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A Scalable Decomposition Algorithm for PMU Placement Under Multiple-Failure Contingencies

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***Abstract*—This paper proposes scalable models and algorithms for the PMU placement problem (PPP). Existing approaches have difficulty scaling to full-scale systems, and are not guaranteed to be resilient to multiple component failures. This paper expands PPP to a more general k-resilient PPP, where any k PMUs and/or lines can fail without jeopardizing the full supervision criterion. Our PPP model — a novel formulation based on maximum-flow network design — is unique in that it is amenable to efficient decomposition, which significantly improves tractability and scalability. We present two cutting plane algorithms to support this decomposition – the first such algorithms for the PPP to our knowledge. The improvements in computational efficiency afforded by the network decomposition suggests that our approach has the potential to solve large-scale systems.**

I. Introduction

Phasor measurement units (PMUs) are instruments that are placed at electric buses in order to provide real-time monitoring, data measurement, and supervision capabilities. These devices are increasingly used by system designers and engineers to measure voltage- and current- phasors across the grid. The data provided by these units support system analysis studies, reliability assessments in normal operating scenarios, and contingency operations during failure scenarios. They can also be utilized for monitoring purposes in security scenarios where the integrity of the grid is under threat by an adversary – for example, in the context of a cyber security attack.

To support these uses, individual PMUs must be placed across a grid at locations necessary to provide the requisite supervision. A PMU provides voltage phasors at the installed bus and current phasors for incident lines. Because electric grids are considered safety-critical systems, it is desirable to ensure that a sufficient number of PMUs are placed within the grid to provide full supervision. This is generically referred to as the *PMU placement problem* (PPP) [4].

Direct measurement of all system states is possible if PMUs are placed at all buses. However, this approach is unnecessarily expensive and cumbersome in that it provides excessive redundant measurement data that must be captured and processed by the supervisory control system.

A more novel approach is to identify a minimum number of PMUs to installed in order to satisfy the condition of complete supervision. Existing approaches to this problem have presented integer programming formulations that are variants of cover problems like the *Dominating Set Problem* (DSP) [4], [9]. Solution approaches vary from search algorithms (e.g., [5], [11]), to heuristics (e.g., [10]), to solving linear integer programs directly (e.g., [2], [6], [7], [15]). Furthermore, because it is desirable to maintain full supervision in the event of failing or compromised PMUs and/or lines, several variants of this problem have been cast as contingency-constraint problems where full supervision is maintained in the event of a single-element contingency scenario (e.g., [2], [12]). However, these existing variants of PPP are only capable of guaranteeing full supervision when no more than one system component fails.

This paper makes several advances to PPP. First, we present a more general *k-failure resilient variant of* PPP, referred to as RPPP. This model advances previous approaches by generating PMU placement schemes that will maintain full supervision in the event of any k PMUs and/or lines failures. Second, we present an alternative formulation, which yields linear second-stage problems, and therefore, convex recourse functions. To the best of our knowledge, this paper is the first to propose a *new* formulation for RPPP based on *maximum-flow network design*, as most existing approaches in literature formulate RPPP (for $k = 1$) as variants of DSP. The maximum-flow formulation supports efficient decomposition, and thus, has the potential to solve large-scale systems, even if k is non-trivial. In contrast, existing models cannot be decomposed efficiently, meaning that the integer programs that must be solved are exponentially large in terms of the number of variables and constraints. Third, this paper presents two cutting plane algorithms to support this decomposition. These algorithms are, to the best of our knowledge, the first decomposition algorithms for the PPP and the RPPP. The empirical results presented show significant improvements in computational efficiency in terms of both speed and memory. This promising result suggests that our approach has the potential to solve regional, or even national, scale systems.

