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Finite-Difference Algorithm for Simulating 3D Electromagnetic Wavefields in Conductive Media

David F. Aldridge, Lewis C. Bartel, Hunter A. Knox, Darin Q. Pike, and Kimberly A. Schramm

Geophysics and Atmospheric Sciences Department, Sandia National Laboratories, Albuquerque, New Mexico

Introduction and Summary

Electromagnetic (EM) wavefields are routinely used in geophysical exploration for detection and characterization of subsurface geological formations of economic interest. Recorded EM signals depend strongly on the current conductivity of geologic media. Hence, they are particularly useful for inferring fluid content of saturated porous bodies.

We have developed a numerical algorithm for simulating three-dimensional (3D) EM wave propagation and diffusion in heterogeneous conductive materials. Maxwell's equations are combined with isotropic constitutive relations to obtain a set of six, coupled, first order, linear partial differential equations governing the electric and magnetic vectors. A particular advantage of the system is that it does not contain spatial derivatives of the three medium parameters electric permittivity, magnetic permeability, and current conductivity. Numerical solution methodology consists of explicit, time-domain finite-differencing (FD) on a 3D staggered rectangular grid. Temporal and spatial FD operators have order 2 and N (= 2, 4, 6, 8, 10). An artificially-large electric permittivity is used to maximize the FD timestep, and thus reduce execution time. For the low-frequencies typically used in geophysical exploration, accuracy is not unduly compromised. Grid boundary reflections are mitigated via convolutional perfectly matched layers (C-PMLs) imposed at the six grid flanks. A shared-memory-parallel code implementation via OpenMP directives enables rapid algorithm execution on a multi-thread computational platform.

Good agreement is obtained in comparisons of numerically-generated data with reference solutions (e.g., point current density or magnetic dipole sources in a homogeneous conductive wholespace). We are particularly interested in accurate representation of high-conductivity sub-grid-scale features common in an industrial environment (borehole casing, pipes, railroad tracks). Present efforts are oriented toward calculating EM responses of these objects via a First Born Approximation (FBA) approach.

Electromagnetic Wave Equations and 3D Finite-Difference Solution

Combine two of *Maxwell's equations* (i.e., the Ampere-Maxwell law and the Faraday law) with three *constitutive relations* (appropriate for linear, time-invariant, and isotropic media) to obtain the ***EH Partial Differential System***:

$$\varepsilon(\mathbf{x})\frac{\partial \mathbf{e}(\mathbf{x},t)}{\partial t} + \sigma(\mathbf{x})\mathbf{e}(\mathbf{x},t) - \mathbf{curl}\,\mathbf{h}(\mathbf{x},t) = -\mathbf{j}_s(\mathbf{x},t) - \frac{\partial \mathbf{d}_s(\mathbf{x},t)}{\partial t},$$

$$\mu(\mathbf{x})\frac{\partial \mathbf{h}(\mathbf{x},t)}{\partial t} + \mathbf{curl}\,\mathbf{e}(\mathbf{x},t) = -\frac{\partial \mathbf{b}_s(\mathbf{x},t)}{\partial t}.$$

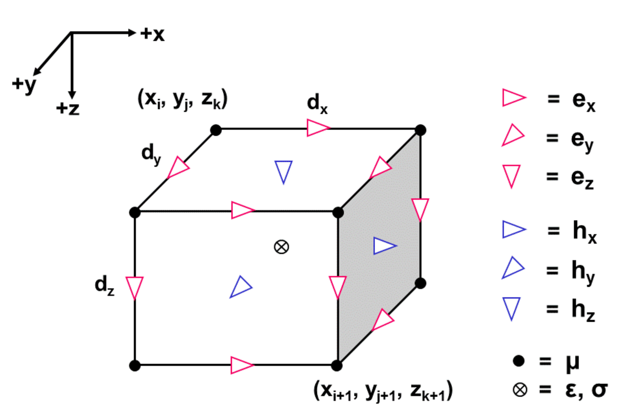
A set of six coupled, linear, first-order, inhomogeneous partial differential equations.

