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THE DERIVATION OF RANDOM VIBRATION SPECIFICATIONS FROM FIELD TEST DATA FOR USE WITH A SIX DEGREE-OF-FREEDOM SHAKER TEST*

Jerome S. Cap
Sandia National Laboratories
Analytical Structural Dynamics Department

D. Gregory Tipton
Sandia National Laboratories
Analytical Structural Dynamics Department

David O. Smallwood
Consultant

Six Degree-of-Freedom (6DOF) shaker tables represent a quantum leap in our ability to perform realistic random vibration testing. In order to take full advantage of a 6DOF shaker table, the underlying test specifications must evolve from the traditional set of three independent, orthogonal autospectra to a fully coupled spectral density matrix. This transition is hindered by the fact that we rarely have sufficient data channels present during a field test to specify the coupled inputs directly from the data. This presentation describes a process that combines field test data with a system level Finite Element Model (FEM) in order to derive a set of Multi-Degree-of-Freedom (MDOF) specifications for a component riding on that system.

INTRODUCTION

The process can be divided into three steps: 1) the generation of the system transfer functions, 2) the generation of the system inputs, and 3) the derivation of the component inputs. While the final process used in the derivation process will be the main focus of this paper, important lessons learned along the way are also discussed. Two side studies are also included showing how well the derived test specifications were reproduced in the test lab, and how to independently scale the different terms in the matrix of input spectra.

DESCRIPTION OF THE SYSTEM

The environment of interest is the random vibration excitation associated with the powered flight maximum dynamic pressure (Max Q) events. The system input points are assumed to be limited to the three pedestals as shown in Figure 1 (the pedestals will be designated as *PA*, *PB*, and *PC*). Since the component is attached to the system using a circumferential bolt pattern it was decided that the component inputs should be defined using X, Y, and Z translations at four points aligned with the cardinal directions (0°, 90°, 180°, and 270°) as is also shown in Figure 1. The response points on the structure corresponding to the component inputs will be designated as *C*1, *C*2, *C*3, and *C*4 respectively.

Because the pedestals are bolted to the launch vehicle's payload plate it was assumed that the local rotations associated with these points were negligible. Therefore, the initial scope for the problem was based on allowing for X, Y, and Z translations at the base of each pedestal (i.e., 9 inputs) and allowing for the X, Y, and Z translations at the four points around the component (i.e., 12 responses). This meant that in its most general form the matrix containing the system transfer functions, H_{CP}, would have size 12x9 as shown in equation (1).

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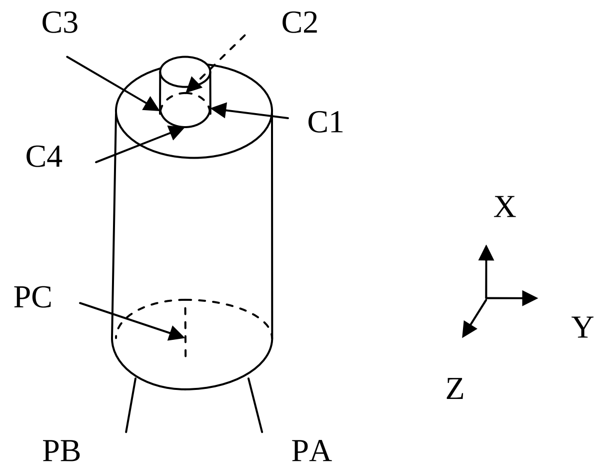

Figure 1: System Layout

$$\begin{Bmatrix} C1X \\ C1Y \\ C1Z \\ C2X \\ C2Y \\ C2Z \\ C3X \\ C3Y \\ C3Z \\ C4X \\ C4Y \\ C4Z \end{Bmatrix} = \begin{bmatrix} H_{C1XPAX} & \cdots & H_{C1XPCZ} \\ \vdots & \ddots & \vdots \\ H_{C4ZPAX} & \cdots & H_{C4ZPCZ} \end{bmatrix}_{size\ 12x9} \begin{Bmatrix} PAX \\ PAY \\ PAZ \\ PBX \\ PBY \\ PBZ \\ PCX \\ PCY \\ PCZ \end{Bmatrix} \tag{1}$$

For random vibration the inputs and responses are presented in the form of Spectral Density Matrices (SDMs). Therefore, equation (1) must be manipulated into the form shown in equation (2) where S_{PP} and S_{CC} are the input and response SDMs respectively.

$$S_{CC(12x12)} = H_{CP(12x9)} S_{PP(9x9)} H'_{CP(12x9)} \tag{2}$$

DEREVATION OF SYSTEM TRANSFER FUNCTIONS

When dealing with flight test data it is often the case that one has internal response data but no input data. Therefore, it was initially intended to use a Multiple-Input-Multiple-Output (MIMO) analysis to derive an optimum set of inputs based on flight test data measured at points on the structure. However, it turned out that in this case the response data measured during the Max Q portion of powered flight was so benign that it was considered too noisy to be of use. Fortunately, we did have a sparse set of acceleration data measured during two flights at the base of the pedestals that could be used to define the inputs. Therefore, the terms in H_{CP} had to be provided as Acceleration/Acceleration Transmissibility Response Functions (TRFs).

