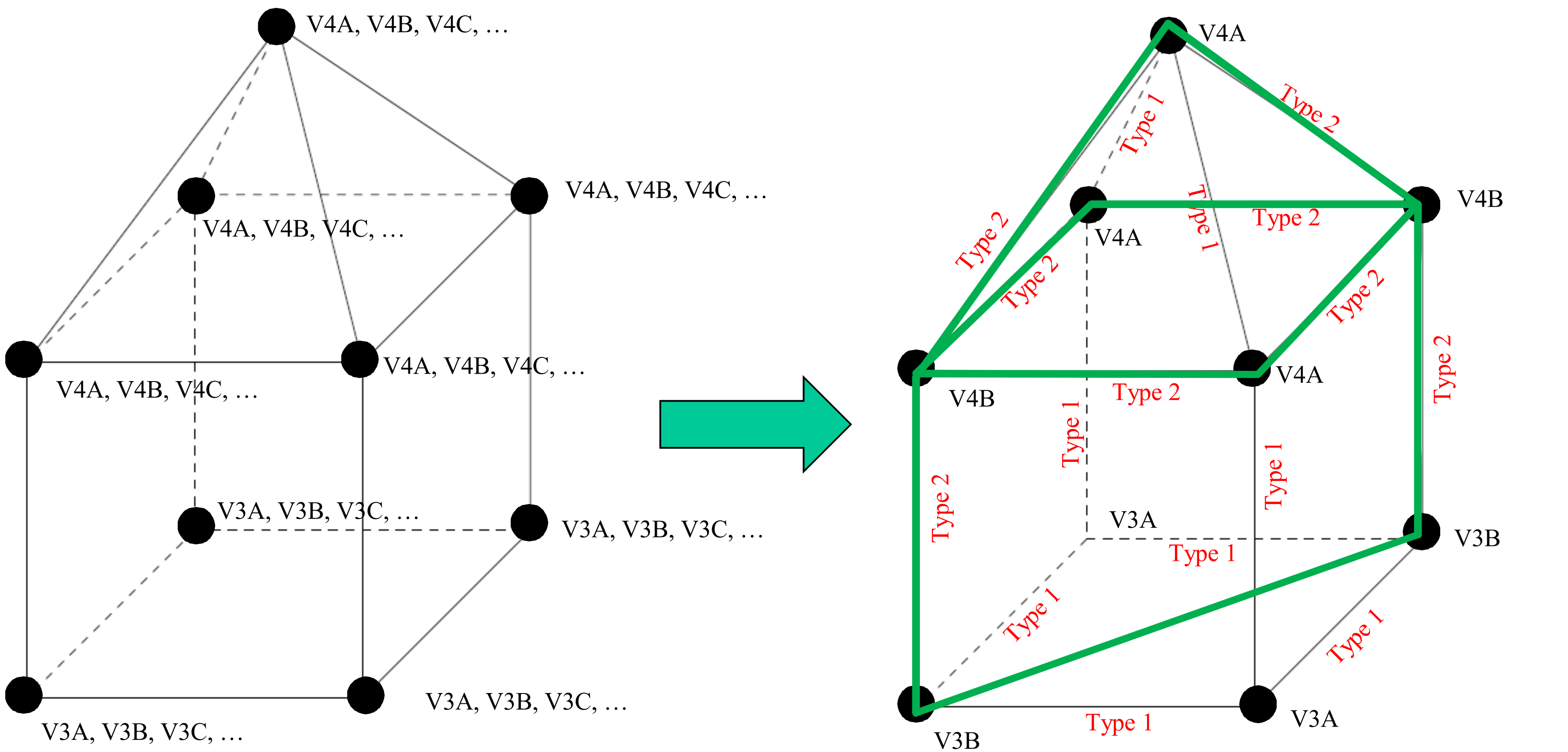



etc.

For an n-valent vertex, there are an infinite number of possible vertex types. However, only a small subset are feasible given the dihedral angles at the adjacent curves.

# Constraint Satisfaction

Each vertex must be assigned a Vertex Type. This can be formulated as a constraint satisfaction problem (CSP) with arc-consistency enforced through the model curves.

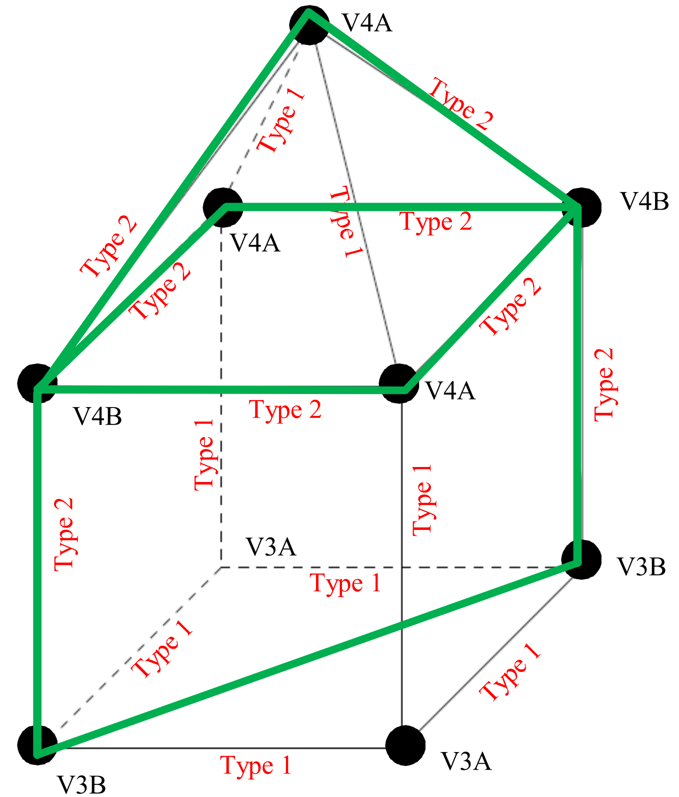



Cycles of Type 2 curves must be closed

# Constraint Satisfaction

Each vertex must be assigned a Vertex Type. This can be formulated as a constraint satisfaction problem with arc-consistency enforced through the model curves.

