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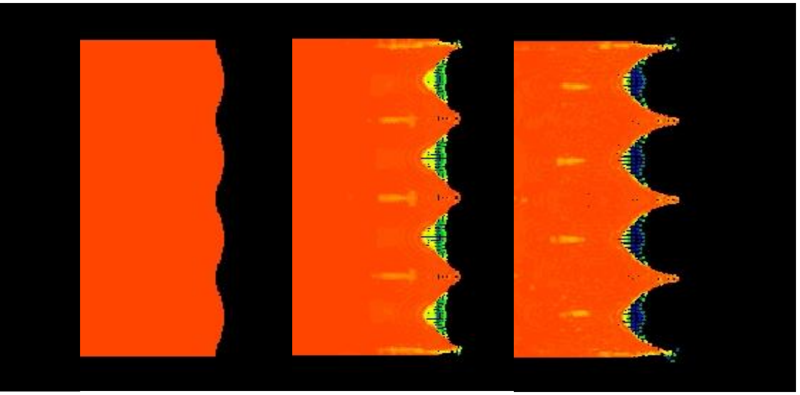

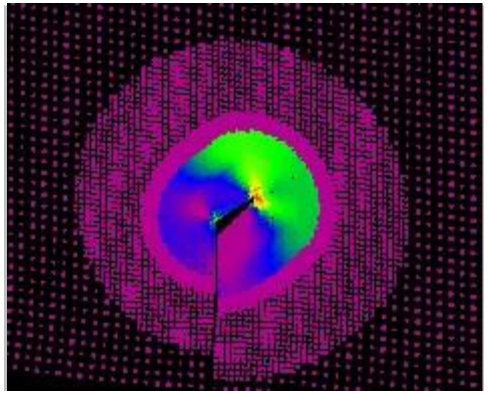

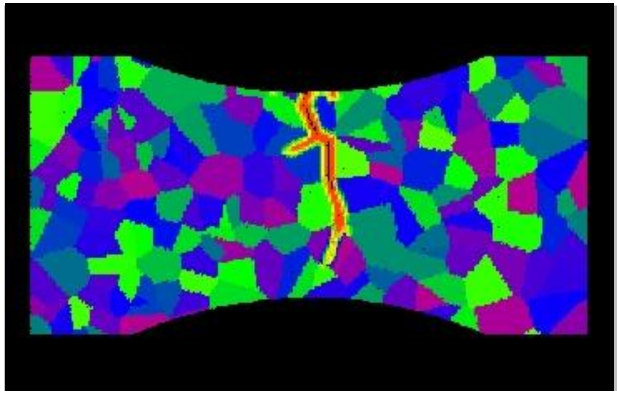

Origin and effect of nonlocality in a composite

Stewart Silling

Sandia National Laboratories

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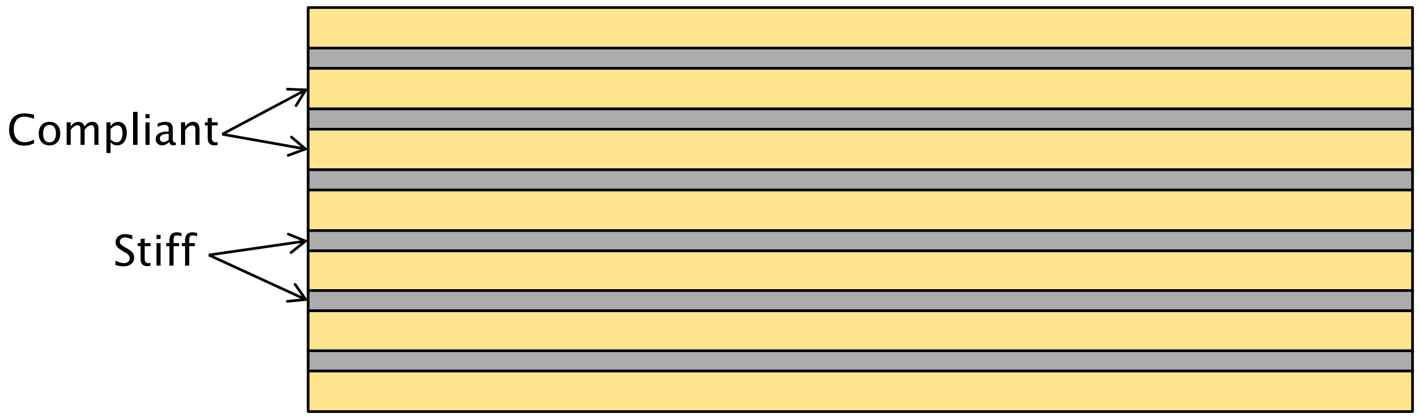
U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

