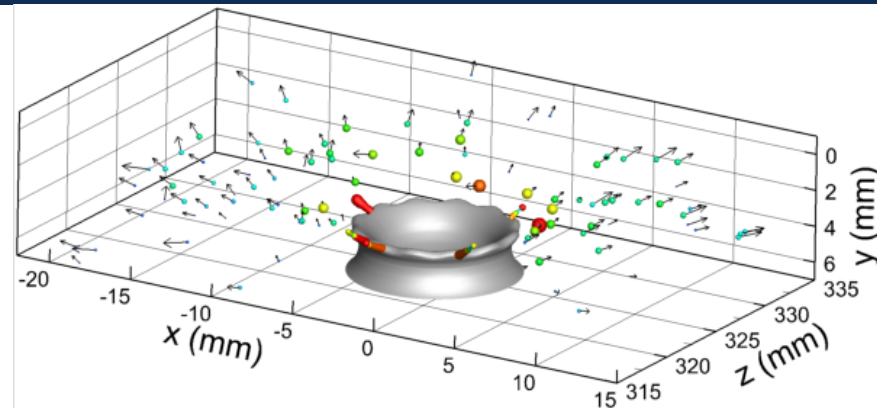
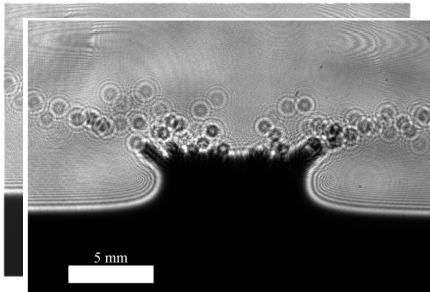


*Exceptional service in the national interest*



Experimental methods to quantify particle positional and displacement uncertainty along the depth direction in digital in-line holography

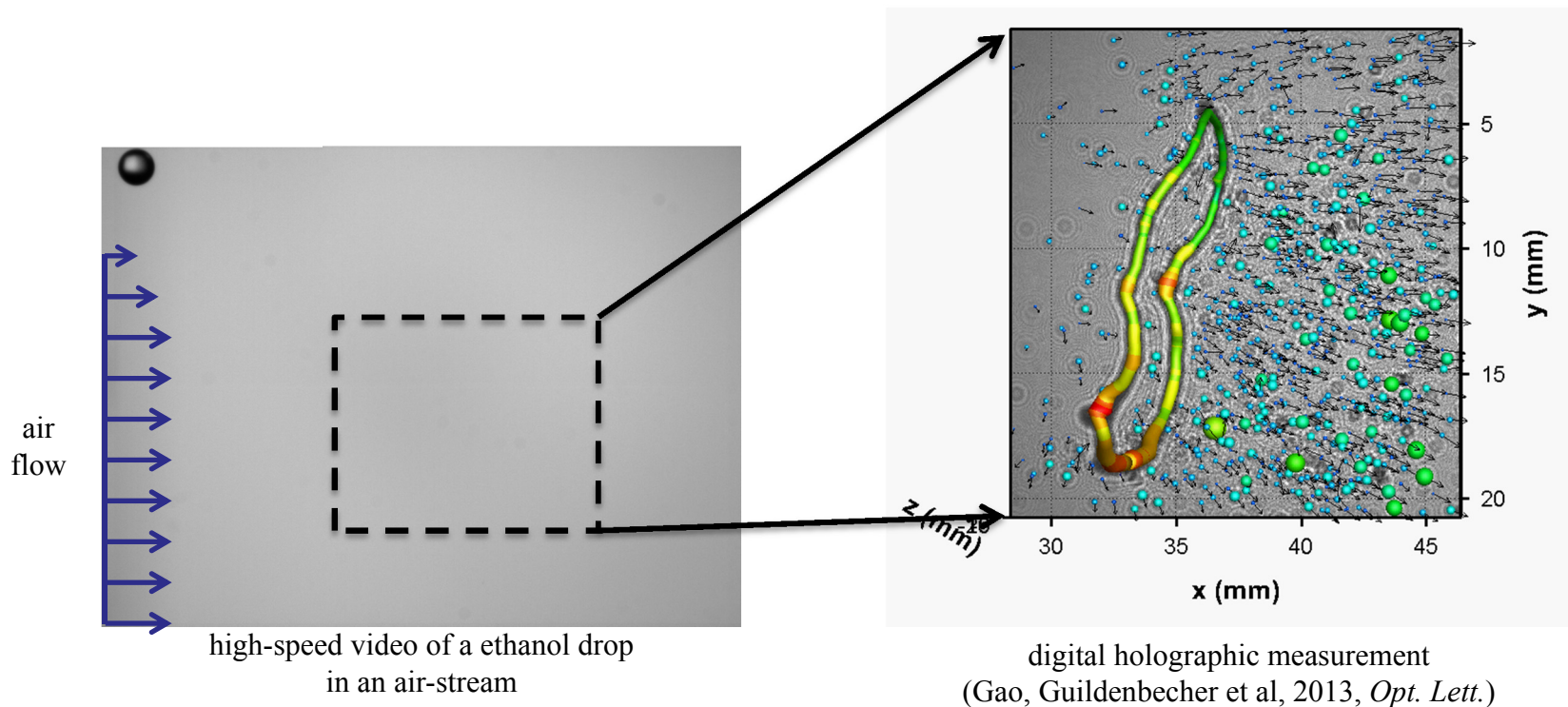
Daniel R. Guildenbecher, Phillip L. Reu, Martin B. Nemer, Jian Gao, Jun Chen

January 14, 2013

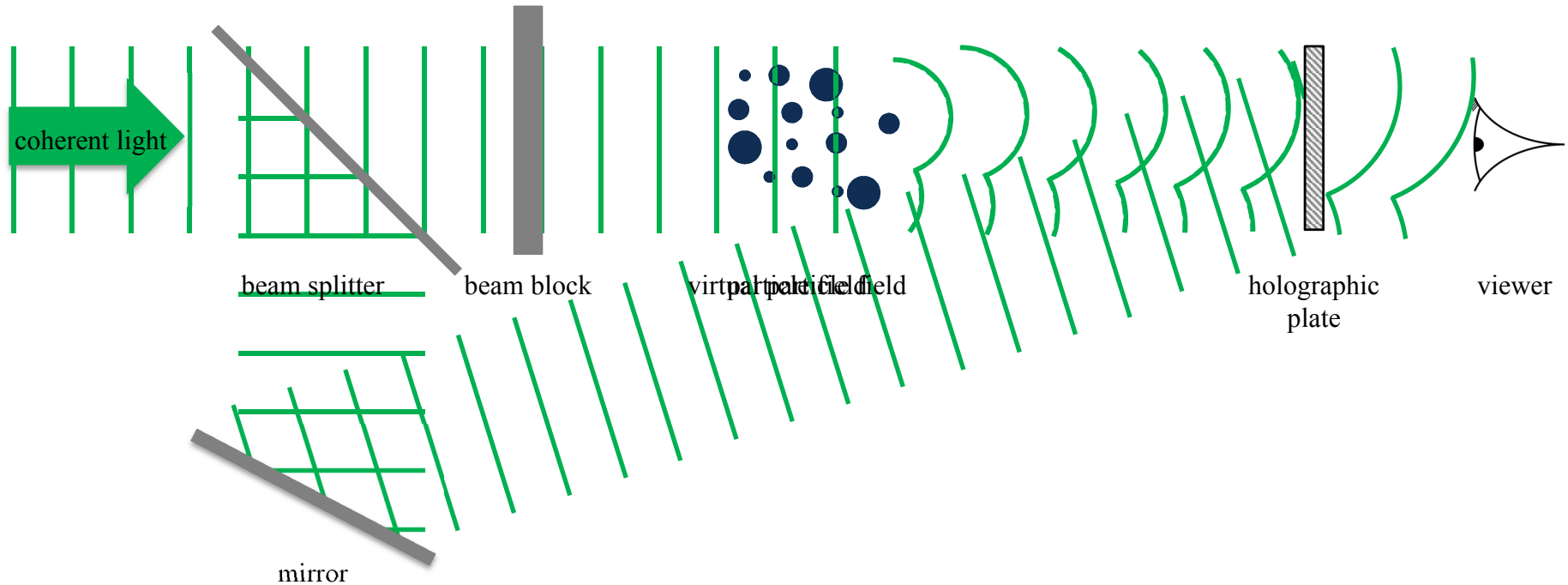
# Motivation: 3D imaging for a 3D world

Challenge: 2D imaging or point-wise measurements cannot resolve 3D flow phenomena