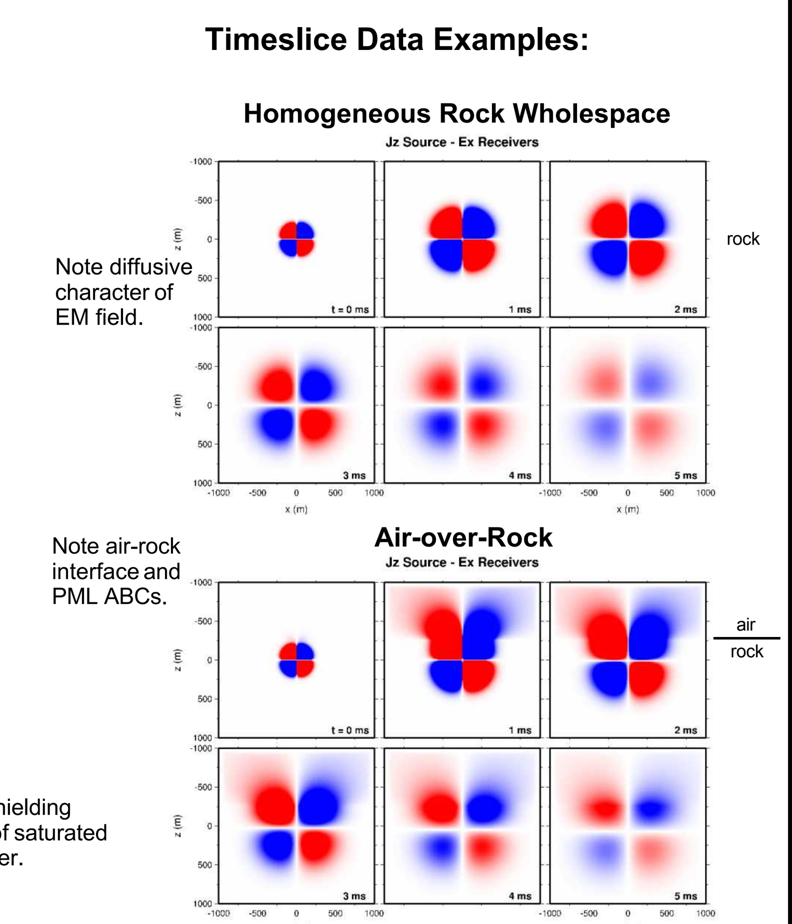
EM wavefield variables: $\mathbf{e}(\mathbf{x},t)$ – electric vector, $\mathbf{h}(\mathbf{x},t)$ – magnetic vector.

EM medium parameters: $\varepsilon(\mathbf{x})$ – electric permittivity, $\sigma(\mathbf{x})$ – current conductivity, $\mu(\mathbf{x})$ – magnetic permeability.

