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<i>Title:</i>	Modeling Compressed Turbulence with BHR
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## Modeling Compressed Turbulence with BHR

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### Motivation

- From ICE to ICF, the effect of mean compression or expansion is important for predicting the state of the turbulence.
- When developing combustion models, we would like to know the mix state of the reacting species.
- To date, research has focused on the effect of compression on the turbulent kinetic energy.
- The current work provides constraints to help development and validation for models of species mixing effects in compressed turbulence.



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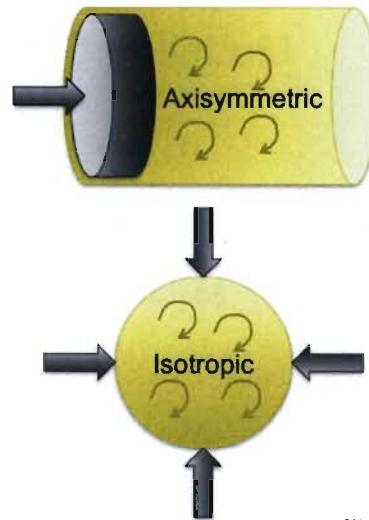
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## Overview

- Consider a homogeneous turbulent field subject to uniform mean compression.
- The flow can be decomposed into a mean flow and turbulent fluctuations.
  - The mean flow must be treated compressibly.
  - The turbulent fluctuations may be amenable to simpler models.
- For example, the DNS of Wu, et al. (1985), assumes the turbulent density fluctuations could be neglected.



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## Approach

- Cambon, et al. (1992) demonstrate (based on an observation of Frisch) that a simple rescaling relates homogeneous isotropic compressed turbulence and decaying turbulence.
- This result assumes we can neglect density fluctuations.
- The current work extends this to the case of low Atwood number, low Mach number buoyancy driven turbulence, where the turbulent density fluctuations can be treated by the Boussinesq approximation.



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## Governing equations

- Consider an infinite fluid with some initial small density perturbation, subject to a body force in the  $x_3$  direction.
- Introducing the standard ensemble and Favre averages, and assuming spatially homogeneous turbulence, the mean equation are

$$\begin{aligned}\frac{1}{\langle \rho \rangle} \frac{\partial \langle \rho \rangle}{\partial t} + S_{ii} &= 0 \\ \frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} &= -\frac{1}{\langle \rho \rangle} \frac{\partial \langle \rho \rangle}{\partial x_i} + g_i \\ \frac{\partial \tilde{c}}{\partial t} &= 0\end{aligned}$$

or,

$$\begin{aligned}\dot{S}_{ij} + S_{ik} S_{kj} &= 0 \\ \frac{\partial \langle \rho \rangle}{\partial x_3} &= \langle \rho \rangle g \\ \tilde{c} &= C_0\end{aligned}$$

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## Re-scaling

- Introduce a coordinate transform to remove the mean compression:

$$x_i^* = J^{-1/3} x_i \quad \frac{dt^*}{dt} = J^{-2/3}$$

where

$$J = \exp\left(\int_0^t S_{ii}(t') dt'\right) = \frac{\langle \rho(t=0) \rangle}{\langle \rho \rangle}$$

and the following re-scaling for the fluctuating components:

$$\begin{aligned}\rho'(t) &= J^{-1}(t) \rho^*(t^*) & u_i''(t) &= J^{-1/3}(t) u_i^*(t^*) \\ p'(t) &= J^{-5/3}(t) p^*(t^*) & c''(t) &= c^*(t^*)\end{aligned}$$

- The resulting equations for the re-scaled fluctuations are identical to the standard Boussinesq equations without compression, provided that
  - The density variations are small
  - The mean strain tensor,  $S_{ij}$ , is isotropic.
  - The uncompressed case has a body-force and viscosity scaled by time.



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## Constraints for RANS

- Homogeneous buoyancy driven turbulence subject to an isotropic compression corresponds to freely evolving buoyancy driven turbulence with a time depended body force.
- Since this re-scaling is an exact analytic result, any RANS model should preserve it.
- Following Cambon, et al., we can neglect the viscosity variation, since in general, RANS models do not account for time variations in viscosity.
- Here the re-scaling will be applied specifically to the BHR model.