Objectives

- Demonstrate how nonlocality arises in a simple material system.
- Derive a peridynamic model from a micromechanical model.
- Show that the local theory is an approximation that is exact for uniform strain.
- Show that nonlocality makes a difference in a macroscopic experiment.

Layered composite



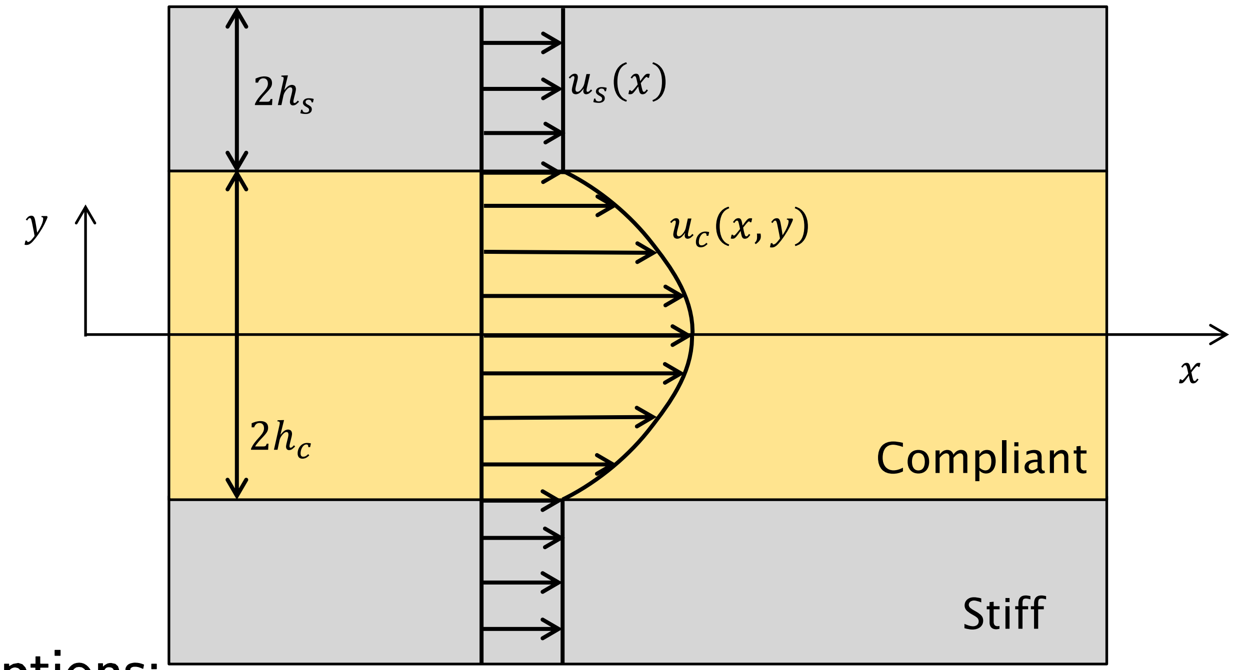
Two phases:

- Compliant: elastic moduli μ_c, E_c.
- Stiff: elastic moduli μ_s, E_s.
- $\mu_s \gg \mu_c, \qquad E_s \gg E_c$
- Interfaces are rigidly connected.
- Layer thicknesses are $2h_s$, $2h_c$.

Traditional approach

- Assume $u_s(x) = u_c(x)$ everywhere.
- Set
$$\bar{E} = \frac{h_s E_s + h_c E_c}{h_s + h + c}.$$
$$\bar{\sigma}'(x) + b(x) = 0, \qquad \bar{\sigma} = \bar{E}u'$$
- This local "homogenized" model neglects possibility of unequal displacements between phases.
- This will turn out to be a special case of the nonlocal model to be derived.