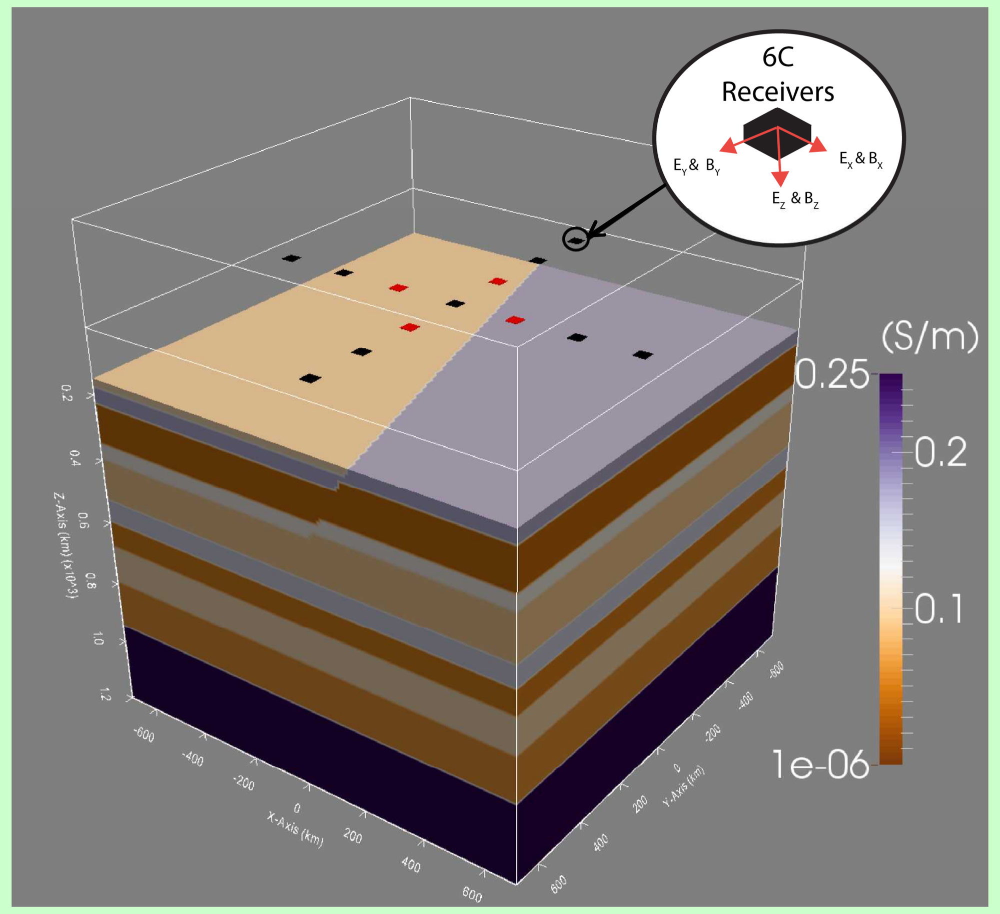
EM sources: $\mathbf{j}_s(\mathbf{x},t)$ – current density, $\mathbf{b}_s(\mathbf{x},t)$ – magnetic induction, $\mathbf{d}_s(\mathbf{x},t)$ – electric displacement.

Numerical solution methodology: explicit, time-domain finite-differencing on a 3D staggered spatial grid (after Yee, 1966):



