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Title: Material point method simulation of dense granular material

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Material point method simulation of dense granular material

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Accurate modeling and simulation of granular flow or deformation requires a numerical method with Lagrangian capability to account for history dependence of the material. However, large deformation or flow of the material requires an Eulerian description. Numerically, different descriptions of the material result in different codes and applications. Unsatisfactory results have been reported by many modelers using both methods. For instance, element deletion scheme is used in the finite element method to eliminate the highly distorted elements, which results in reduction of inertia from the problem. In the codes using Eulerian description, how to advect brittle damage of the material is a significant issue. To address these issues we use the material point method, which uses both Lagrangian material points and Eulerian mesh simultaneously. Improvements are made to the original material point method for our applications. It is found the improvements are critically important to granular flows results from brittle damage of the material, while it is marginally important to ductile materials.

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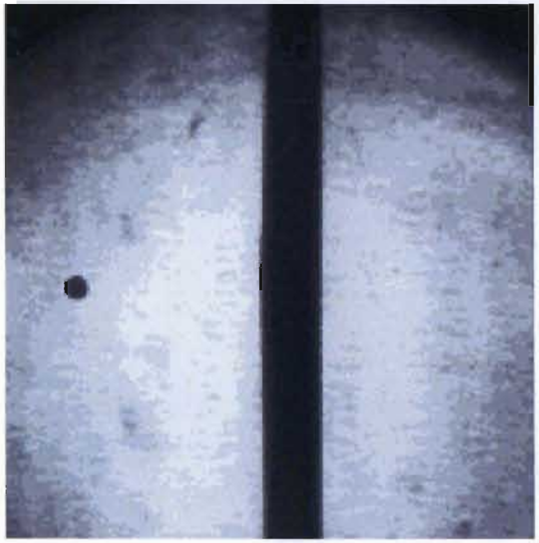
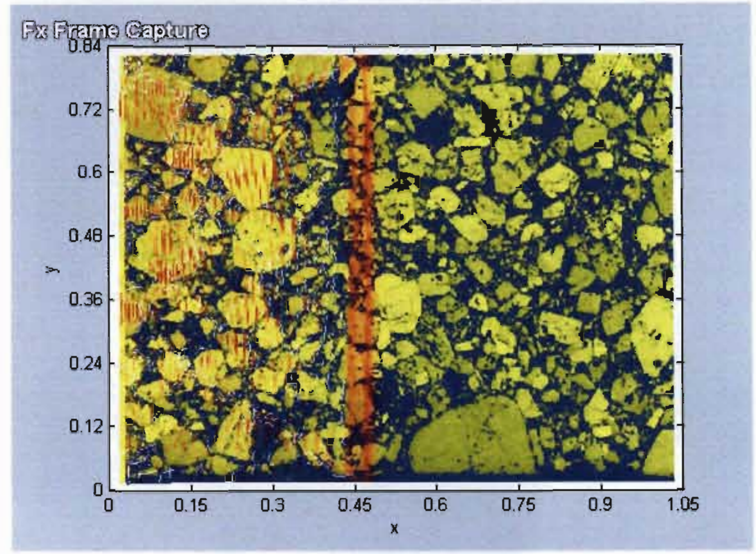
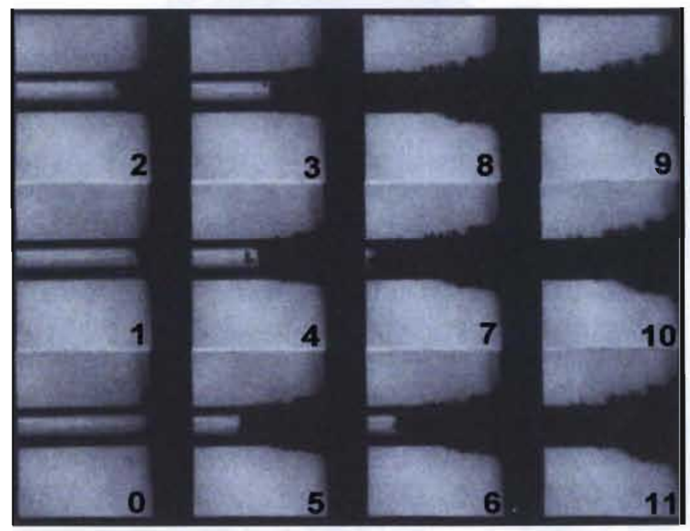
Dense granular material and debris flows