The system TRFs were generated using a FEM analysis. The model was excited by applying a force to a large inertial mass attached to the base of each pedestal in succession. In this manner the input acceleration could be obtained. The full suite of TRFs were generated from each pedestal to each of the four component input points.

DERIVATION OF SYSTEM INPUTS

Data from two flights were used as the data base. In this paper the acceleration measurement with the largest overall root mean square (rms) acceleration has been normalized to have a root mean square (rms) value of 1g. All the other measurements were scaled by the same factor.

At this point it is useful to introduce the concept of the normalized SDM (NSDM). The NSDM retains the diagonal terms, C_{II}, "as is" but scales the off-diagonal terms, C_{IJ}, as shown in equation (3). The magnitude squared of each off-diagonal term, C_{IJN} is the ordinary coherence, γ^2, and the arctangent of the ratio of the imaginary part divided by the real part of an off-diagonal term is the phase, ϕ.

$$C_{IJN} = \frac{C_{IJ}}{\sqrt{C_{II}C_{JJ}}} \tag{3}$$

The first flight provided data from a tri-axial accelerometer designated as VR10 X, Y & Z. The NSDM for the three acceleration signals obtained from the first flight is shown as Figure 2.

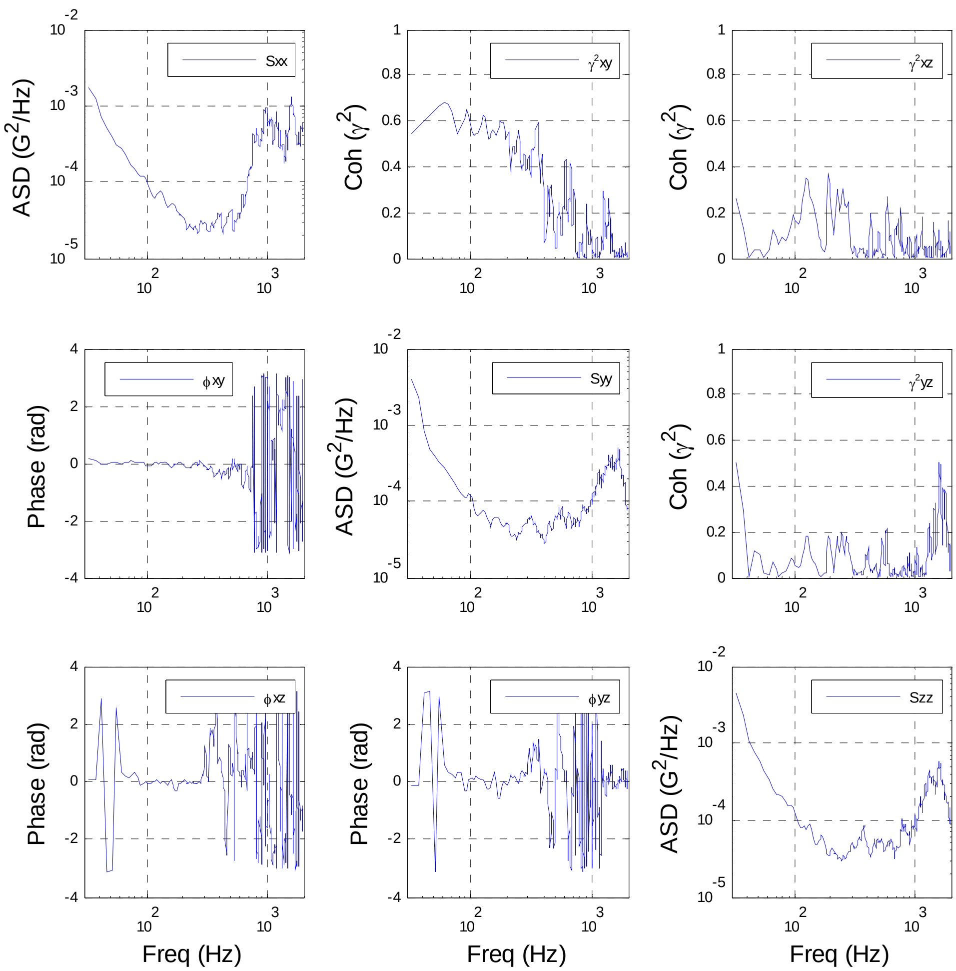

Figure 2: Normalized SDM, 1st flight, VR10X, VR10Y, and VR10Z

The second flight provided data from a set of three X-axis accelerometers with one accelerometer located at each pedestal base. The NSDM for this flight is shown as Figure 3. As can be seen the coherence was quite high over significant frequency ranges while the phase was not always zero. This suggests that there is some amount of rigid body rotations combined with the rigid body translations.