- Experimental repetition needed to capture spatial statistics



# What is holography?

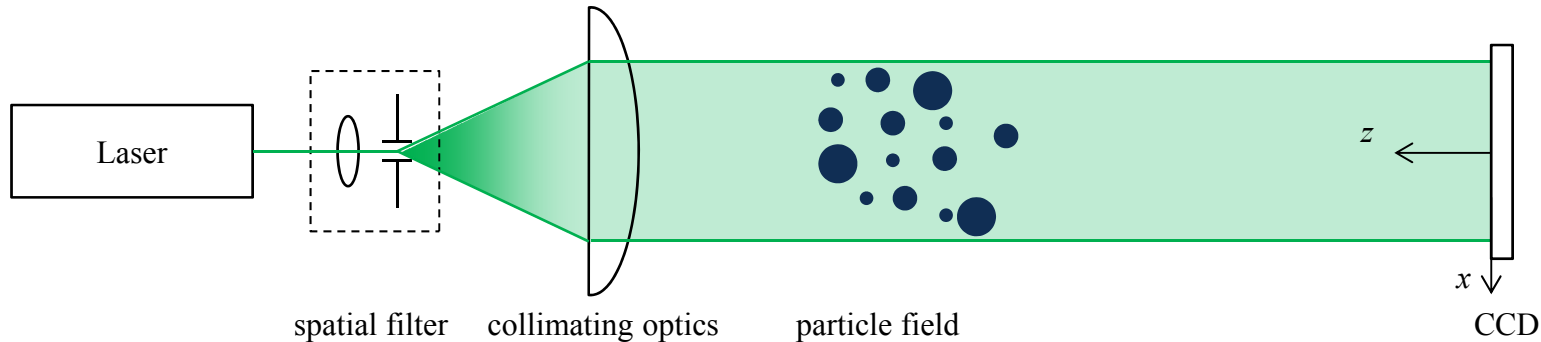


Optical method first proposed by Gabor in 1948

1. Coherent light scattered by particle field forms the object wave,  $E_o$
2. Interference with a reference wave,  $E_r$ , forms the hologram:  $h = |E_o + E_r|^2$
3. Reconstruction with  $E_r$  forms virtual images at original particle locations

$$h \cdot E_r = \underbrace{(|E_o|^2 + |E_r|^2)E_r}_{\text{DC term}} + \underbrace{|E_r|^2 E_o}_{\text{virtual image}} + \underbrace{E_r^2 E_o^*}_{\text{real image}}$$

# Digital in-line holography (DIH)



Holographic plate and cumbersome wet-chemical processing replaced with digital sensor (CCD or CMOS)

- Resolution of digital sensors (order 100 line pairs/mm) is much less than resolution of photographic emulsions (order 5,000 line pairs/mm)
  - For suitable off axis angles,  $\theta$ , the fringe frequency,  $f$ , is typically too large to resolve with digital sensors ( $f = 2\sin(\theta/2)/\lambda$ )
- Rather, the in-line configuration ( $\theta = 0$ ) is typically utilized
  - Reference wave is that portion of the beam which passes through the particle field undisturbed
  - Consequently, the real image overlaps with an out-of-focus virtual image and DC term



# Digital in-line holography (DIH)

- In the computer, we multiply the digitally recorded hologram  $h$  by an estimate of the complex conjugate of the reference wave  $E_r^*$

$$h \cdot E_r^* = \underbrace{(|E_o|^2 + |E_r|^2)E_r^*}_{\text{DC term}} + \underbrace{E_r^{*2}E_o}_{\text{virtual image}} + \underbrace{|E_r|^2E_o^*}_{\text{real image}}$$

- This complex amplitude can be numerically propagated to any distance along the optical axis,  $z$ , using the diffraction equations

$$E(x, y, z) = h(x, y) \cdot E_r^*(x, y) \otimes g(x, y, z)$$

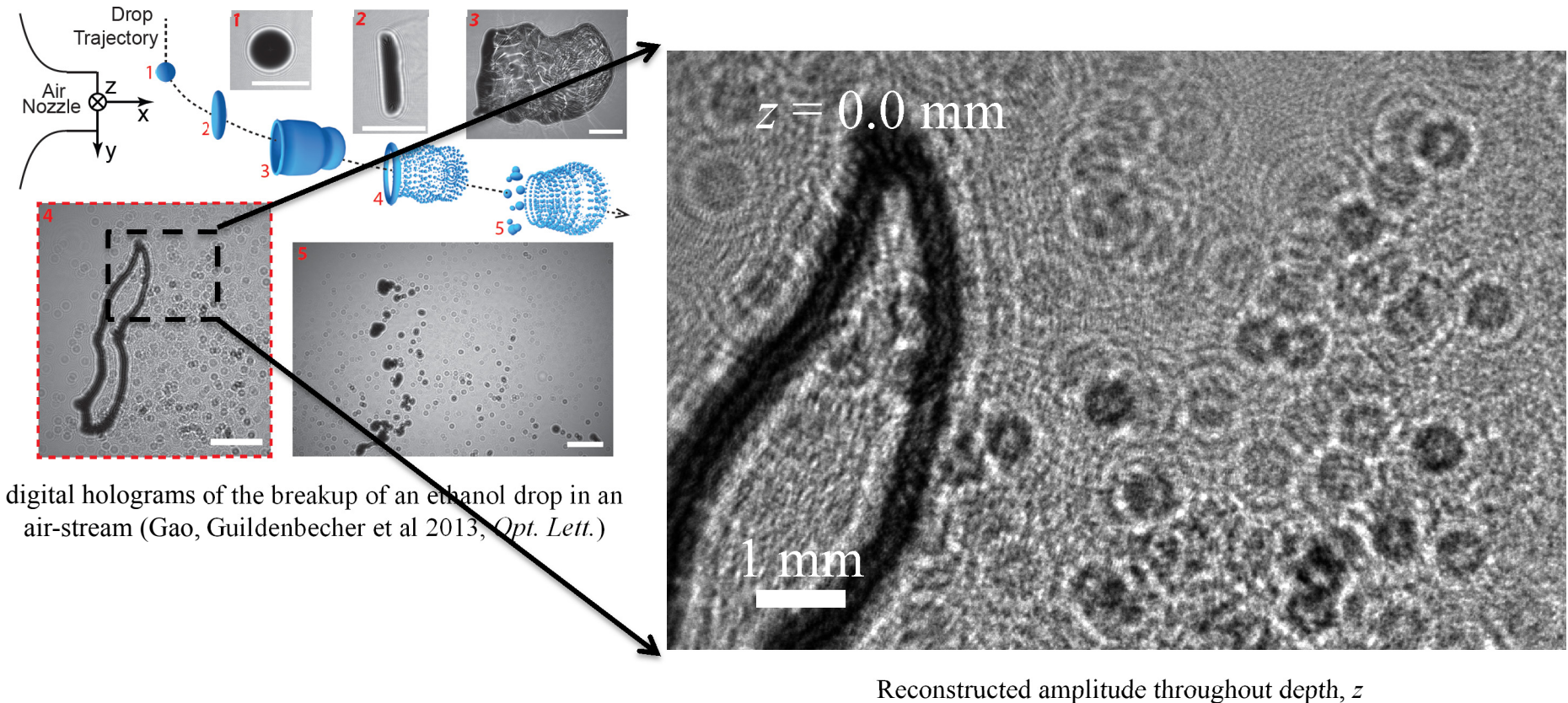
- Rayleigh-Sommerfeld:  $g(x, y, z) = e^{jk\sqrt{x^2+y^2+z^2}} / j\lambda\sqrt{x^2+y^2+z^2}$
- Fresnel-Kirchhoff:  $g(x, y, z) = \frac{e^{jkz}}{j\lambda z} e^{jk(x^2+y^2)/2z}$