II. PMU Placement Formulations

A power system instance is represented by an undirected graph $G(V, E)$. For notational convenience, we may write $e = \{u, v\}$ to denote the fact that lines are defined between buses $u, v \in V$. Since a PMU placed at bus u can measure the current phasors of an incident line $e = \{u, v\}$, we define

$G(V,A)$ to be the digraph that is obtained by introducing a pair of antiparallel arcs $e^+ = (u,v)$ and $e^- = (v,u)$ for every line $e = \{u,v\} \in E$ and self-joining arcs (v,v) for all $v \in V$. Therefore, $A := \{e^+ = (u,v), e^- = (v,u) | e = \{u,v\} \in E\} \cup \{(v,v) | v \in V\}$ and let a be its index.

In consideration of contingency constraints, let the set of all size k contingencies be defined as follows.

$$C = \left\{ \tilde{\boldsymbol{d}} \in \{0,1\}^{|E|+|V|} \middle| \sum_{e\in E} \tilde{d}_e + \sum_{v \in V} \tilde{d}_v = k \right\} \quad (1)$$

Let $\tilde{\boldsymbol{d}}^c \in \{0,1\}^{|E|+|V|}$ be the contingency vector, for all $c = 1, \cdots, |C|$ and let $\mathcal{C}$ be the contingency-scenario index set.

For all models, $\delta^+_{G(v)}$ and $\delta^-_{G(v)}$ denote arcs of graph G that have v as their source and target node, respectively. The measurement limitation of bus v is given by $\tilde{m}_v$. $\tilde{z}_v$ is a binary parameter that takes value 1 if bus v is a zero-injection bus. $\tilde{d}^c_v$ and $\tilde{d}^c_e$ are binary parameters that takes value 1 if the PMU at bus v and line e are part of contingency c and 0 otherwise, respectively. Finally let x_v be binary variable that takes value 1 if a PMU is installed at node v and 0 otherwise. Let y_a be binary variable that takes value 1 if the PMU installed at the source node of arc a is used to observe the target node of arc a and 0 otherwise. Let w_a be binary variable that takes value 1 if zero-injection property of the source node of arc a is used to observed the target node of arc a and 0 otherwise.

The contingency-constrained formulation with consideration for zero-injection buses and measurement limitations, *Resilient Dominating Set Problem* (RDS), is as follows.

$$\min_{\boldsymbol{w},\boldsymbol{x},\boldsymbol{y}} \quad \sum_{v\in V} x_v \quad (2a)$$

$$\text{s.t.} \quad \sum_{a \in \delta^-_{G(v)}} w^c_a + \sum_{a \in \delta^-_{G(v)}} y^c_a \geq 1 \quad \forall v \in V, c \in \mathcal{C}, \quad (2b)$$

$$\sum_{a \in \delta^+_{G(v)}} y^c_a \leq \tilde{z}_v \quad \forall v \in V, c \in \mathcal{C}, \quad (2c)$$

$$\sum_{a \in \delta^+_{G(v)}} w^c_a \leq \tilde{m}_v x_v (1 - \tilde{d}^c_v) \quad \forall v \in V, c \in \mathcal{C}, \quad (2d)$$

$$\left.\begin{matrix} w^c_{e+}, w^c_{e-} \\ y^c_{e+}, y^c_{e-} \end{matrix}\right\} \leq (1 - \tilde{d}^c_e) \quad \forall e \in E, c \in \mathcal{C}, \quad (2e)$$

$$w^c_a \in \{0,1\} \quad \forall a \in A, c \in \mathcal{C}, \quad (2f)$$

$$y^c_a \in \{0,1\} \quad \forall a \in A, c \in \mathcal{C}, \quad (2g)$$

$$x_v \in \{0,1\} \quad \forall v \in V. \quad (2h)$$

The objective (2a) is to minimize the number of PMUs installed. For each contingency scenario $c \in \mathcal{C}$, a set of constraints enforce the full system supervision criterion. Constraints (2b) are observability constraints for each bus-contingency pair. Bus v is observable by a local PMU, a PMU installed at an incident bus via w^c_a, or through the zero-injection bus property via y^c_a. Constraints (2d) enforce measurement limitations at each bus. If a line fails, constraints (2e) state that PMU(s) installed at the *from bus* and/or *to bus* of e cannot be used to measure the current phasors of the line. Analogously, the zero injection property cannot be used to infer the voltage phasors of an incident bus.