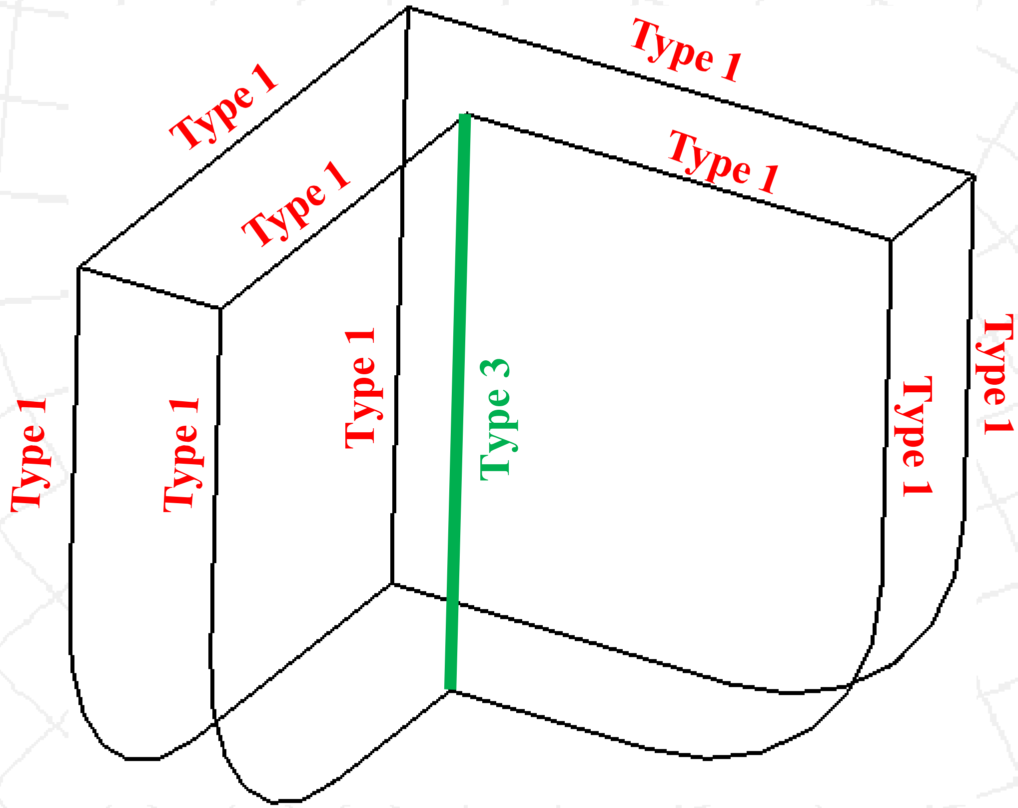
Type classification for each curve, with closed curve cycles, uniquely define the topology of the required boundary sheets. Insertion into the THex mesh and extraction of the THex sheets is straightforward.




Cycles of Type 2 curves must be closed

# Model Concavities

Model concavities will result in curve classifications of Type 3 or higher. Each curve, of Type N, must be part of (N-1) closed cycles.