Source: NASA





Willmott and Radford, J. Appl Phys. **97**, 2005

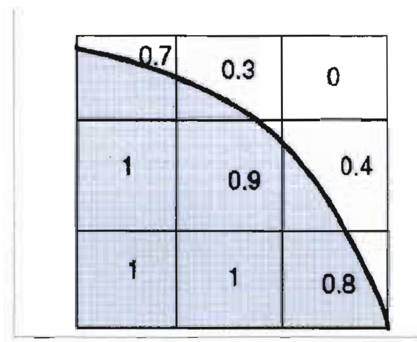
- Often dense granular material is generated by impact and fragmentation of a solid material.
- Accompanied by large deformation or flow of the original material.
- The material response is often history dependent.

Numerical issues in modeling such material

- Traditionally finite element methods are used.
- Because of fragmentation and debris flow, often mesh distortion is an issue.
- Eulerian method, such as finite difference, finite volume, cannot be efficiently used because of failure flags need to follow the motion of the material and cannot be averaged.
  - For brittle material, at a point the material is either failed (failure = 1) or not failed (failure = 0).

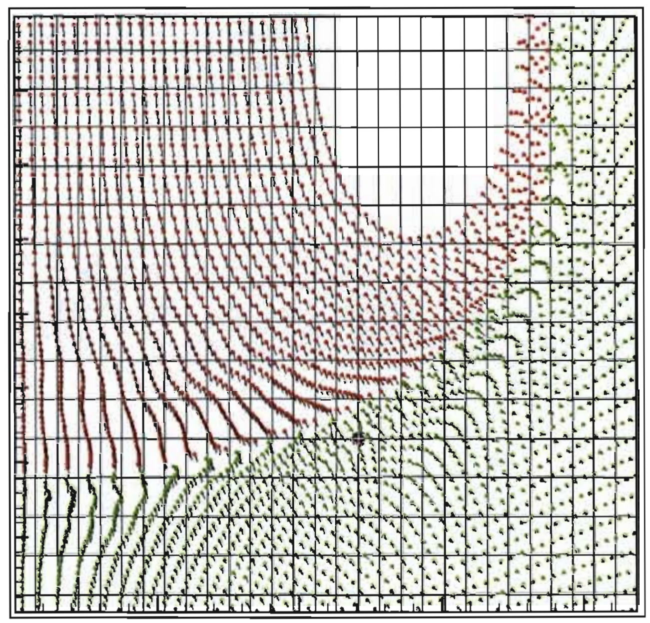
FEM, (Lagrangian Method)

Cline and Reaugh, J. De. Physique IV, 1991



failure = 0.9 ?!

Eulerian method



Material point method

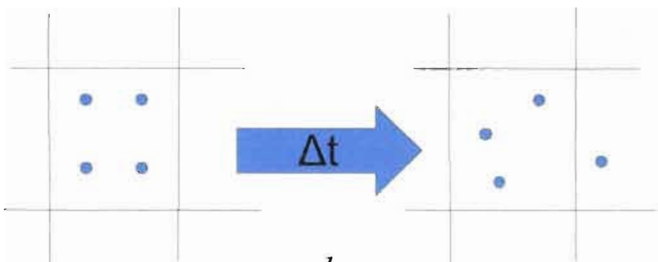
Material point method (MPM) vs. finite element method (FEM)

$$m_{ij}\frac{d\boldsymbol{u}_j}{dt} = -\int \sigma\cdot\nabla S_i(\boldsymbol{x})\,dv + \int \rho \boldsymbol{g}\, S_i(\boldsymbol{x})\,dv + \int_{\partial v} S_i(\boldsymbol{x})\,\square\cdot \boldsymbol{n}\,dS,$$ (the virtual work principle).