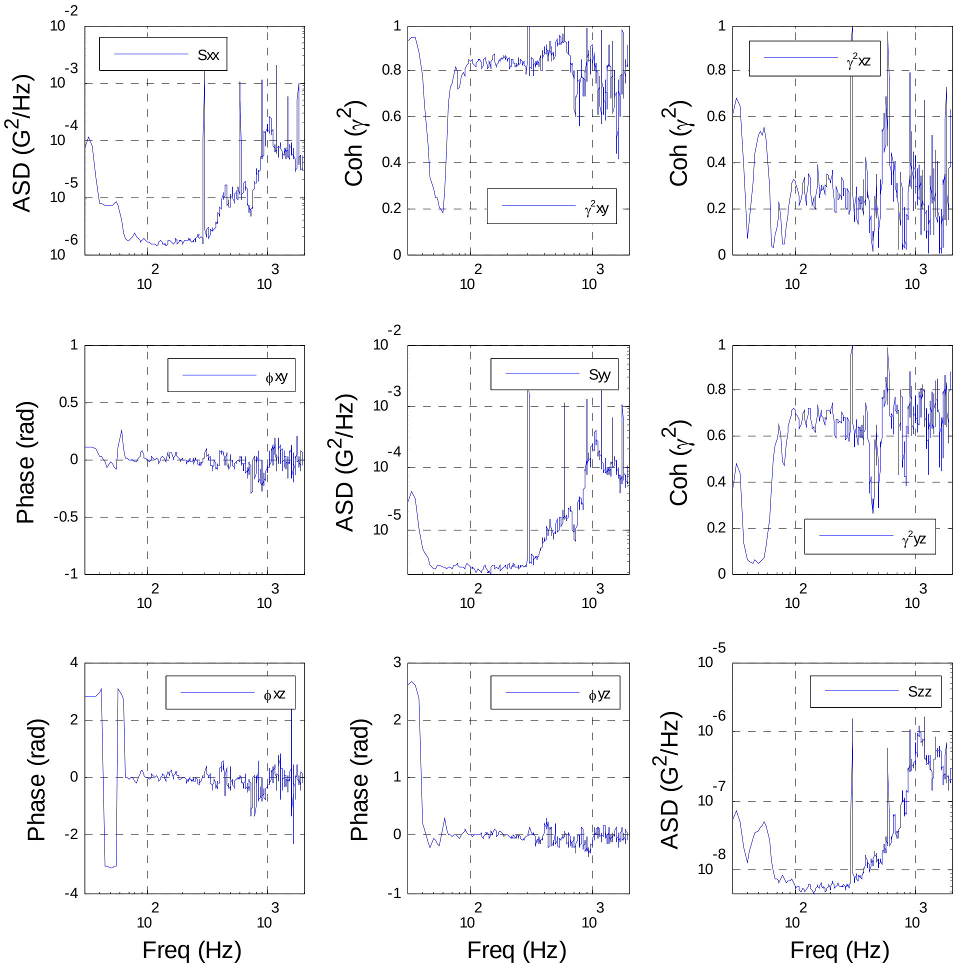

Figure 3: Normalized SDM, 2nd flight, VX10, VX11X, and VX12X

In theory, combining the data from the two flights could define 5 rigid body modes (only rotation about the X-axis would be undefined). Unfortunately, two potential problems exist. These problems are best explained by comparing the auto spectra from the three X-axis accelerometers present for the second flight (V10X, V11X, and V12X) and the lone X-axis accelerometer present for the first flight (V10X) as shown in Figure 4.

The first problem is that the data from the second flight are clearly contaminated by noise at 300 Hz and the corresponding harmonics. The second problem is that the magnitudes of the V10X signals are quite different for the two flights, which could make the combination of the data quite difficult. Each of these problems could be overcome, but the objective of this study was just to demonstrate the technology and not to perform an actual qualification test. Therefore, it was decided to leave these problems for future study. Hence, for this study the input spectra will be determined solely from the first flight data. The low frequency portion of the data (< 30 Hz) was not omitted from the analysis so the rigid body rotations will not be a significant factor for this study. It was also decided to generate test specifications using straight line segments as a practical consideration for the transmission of the test specifications. Figure 5 compares the raw NSDM and straight line segment test specifications for the system inputs.

