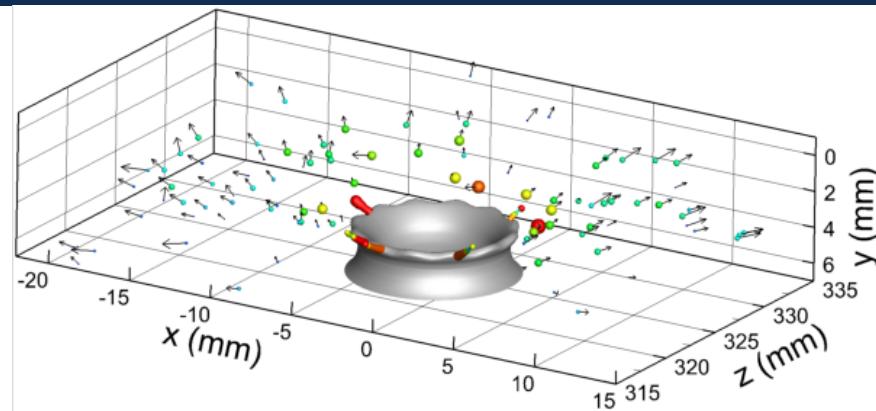
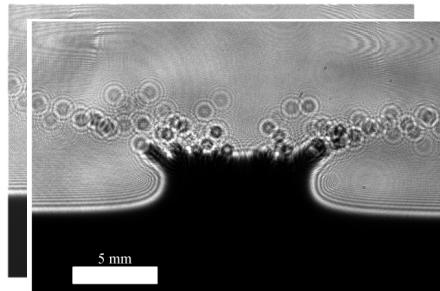


Exceptional service in the national interest



Experimental methods to quantify particle positional and displacement uncertainty along the depth direction in digital in-line holography

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January 14, 2013

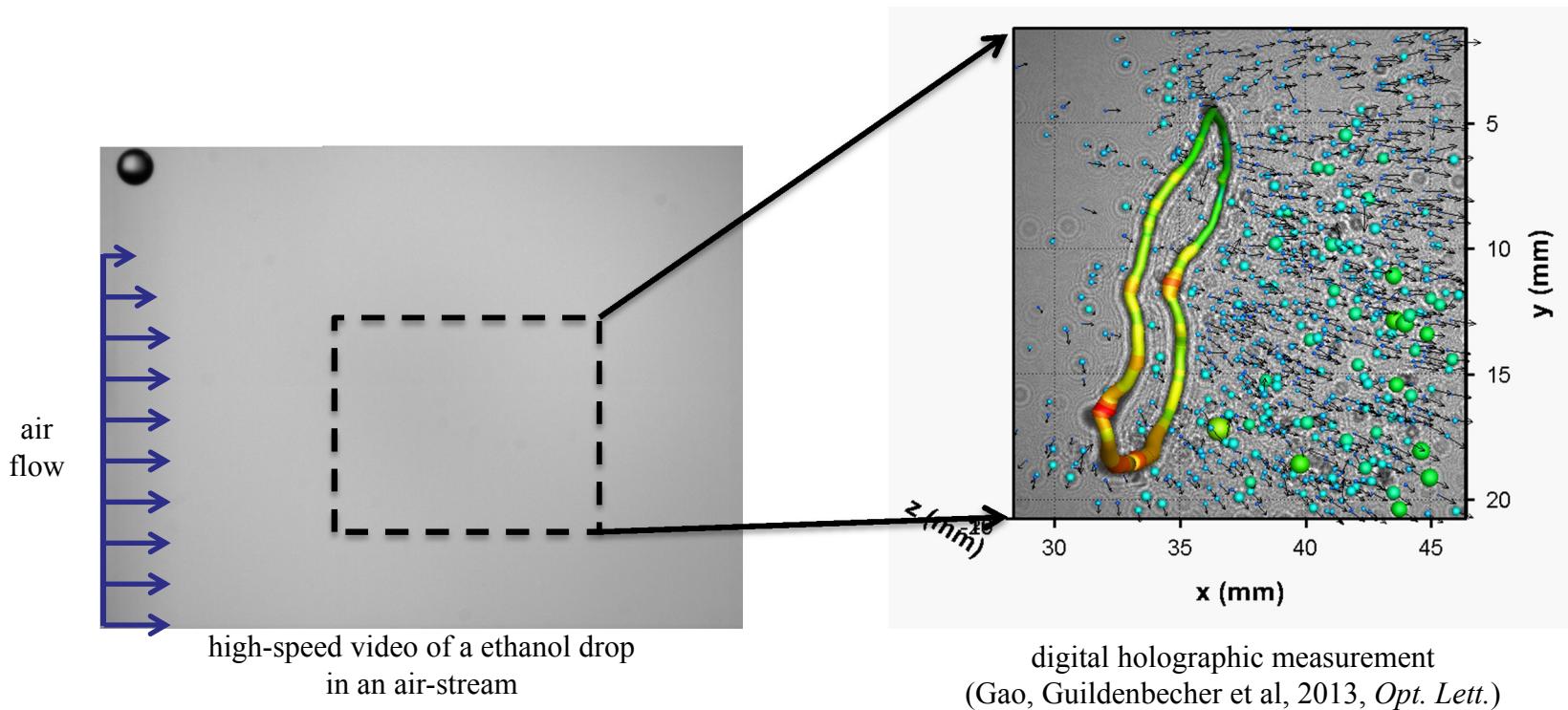


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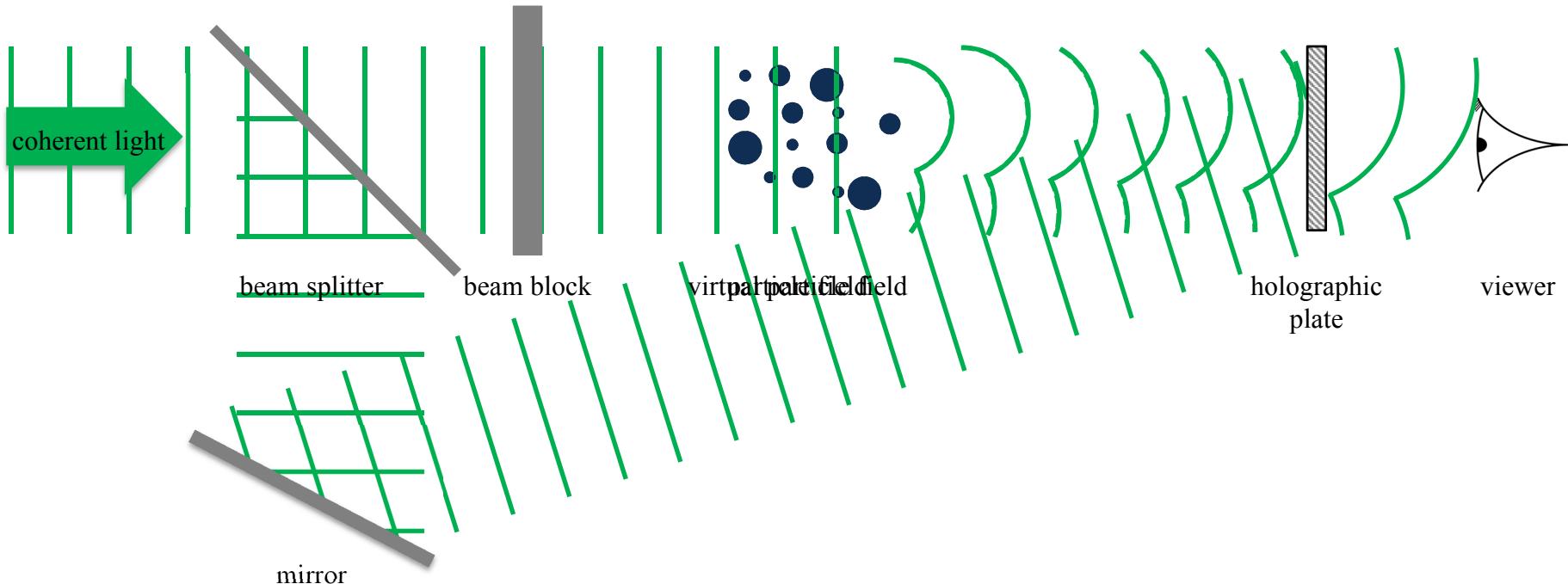
Motivation: 3D imaging for a 3D world