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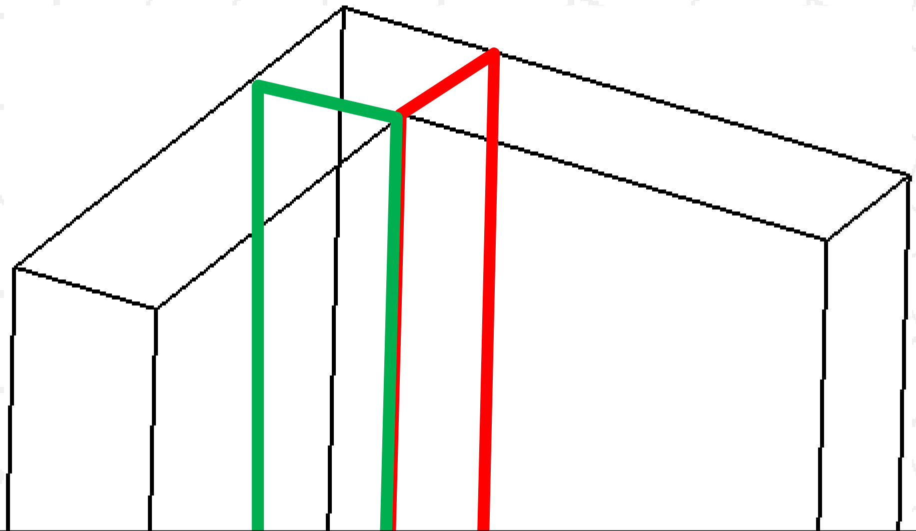
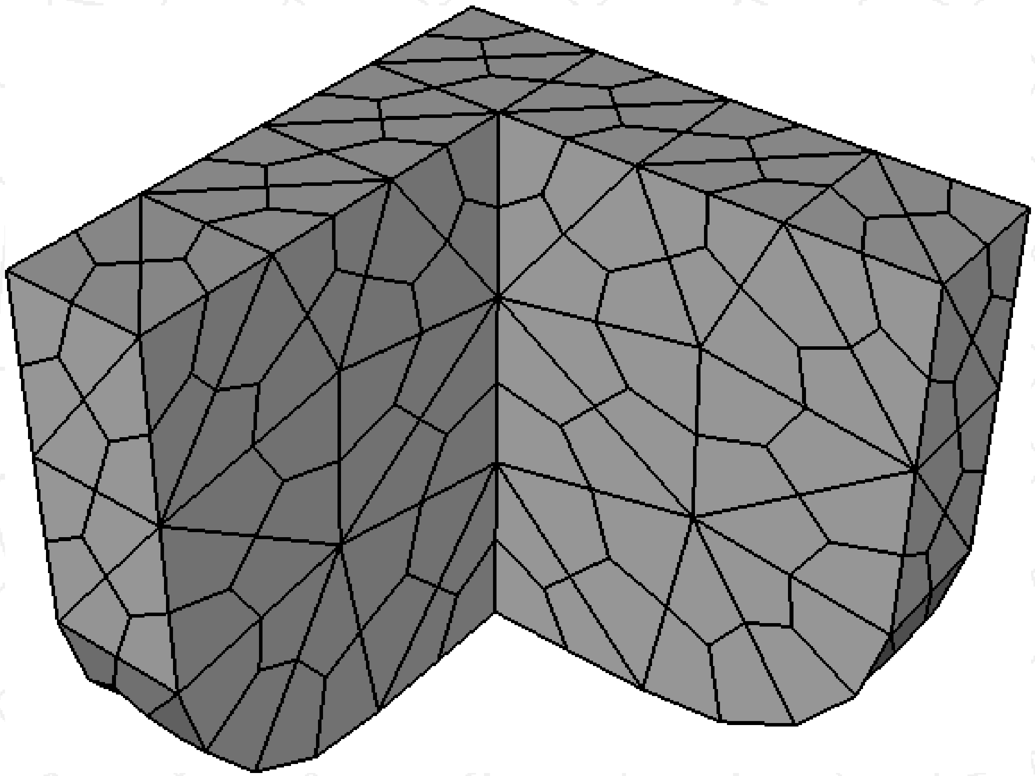


Completion of closed cycles requires traversing across the volume … similar to generalized geometry decomposition. However, the tolerances are much higher. As long as the topology is correct, mesh optimization can determine spatial node locations later.

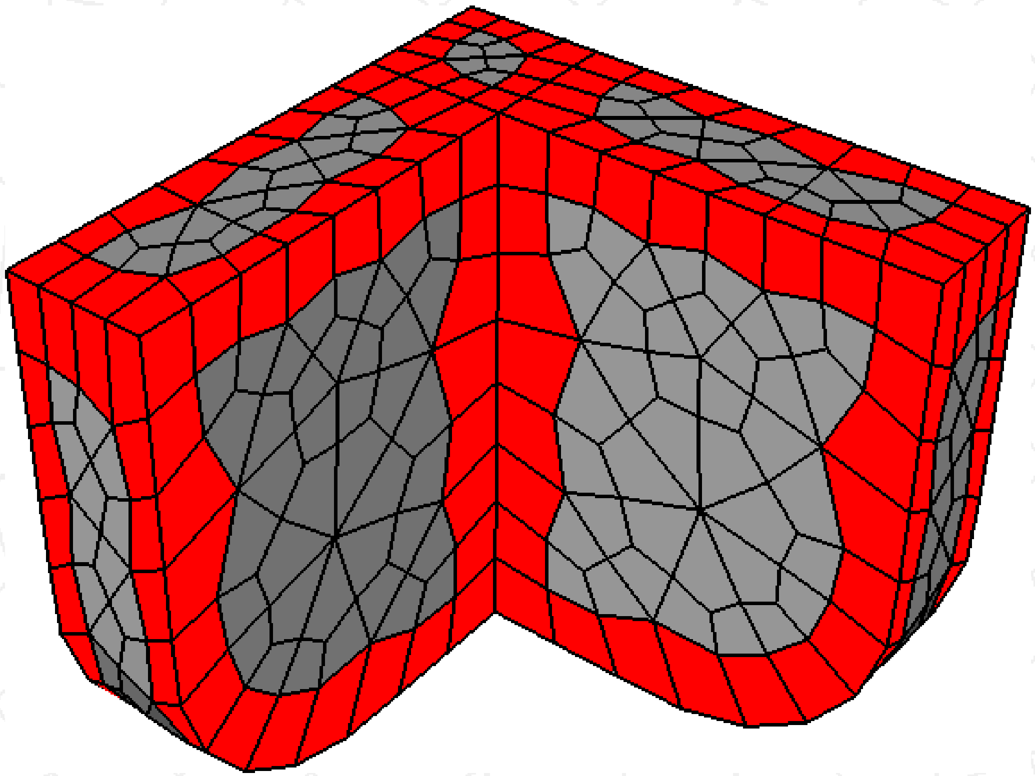
We are currently using projection of planes defined by boundary normals. Tensor fields could also be used to guide traversal.