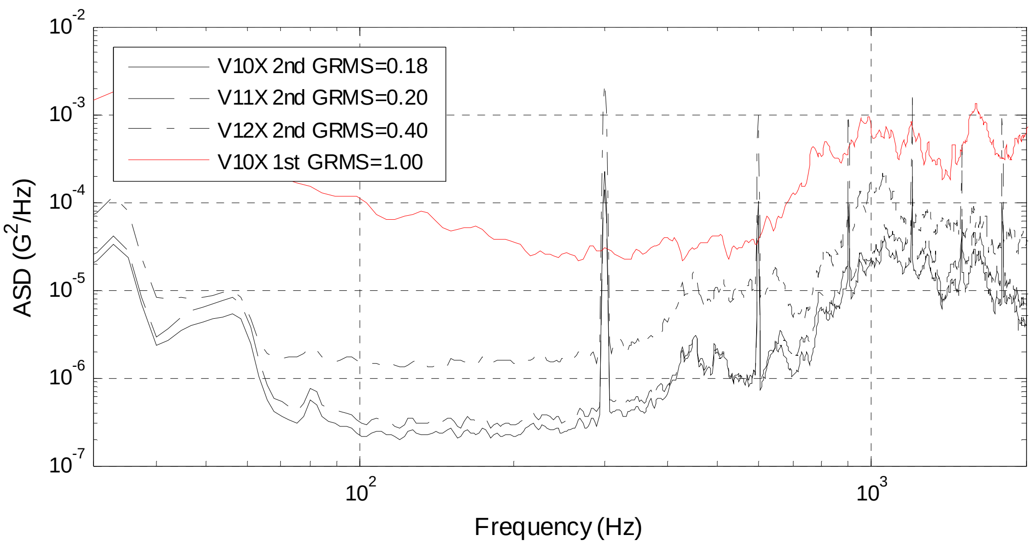

Figure 4: X-Axis Auto Spectra from 1st and 2nd Flights

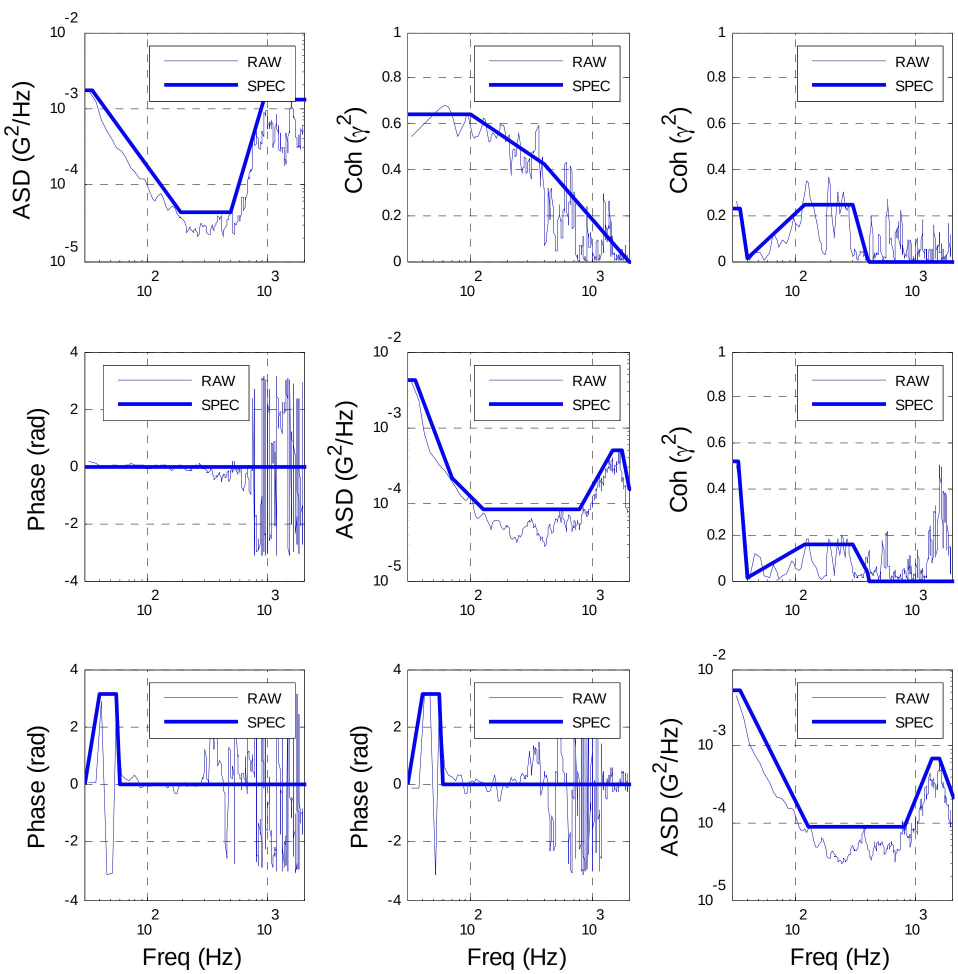

Figure 5: Comparison of System Input Raw Data and Straight Line Segment Test Specifications