Challenge: 2D imaging or point-wise measurements cannot resolve 3D flow phenomena

- Experimental repetition needed to capture spatial statistics



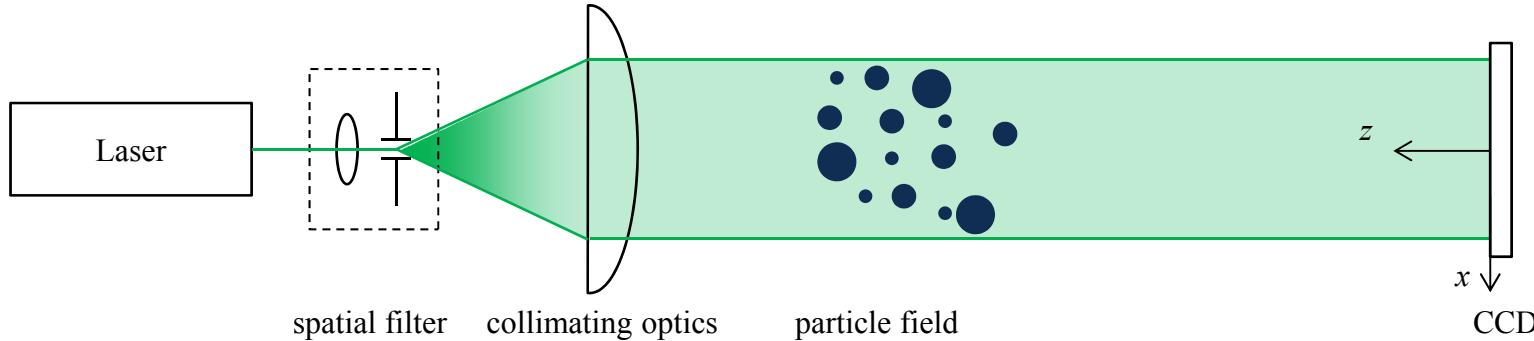
What is holography?



Optical method first proposed by Gabor in 1948

1. Coherent light scattered by particle field forms the object wave, E_o
2. Interference with a reference wave, E_r , forms the hologram: $h = |E_o + E_r|^2$
3. Reconstruction with E_r forms virtual images at original particle locations
$$h \cdot E_r = \underbrace{(|E_o|^2 + |E_r|^2)E_r}_{\text{DC term}} + \underbrace{|E_r|^2 E_o}_{\text{virtual image}} + \underbrace{E_r^2 E_o^*}_{\text{real image}}$$

Digital in-line holography (DIH)



Holographic plate and cumbersome wet-chemical processing replaced with digital sensor (CCD or CMOS)

- Resolution of digital sensors (order 100 line pairs/mm) is much less than resolution of photographic emulsions (order 5,000 line pairs/mm)
 - For suitable off axis angles, θ , the fringe frequency, f , is typically too large to resolve with digital sensors ($f = 2\sin(\theta/2)/\lambda$)
- Rather, the in-line configuration ($\theta = 0$) is typically utilized
 - Reference wave is that portion of the beam which passes through the particle field undisturbed
 - Consequently, the real image overlaps with an out-of-focus virtual image and DC term

Digital in-line holography (DIH)

- In the computer, we multiply the digitally recorded hologram h by an estimate of the complex conjugate of the reference wave E_r^*

$$h \cdot E_r^* = \underbrace{(|E_o|^2 + |E_r|^2)E_r^*}_{\text{DC term}} + \underbrace{E_r^{*2}E_o}_{\text{virtual image}} + \underbrace{|E_r|^2E_o^*}_{\text{real image}}$$

- This complex amplitude can be numerically propagated to any distance along the optical axis, z , using the diffraction equations

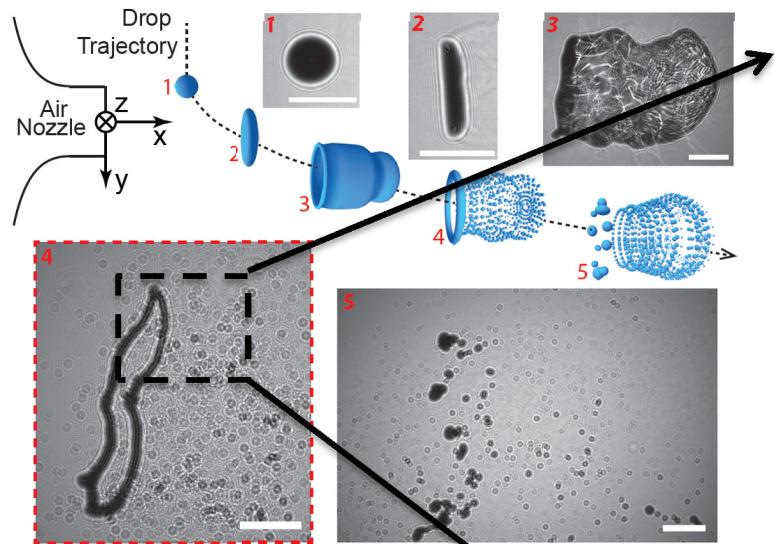
$$E(x, y, z) = h(x, y) \cdot E_r^*(x, y) \otimes g(x, y, z)$$

- Rayleigh-Sommerfeld: $g(x, y, z) = e^{jk\sqrt{x^2 + y^2 + z^2}} / j\lambda\sqrt{x^2 + y^2 + z^2}$
- Fresnel-Kirchhoff: $g(x, y, z) = \frac{e^{jxz}}{j\lambda z} e^{jk(x^2 + y^2)/2z}$
- Numerically, the convolution is computed using the fast Fourier transform (FFT)