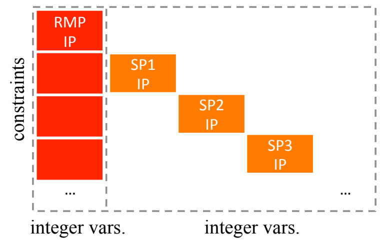

Fig. 1. L-shaped block structure with integer contingency scenario blocks.

Practically, the number of contingencies grows so quickly that RDS (2) is likely to be intractable, even for modest-sized systems and k . Additionally, despite the L-shaped block structure of its constraint matrix, Figure (1), the integer second-stage problems associated with each scenario yields a non-convex recourse function and standard decomposition methods are not directly applicable.

Next, we present an alternative formulation, based on network design, with a linear recourse function.

A. Maximum-Flow Network Design

The network design problem is defined on a directed graph $D(V', A')$. For clarity of exposition, we first demonstrate the reformulation using a simple 4-bus system (Figure 2). We

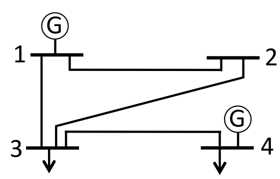

Fig. 2. A 4-bus power system with zero-injection bus 2.

begin by defining the key components of this reformulation, two special nodes, a source node s and a terminal node t, and two sets of directed arcs. For each bus $v \in V$ in the original power system, define arc (s, p_v) with a flow capacity of $\tilde{m}_v$ and arc (t_v, t) with unit flow capacity. Arc (s, p_v) represents PMU installation at bus $v \in V$. A unit flow on arc (t_v, t) represents observability of bus v. The augmented maximum-flow network of the 4-bus system is shown in Figure 3. Due to space constraints, we summarize the descriptions of all other arcs and nodes in Table I. In the augmented network (3), each arc into terminal node t has unit capacity, thus, if the cumulative flow into node t equals 4 units then all buses are observable. If arc (s, p_1) is selected (exits), up to m_1 units can flow from s to p_1. A unit flow on path $s - p_1 - t_1 - t$ corresponds to using the PMU installed at bus 1 to measure bus 1. A unit flow on the path $s - p_1 - o_{12} - d_{12} - t_2 - t$ corresponds to using the PMU installed at bus 1 to measure bus 2. However, if transmission line $\{1,2\}$ fails, that is if arc (o_{12}, d_{12}) fails, path $s - p_1 - o_{12} - d_{12} - t_2 - t$ is no longer viable. A unit flow on path $s - z_2 - p_2 - t_2 - t$ corresponds to using the zero-injection bus property of bus

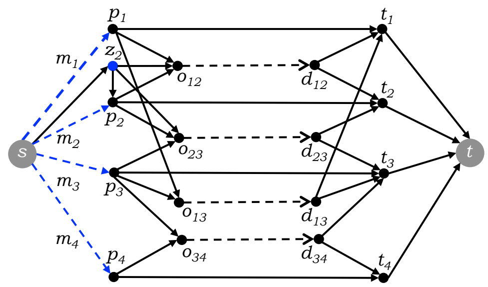

Fig. 3. Maximum-flow network expansion for 4-bus example. With the exception of arcs (s, p_v) for all $v \in V$, all other arcs have unit flow capacity.

2 to infer the voltage phasors of bus 2. A unit flow along path $s - z_2 - o_{12} - d_{12} - t_1 - t$ corresponds to using the zero-injection bus property of bus 2 to infer the voltage phasors of bus 1. Let $D(V', A')$ represent the augmented network and let