- Numerically, the convolution is computed using the fast Fourier transform (FFT)

$$E(x, y, z) = FFT^{-1} \left\{ FFT \left\{ I_o(x, y) E_r^*(x, y) \right\} FFT \left\{ g(x, y, z) \right\} \right\}$$

- Visualized via the reconstructed amplitude,  $A = |E|$ , or intensity,  $I = |E|^2$

# Digital in-line holography (DIH)



- In-focus structures are clearly observed at different depths,  $z$
- “Rings” around the in-focus structures are the out-of-focus virtual images

# The depth-of-focus problem

The spatial extent of the diffraction pattern limits the angular aperture,  $\Omega$ , from which a particle is effectively reconstructed (Meng et al, 2004, *Meas. Sci. Technol.*):

- From the central diffraction lobe  $\rightarrow \Omega \approx 2\lambda/d$
- Using the traditional definition of depth-of-focus,  $\delta$ , based on change of intensity within the particle center  $\rightarrow \delta \approx 4\lambda/\Omega^2$
- Therefore: for in-line holography,  $\delta \approx d^2/\lambda$ 
  - Example:  $d = 465 \mu\text{m}$ ,  $\lambda = 532 \text{ nm}$   $\rightarrow \delta \approx 400 \text{ mm}$ !

Literature contains two basic methods to find the focal plane with improved accuracy:

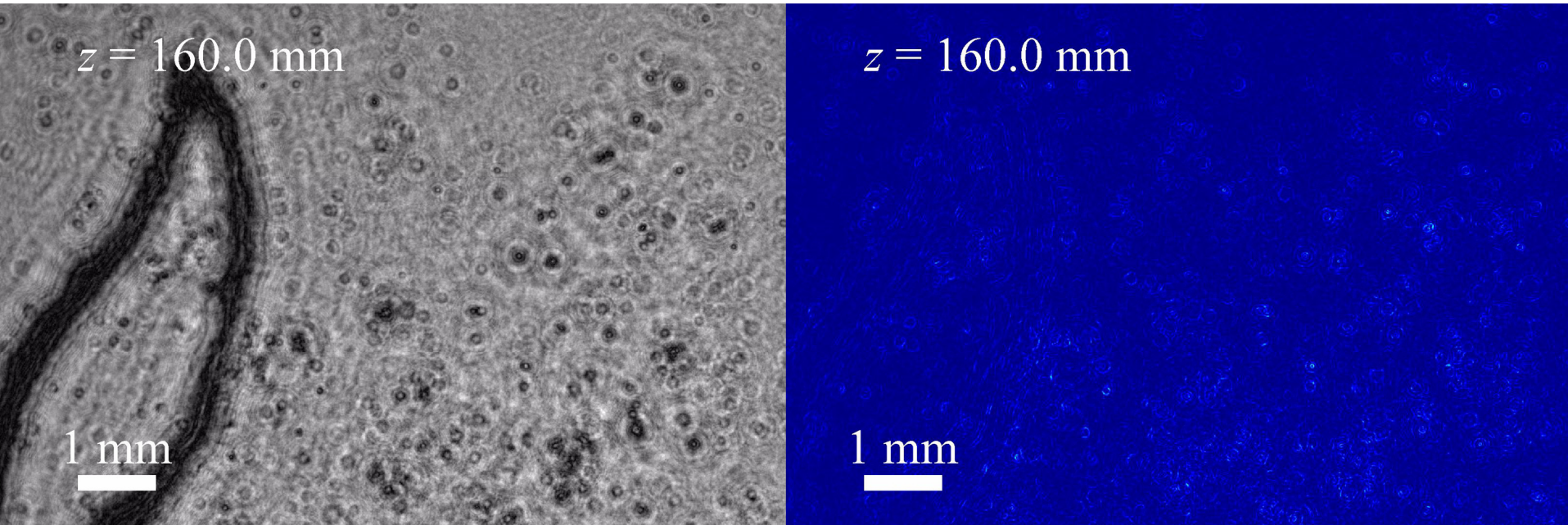
1. Fit a model to the observed diffraction patterns (inverse method)
  - Generally accurate with small depth uncertainty
  - Limited to objects with known diffraction patterns (spheres)
2. Reconstruct the amplitude (or intensity) throughout depth and apply a focus metric to find “in-focus” objects
  - No *a-priori* knowledge of particle shape required
  - Accuracy is a strong function of the chosen focus metric



# Hybrid particle extraction method

Basic idea: In-focus regions display a minimum amplitude within the particle interior and a maximum sharpness at the particle edges

- Validity of this assumption has been verified through simulation



Reconstructed amplitude throughout depth,  $z$

Reconstructed edge sharpness throughout depth,  $z$

- Optimum threshold for particle extraction is automatically extracted from the threshold of the amplitude which displays maximum edge sharpness
  - Further details in Guildenbecher et al, 2013, *Appl. Opt.* and Gao, Guildenbecher, et al, 2013, *Opt. Express*.

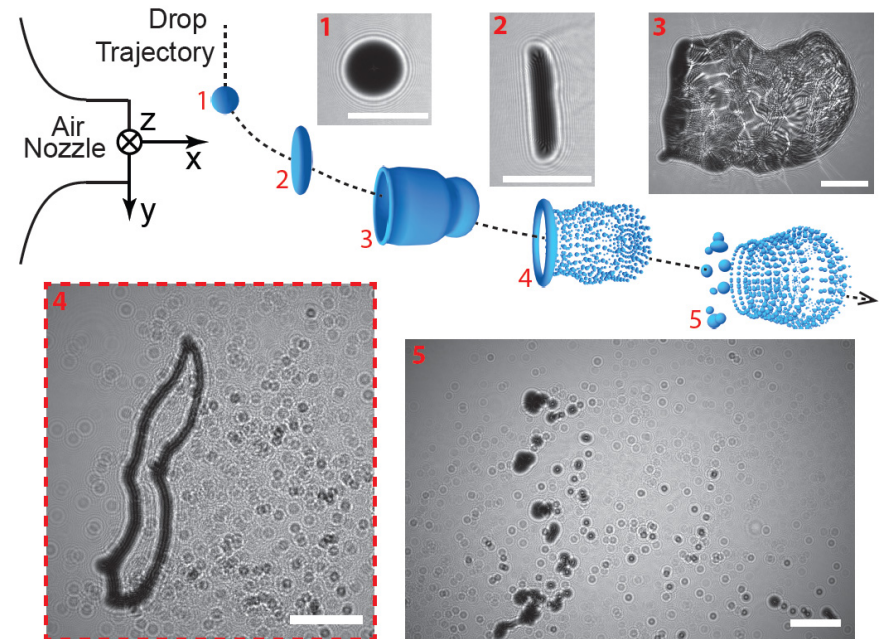
# Aerodynamic drop fragmentation

Motivation: fundamental spray process and an important canonical problem for multiphase simulations