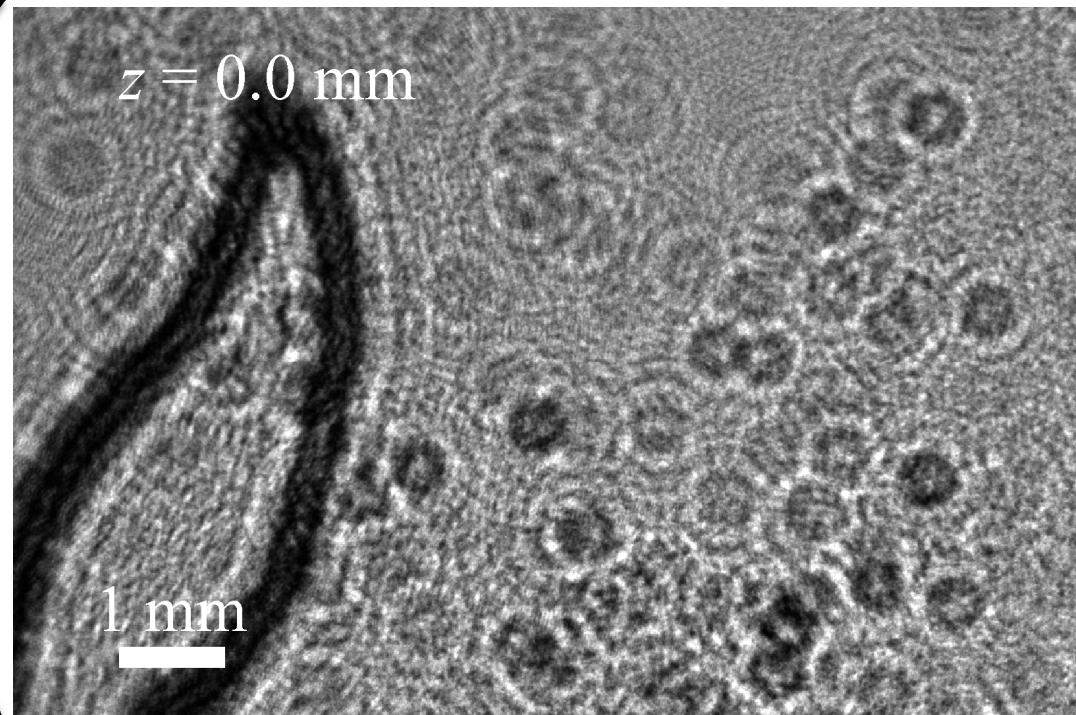
$$E(x, y, z) = \text{FFT}^{-1} \left\{ \text{FFT} \left\{ I_0(x, y) E_r^*(x, y) \right\} \text{FFT} \left\{ g(x, y, z) \right\} \right\}$$

- Visualized via the reconstructed amplitude, $A = |E|$, or intensity, $I = |E|^2$

Digital in-line holography (DIH)



digital holograms of the breakup of an ethanol drop in an air-stream (Gao, Guildenbecher et al 2013, *Opt. Lett.*)



Reconstructed amplitude throughout depth, z

- In-focus structures are clearly observed at different depths, z
- “Rings” around the in-focus structures are the out-of-focus virtual images

The depth-of-focus problem

The spatial extent of the diffraction pattern limits the angular aperture, Ω , from which a particle is effectively reconstructed (Meng et al, 2004, *Meas. Sci. Technol.*):

- From the central diffraction lobe $\rightarrow \Omega \approx 2\lambda/d$
- Using the traditional definition of depth-of-focus, δ , based on change of intensity within the particle center $\rightarrow \delta \approx 4\lambda/\Omega^2$
- Therefore: for in-line holography, $\delta \approx d^2/\lambda$
 - Example: $d = 465 \mu\text{m}$, $\lambda = 532 \text{ nm} \rightarrow \delta \approx 400 \text{ mm}!$

Literature contains two basic methods to find the focal plane with improved accuracy:

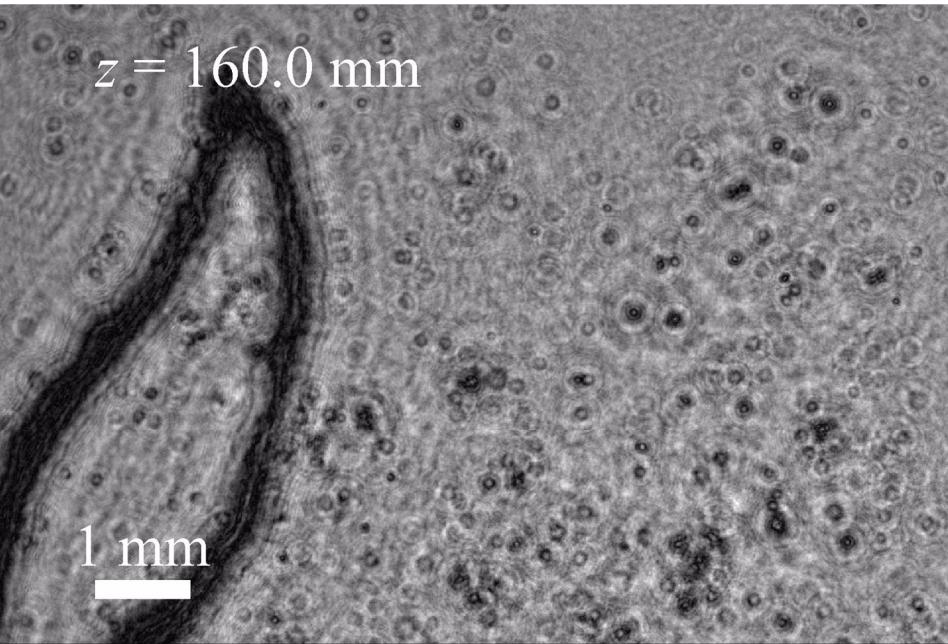
1. Fit a model to the observed diffraction patterns (inverse method)
 - Generally accurate with small depth uncertainty
 - Limited to objects with known diffraction patterns (spheres)
2. Reconstruct the amplitude (or intensity) throughout depth and apply a focus metric to find “in-focus” objects
 - No *a-priori* knowledge of particle shape required
 - Accuracy is a strong function of the chosen focus metric

Hybrid particle extraction method

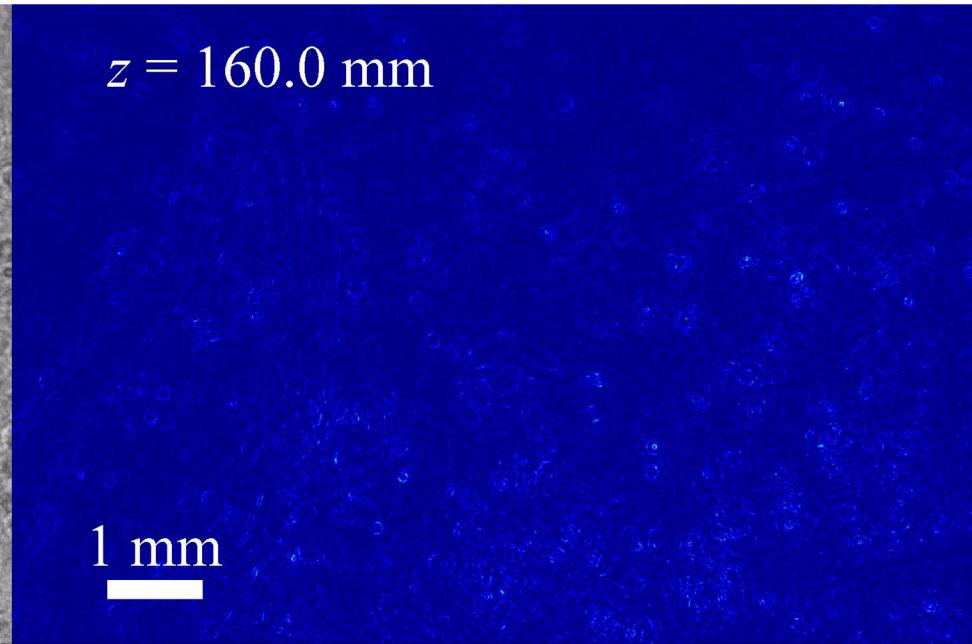
Basic idea: In-focus regions display a minimum amplitude within the particle interior and a maximum sharpness at the particle edges

- Validity of this assumption has been verified through simulation

$z = 160.0 \text{ mm}$



$z = 160.0 \text{ mm}$



Reconstructed amplitude throughout depth, z

Reconstructed edge sharpness throughout depth, z

- Optimum threshold for particle extraction is automatically extracted from the threshold of the amplitude which displays maximum edge sharpness
 - Further details in Guildenbecher et al, 2013, *Appl. Opt.* and Gao, Guildenbecher, et al, 2013, *Opt. Express*.