With regards to the autospectra the enveloping process was straight forward with the test specification designed to exceed the raw NSDM for all frequencies. With regards to the coherence, the hashy nature of the coherence made it seem inappropriate to envelop the raw spectra. Experience has shown that lower coherence values produce a more stable test. Therefore, the test specifications were defined to envelop the raw coherence levels at low frequency where the coherence was relative smooth, to fair through the raw coherences in the mid frequency range, and to set the coherence to zero at high frequency for the XZ (1,3) and YZ (2,3) terms. The phase tended to vary even more wildly than the coherence so the phases were set to zero (which is technically the faired value) for all but the low frequency portion of the XZ and YZ terms.

DERIVATION OF COMPONENT INPUTS

The next step was to compute the projected inputs to the component based on the system inputs and the TRFs. The fact that it was only possible to define a common X, Y, and Z motion for all three pedestals by definition led to two simplifications in the complexity of the problem. The first simplification was that the TRF matrix had to collapse to size 12x3 where the columns in H_{CP} associated with a given translational direction for PA, PB, and PC were simply added together. Equation (4) shows the resulting input/response formulation.

$$S_{CC(12x12)} = H_{CP(12x3)} S_{PP(3x3)} H'_{CP(12x3)} \tag{4}$$

However, a somewhat less obvious consequence of using a 3x3 system input SDM is the fact that the computed 12x12 component input SDM was not positive definite, which is incompatible with performing a valid test. The problem was attributed to the fact that one cannot generate more responses than one has independent inputs. Therefore, it was necessary to define the response of the component using only three orthogonal “global” translations. This was accomplished by applying the transformation, H_T, shown in equation (5), to the 12x12 component input SDM using the equation shown in equation (6).

$$H_T = 0.25 \begin{bmatrix} 1\,0\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0 \\ 0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,1\,0 \\ 0\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,1 \end{bmatrix} \tag{5}$$

$$S_{CC(3x3)} = H_T S_{CC(12x12)} H'_T \tag{6}$$

The test was now technically a 3DOF test, but the rotational degrees of freedom were allowed to respond in whatever manner the shaker control algorithms determined would produce the best match for the 3x3 component input SDM (as opposed to constraining them to be zero).

Figure 6 presents the corresponding component input SDM using the same format as was used to present the system input SDM in Figure 5. While the autospectra are credible, the phase and coherence are considerably wilder than had been expected given the fact that the input SDM was smoothed.

LABORATORY SIMULATION

The goal of this study was to develop a set of Multi-Degree-of-Freedom (MDOF) test specifications that could be run on the 6DOF shaker and that was accomplished. Figure 7 compares the desired and measured magnitudes for the diagonal Auto spectral Densities (ASDs) and off-diagonal Cross Spectral Densities (CSDs) for the component input SDMs (denoted “DES” and “MEAS” respectively). In general the agreement was excellent, but there are still frequency bands where the measured response was considerably higher than the desired spectra. These points appear to correspond to frequencies where the desired spectra are deeply notched so it is assumed that the control system may simply have reached its maximum dynamic range.