Micromechanical model



Assumptions:

- Set $h_s = h_c = 1$ for simplicity.
- u_s is independent of y.
- $u_c(x, y) = u_s(x) + (1 - y^2)w(x)$.

Equilibrium between phases

- Shear traction on a stiff layer:

$$\tau = \mu_c \frac{\partial u_c}{\partial y}(x, -1) = 2\mu_c w(x).$$

- Resulting force balance on a stiff layer:

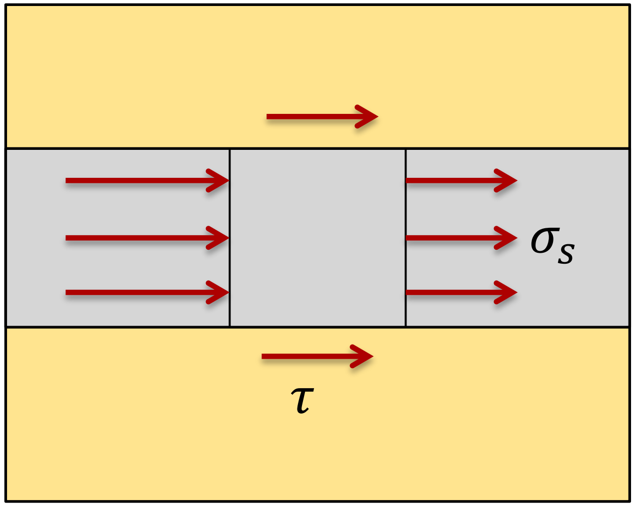
$$h_s \sigma_s' + \tau = 0$$

where σ_s is the normal stress in a stiff layer.

- Leads to

$$E_s u_s''(x) + 2\mu_c w(x) = 0.$$

- Now have two coupled fields: $u_c(x)$ and $w(x)$.



Smoothed displacement field

- Define

$$\bar{u}(x) = \frac{u_s(x) + u_c(x)}{2} = u_s(x) + \frac{w(x)}{3}.$$

- Recall the force balance

$$E_s u_s'' + 2\mu_c w(x) = 0.$$

- Combine the last two equations to get rid of u_s:

$$\bar{u}'' = \frac{1}{3} w''(x) - \frac{2\mu_c}{E_s} w(x).$$

Solve ODE for w

- For prescribed $\bar{u}$, now have a 2nd order linear ODE for w:

$$\bar{u}'' = \frac{1}{3} w''(x) - \frac{2\mu_c}{E_s} w(x).$$

- Solution is

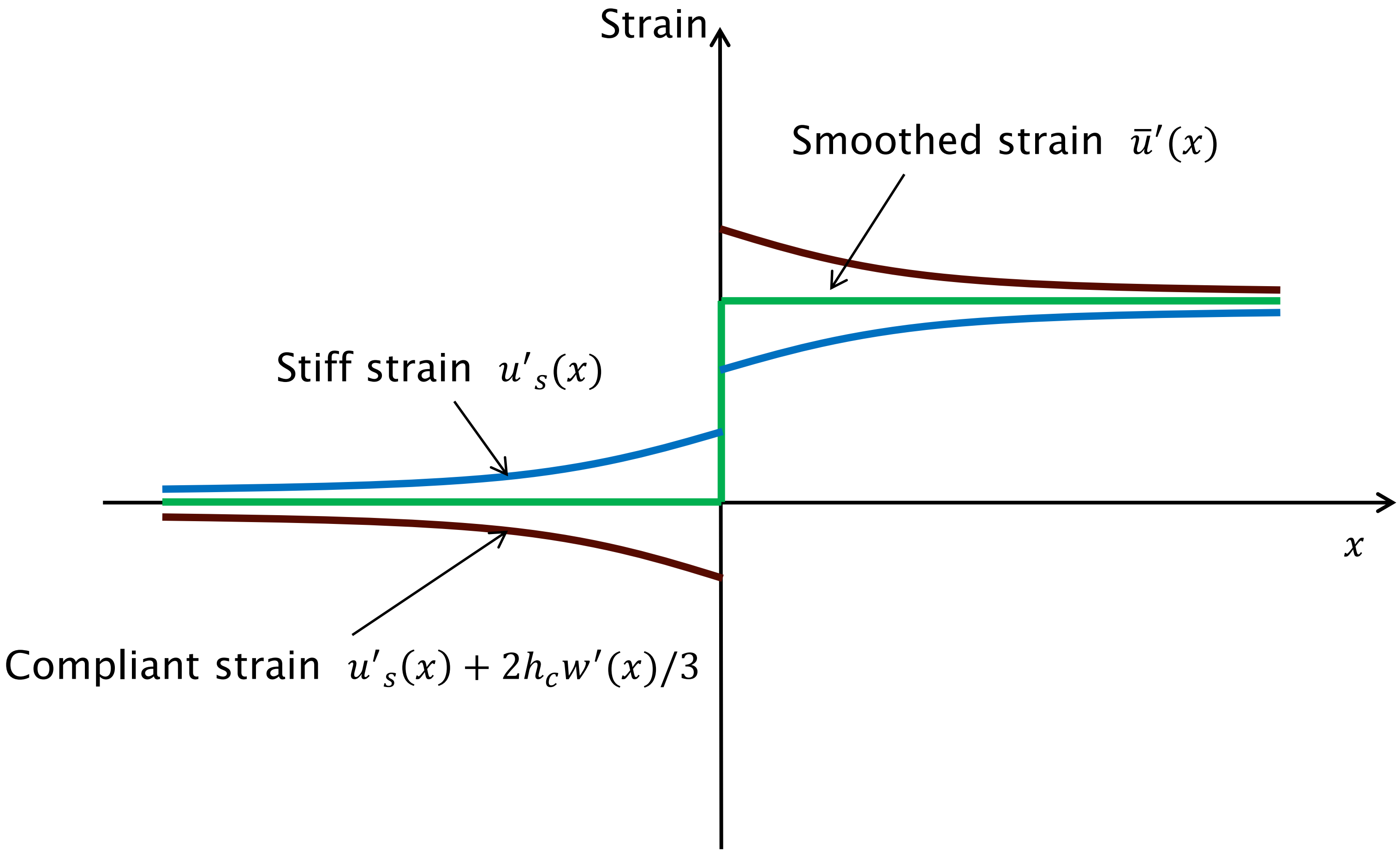
$$w(x) = -\int_{-\infty}^{\infty} \bar{u}''(p) G(x-p)\, dp$$

Where

$$G(x) = k e^{-\lambda |x|}, \qquad \lambda = \sqrt{\frac{6\mu_c}{E_s}}, \qquad k = \frac{1}{2\lambda}.$$

Jump in smoothed strain

Set $\bar{u}''(x) = \Delta(x)$ where Δ is the Dirac delta function.



Find smoothed stress

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- Define

$$\bar{\sigma}(x) = \frac{1}{2}\left(\int_0^1 \sigma_s \, dy + \int_0^1 \sigma_c(x,y) \, dy\right)$$

where

$$\sigma_s = E_s \frac{\partial u_s}{\partial x}, \qquad \sigma_c = E_c \frac{\partial u_c}{\partial x}.$$

- After using previous results, find

$$\bar{\sigma}(x) = \bar{E}\bar{u}'(x) + \gamma \int_{-\infty}^{\infty} \bar{u}''(p) G'(x-p) \, dp,$$

Local term

Nonlocal term

$$\bar{E} = \frac{E_s + E_c}{2}, \qquad \gamma = \frac{E_s - E_c}{6}.$$

Equation of motion in terms of smoothed displacement

- Get rid of $\bar{\sigma}$, using

$$\rho\ddot{\bar{u}}(x,t) = \bar{\sigma}'(x,t) + b(x,t).$$

- After some manipulations, find

$$\rho\ddot{\bar{u}}(x,t) = E_c\bar{u}''(x,t) + \gamma k\lambda^4 \int_{-\infty}^{\infty} (\bar{u}(p,t) - \bar{u}(x,t))e^{-\lambda|x-p|}\, dp + b(x,t),$$

- This contains both local and nonlocal terms, similar to Kroner's nonlocal model (also DiPaola's).

Can we avoid the local term?

- Observe that for any function $v(x)$,

$$v''(x) = \int_{-\infty}^{\infty} v(p)\Delta''(x-p)\, dp.$$

- Approximate

$$\Delta(x) \approx \frac{\tau e^{-\tau|x|}}{2}$$

  where τ is a large parameter.