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## BHR equations

- The BHR equations (Besnard, et al., 1992) for homogeneous buoyancy driven turbulence undergoing mean compression reduce to the following set of ODEs:

$$\begin{aligned}\frac{\partial k}{\partial t} &= a_3 g - R_{ij} S_{ij} - \varepsilon \\ \frac{\partial \varepsilon}{\partial t} &= -C_{\varepsilon 1} R_{ij} S_{ij} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + C_{\varepsilon 3} \varepsilon S_{kk} - C_{\varepsilon 4} \frac{\varepsilon}{k} a_3 g \\ \frac{\partial a_3}{\partial t} &= b g - C_{a1} \frac{\varepsilon}{k} a_3 + (C_{a2} - 1) a_3 \frac{\partial \bar{u}_3}{\partial x_3} \\ \frac{\partial b}{\partial t} &= -C_{b2} \frac{\varepsilon}{k} b\end{aligned}$$



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## Re-scaling the turbulence quantities

- Based on the exact re-scaling for the turbulent fluctuations, we can write the following re-scalings for the turbulence model quantities:

$$\begin{aligned} k(t) &= \frac{1}{2} \langle u_i''(t) u_i''(t) \rangle = J^{-2/3} k^*(t^*) \\ \varepsilon(t) &= \nu \left\langle \frac{\partial u_i''(t)}{\partial x_j} \frac{\partial u_j''(t)}{\partial x_i} \right\rangle = J^{-4/3} \varepsilon^*(t^*) \\ a_3(t) &= -\langle u_3''(t) \rangle = J^{-1/3} a_3^*(t^*) \\ b(t) &\approx \frac{\langle \rho'(t)^2 \rangle}{\langle \rho \rangle^2} = b^*(t^*) \end{aligned}$$

- Inserting these into the BHR equations, and making use of  $S_{ij} = (d/3)\delta_{ij}$  and  $R_{ii} = 2k$  we find the following:



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## Re-scaled BHR equations

- Turbulent kinetic energy

$$\frac{\partial k^*}{\partial t^*} = a_3^* g - \varepsilon^*$$

- Dissipation Rate

$$\frac{\partial \varepsilon^*}{\partial t^*} = -C_{\varepsilon 2} \frac{\varepsilon^{*2}}{k^*} - C_{\varepsilon 4} \frac{\varepsilon^*}{k^*} a_3^* g^* + J^{2/3} \left( \frac{4}{3} - \frac{2}{3} C_{\varepsilon 1} - C_{\varepsilon 3} \right)$$

- So, to preserve the scaling we must require

$$C_{\varepsilon 3} = \frac{2}{3} (2 - C_{\varepsilon 1})$$

- This is consistent with the result found by Cambon, et al., for the simpler case, and assuming  $C_{\varepsilon 1} = 1$ , following Reynolds.



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## Re-scaled BHR equations

- Density-velocity correlation:

$$\frac{\partial a_3^*}{\partial t^*} = b^* g^* - C_{a1} \frac{\varepsilon^*}{k^*} a_3^* + J^{2/3} \frac{1}{3} (C_{a2} + 3) a_3^* d$$

- To preserve the scaling:

$$C_{a2} = -3$$

- Density self-correlation:

$$\frac{\partial b^*}{\partial t^*} = -C_{b2} \frac{\varepsilon^*}{k^*} b^*$$

- Does not impose further constraints



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## Conclusions

- The Cambon, et al., re-scaling has been extended to buoyancy driven turbulence, including the fluctuating density and concentration equations.
- The new scalings give us helpful constraints for developing and validating RANS turbulence models.
- The technique has been demonstrated for the BHR model, yielding the constraints:

$$C_{\varepsilon 3} = \frac{2}{3} (2 - C_{\varepsilon 1})$$

$$C_{a2} = -3$$

- Extensions and additional applications of the approach are under investigation.



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