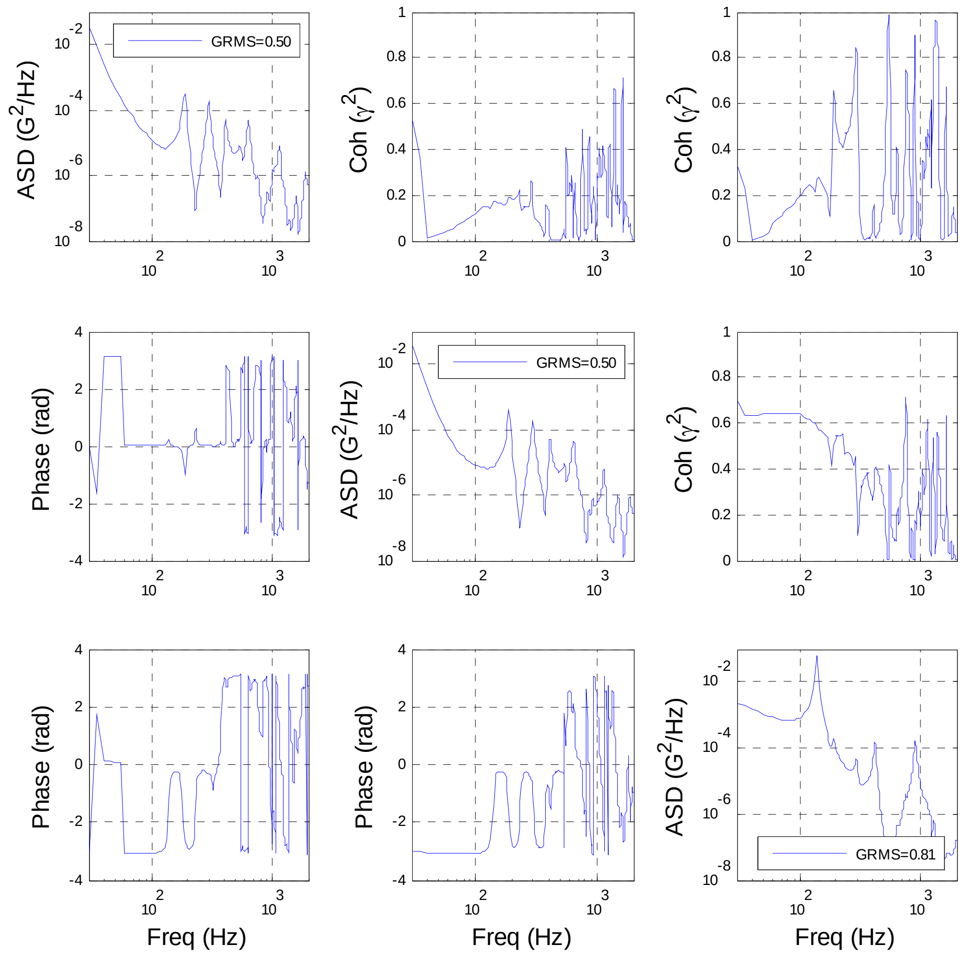

Figure 6: Raw Component Input SDM

SCALING OF TEST LEVELS

For the purposes of this study, the raw computed component inputs were applied "as is" in the 6DOF shaker test. However, it is often desirable to scale the inputs in order to provide margin and to utilize straight-line segment test specifications in order to simplify the amount of information that must be transmitted to define the test. The scaling must not violate the need for the SDM to be positive definite. This requirement was met for the enveloping of the system inputs, but the process was based on engineering judgment rather than on a rigorous procedure.

In order to solve this problem in a systematic manner, let C_{II} and C_{JJ} be two original diagonal terms and C_{IIN} and C_{JJN} be the corresponding scaled diagonal terms. Our working hypothesis is that if the off diagonal terms in the SDM were scaled according to the formula shown in equation (7) the phase and coherence would be preserved, which in turn would keep the SDM positive definite. Appendix A provides proof that this assumption was indeed valid.

$$C_{IJN} = C_{IJ}\sqrt{\frac{C_{IIN}}{C_{II}}}\sqrt{\frac{C_{JJN}}{C_{JJ}}} \quad (7)$$

Figure 8 shows a set of straight line autospectra based on the underlying raw component inputs. Figure 9 shows the resulting SDM.