TABLE I
TRANSFORMATION FROM THE ORIGINAL POWER SYSTEM TO THE AUGMENTED NETWORK

	Original System $G(V, E)$	Augmented Network $D(V', A')$	Capacity	Faillable
Nodes	$v \in V$	s, t	-	No
		$p_v \ \forall v \in V$	-	No
		$t_v \ \forall v \in V$	-	No
		$o_e \ \forall e \in E$	-	No
		$d_e \ \forall e \in E$	-	No
		$z_v \ \forall v \in Z$	-	No
Edges/Arcs	$e \in E$	$(s, p_v) \ \forall v \in V$	$\tilde{m}_v$	Yes
		$(t_v, t) \ \forall v \in V$	1	No
		$(p_v, t_v) \ \forall v \in V$	1	No
		$(o_e, d_e) \ \forall e \in E$	1	Yes
		$(p_u, o_e) \ \forall e = \{u, v\} \in E$	1	No
		$(p_v, o_e) \ \forall e = \{u, v\} \in E$	1	No
		$(d_e, t_u) \ \forall e = \{u, v\} \in E$	1	No
		$(d_e, t_v) \ \forall e = \{u, v\} \in E$	1	No
		$(s, z_v) \ \forall v \in Z$	1	No
		$(z_v, o_e) \ \forall e \in \rho_{G(v)}, v \in Z$	1	No
		$(z_v, p_v) \ \forall v \in Z$	1	No

A'_p, A'_d and A'_r represent the sets of PMU placement arcs, line arcs, and residual arcs, $A'_r := A' \setminus \{A'_p \cup A'_d\}$, respectively. Let f_a be the flow on arc a. Then, RPPP can be formulated as a *resilient maximum-flow network design* problem (RND).

$$\min_{\boldsymbol{x}, \boldsymbol{f} \geq \boldsymbol{0}} \sum_{a \in A^p} x_a \tag{3a}$$

$$\text{s.t. } f_a^c = 1 \quad \forall a \in \delta^-_{D(t)}, c \in \mathcal{C}, \tag{3b}$$

$$\sum_{a \in \delta^+_{D(v)}} f_a^c = \sum_{a \in \delta^-_{D(v)}} f_a^c \ \forall v \in V' \setminus \{s, t\}, c \in \mathcal{C}, \tag{3c}$$

$$f_a^c \leq \tilde{m}_a x_a (1 - \tilde{d}_a^c) \quad \forall a \in A'_p, c \in \mathcal{C}, \tag{3d}$$

$$f_a^c \leq 1 - \tilde{d}_a^c \quad \forall a \in A'_d, c \in \mathcal{C}, \tag{3e}$$

$$f_a^c \leq 1 \quad \forall a \in A'_r, c \in \mathcal{C}, \tag{3f}$$

$$x_a \in \{0, 1\} \quad \forall a \in A'_p. \tag{3g}$$

The objective (3a) is minimize the number of PMU "arcs" built. For each contingency scenario c, define a set of constraints to enforce observability of all buses by forcing a unit flow on arcs into terminal node t (3b). Constraints (3c) are flow balance constraints for all nodes other than nodes s and t. Constraints (3d) enforce measurement limitations at each bus. If a transmission line fails, constraints (3e) state that flow on arc $a \in A'_d$ must be zero. Finally, constraints (3f) enforce capacity constraints on the rest of the arcs. Figure 4 depicts

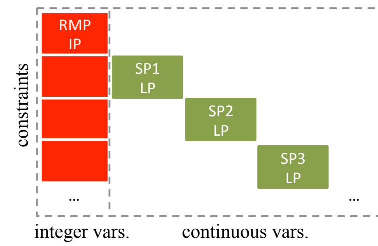

Fig. 4. L-shaped block structure with linear contingency blocks.

the structure of the constraint matrix of RND (3). In contrast to the RDS structure (Figure 1), the new formulation yields linear second-stage problems.

III. SOLUTION APPROACHES

RDS and RND will typically have extremely large number of variables and constraints because it grows with the number of contingency scenarios, which increases exponentially with $|V| + |E|$ and k. For large power systems and/or a contingency budget k greater than one, RDS and RND rapidly become computationally intractable for increasing system size.