- No viable methods to measure secondary drop size/velocity statistics or the 3D morphology of the ring shaped ligament

Experimental configuration: Double-pulsed laser and imaging hardware as typically used in PIV

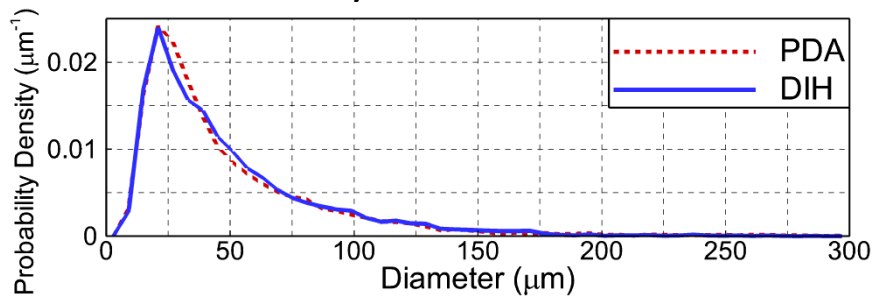
- $\lambda = 532 \text{ nm}$ , 5 ns pulsewidth
- Interline transfer CCD ( $4008 \times 2672$ ,  $9 \mu\text{m}$  pixel pitch)
- Temporal separation,  $\Delta t = 62 \mu\text{s}$ , determined by laser timing
  - Note: experiments in Guildenbecher et al, 2013, *Proceedings of Digital Holography and 3-D Imaging* confirm no loss of accuracy due to the reduced coherence length of these lasers



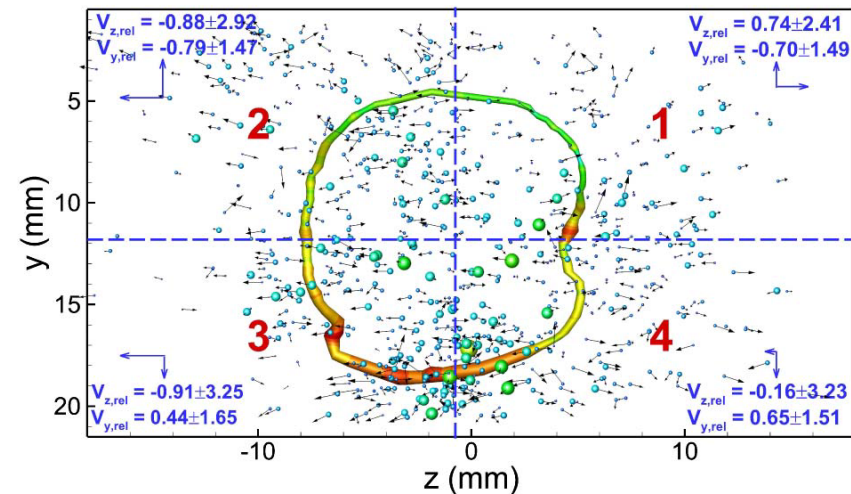
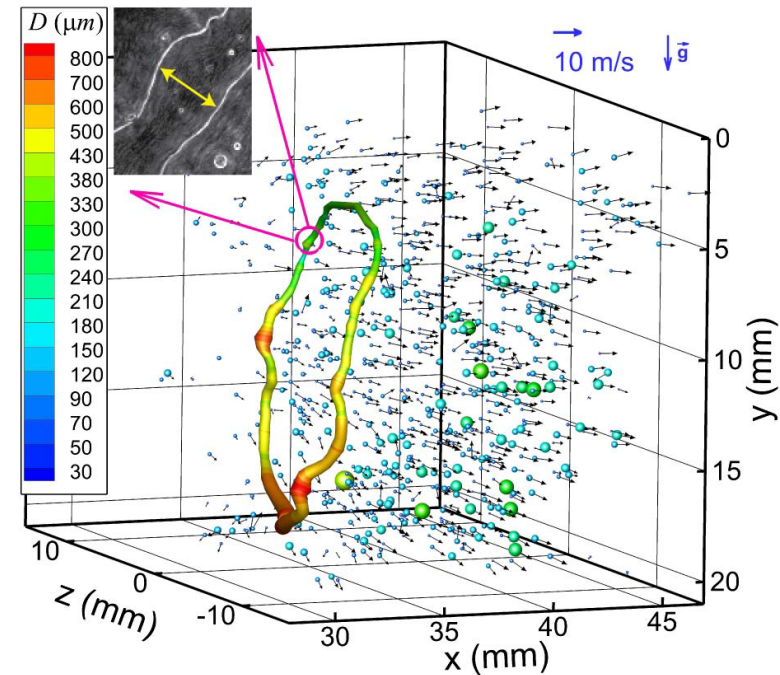
digital holograms of the breakup of an ethanol drop in an air-stream (Gao, Guildenbecher et al 2013, *Opt. Lett.*)

# Aerodynamic drop fragmentation

- Secondary drop sizes/positions extracted by the hybrid method
  - Comparison with phase Doppler anemometer (PDA) data confirms accuracy of measured sizes



- Ring measured from z-location of maximum edge sharpness
  - Total volume of ring + secondary drops is within 2.2% of the initial volume
- 3C velocity measured by particle matching between successive frames
  - Expected symmetry observed with higher uncertainty in z-direction





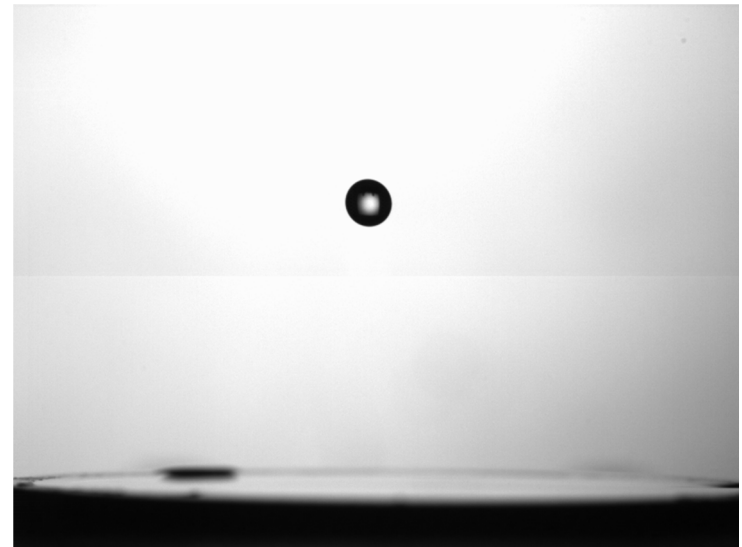
# Drop impact on a thin film

Motivation: measurement of secondary droplet by other methods requires significant experimental repetition