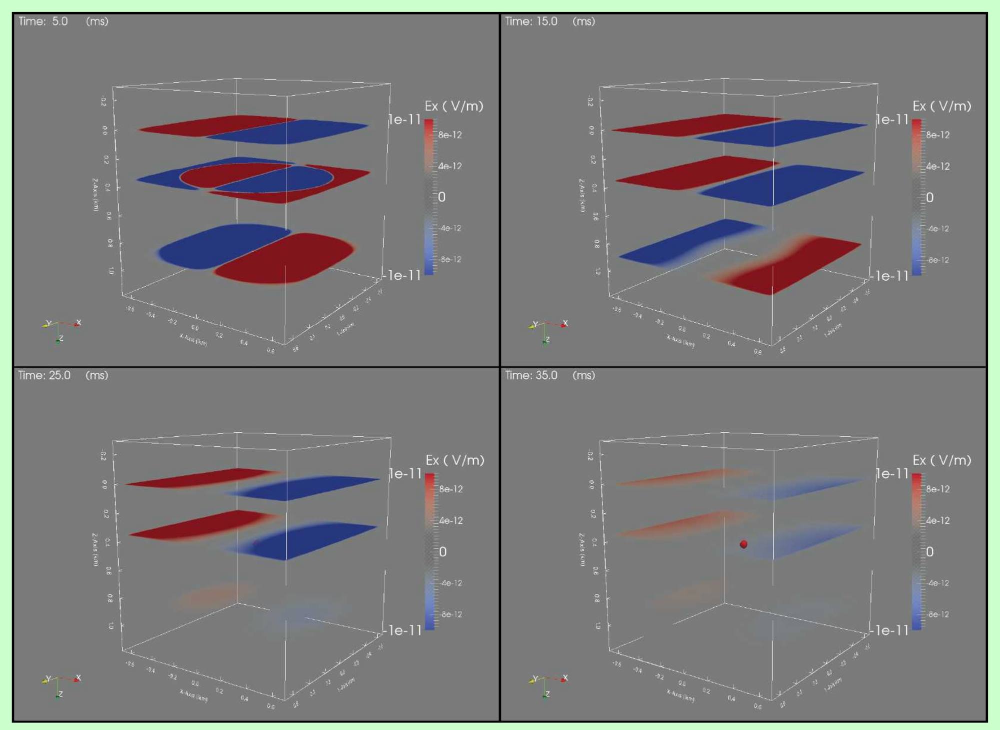


3D Conductivity Model and Surface Acquisition Geometry



Cubit model construction

Ex Electric Field: Layer Model; 3D Volume Visualizations

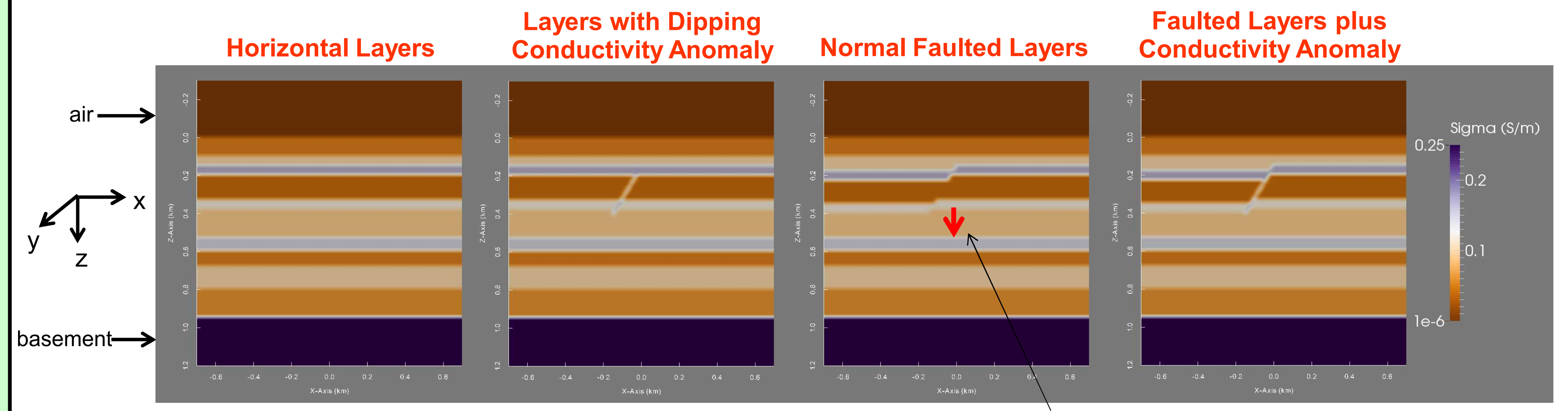


ParaView visualizations

Modeling Conclusions

1) Multi-component magnetic and electric trace data display amplitude, waveform, and polarity variations depending on receiver station location relative to sub-surface geologic features.
   - amplitudes (per unit source current) are small, but considered detectable.
2) Different geologic expressions of subsurface faulted zone (conductivity anomaly, normal faulted layers, and both) imply different characteristics in surface-recorded EM data. Interpretation is ambiguous!
3) 3D volume visualizations of time-evolving subsurface EM wavefields (both physical fields and wavefield differences) provide clues for proper data interpretation.
4) Air-earth interface (where conductivity changes by several orders of magnitude) handled accurately by FD algorithm.
5) More realistic EM wavefield sourcing (i.e., current-carrying wire loops) now needed.
6) Accurate modeling of sub-grid-scale features always a challenge!