MPM

$$\int \sigma\cdot\nabla S_i(\boldsymbol{x})\,dv = \sum_p v_p \sigma_p \cdot \nabla S(\boldsymbol{x}_p)$$

where subscript p denotes material points that move across the Eulerian mesh.



$$\boldsymbol{u}_p^{n+1} = \boldsymbol{u}_p^n + \Delta t \sum_i \frac{d\boldsymbol{u}_i}{dt} S_i(\boldsymbol{x}_p)$$

$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \quad t \sum_i \left(\boldsymbol{u}_i + 0.5\frac{d\boldsymbol{u}_i}{dt} \quad t\right) S_i(\boldsymbol{x}_p)$$

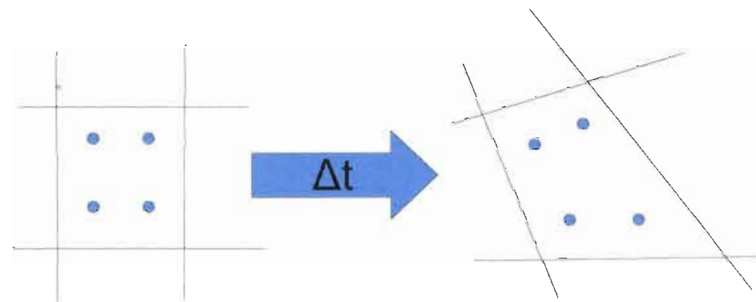
$$m_i \boldsymbol{u}_i^{n+1} = \sum_p m_p \boldsymbol{u}_p^{n+1} S_n\left(\boldsymbol{x}_p^{n+1}\right)$$

Mesh cells or elements are Eulerian. They are fixed.

FEM

$$\int \sigma\cdot\nabla S_i(\boldsymbol{x})\,dv = \sum_g w_g J_g\, \sigma_g \cdot \nabla S(\boldsymbol{x}_g),$$

where subscript g denotes Gauss integration points.



Gauss points are fixed on elements.

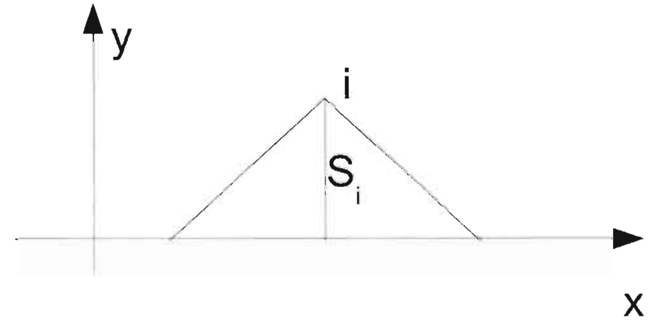
Elements are Lagrangian. They can become distorted for large material deformation.

Both the material points and Gauss points are Lagrangian points and can be used to track deformation history of the material. However, FEM has the difficulty of mesh distortion.

Cell Crossing Noise and its treatment in MPM

$$m_i \frac{d\boldsymbol{u}_i}{dt} = -\sum_p v_p \boldsymbol{\sigma}_p \cdot \nabla S_i(\boldsymbol{x}_p) + \int \rho \boldsymbol{g} S_i(\boldsymbol{x}) dv + \int_{\partial v} S_i(\boldsymbol{x}) \boldsymbol{\sigma} \cdot \boldsymbol{n} dS,$$

The discontinuity of shape function gradient causes an instability (Bardenhagen and Kober, 2004).