# Concavity Example



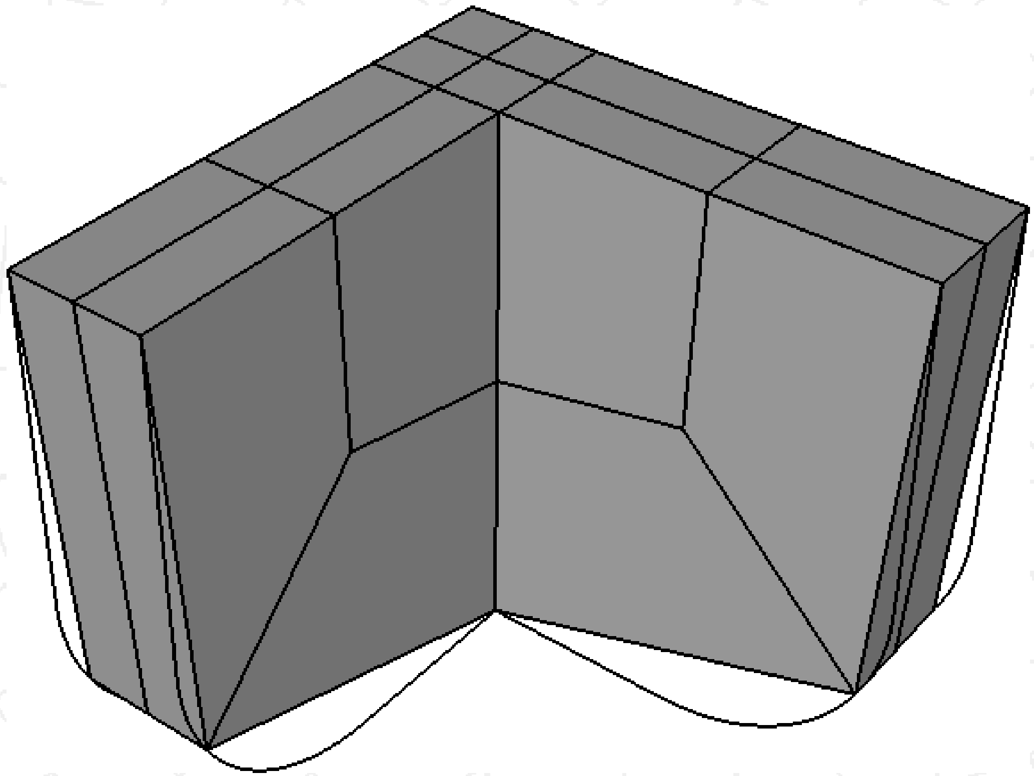
We start out with a Thex mesh.

# Concavity Example



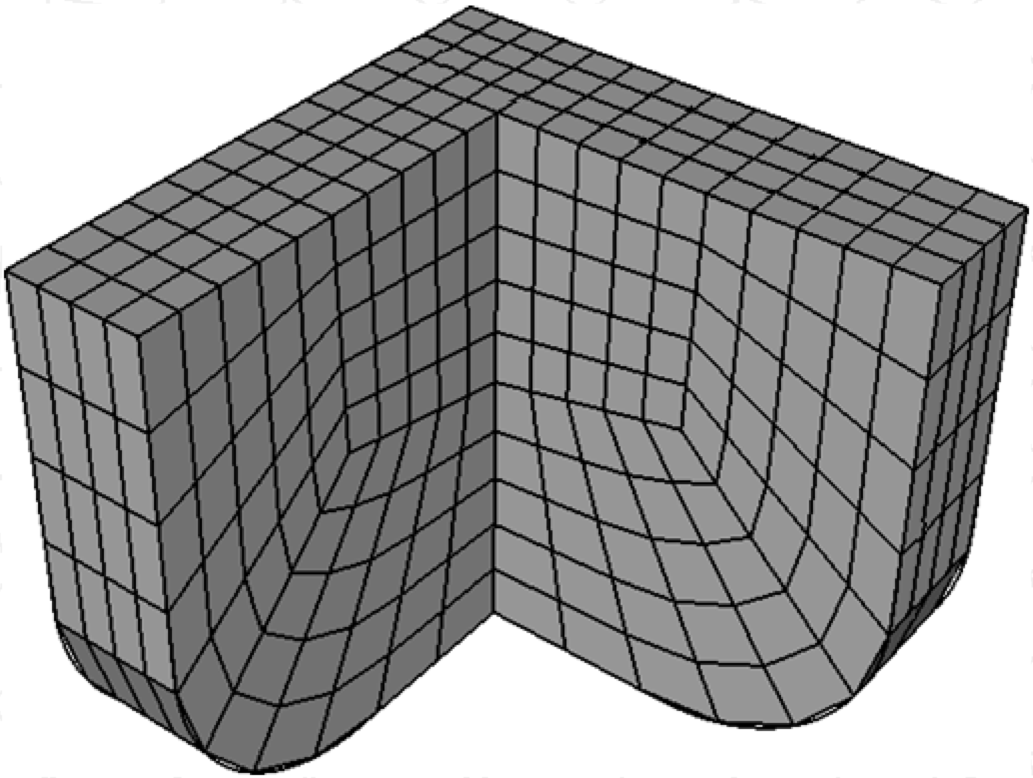
Insertion of boundary fundamental sheets

# Concavity Example



Extracting all the THex sheets gives us this block decomposition, which is just a midpoint subdivision of the partitions separated by the sheets we inserted at the concavities.