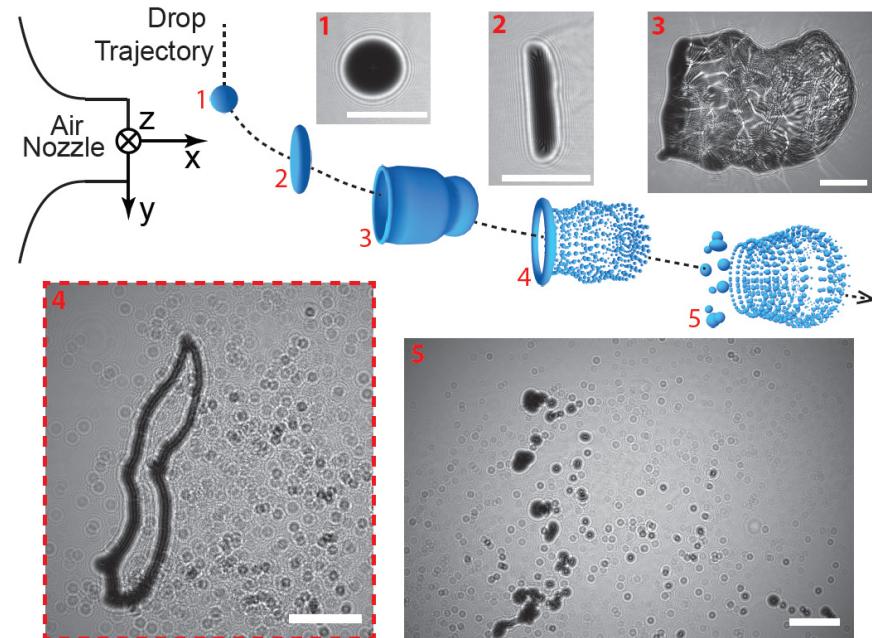
Aerodynamic drop fragmentation

Motivation: fundamental spray process and an important canonical problem for multiphase simulations

- No viable methods to measure secondary drop size/velocity statistics or the 3D morphology of the ring shaped ligament

Experimental configuration: Double-pulsed laser and imaging hardware as typically used in PIV

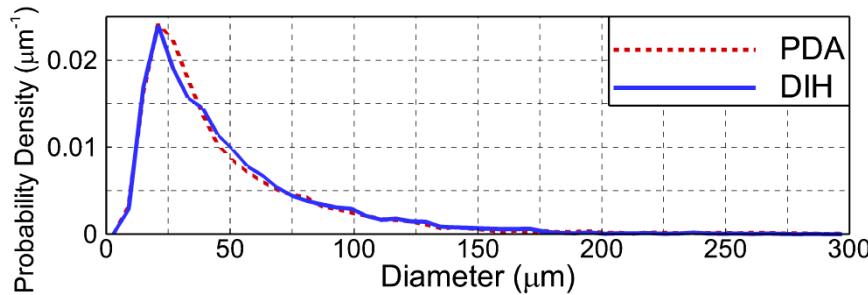
- $\lambda = 532 \text{ nm}$, 5 ns pulsed width
- Interline transfer CCD (4008×2672 , 9 μm pixel pitch)
- Temporal separation, $\Delta t = 62 \mu\text{s}$, determined by laser timing
 - Note: experiments in Guildenbecher et al, 2013, *Proceedings of Digital Holography and 3-D Imaging* confirm no loss of accuracy due to the reduced coherence length of these lasers



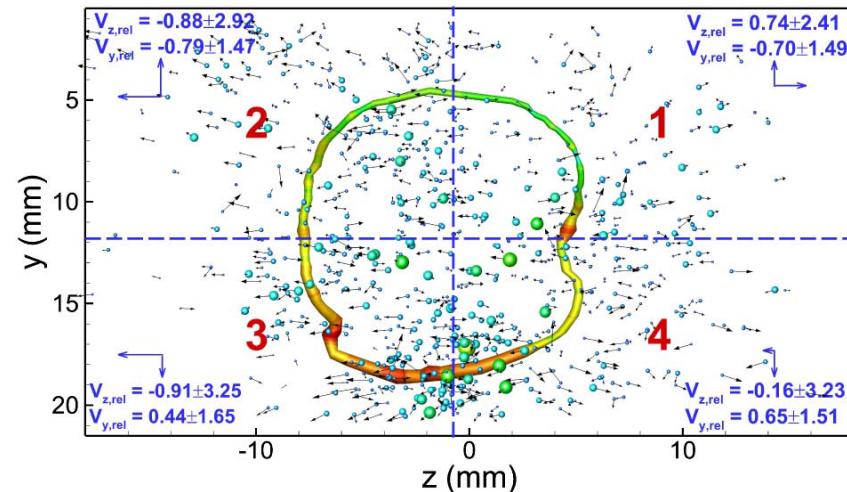
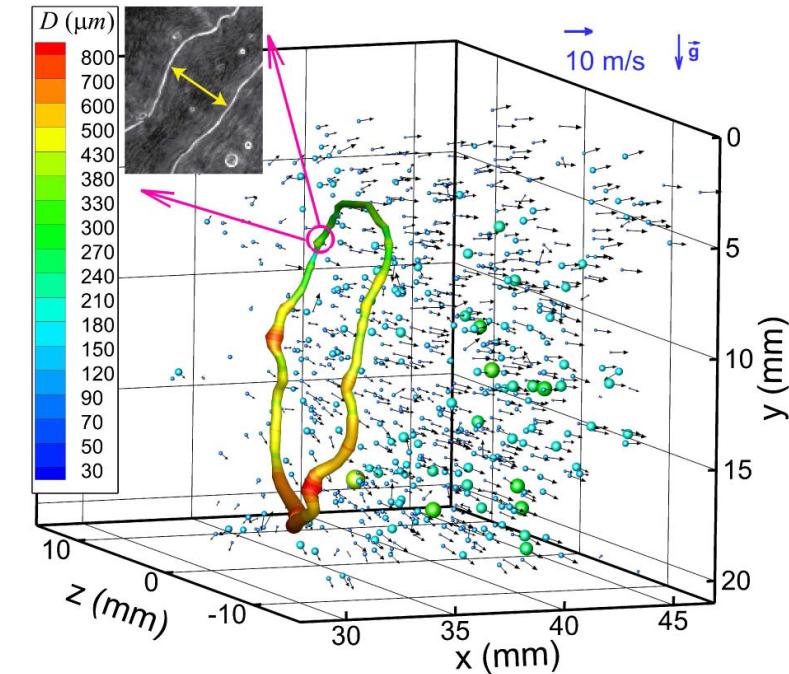
digital holograms of the breakup of an ethanol drop in an air-stream (Gao, Guildenbecher et al 2013, *Opt. Lett.*)

Aerodynamic drop fragmentation

- Secondary drop sizes/positions extracted by the hybrid method
 - Comparison with phase Doppler anemometer (PDA) data confirms accuracy of measured sizes