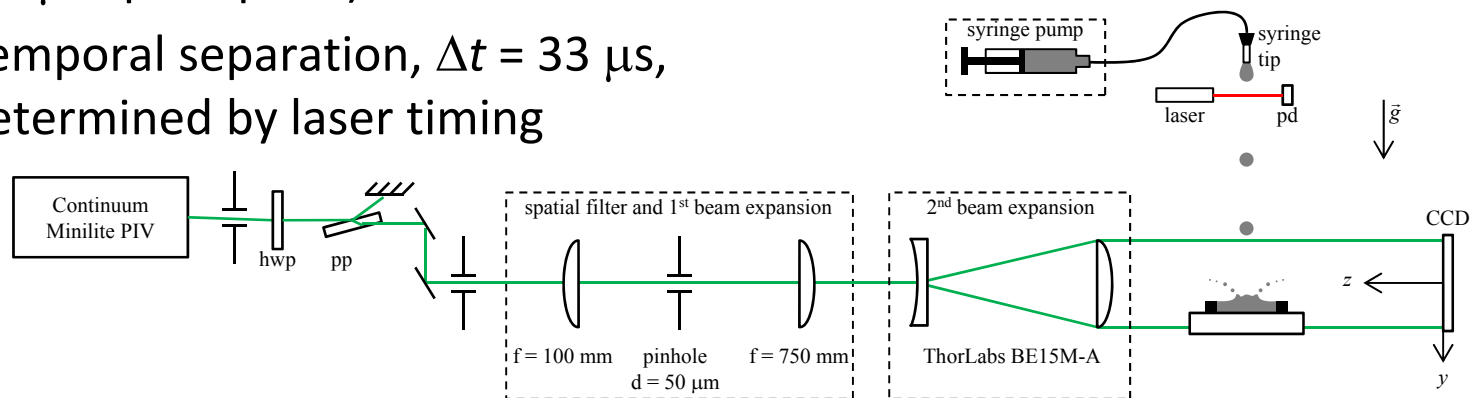
- Process symmetry provides opportunities to validate accuracy

Experimental configuration:

- Double pulsed laser ( $\lambda = 532$  nm, 5 ns pulsewidth)
- Interline transfer CCD ( $4872 \times 3248$ ,  $7.4 \mu\text{m}$  pixel pitch)
- Temporal separation,  $\Delta t = 33 \mu\text{s}$ , determined by laser timing



impact of a 3 mm water drop on a 2 mm water film  
(Guildenbecher et al, 2013, *Exp. Fluids*.)

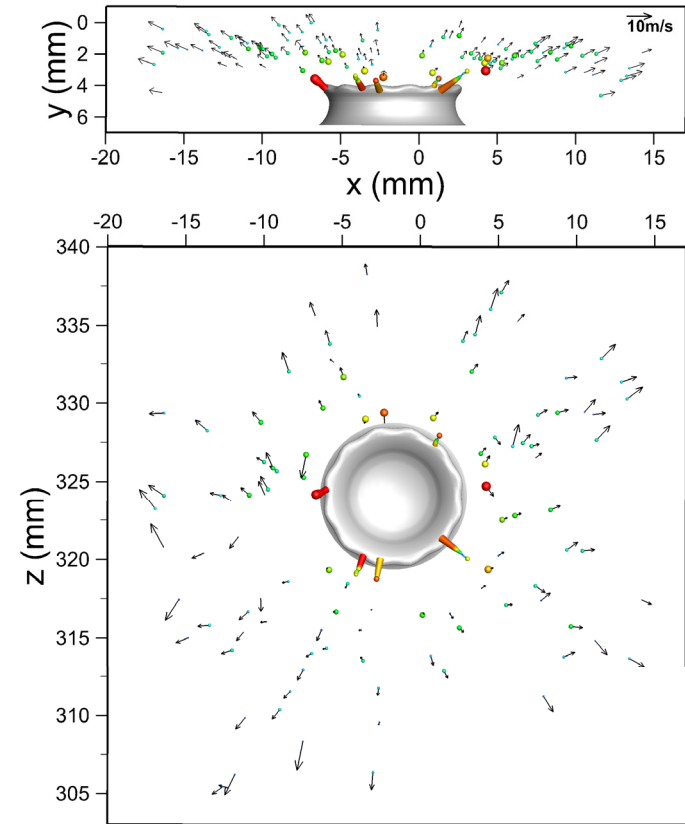
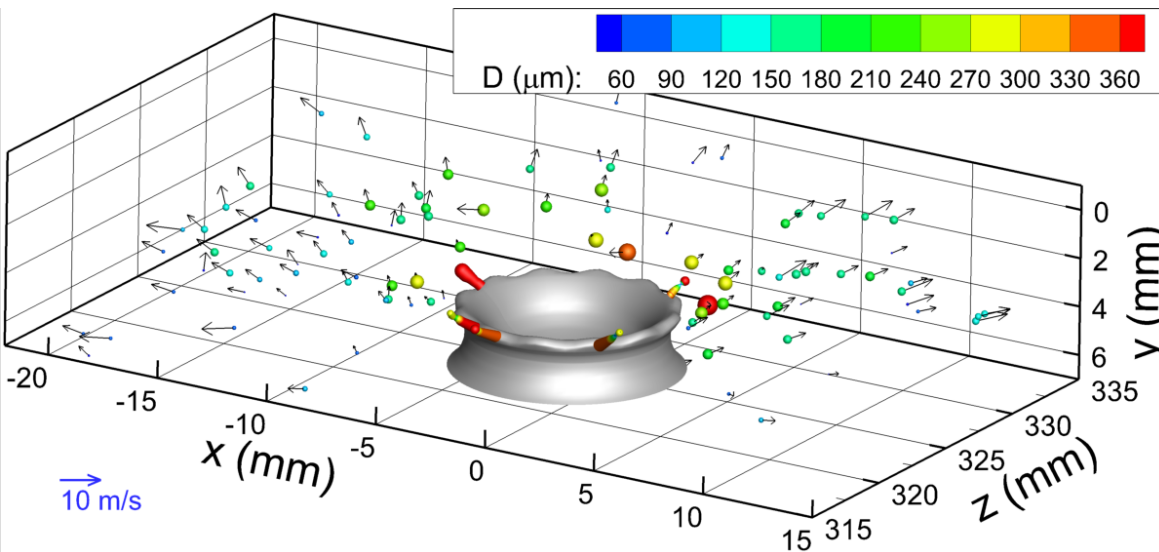
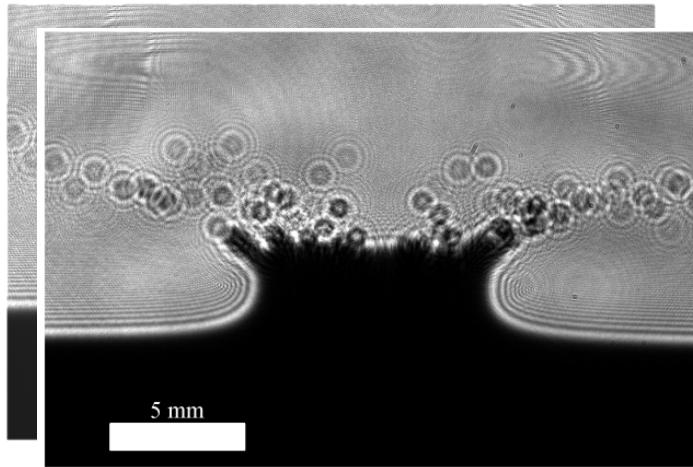


experimental configuration of holographic recording of drop impact on a thin film  
(Guildenbecher et al, 2013, *Exp. Fluids*.)