A. Benders Decomposition

We now briefly discuss an alternative formulation with only $|V|$ binary variables but possibly an exponential number of constraints. We use *linear programming* duality to generate valid inequalities for the projection of the natural formulation onto the space of the PMU placement variables $\boldsymbol{x}$.

Let $z(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{d}}^c)$ represent the maximum-flow given candidate PMU placement $\tilde{\boldsymbol{x}}$ and contingency scenario prescribed by $\tilde{\boldsymbol{d}}^c$. $z(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{d}}^c) \geq |V|$ should be satisfied for all $c \in \mathcal{C}$. Thus contingency feasibility conditions can be restated as follows.

$$\sum_{a \in A'_p} m_a (\tilde{x}_a - \tilde{d}_a^c)^+ \beta_a^\ell + \sum_{a \in A'_d} m_a (1 - \tilde{d}_a^c) \gamma_a^\ell + \sum_{a \in \delta^-_{D(t)}} m_a \pi_a^\ell + \sum_{a \in A'_r} m_a \varepsilon_a^\ell \geq |V|, \quad \forall \ell \in \mathcal{L}^c, c \in \mathcal{C}. \tag{4}$$

where $\mathcal{L}^c$ is the set of extreme points of the dual of constraints (3c) - (3f). We enforce all constraints for the no failure scenario explicitly. For all contingency scenarios $c \in \mathcal{C}$, constraint set (4) ensures that the system is fully observable. By using a Benders decomposition (BD), we are able to decompose the extremely large formulation (3) into a master problem and multiple subproblems. In theory, this enables us to solve larger instances, which would not be possible by a direct solution of RDS and RND. For a detailed treatment of BD please refer to [3].

B. Implicit Contingency Screening

Given a non-trivial k, the sizes of most systems in operation may preclude direct solution of (3). Even using BD may not be tractable because each contingency scenario must be considered *explicitly*. Our goal is to instead use a separation oracle that *implicitly* evaluates all contingency scenarios and either identifies a violated one, a contingency with k failures that renders a part of the system unobservable, or certificatifies that no such contingency scenario exists.

1) Maximum Flow Separation Problem: Given $\tilde{\boldsymbol{x}}$, we solve a *Maximum-Flow Separation Problem* (MSP) to determine the maximum observability under the worst-case contingency scenario with k failures. In this bilevel program, the upper-level decisions $\boldsymbol{d}$ correspond to binary contingency-selection decisions and the lower-level decisions $\boldsymbol{f}$ correspond to maximum flows relative to $\tilde{\boldsymbol{x}}$ and $\boldsymbol{d}$. $\mathrm{MSP}(\tilde{\boldsymbol{x}})$ is given as follows.

$$\min_{\boldsymbol{d}} \max_{\boldsymbol{f}\geq \boldsymbol{0}} \sum_{a\in\delta_{D(t)}^-} f_a \tag{5a}$$

$$\text{s.t.} \sum_{a\in A'_p} d_a + \sum_{a\in A'_d} d_a = k, \tag{5b}$$

$$(\alpha)\ \sum_{a\in\delta^+_{D(t)}} f_{uv} = \sum_{a\in\delta^-_{D(t)}} f_{vu} \quad \forall v\in V'\setminus\{s,t\}, \tag{5c}$$

$$(\beta)\ f_a \leq \tilde{m}_a \tilde{x}_a(1-d_a) \quad \forall a\in A'_p, \tag{5d}$$

$$(\gamma)\ f_a \leq 1-d_a \quad \forall a\in A'_d, \tag{5e}$$

$$(\pi)\ f_a \leq 1 \quad \forall a\in \delta^-_{D(t)}, \tag{5f}$$

$$(\varepsilon)\ f_a \leq 1 \quad \forall a\in A'_r, \tag{5g}$$

$$\boldsymbol{d}\in\{0,1\}^{|E|+|V|}. \tag{5h}$$

The objective (5a) is to minimize the maximum flow into terminal node t. Given $\boldsymbol{d}$, the goal of the system operator (inner minimization) is to determine the optimal flow such that system observability is maximized. Constraint (5b) limits the total number of system elements that can be in a contingency. Next, we outline an algorithm for solving RND that combines a BD with the aid of the separation oracle given by (5).