- Ring measured from z-location of maximum edge sharpness
 - Total volume of ring + secondary drops is within 2.2% of the initial volume
- 3C velocity measured by particle matching between successive frames
 - Expected symmetry observed with higher uncertainty in z-direction



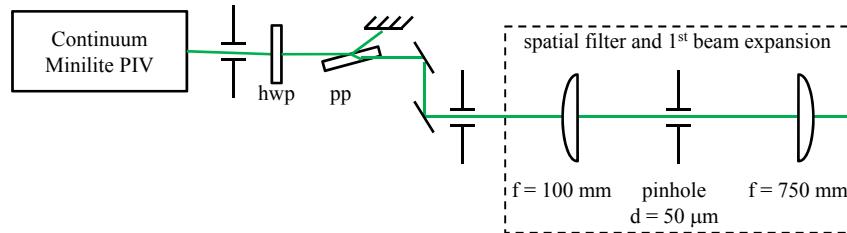
Drop impact on a thin film

Motivation: measurement of secondary droplet by other methods requires significant experimental repetition

- Process symmetry provides opportunities to validate accuracy

Experimental configuration:

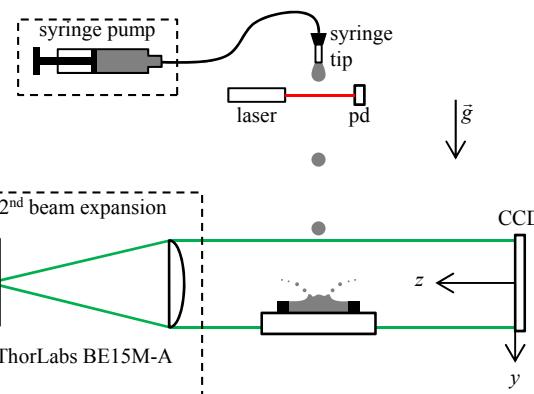
- Double pulsed laser ($\lambda = 532$ nm, 5 ns pulselength)
- Interline transfer CCD (4872 \times 3248, 7.4 μm pixel pitch)
- Temporal separation, $\Delta t = 33 \mu\text{s}$, determined by laser timing



experimental configuration of holographic recording of drop impact on a thin film
(Guildenbecher et al, 2013, *Exp. Fluids.*)

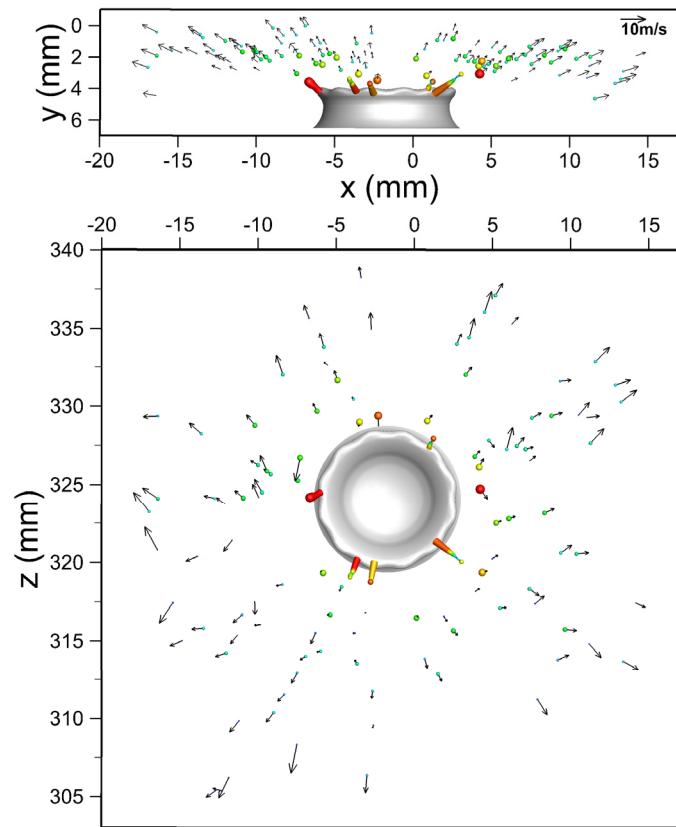
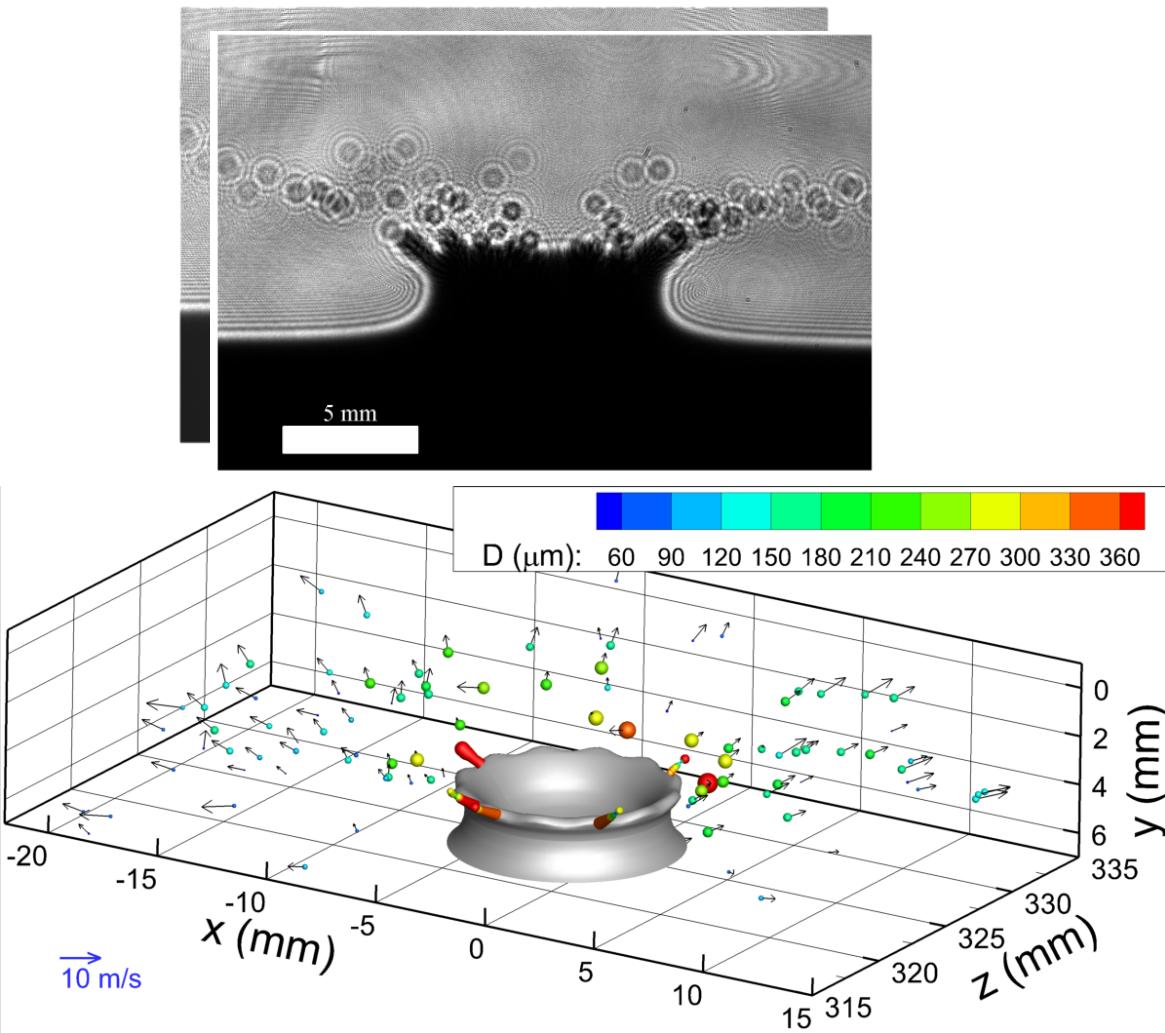


impact of a 3 mm water drop on a 2 mm water film
(Guildenbecher et al, 2013, *Exp. Fluids.*)