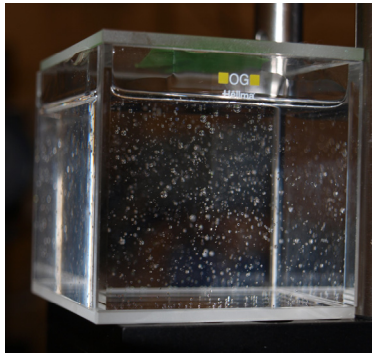
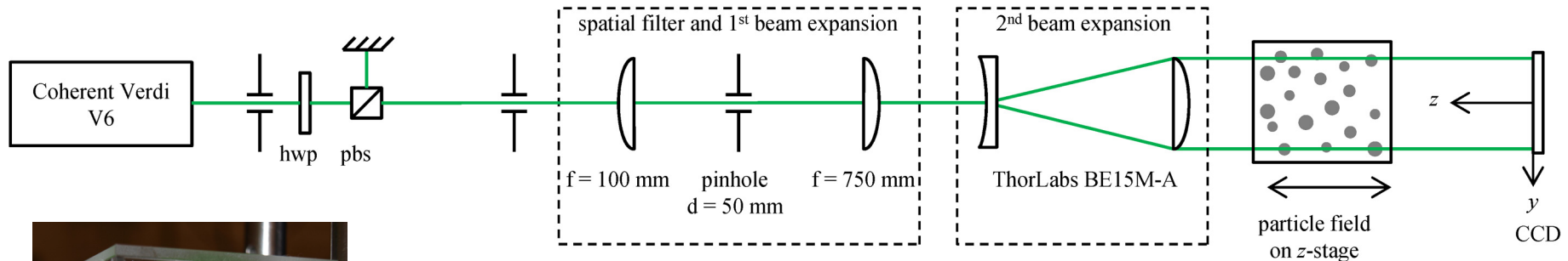
# Drop impact on a thin film

Again processed with the hybrid method



holographic reconstruction of  
drop impact on a thin film  
(Guildenbecher et al, 2013, *Exp. Fluids*.)

# Experimental validation

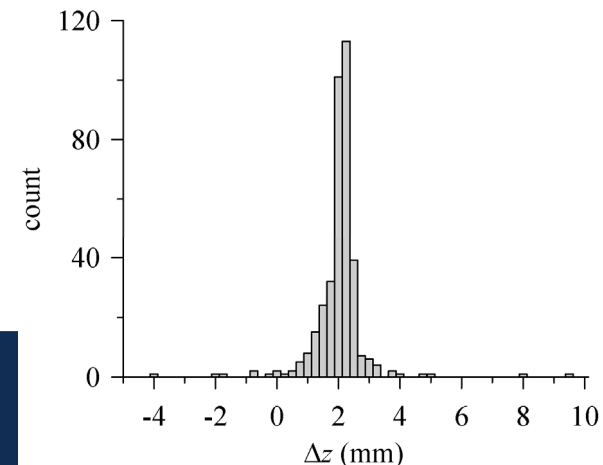


particle field

- Quasi-stationary particle field
  - Polystyrene beads ( $\bar{d} \approx 465 \mu\text{m}$ ) in 10,000 cSt silicone oil
  - Settling velocity  $\approx 1 \mu\text{m/s}$
- Multiple holograms recorded, displacing the particle field 2 mm in the z-direction between each acquisition

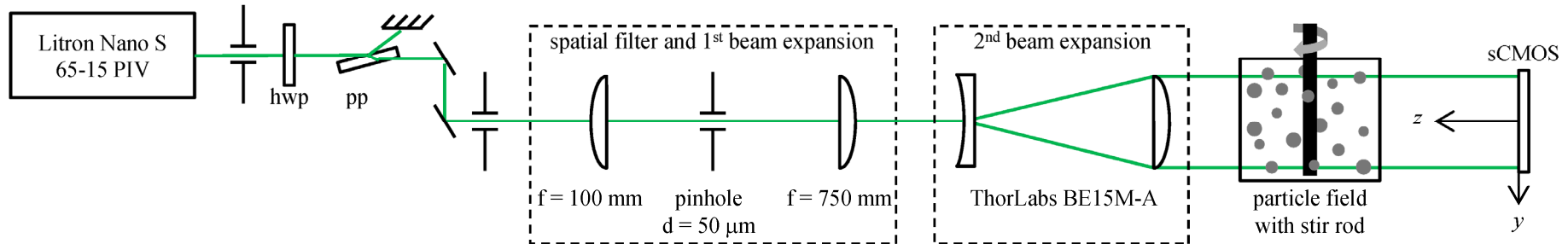
- Displacement found by particle matching between successive holograms
  - Actual displacement,  $\Delta z = 2.0$  mm
  - Measured displacement,  $\Delta z = 1.91 \text{ mm} \pm 0.81 \text{ mm}$ 
    - Standard deviation of 1.74 times mean diameter

**Issue: overall z-positions are not validated and effects due to particle motion are not considered**

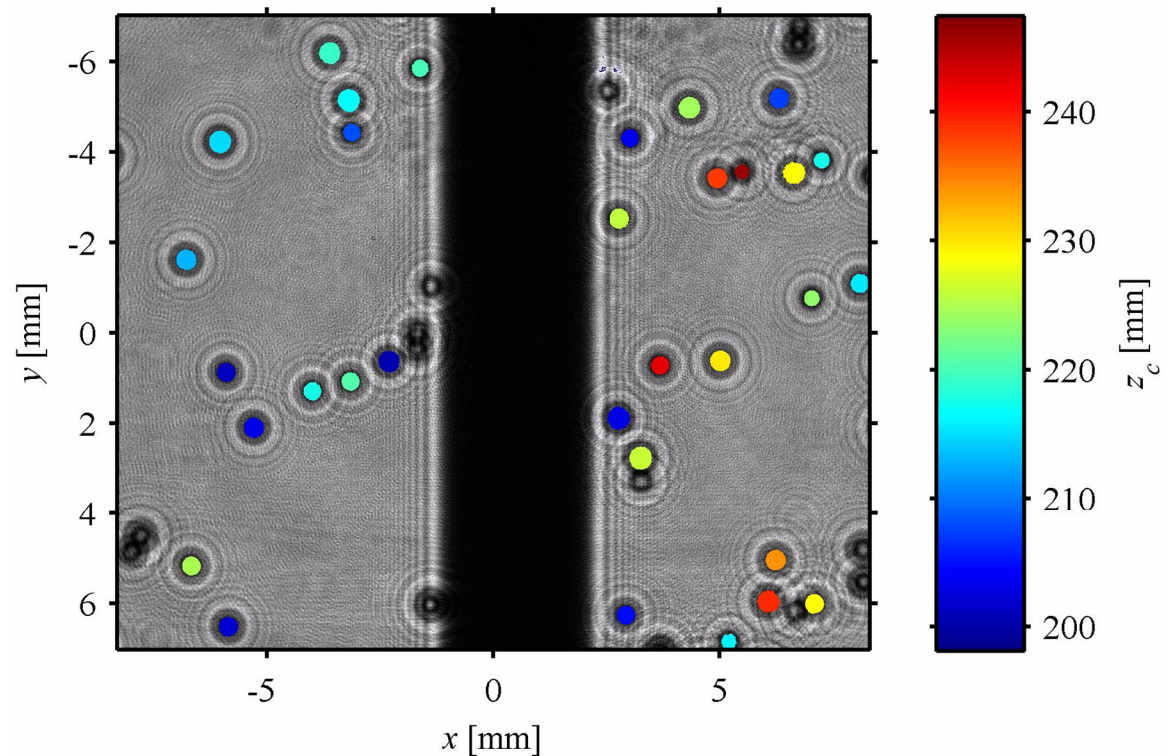


histogram of measured displacements

# Experimental configuration for z validation



- Particles stirred by a rotating rod ( $r_0 = 1.58$  mm,  $\omega_0 = 100$  rpm)
- Recorded at 15Hz with a LaVision sCMOS camera ( $2560 \times 2160$ ,  $6.5$   $\mu\text{m}$  pixel pitch)