Algorithm 1 *Implicit Contingency Screening* (ICS)

1: $t \leftarrow 0$
2: Solve RMP and let $\tilde{\boldsymbol{x}}^t$ be opt. solution if feasible
3: **if** RMP is infeasible
4: EXIT, RPPP is infeasible
5: **else**
6: Solve $\mathrm{MSP}(\tilde{\boldsymbol{x}}^t)$, let w^t be opt. obj. value and $\tilde{\boldsymbol{d}}^t$
7: be opt. contingency vector
8: **if** $w^t < |V|$ then
9: Solve $\mathrm{PSP}(\tilde{\boldsymbol{x}}^t, \tilde{\boldsymbol{d}}^t)$, add feasibility cut to RMP
10: $t \leftarrow t+1$, go to step 2
11: **else**
12: $\tilde{\boldsymbol{x}}^t$ is optimal, EXIT

At each iteration t, we iterate between solving a relaxed master problem (RMP) to find a candidate PMU placement $\tilde{\boldsymbol{x}}^t$, $\mathrm{MSP}(\tilde{\boldsymbol{x}}^t)$ to find the worst-case contingency $\tilde{\boldsymbol{d}}^t$ and $\mathrm{PSP}(\tilde{\boldsymbol{x}}^t, \boldsymbol{d}^t)$ to generate a feasibility cut. Either a contingency that renders a part of the system unobservable is identified or no such contingency exists, which means that the current PMU placement is optimal and the algorithm terminates.

IV. Numerical Experiments

We implemented our approach in C++ and CPLEX 12.4-ILOG Concert Technology 2.9. All experiments ran on a machine with a 2.93 GHz Xeon processor and 32 GB memory. We tested our approach on five systems: 30-bus, 57-bus, RTS 96, 118-bus, and WECC 240-bus [13] using $\tilde{m}_v = 3\ \forall v\in V$. For each system, we considered contingency budgets $k = 0,1,2$ and enforced a maximum runtime of 7200 seconds.

In order to ensure feasibility for $k=2$, we made the simplifying assumption that PMUs installed at buses with less than $k+1$ incident lines cannot fail. We also added sets of valid inequalities derived from connectivity requirements.

TABLE II
RUNTIMES FOR DIFFERENT SOLUTION APPROACHES

System	m	k	Solution time (secs) RDS	RND	BD	ICS	No. PMUs
30	0	0	0	0	0	0	8
	71	1	0	1	1	0	16
	$2K+$	2	2	5	6	0	26
57	0	0	0	0	0	0	14
	137	1	6	50	13	4	22
	$9K+$	2	12	249	122	2	47
96	0	0	0	0	0	0	22
	193	1	30	120	13	2	34
	$18K+$	2	72	OM	98	6	57
118	0	0	0	0	0	0	36
	304	1	10	74	19	2	61
	$46K+$	2	OM	OM	802	14	104
240	0	0	0	0	0	0	65
	588	1	9	221	124	6	132
	$172K+$	2	OM	OM	(239)	64	180

Table II provides the runtime, in CPU seconds, for each instance under the four different approaches. The first two approaches, RDS and RND, solve extensive forms of (2) and (3), respectively. The latter two approaches solve RND using BD and ICS algorithms. OM means that the approach exited with a status of *out-of-memory*. Note that RDS and RND can only solve the smaller instances. This is because of the sheer size of the problem, in which a full recourse problem must be embedded within the formulations for each scenario. As the number of scenarios grows, these formulations quickly becomes intractable. We are working with moderate-sized systems here and target systems will be in the order of thousands of elements, which will make monolithic math programs such as RDS and RND intractable even sooner.

A BD algorithm bypasses the size problem via a delayed cut generation. However, it still suffers from the combinatorial growth in the number of contingency scenarios – for each contingency a PSP must be solved to check for violated feasibility cuts. We see that larger problem instances can be solved, relative to RDS and RND, but the BD algorithm nonetheless cannot solve the largest problem instances. Paranthetical number under column BD indicate the maximum number of observable buses under the worst-case contingency scenario at the end of the two-hour time limit.