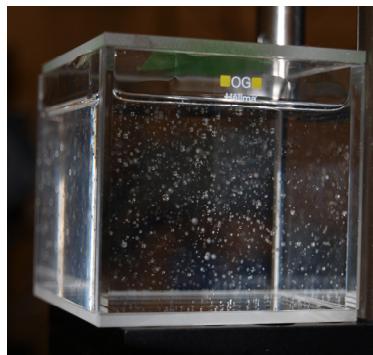
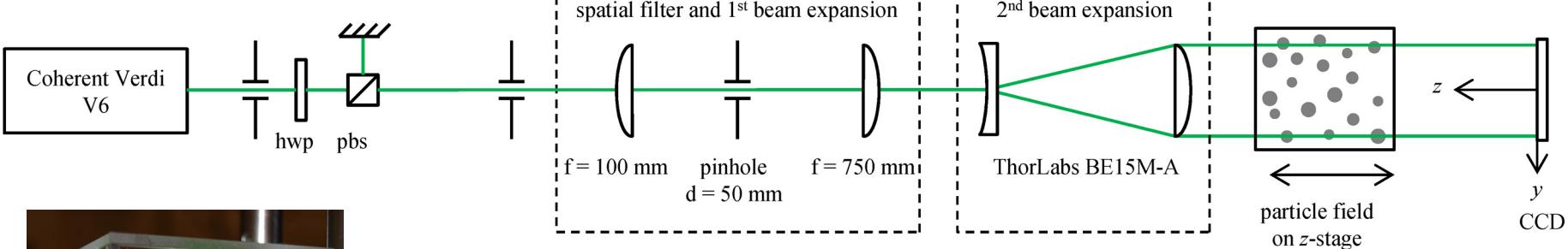
Drop impact on a thin film

Again processed with the hybrid method



holographic reconstruction of
drop impact on a thin film
(Guildenbecher et al, 2013, *Exp. Fluids.*)

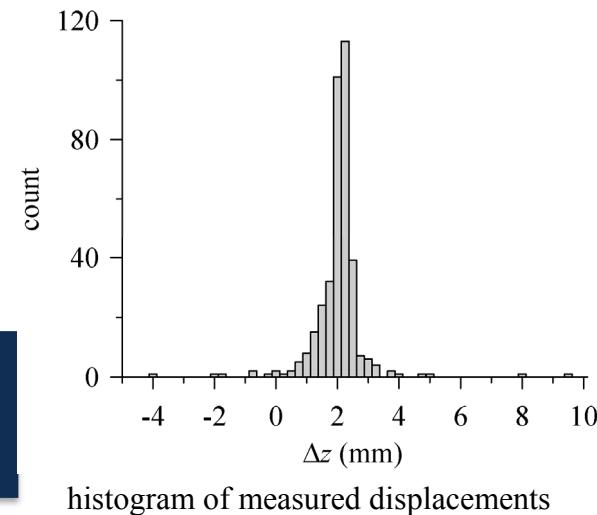
Experimental validation



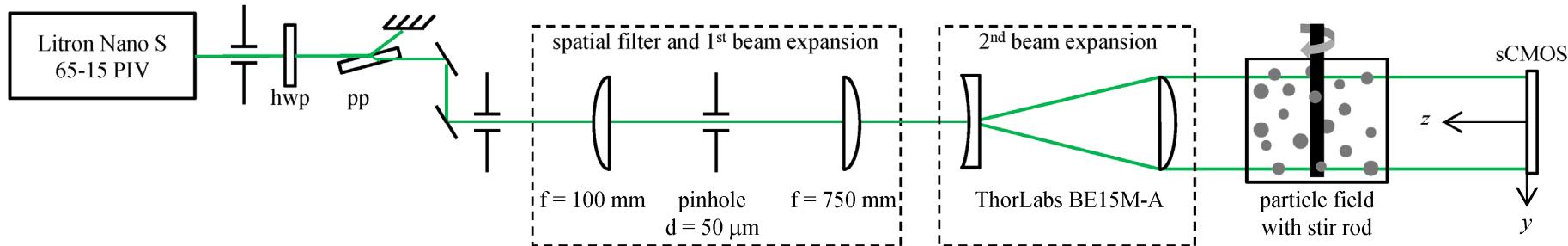
particle field

- Quasi-stationary particle field
 - Polystyrene beads ($\bar{d} \approx 465 \mu\text{m}$) in 10,000 cSt silicone oil
 - Settling velocity $\approx 1 \mu\text{m/s}$
- Multiple holograms recorded, displacing the particle field 2 mm in the z-direction between each acquisition
- Displacement found by particle matching between successive holograms
 - Actual displacement, $\Delta z = 2.0 \text{ mm}$
 - Measured displacement, $\Delta z = 1.91 \text{ mm} \pm 0.81 \text{ mm}$
 - Standard deviation of 1.74 times mean diameter

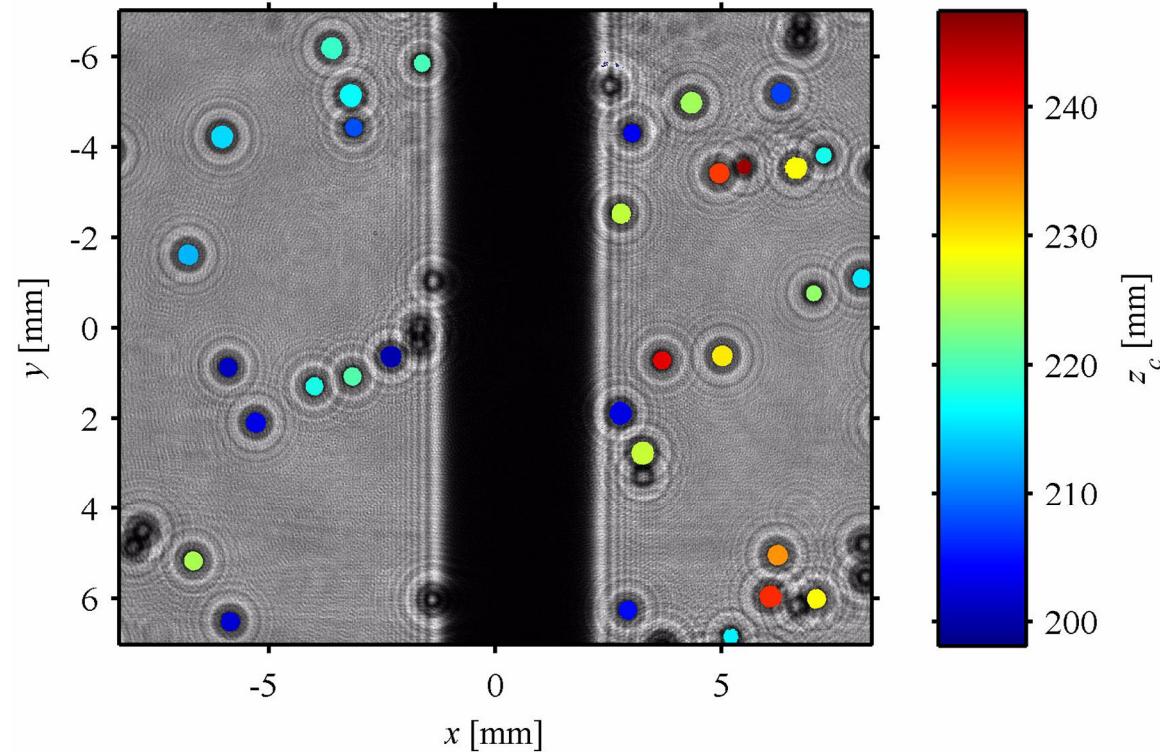
Issue: overall z-positions are not validated and effects due to particle motion are not considered