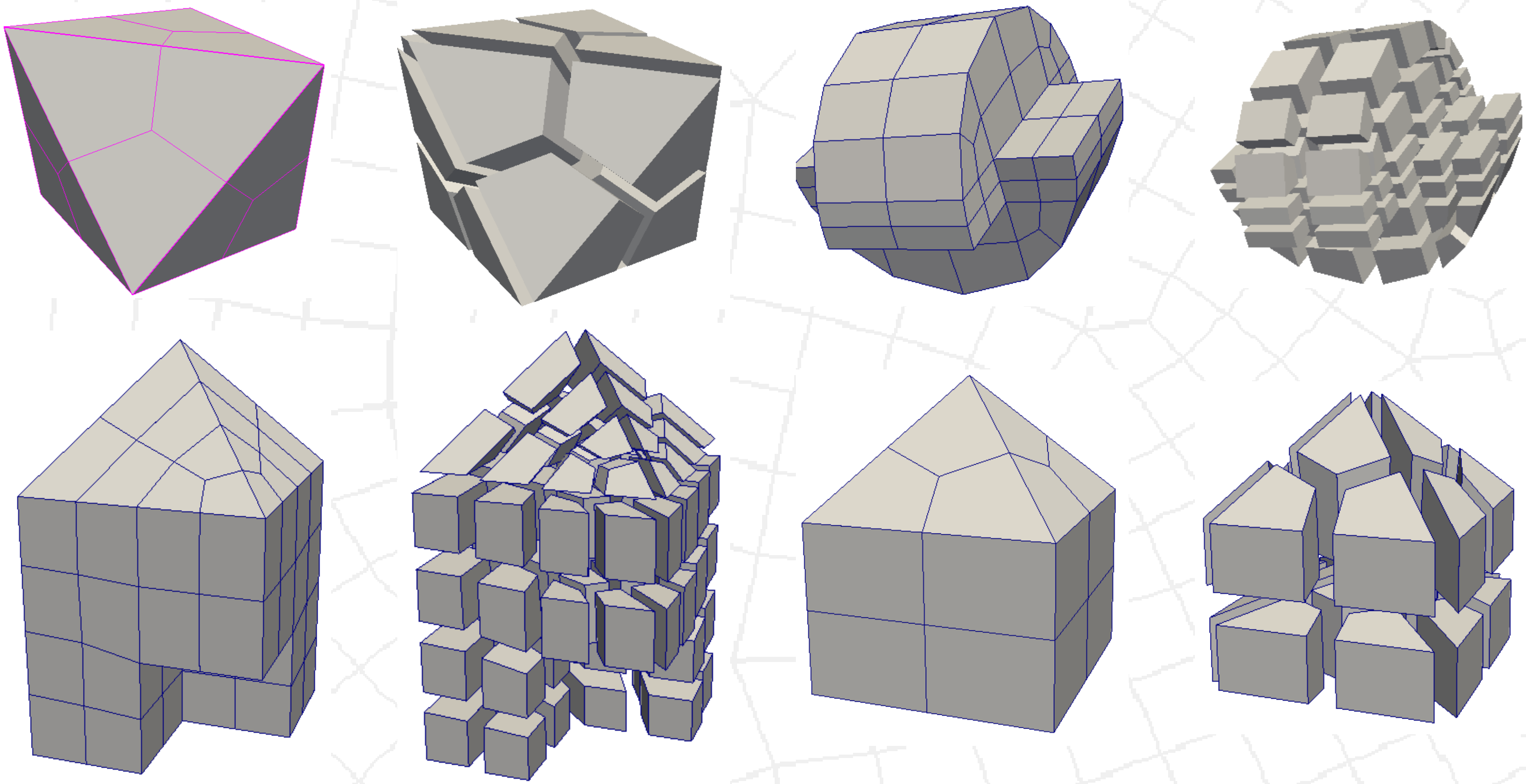
# Concavity Example



We can then refine each block as desired.

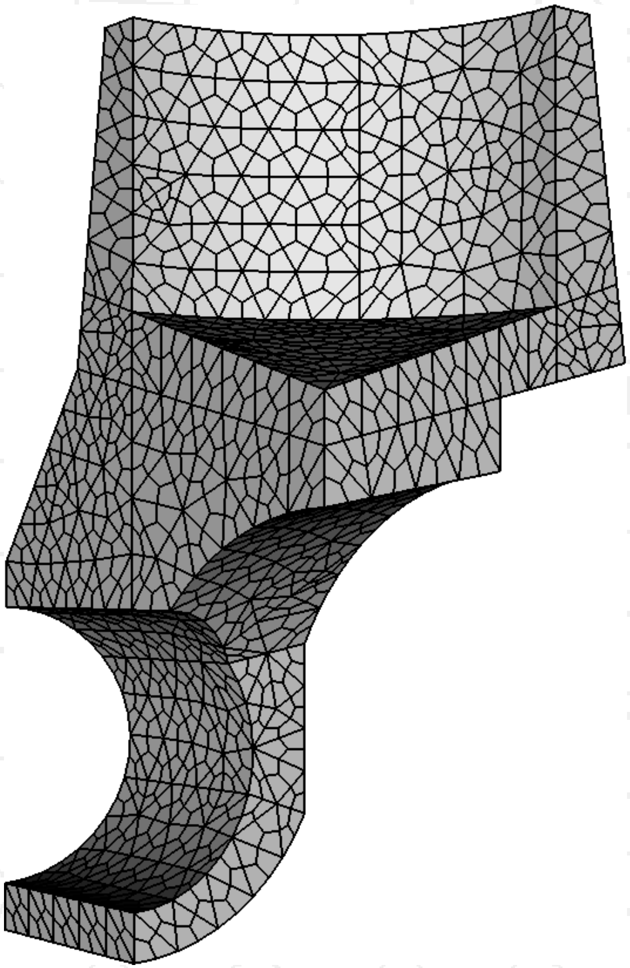
# Examples from Automated Implementation