- Replace the local term in the equation of motion to arrive at

$$\rho\ddot{\bar{u}}(x,t) = \int_{-\infty}^{\infty} \left(\frac{E_c \tau}{2} e^{-\tau|x-p|} + \gamma k \lambda^4 e^{-\lambda|x-p|} \right) (\bar{u}(p,t) - \bar{u}(x,t))\, dp + b(x,t),$$

- This is the equation of motion in linearized peridynamics.

Nonlocal length scale

- Recall

$$\rho\ddot{\bar{u}}(x,t) = \int_{-\infty}^{\infty} \left(\frac{E_c\tau}{2} e^{-\tau|x-p|} + \gamma k \lambda^4 e^{-\lambda|x-p|} \right) (\bar{u}(p,t) - \bar{u}(x,t))\, dp + b(x,t),$$

Nonlocal term

$$\lambda = \sqrt{\frac{6\mu_c}{E_s}} = \frac{1}{\text{length scale}}$$

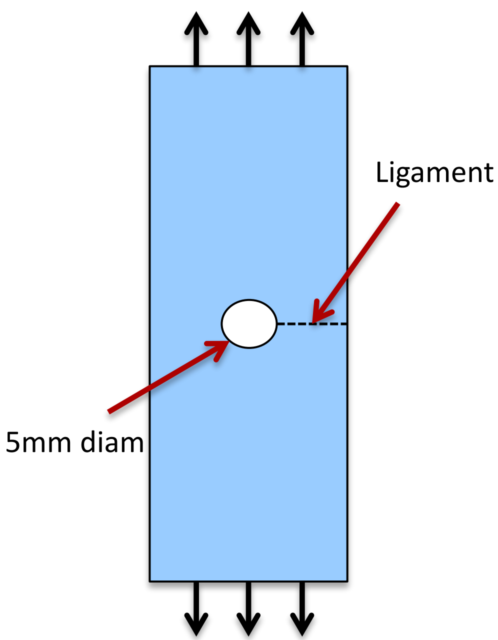
More generally

$$\lambda = \sqrt{\frac{3\mu_c(h_s+h_c)}{E_s h_s h_c^2}} = \frac{1}{\text{length scale}}$$

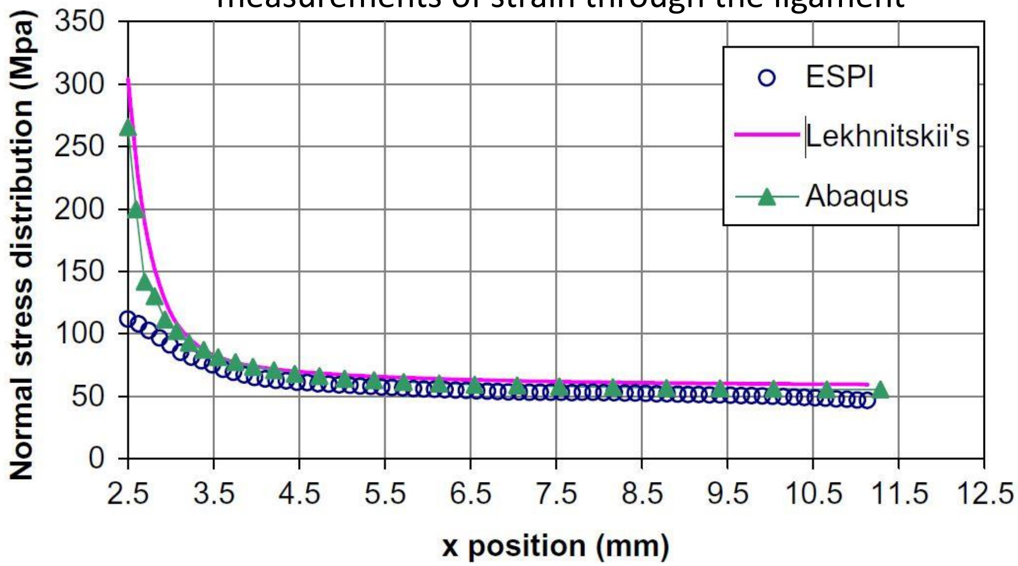
- The softer the compliant material is, the greater the length scale.
- The nonlocal length scale is not determined only by the geometrical scales.