Experimental configuration for z validation



- Particles stirred by a rotating rod ($r_0 = 1.58 \text{ mm}$, $\omega_0 = 100 \text{ rpm}$)
- Recorded at 15Hz with a LaVision sCMOS camera (2560×2160 , $6.5 \mu\text{m}$ pixel pitch)



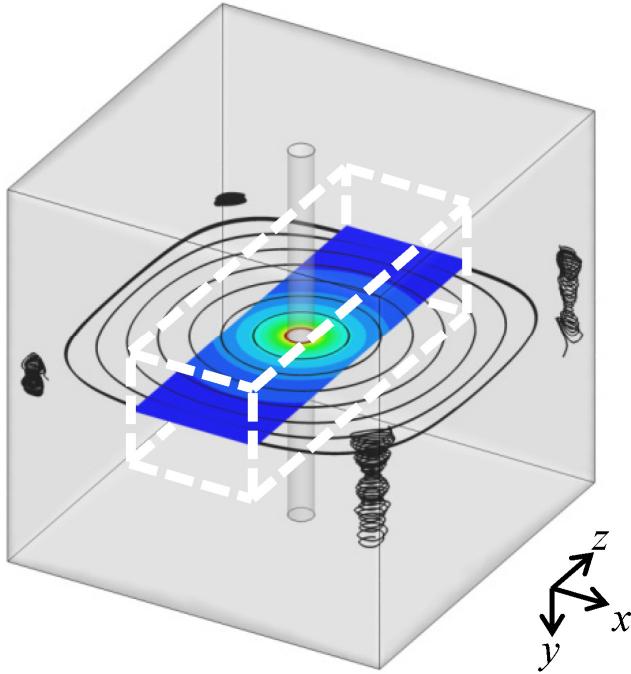
particles measured with the hybrid method, background shows the recorded holograms

Extraction of theoretical trajectory

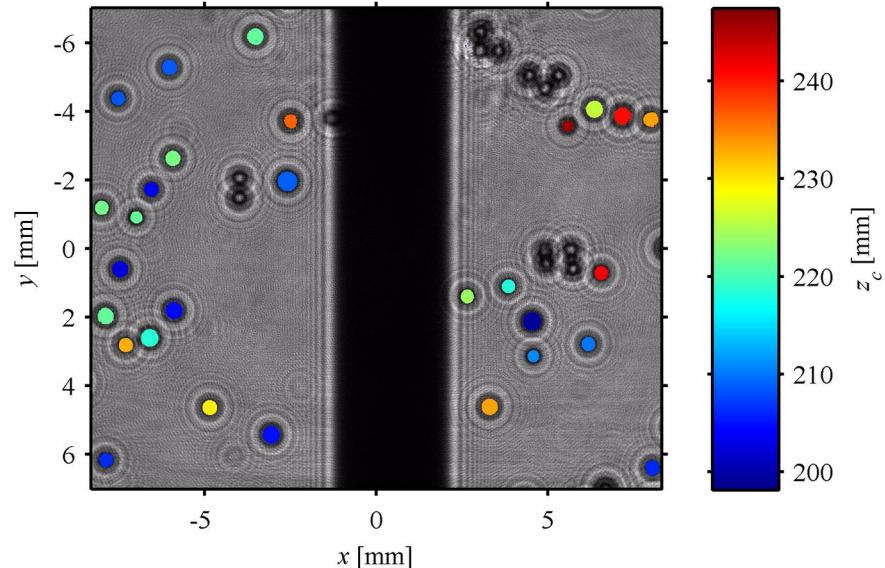
In the x - z plane, particles are expected to travel in near perfect circles

$$x(t) = r \cos(\omega t + \theta_0)$$

- Assuming measured x -positions have minimal error, curve fit $\rightarrow r, \omega, \theta_0$



simulated flow field showing streamlines and total velocity contour within the center x - z plane of the field of view (dotted lines)

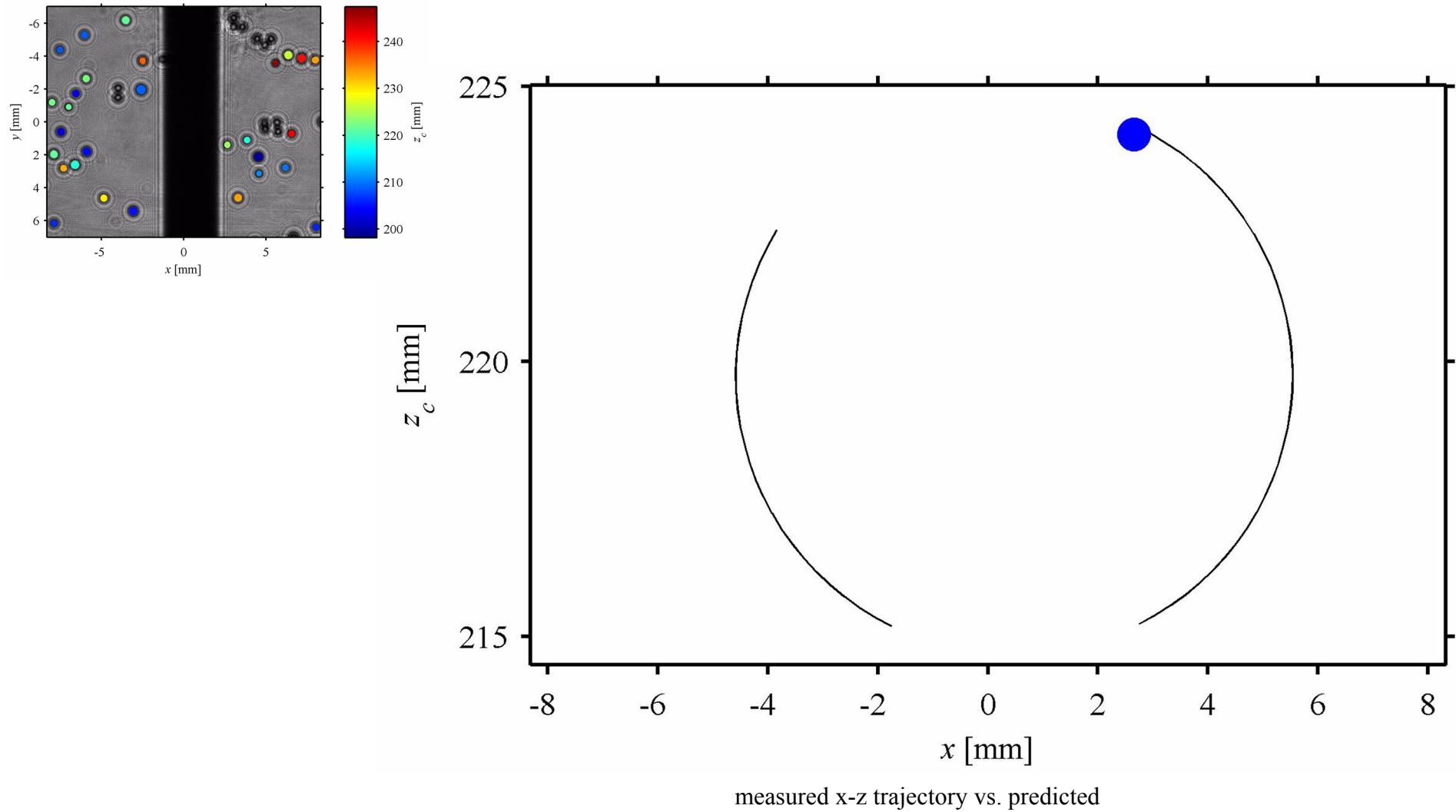


example in-plane trajectory

- Measured $r = 5.04$ mm, $\omega = 9.414$ rpm
- At this r , simulation gives $\omega = 9.406$ rpm