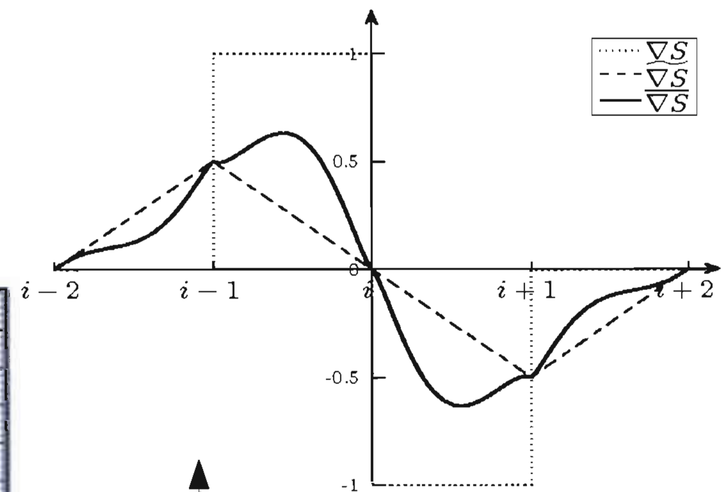


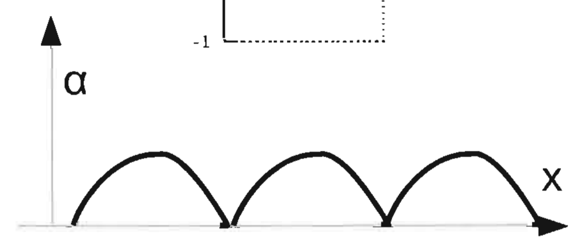
Solution: Replace ∇S_i by

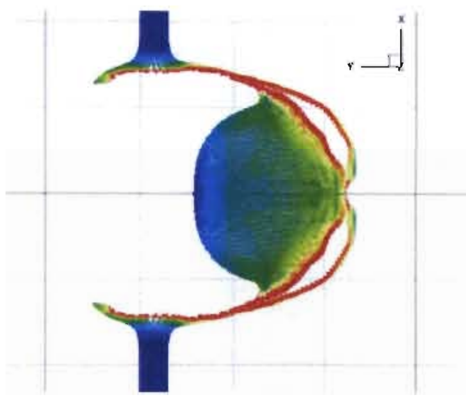
$$\overline{\nabla S_i}(\boldsymbol{x}) = \alpha(\boldsymbol{x}) \nabla S_i(\boldsymbol{x}) + [1-\alpha(\boldsymbol{x})] \widetilde{\nabla S_i}(\boldsymbol{x}),$$

$$\widetilde{\nabla S_i}(\boldsymbol{x}) = \sum_{j=1}^{N} \frac{1}{V_j} (S_j, \nabla S_i) S_j(\boldsymbol{x}),$$

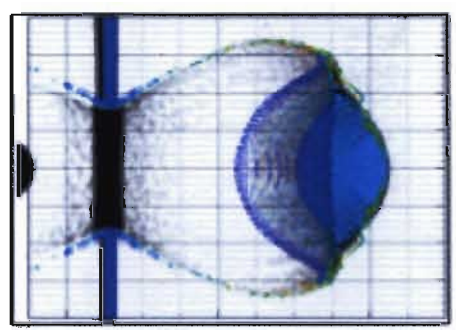
where α(x) = 0 on cell boundary.