Are composites nonlocal?

- Open hole tension in a fabric laminate:
  - FEM agrees with anisotropic local theory in predicting the stress concentration near the hole.
  - Both overpredict the stress concentration.



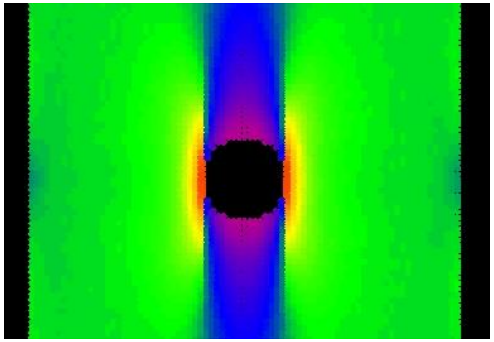
Electronic speckle interferometry (ESPI) measurements of strain through the ligament



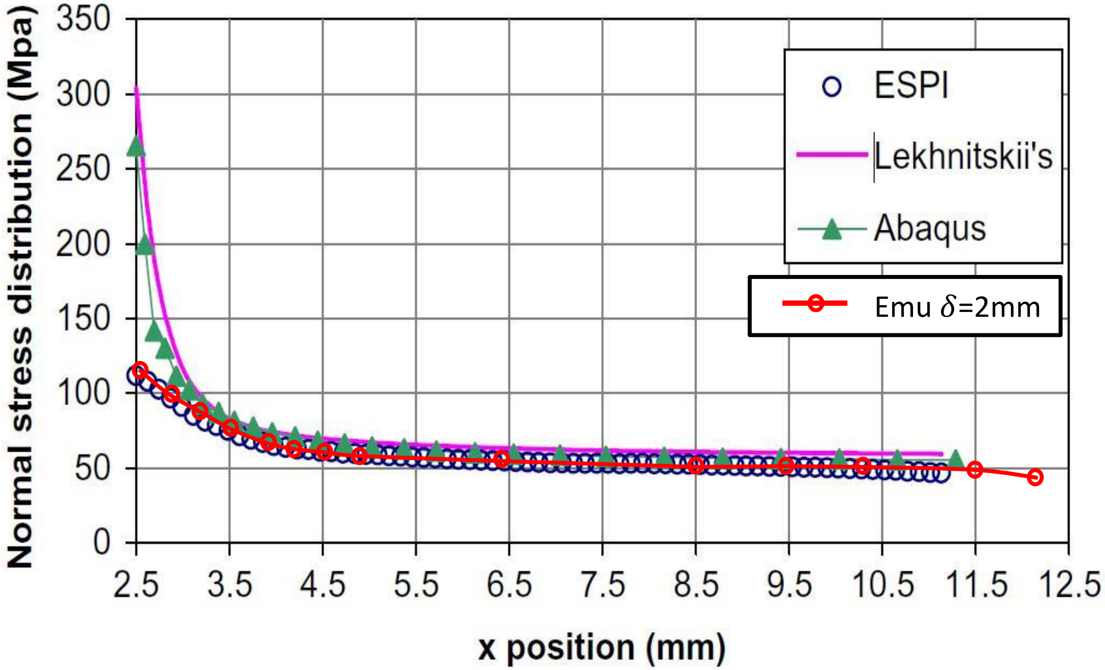
Source: Toubal, Karama, and Lorrain, Composite Structures 68 (2005) 31-36

Are composites nonlocal?

- Peridynamic model is more accurate than the local model for predicting stress concentration in a laminate.
- $h_s = h_c$ =0.4mm, $E_s = 150$GPa, μ_c =4GPa.
- $\Rightarrow 1/\lambda = 1.41$mm.



EMU: contours of longitudinal stress
Horizon = 2mm



Data of Toubal, Karama, and Lorrain, Composite Structures 68 (2005) 31-36

Discussion

- The modeling decision to use a smoothed displacement field introduced nonlocality.
  - This is even though the micromechanical model used only local interactions.
- The nonlocal term vanishes if the strain is uniform.
  - It becomes important if strong gradients are present.
- The length scale for nonlocality involves the material properties as well as the geometrical length scales.