Comparison with measured results

Predicted z -trajectory: $z(t) = r \sin(\omega t + \theta_0)$ and $\Delta z(t) = r \omega \cos(\omega t + \theta_0) \cdot \Delta t$



Conclusions

For all trajectories

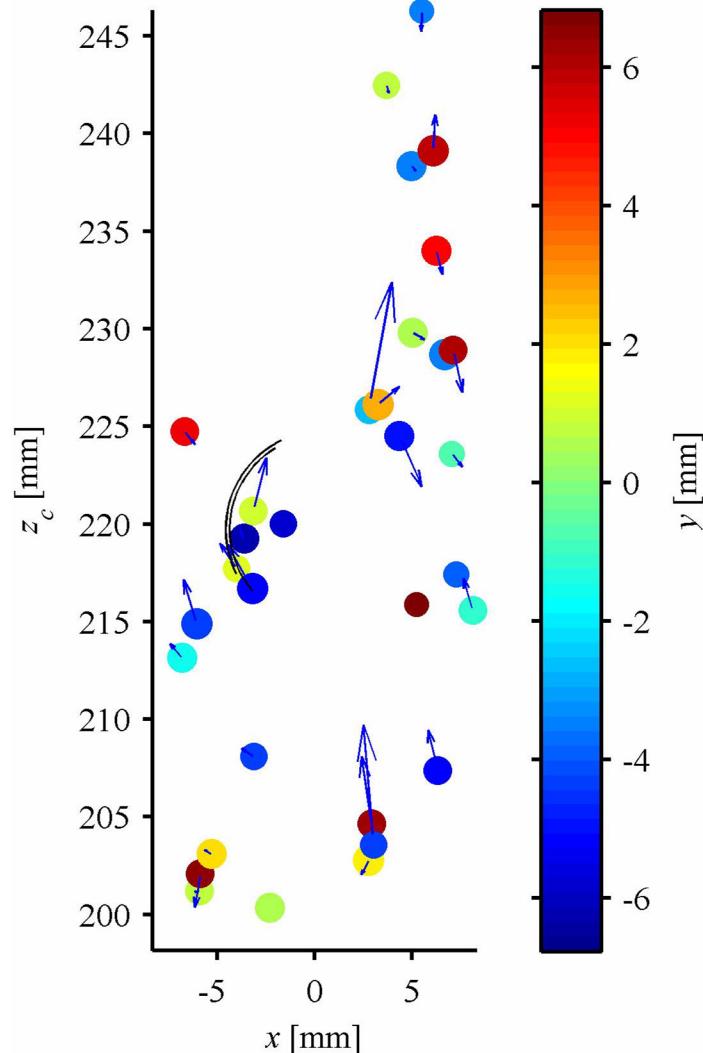
- Error in measured $z = -0.04 \pm 1.51$ mm
- Error in measured $\Delta z = -0.03 \pm 1.05$ mm
 - Standard deviation of $2.3 \cdot \bar{d}$

Experiments repeated with smaller particles
($\bar{d} = 118 \mu\text{m}$, see paper for details)

- Error in measured $z = -0.003 \pm 0.379$ mm
- Error in measured $\Delta z = -0.001 \pm 0.302$ mm
 - Standard deviation of $2.6 \cdot \bar{d}$

Next steps:

- Compare results with alternative particle detection methods
- Use results to quantify effects of particle overlap and other experimental noise sources



all measured x - z trajectories vs. predicted

3D, 3C fluid velocity measurements?

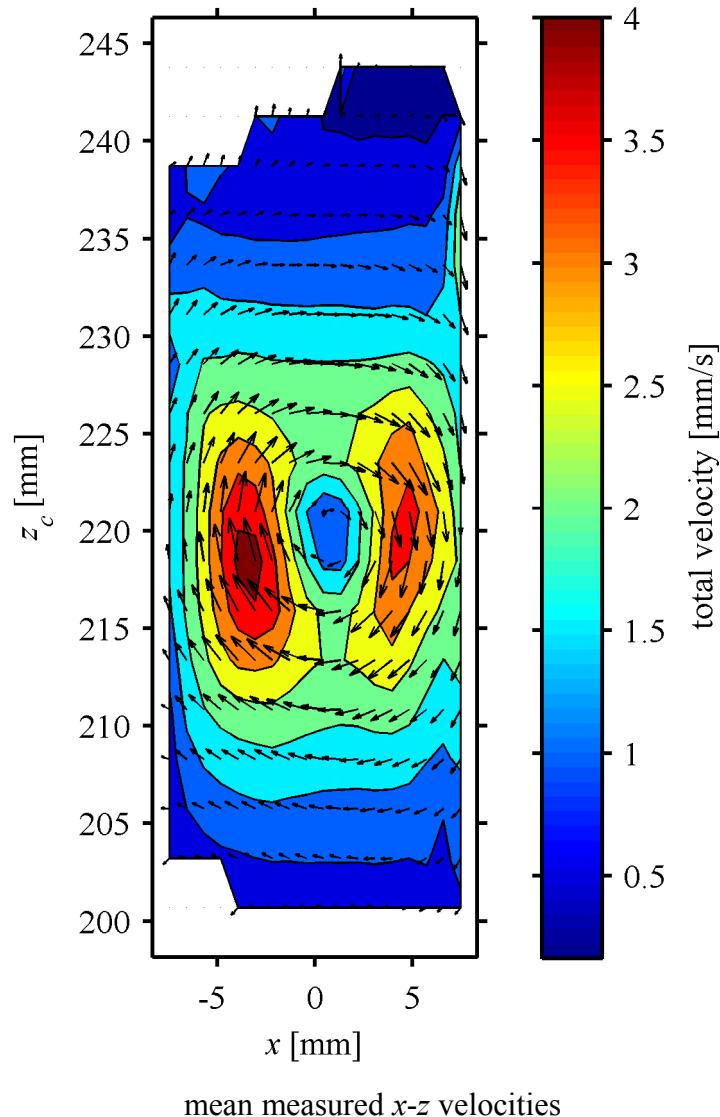
Advantages:

- Simple optical setup requiring only one line-of-sight view
- Large depth of field (hundreds of mm possible)
- Particle sizes can be measured (if desired)

Challenges:

- High uncertainty in the z-direction
- Particle field must be relatively spare providing only limited vectors
- Vectors at random positions
- Methods not as mature as PIV or even tomographic-PIV

Note: the literature contains many works on holographic-PIV. My own work has not been focused on these applications



Questions?