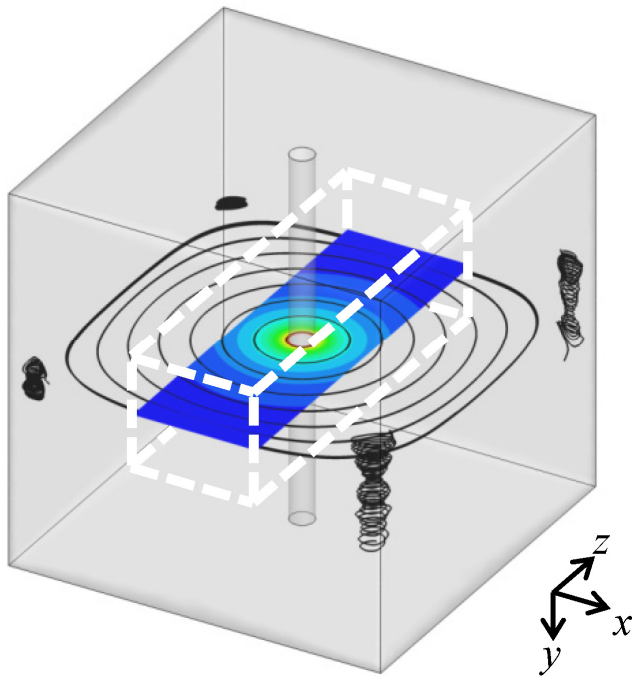
particles measured with the hybrid method, background shows the recorded holograms

# Extraction of theoretical trajectory

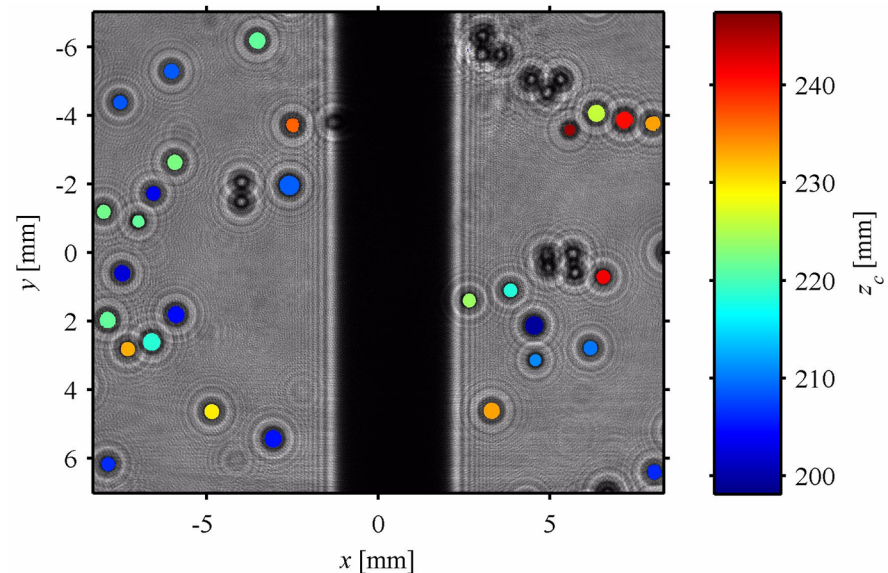
In the  $x$ - $z$  plane, particles are expected to travel in near perfect circles

$$x(t) = r \cos(\omega t + \theta_0)$$

- Assuming measured  $x$ -positions have minimal error, curve fit  $\rightarrow r, \omega, \theta_0$



simulated flow field showing streamlines and total velocity contour within the center  $x$ - $z$  plane of the field of view (dotted lines)

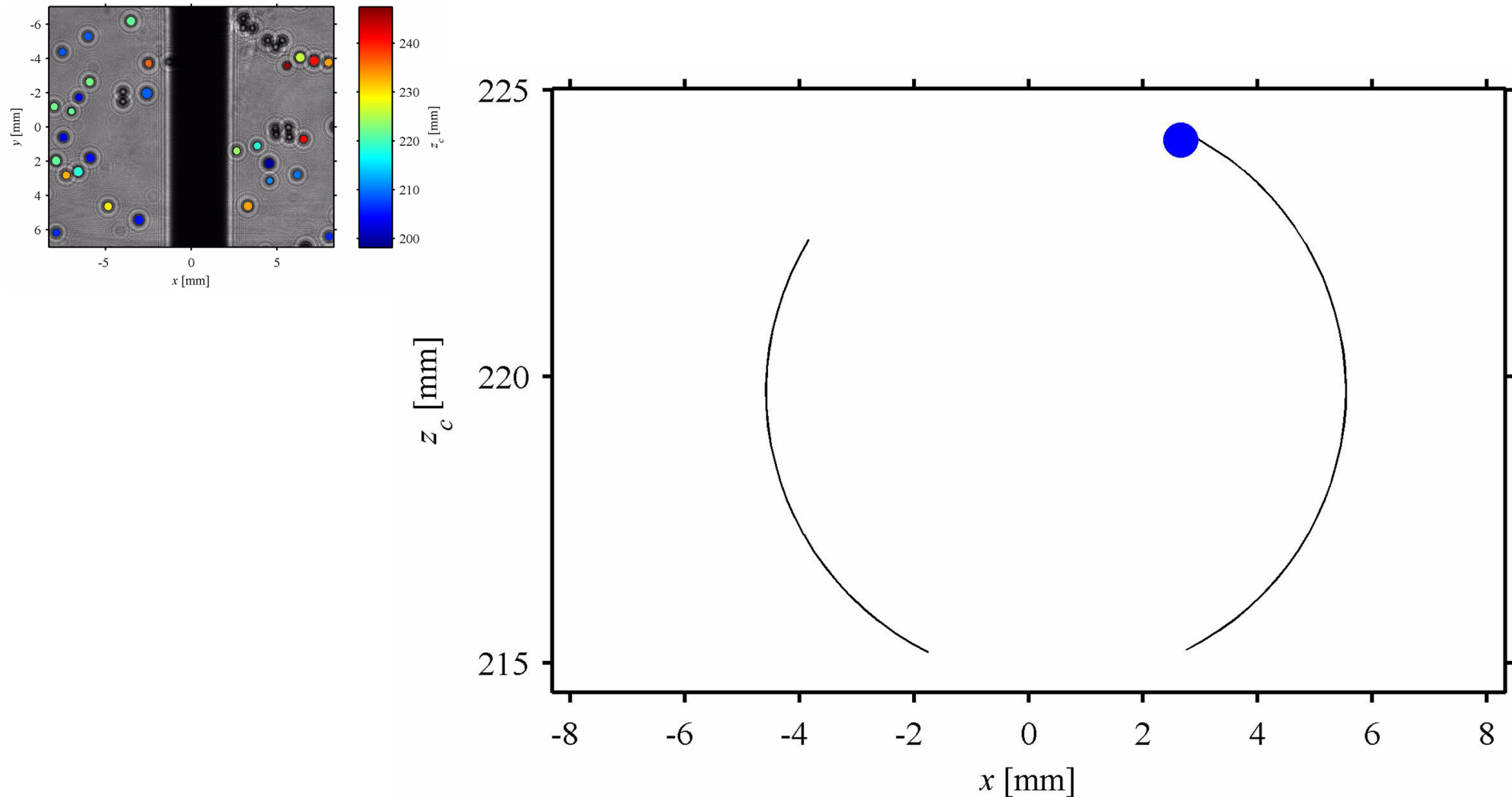


example in-plane trajectory

- Measured  $r = 5.04$  mm,  $\omega = 9.414$  rpm
- At this  $r$ , simulation gives  $\omega = 9.406$  rpm