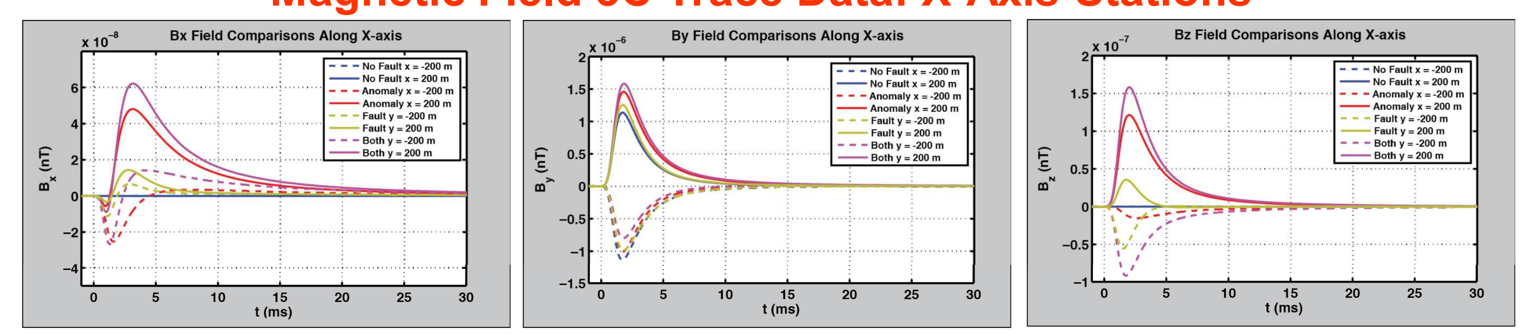
Four Model Variants (vertical xz sections at y = 0 km)



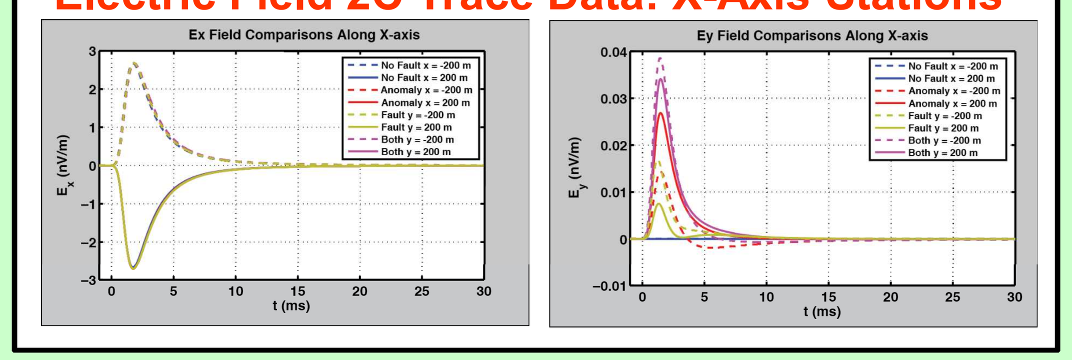
Δσ = +0.174 S/m (= 2x background ave) vertical point current density source, 450 m deep, 1 A magnitude

Z-component electric field (Ez) *not* routinely recorded by surface receivers.

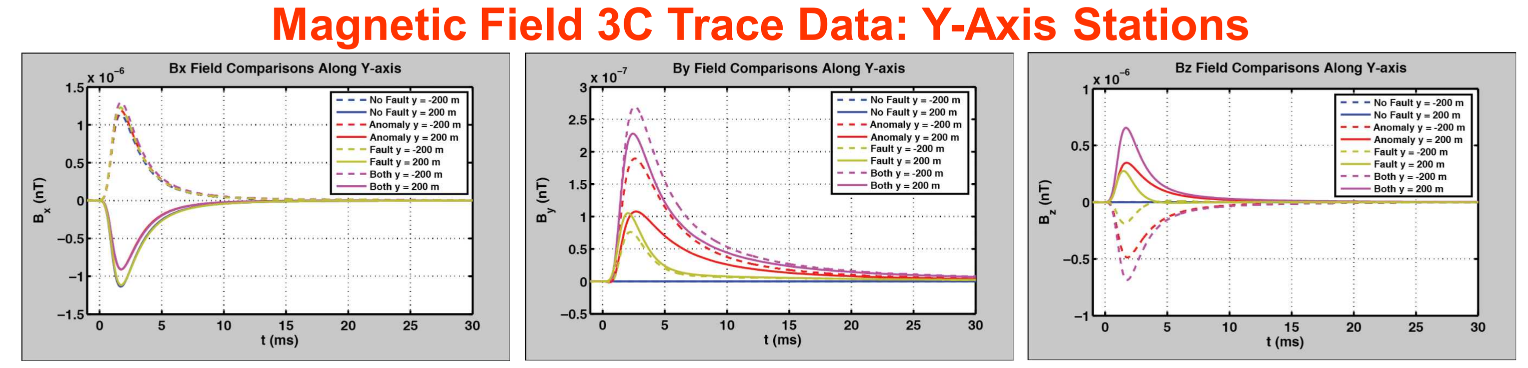
Magnetic Field 3C Trace Data: X-Axis Stations