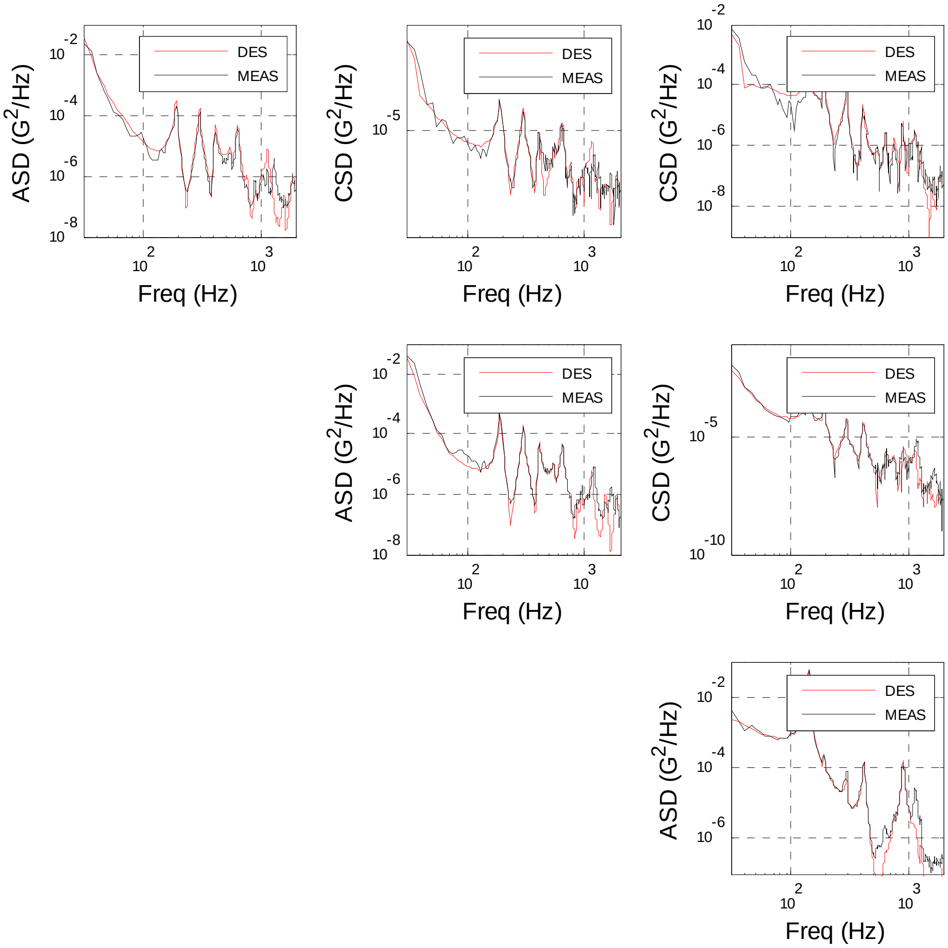

Figure 7: Comparison of Desired and Achieved Component Input SDM

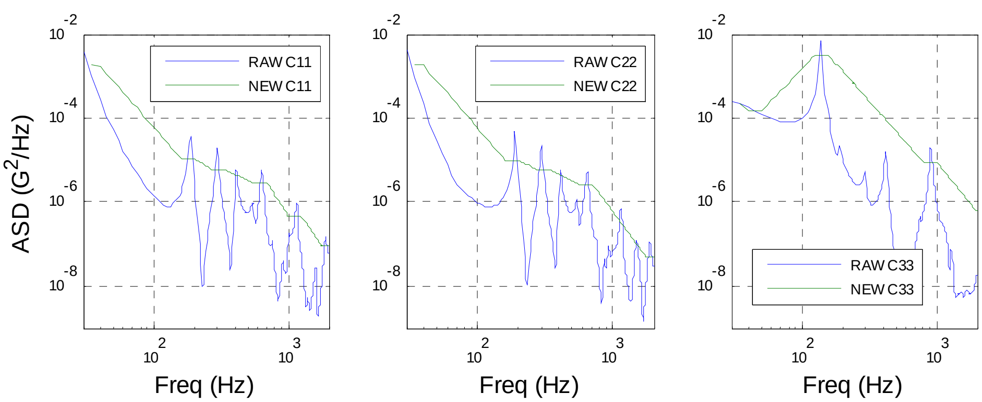

Figure 8: Derivation of Straight-Line Segment Component Input Autospectra

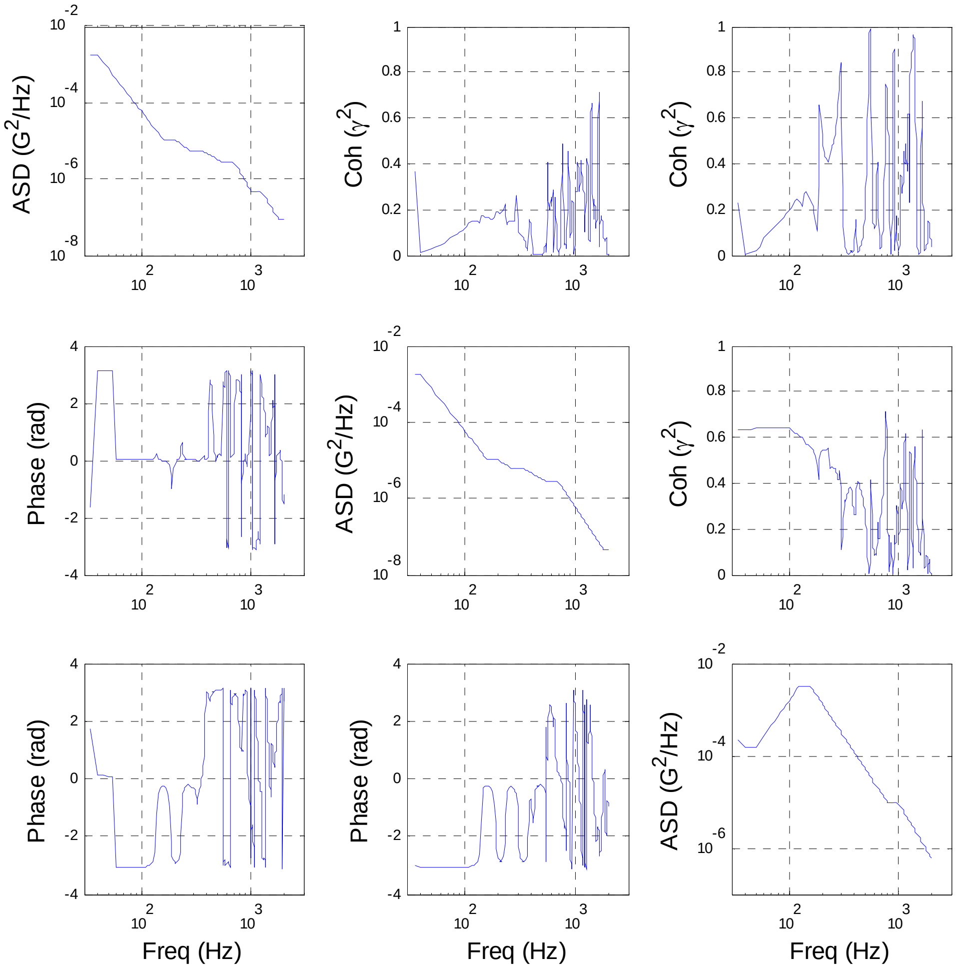

Figure 9: Straight-Line Segment Component Input SDM

LESSONS LEARNED

The main lesson, as discussed earlier in this paper was the need to limit the DOF of the component inputs to less than or equal to the number of DOF associated with the system input.

The order in which the component and system axes were specified was different. As a result of this the axial inputs were originally assigned to the first row in the component SDM while the shaker had initially been configured to expect the axial inputs to be in the third row of the SDM. While the correction involves a simple flipping of the first and third rows of the matrix, this could have resulted in a crossing of the test inputs. The lesson here is that the chances for this type of mistake are much greater with multi-axis testing than for traditional single-axis testing.

The original FEA results were presented using 4 Hz spacing while the nearest spacing that was compatible with the shaker control system was 5 Hz. A simple interpolation routine was used to adjust the spacing but the resulting SDM was no longer positive definite. The short term fix was to have the FEA results exported using 1 Hz resolution and then decimate the TRFs. However, a methodology for interpolating the SDM while keeping the SDM positive definite would seem to be a tractable problem for future investigation.

CONCLUSIONS

Within the constraints associated with the raw system input data, we were successful in our efforts to develop a workable MDOF component input specification.

It is assumed that by adjusting the scaling on the data from the second flight it should be possible to combine it with the data from the first flight to increase the number of DOF from 3 to 5 for the system inputs by defining a distinct out-of-plane (X-axis) DOF for each pedestal. Since the in-plane motions at each pedestal are more likely to be slaved together than the out-of-plane motions, this should come very close to capturing the salient features of the true input.