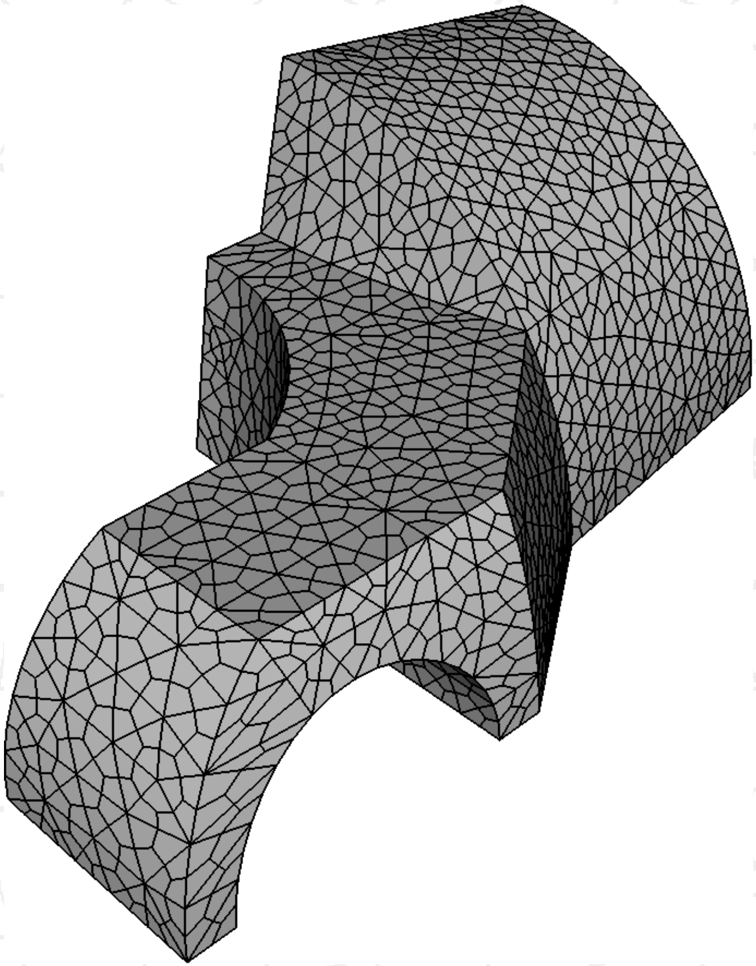
Models generated by Nicolas Kowalski, PhD. candidate, CEA, France.



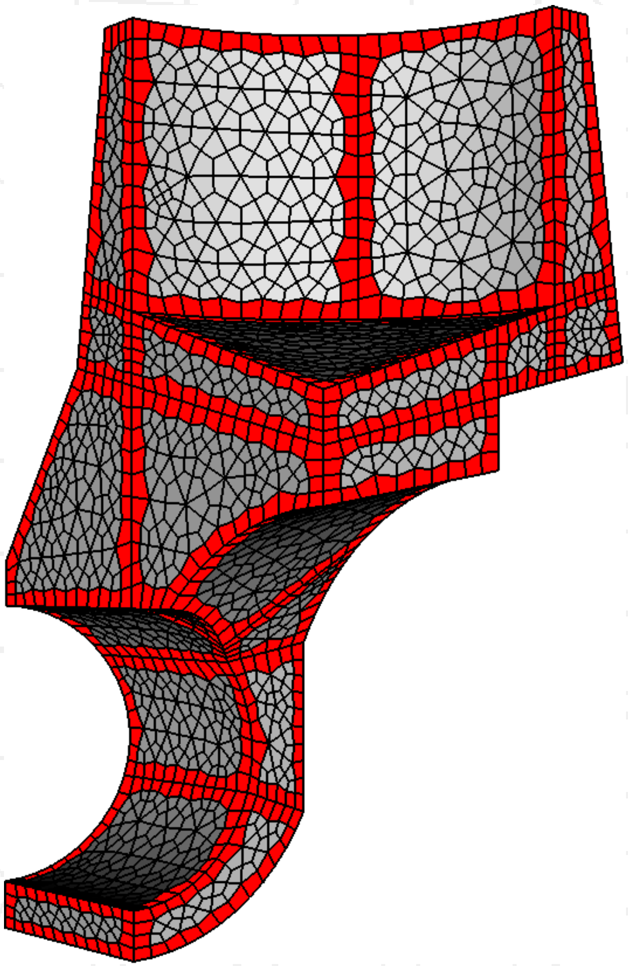
Midpoint subdivision on steroids … accounts for model concavities and vertices with valence greater than 3.