Original MPM



Dual domain material point (DDMP) method

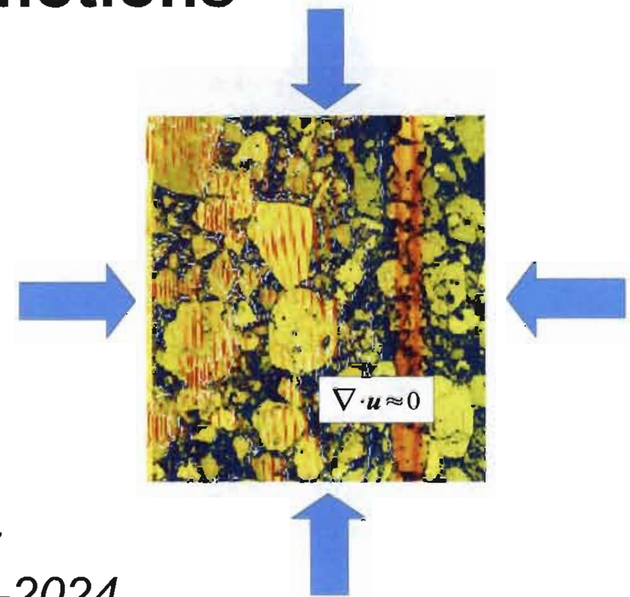
Meso and macro scale motions

Let < > denote averaged macroscopic quantity, then

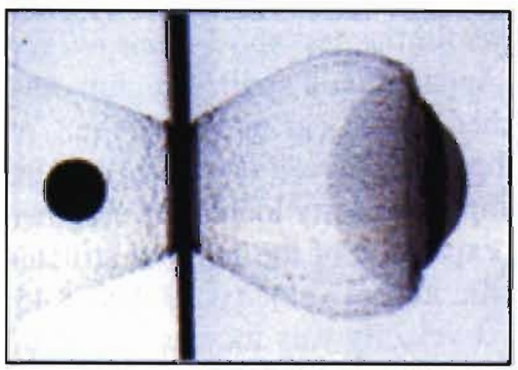
$$\langle \nabla \cdot \boldsymbol{u} \rangle \neq \nabla \cdot \langle \boldsymbol{u} \rangle$$

$$\langle \nabla \cdot \boldsymbol{u} \rangle = \alpha \nabla \cdot \langle \boldsymbol{u} \rangle + B$$

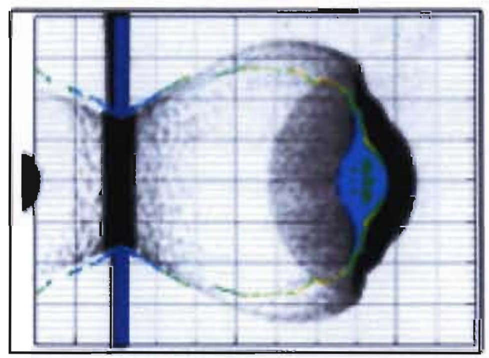
where B is related to other effects resulting in the change of the material density, such as relative compressibilities, thermal expansion, phase change, etc.



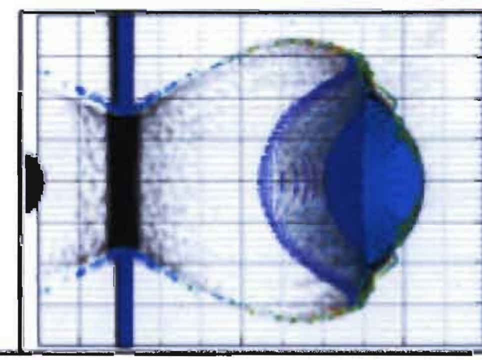
Debris flow from an Impact:
Velocity: 6.71km/s. Materials: Al-2024