A logical next topic for study will be the development of a systematic way in which to smooth the phase and coherence while still keeping the SDM positive definite.

Future efforts will be directed towards an evaluation of the conservatism of a traditional single-axis shaker test based on the assumption that the 6DOF test is the best measure of the true field environment.

APPENDIX A: VERIFICATION OF PROCESSS FOR SCALING POSITIVE DEFINITE MATRICES

The rules by which an SDM can be scaled and still be positive definite can be expressed using a simple exercise in matrix algebra. Let $\{X\}$ and $\{Y\}$ be two vectors of random variables. The corresponding spectral densities (which are by definition positive definite) are given by equation (8).

$$S_{XX} = \mathrm{E}[XX'] \; and \; S_{YY} = \mathrm{E}[YY'] \tag{8}$$

Assume that *X* and *Y* are related by a system of linear transfer functions as shown in equation (9).

$$\{Y\} = H\{X\} \tag{9}$$

Then the spectral density of *Y* with respect to the spectral density of *X* is given by equation (10).

$$S_{YY} = \mathrm{E}[HXX'H'] = H\mathrm{E}[XX']H' = HS_{XX}H' \tag{10}$$

Because S_{YY} is positive is positive definite, $H\mathbf{S}_{XX}H'$ must also be positive definite. Now let *H* be defined as a diagonal matrix *A* as shown in equation (11).

$$H = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \tag{11}$$

Substituting the value of *H* shown in equation (11) into equation (10) yields the expression shown in Equation (12).

$$S_{YY} = [A][S_{XX}][A]' = \begin{bmatrix} A_1^2 S_{11} & A_1 A_2 S_{12} & A_1 A_3 S_{13} \\ A_2 A_1 S_{21} & A_2^2 S_{22} & A_2 A_3 S_{23} \\ A_3 A_1 S_{31} & A_3 A_2 S_{32} & A_3^2 S_{33} \end{bmatrix} \tag{12}$$

If one defines $A_1 = \sqrt{C_{11N}/C_{11}}$, $A_2 = \sqrt{C_{22N}/C_{22}}$, and $A_3 = \sqrt{C_{33N}/C_{33}}$, then the reader can see that the scale factor defined in equation (7) will produce a positive definite matrix.