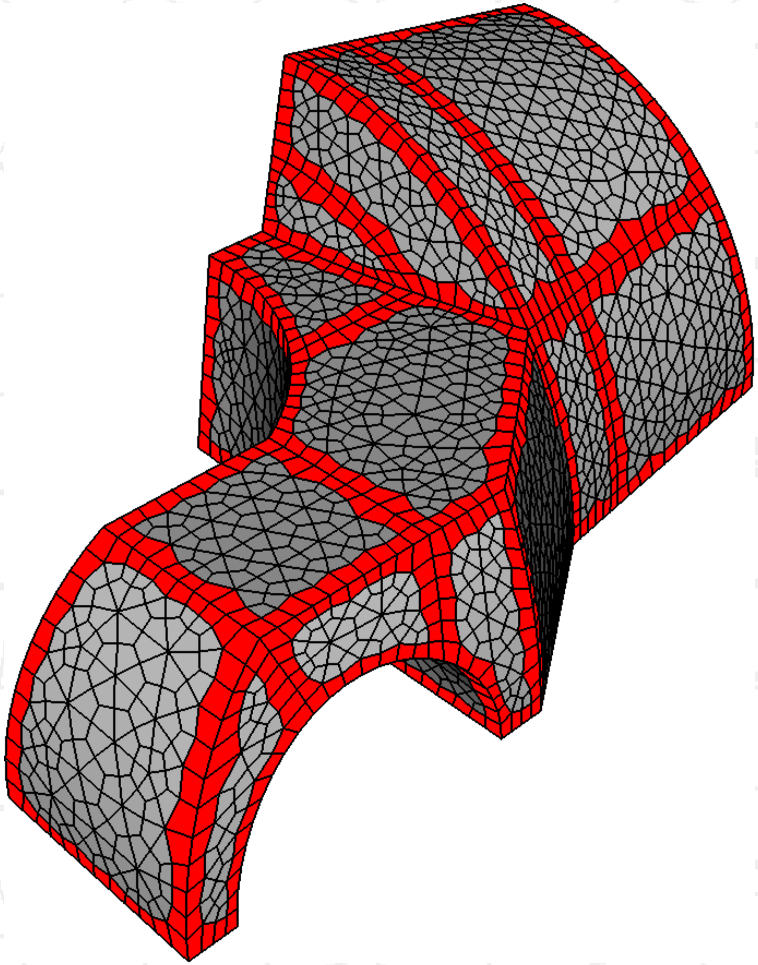
# Complex Manual Example



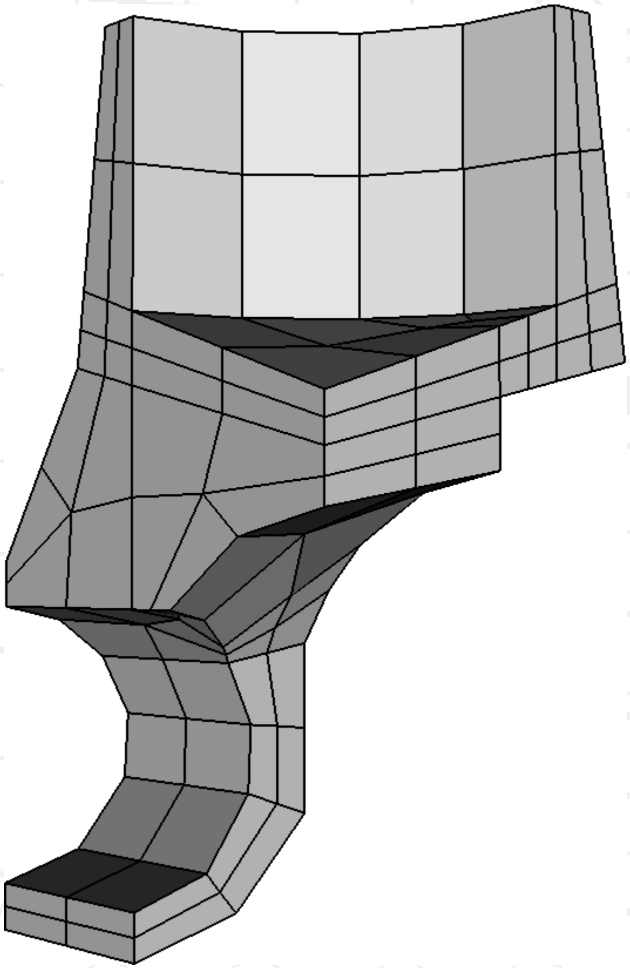


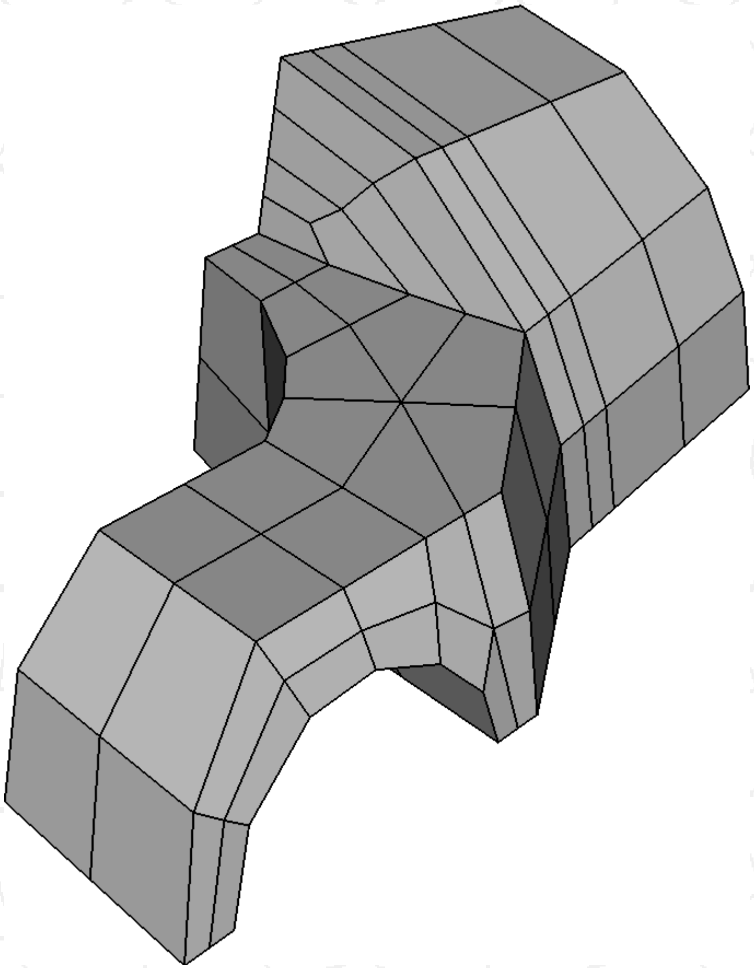
# Example #3





# Example #3





LOCKHEED MARTIN

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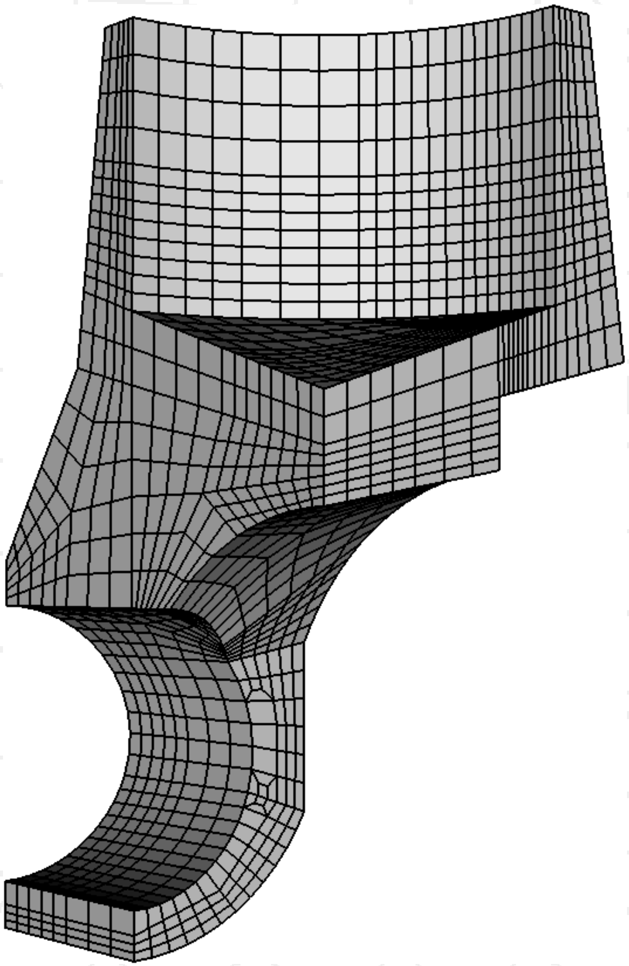
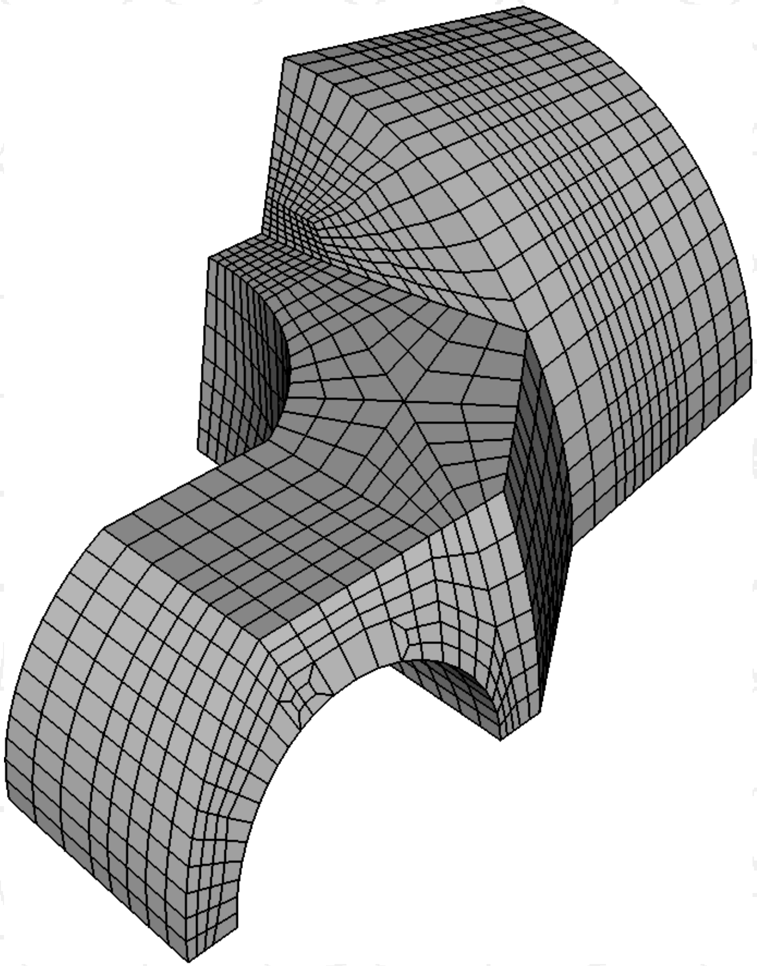
# Example #3





# Conclusions

- Direct manipulation sheet manipulation provide mechanism to convert any input mesh into any desired mesh.
  - Sheet Insertion (pillowing)
  - Sheet Extraction

- Goal mesh can be any desired mesh:
  - Demonstrated: Coarsening, Mesh Matching, THex Cleanup
  - Other: Refinement, Hex Quality Improvement, and more …

- Proof of transformation existence requires modification of mesh boundary.