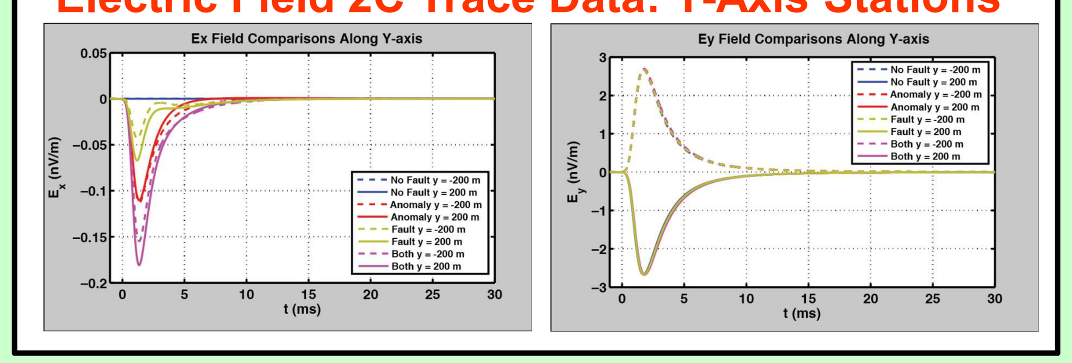
Electric Field 2C Trace Data: X-Axis Stations



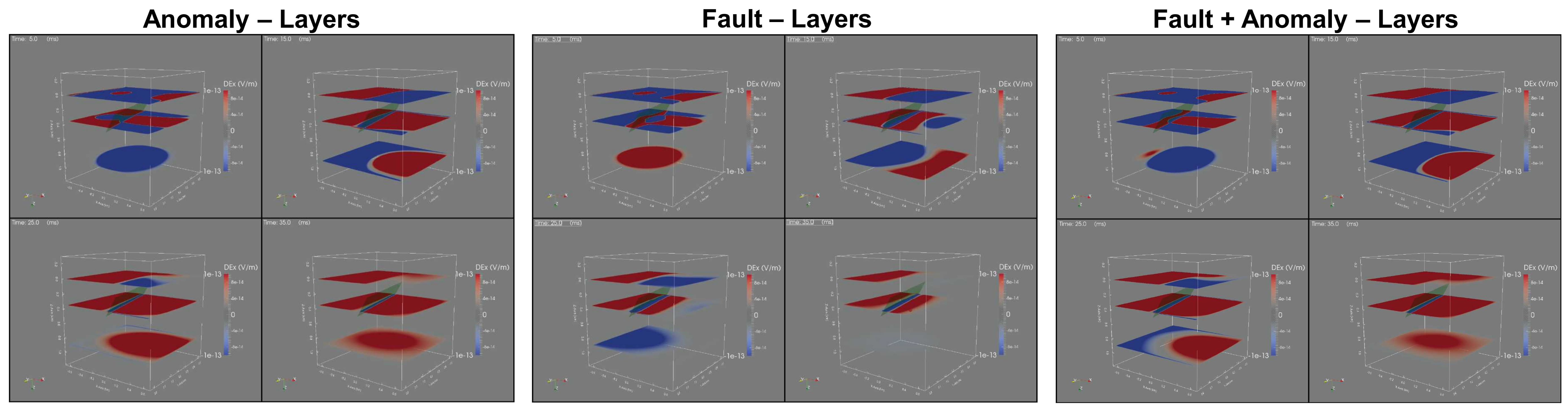
Magnetic Field 3C Trace Data: Y-Axis Stations



Electric Field 2C Trace Data: Y-Axis Stations



3D Volume Visualizations of *Difference* Electric Wavefields (Ex-component)



Acknowledgements

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