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Title: Modeling Compressed Turbulence with BHR

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Modeling Compressed Turbulence with BHR

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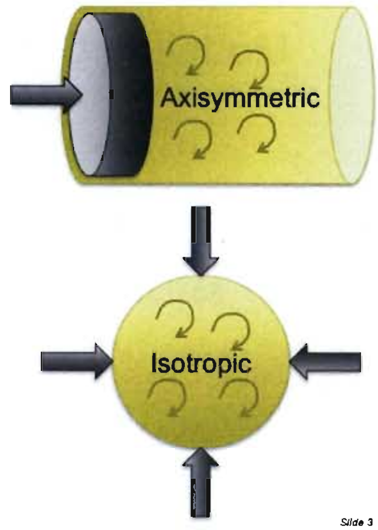
Motivation

- From ICE to ICF, the effect of mean compression or expansion is important for predicting the state of the turbulence.
- When developing combustion models, we would like to know the mix state of the reacting species.
- To date, research has focused on the effect of compression on the turbulent kinetic energy.
- The current work provides constraints to help development and validation for models of species mixing effects in compressed turbulence.

Overview

- Consider a homogeneous turbulent field subject to uniform mean compression.
- The flow can be decomposed into a mean flow and turbulent fluctuations.
  - The mean flow must be treated compressibly.
  - The turbulent fluctuations may be amenable to simpler models.
- For example, the DNS of Wu, et al. (1985), assumes the turbulent density fluctuations could be neglected.



Approach

- Cambon, et al. (1992) demonstrate (based on an observation of Frisch) that a simple rescaling relates homogeneous isotropic compressed turbulence and decaying turbulence.
- This result assumes we can neglect density fluctuations.
- The current work extends this to the case of low Atwood number, low Mach number buoyancy driven turbulence, where the turbulent density fluctuations can be treated by the Boussinesq approximation.

Governing equations

- Consider a infinite fluid with some initial small density perturbation, subject to a body force in the x_3 direction.
- Introducing the standard ensemble and Favre averages, and assuming spatially homogeneous turbulence, the mean equation are

$$\frac{1}{\langle\rho\rangle}\frac{\partial\langle\rho\rangle}{\partial t} + S_{ii} = 0$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\langle\rho\rangle}\frac{\partial\langle p\rangle}{\partial x_i} + g_i$$

$$\frac{\partial \tilde{c}}{\partial t} = 0$$

or,

$$\dot{S}_{ij} + S_{ik}S_{kj} = 0$$

$$\frac{\partial\langle p\rangle}{\partial x_3} = \langle\rho\rangle g$$

$$\tilde{c} = C_0$$

Re-scaling

- Introduce a coordinate transform to remove the mean compression:

$$x_i^* = J^{-1/3}x_i \qquad \frac{dt^*}{dt} = J^{-2/3}$$

where

$$J = \exp\left(\int_0^t S_{ii}(t')dt'\right) = \frac{\langle\rho(t=0)\rangle}{\langle\rho\rangle}$$

and the following re-scaling for the fluctuating components:

$$\rho'(t) = J^{-1}(t)\rho^*(t^*) \qquad u_i''(t) = J^{-1/3}(t)u_i^*(t^*)$$

$$p'(t) = J^{-5/3}(t)p^*(t^*) \qquad c''(t) = c^*(t^*)$$

- The resulting equations for the re-scaled fluctuations are identical to the standard Boussinesq equations without compression, provided that
  - The density variations are small
  - The mean strain tensor, S_{ij}, is isotropic.
  - The uncompressed case has a body-force and viscosity scaled by time.

Constraints for RANS

- Homogeneous buoyancy driven turbulence subject to an isotropic compression corresponds to freely evolving buoyancy driven turbulence with a time depended body force.
- Since this re-scaling is an exact analytic result, any RANS model should preserve it.
- Following Cambon, et al., we can neglect the viscosity variation, since in general, RANS models do not account for time variations in viscosity.
- Here the re-scaling will be applied specifically to the BHR model.

BHR equations

- The BHR equations (Besnard, et al., 1992) for homogeneous buoyancy driven turbulence undergoing mean compression reduce to the following set of ODEs:

$$\frac{\partial k}{\partial t} = a_3 g - R_{ij} S_{ij} - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} = -C_{\varepsilon 1} R_{ij} S_{ij} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + C_{\varepsilon 3} \varepsilon S_{kk} - C_{\varepsilon 4} \frac{\varepsilon}{k} a_3 g$$

$$\frac{\partial a_3}{\partial t} = bg - C_{a1} \frac{\varepsilon}{k} a_3 + (C_{a2} - 1) a_3 \frac{\partial \tilde{u}_3}{\partial x_3}$$

$$\frac{\partial b}{\partial t} = -C_{b2} \frac{\varepsilon}{k} b$$

Re-scaling the turbulence quantities

- Based on the exact re-scaling for the turbulent fluctuations, we can write the following re-scalings for the turbulence model quantities:

$$k(t) = \frac{1}{2}\langle u_i''(t) u_i''(t)\rangle = J^{-2/3} k^*(t^*)$$

$$\varepsilon(t) = \upsilon\left\langle \frac{\partial u_i''(t)}{\partial x_j}\frac{\partial u_j''(t)}{\partial x_i}\right\rangle = J^{-4/3}\varepsilon^*(t^*)$$

$$a_3(t) = -\langle u_3''(t)\rangle = J^{-1/3} a_3^*(t^*)$$

$$b(t) \approx \frac{\langle \rho'(t)^2\rangle}{\langle\rho\rangle^2} = b^*(t^*)$$

- Inserting these into the BHR equations, and making use of $S_{ij}=(d/3)\delta_{ij}$ and $R_{ii}=2k$ we find the following:

Re-scaled BHR equations

- Turbulent kinetic energy

$$\frac{\partial k^*}{\partial t^*} = a_3^* g - \varepsilon^*$$

- Dissipation Rate

$$\frac{\partial \varepsilon^*}{\partial t^*} = -C_{\varepsilon 2}\frac{\varepsilon^{*2}}{k^*} - C_{\varepsilon 4}\frac{\varepsilon^*}{k^*}a_3^* g^* + J^{2/3}\left(\frac{4}{3} - \frac{2}{3}C_{\varepsilon 1} - C_{\varepsilon 3}\right)$$

- So, to preserve the scaling we must require

$$C_{\varepsilon 3} = \frac{2}{3}(2 - C_{\varepsilon 1})$$

- This is consistent with the result found by Cambon, et al., for the simpler case, and assuming $C_{\varepsilon 1}=1$, following Reynolds.

Re-scaled BHR equations

- Density-velocity correlation:

$$\frac{\partial a_3^*}{\partial t^*} = b^* g^* - C_{a1} \frac{\varepsilon^*}{k^*} a_3^* + J^{2/3} \frac{1}{3} (C_{a2} + 3) a_3^* d$$

- To preserve the scaling:

$$C_{a2} = -3$$

- Density self-correlation:

$$\frac{\partial b^*}{\partial t^*} = -C_{b2} \frac{\varepsilon^*}{k^*} b^*$$

- Does not impose further constraints

Conclusions

- The Cambon, et al., re-scaling has been extended to buoyancy driven turbulence, including the fluctuating density and concentration equations.
- The new scalings give us helpful constraints for developing and validating RANS turbulence models.
- The technique has been demonstrated for the BHR model, yielding the constraints:

$$C_{\varepsilon 3} = \frac{2}{3}(2 - C_{\varepsilon 1})$$

$$C_{a2} = -3$$

- Extensions and additional applications of the approach are under investigation.