# Comparison with measured results

Predicted z-trajectory:  $z(t) = r \sin(\omega t + \theta_0)$  and  $\Delta z(t) = r \omega \cos(\omega t + \theta_0) \cdot \Delta t$



measured x-z trajectory vs. predicted



# Conclusions

For all trajectories

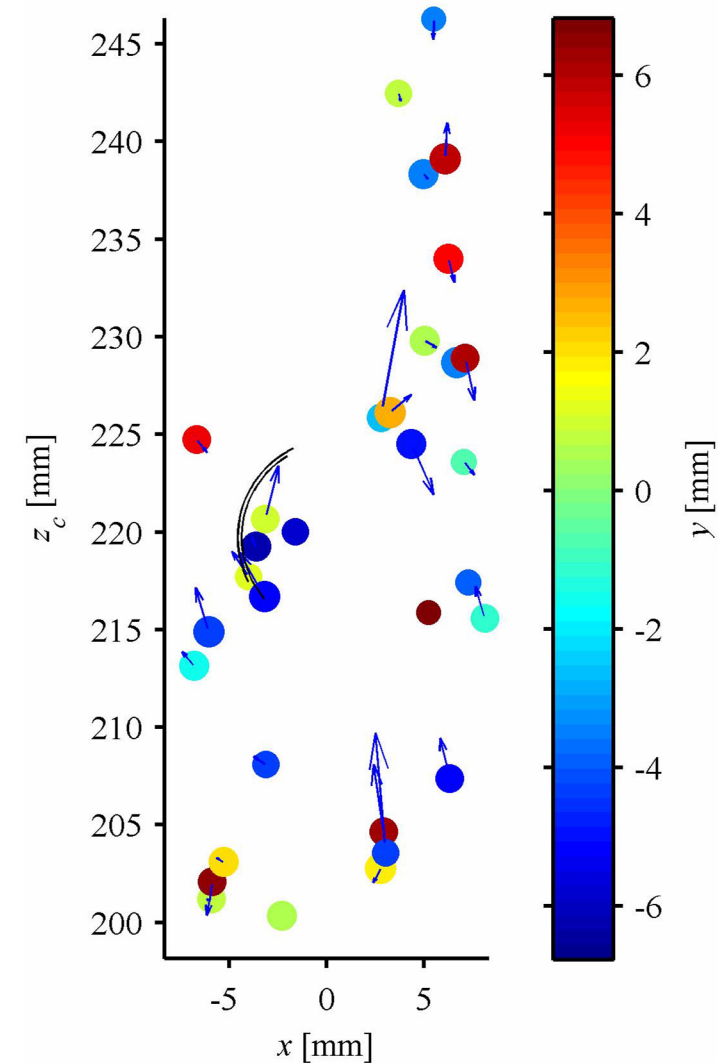
- Error in measured  $z = -0.04 \pm 1.51$  mm
- Error in measured  $\Delta z = -0.03 \pm 1.05$  mm
  - Standard deviation of  $2.3 \cdot \bar{d}$

Experiments repeated with smaller particles  
( $\bar{d} = 118 \mu\text{m}$ , see paper for details)

- Error in measured  $z = -0.003 \pm 0.379$  mm
- Error in measured  $\Delta z = -0.001 \pm 0.302$  mm
  - Standard deviation of  $2.6 \cdot \bar{d}$

Next steps:

- Compare results with alternative particle detection methods
- Use results to quantify effects of particle overlap and other experimental noise sources



all measured  $x$ - $z$  trajectories vs. predicted

# 3D, 3C fluid velocity measurements?

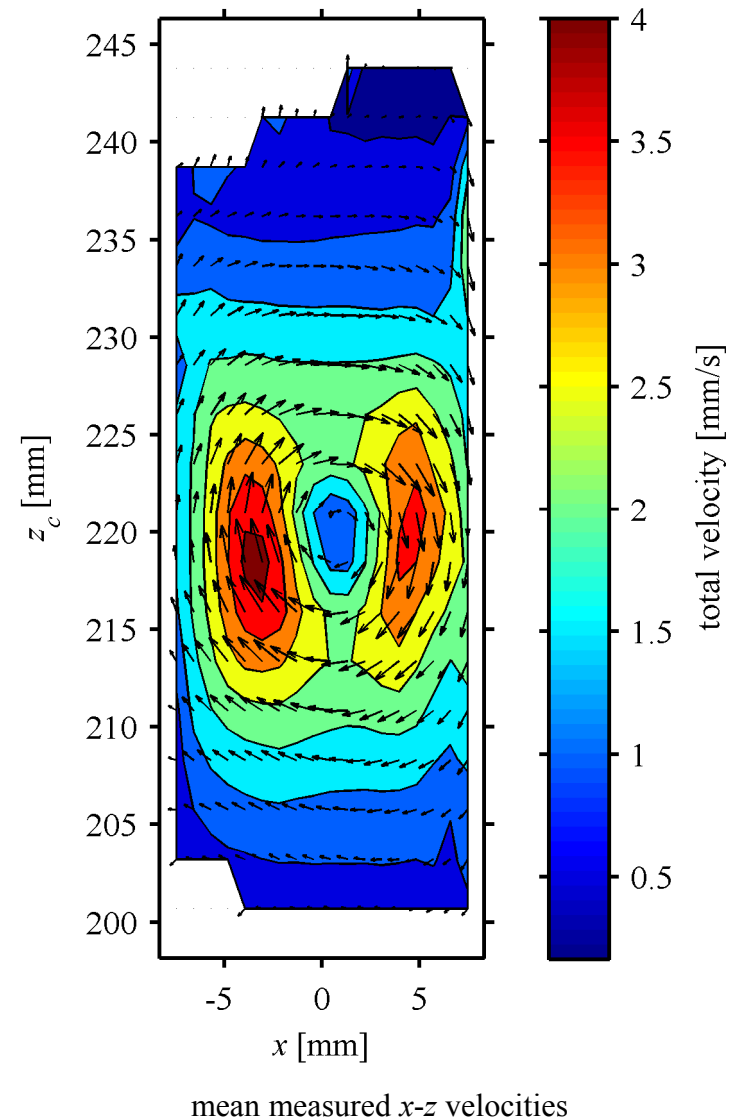
## Advantages:

- Simple optical setup requiring only one line-of-sight view
- Large depth of field (hundreds of mm possible)
- Particle sizes can be measured (if desired)

## Challenges:

- High uncertainty in the z-direction
- Particle field must be relatively sparse providing only limited vectors
- Vectors at random positions
- Methods not as mature as PIV or even tomographic-PIV

**Note: the literature contains many works on holographic-PIV. My own work has not been focused on these applications**





Questions?