With the ICS approach, we see that all instances can be solved, in all cases in approximately 1 minute and frequently in only a few seconds. This is the result of the combination of the strength of the Benders cuts and the strengthening inequalities, enabling the problem to be solved in a small number of iterations, and also the fact that we are able to implicitly evaluate all contingencies quickly and then find feasibility cuts by solving a PSP. The final column of Table II show the number of installed PMUs for each instance. Table III shows the optimal

TABLE III
DETAILED SOLUTIONS FOR 57-BUS AND RTS 96 SYSTEMS

System	k	no. PMUs	PMU locations
	0	14	1,6,12,13,15,19,25,29,32,38,41,51,54,56
57	1	22	1,2,6,9,12,15,18,20,25,27,29,30,32,33,38,41,47,50, 53,54,56
	2	47	1,2,3,5,6,9,10,12,14,15,16,17,18,19,20,21,23,25,26, 27,28,29,30,31,32,33,34,35,38,39,40,41,42,43,44, 45,46,47,49,50,51,52,53,54,55,56,57
	0	22	2,5,8,9,15,16,17,25,26,32,36,39,41,43,49,50,56,60, 64,67,69
96	1	34	1,2,3,7,9,10,15,16,17,20,21,26,27,28,29,31,32,34, 38,39,41,43,44,45,49,50,55,56,57,58,64,67,69,70
	2	57	1,2,3,4,5,6,7,8,9,10,14,15,16,17,18,19,20,21,22,24, 25,26,27,28,29,30,31,33,34,37,38,39,40,41,42,43, 44,45,46,48,49,50,52,53,54,55,57,58,61,62,64,65, 66,67,68,69,70,72,73

PMU placements for the 57-bus and RTS 96 systems. In going from the no-contingency to the single-element contingency, there is a moderate increase in the number of PMUs installed. However, to protect against two-element failures the number of PMUs installed increased considerable. These results follow intuition since power system topologies are typically sparse [14], thus as contingency budget increases more PMUs must be installed locally. Finally, Figure 5 shows that to maintain full observability under two-element failures, PMUs must be installed at over 86% of the buses. The four buses with no PMUs are buses 6, 9, 25 and 28.

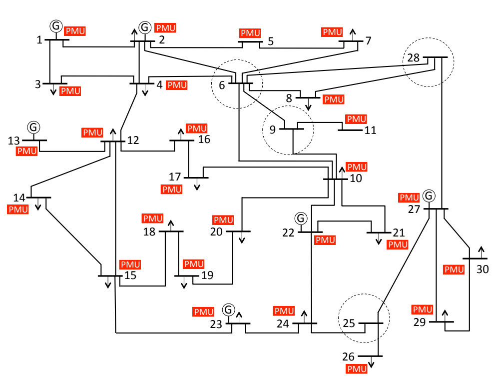

Fig. 5. An optimal 2-failure resilient PMU placement for 30-bus system.

V. CONCLUSION

The ability to solve large-scale RPPP is of high consequence – especially as attention on modernizing electric grids has increased. However, the best exact approaches to date consider only single-element failures and cannot scale to practical power-system sizes. This paper makes three key contributions. First, we present a more general k-failure resilient model for PPP called RDS. By supporting PMU layouts that are provably resilient to any k component failures, critical electric grids can be operated with more flexible margins of safety and security, per the needs of individual system operator. Second, we present an alternative formulation for RDS called RND. This formulation, which to the best our knowledge is the first formulation for PPP based on network design rather than variants of cover problems, permits efficient decomposition. Third, we present two cutting plane algorithms for solving RND, which are the first such decomposition algorithms in this domain. Computational results show that systems of moderate sizes can be solved efficiently, in all cases in about one minute, which provide evidence that the approach may be tractable for large-scale regional and national systems.

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