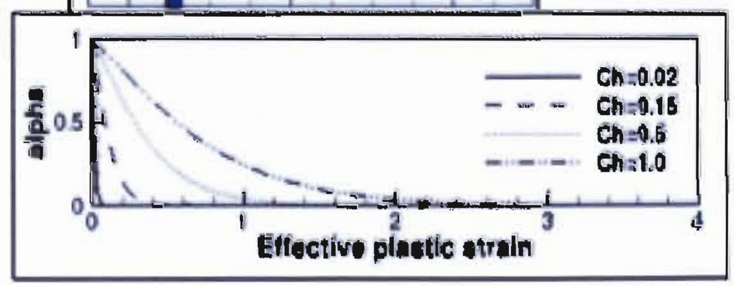


Experimental X-ray image
Piekutowski, 1993

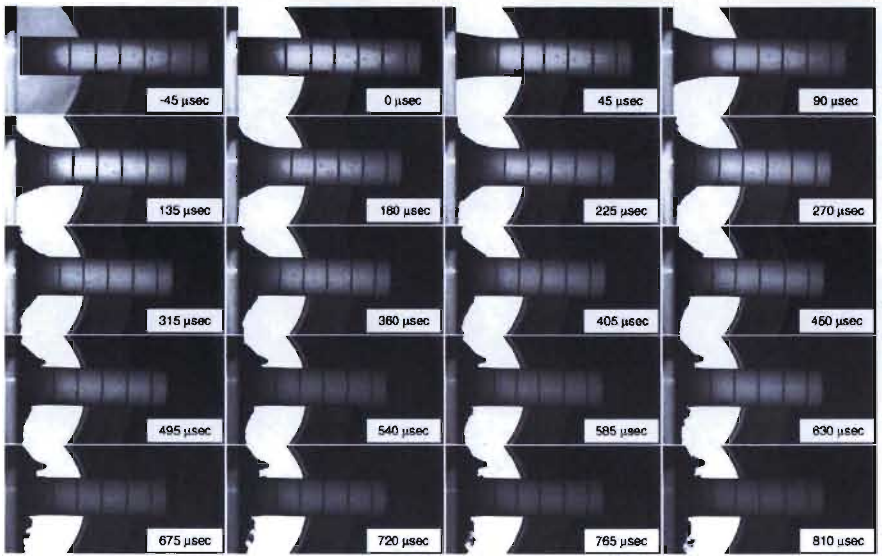


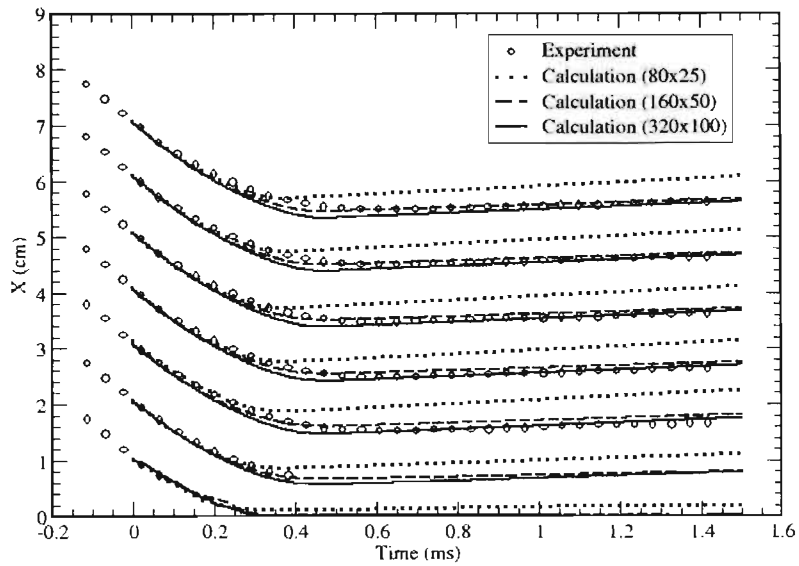
α = 1



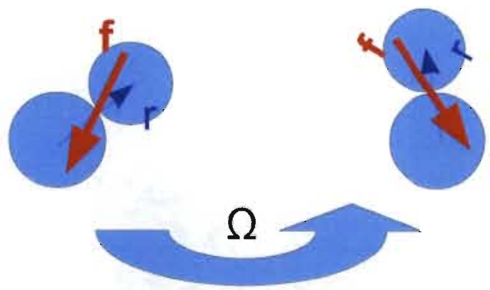


Taylor impact of brittle material





$$\sigma = \frac{1}{V} \sum \boldsymbol{f} \otimes \boldsymbol{r}$$



$$\frac{d\sigma}{dt} - \boldsymbol{\Omega} \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot \boldsymbol{\Omega} = -\frac{\sigma}{\tau} + \lambda \, tr(\dot{\varepsilon}) \boldsymbol{I} + 2G \dot{\varepsilon} ,$$

$$\lambda = \frac{E \nu}{(1+\nu)(1-2\nu)},$$

$$G = \frac{E}{2(1+\nu)}$$

Conclusions

- In numerical calculation of large material deformation, especially granular flows, a Lagrangian capability is important.

- We have developed a statistical tool to relate the macroscopic deformation to mesoscale material deformation.

- The recently developed dual domain material point (DDMP) method, is the only method currently suitable for such calculations.