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A Parallel Helmholtz Solver for Acoustics and Structural Acoustics

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Outline

- **Background**
  - Problem statement
  - Some challenges
  - Solver basics
- **Approach & Examples**
  - Structural damping
  - Solution reuse, inexact solvers
  - Fixed & variable frequency problems
- **Higher Order Elements**
  - Preconditioning strategy
  - Numerical results
- **Closing Remarks**

Background

- **Equilibrium equations:** $M\ddot{u} + C\dot{u} + Ku = f$

Special Cases

- **Statics:** $Ku = f$
- **Implicit dynamics:** $(a_1 M + a_2 C + K)u = \hat{f}, \quad a_1 > 0, a_2 > 0$
- **Modal analysis:** $(K - \sigma M)u = \lambda M u, \;\; \sigma \leq 0$
- **Helmholtz: acoustic, structural or structural-acoustic frequency domain analysis,** $u = e^{i\omega t}x, \;\; f = e^{i\omega t}b \;\; \Rightarrow$

$$(K + i\omega C - \omega^2 M)x = b$$

Q: What’s so different about Helmholtz problems?

Background

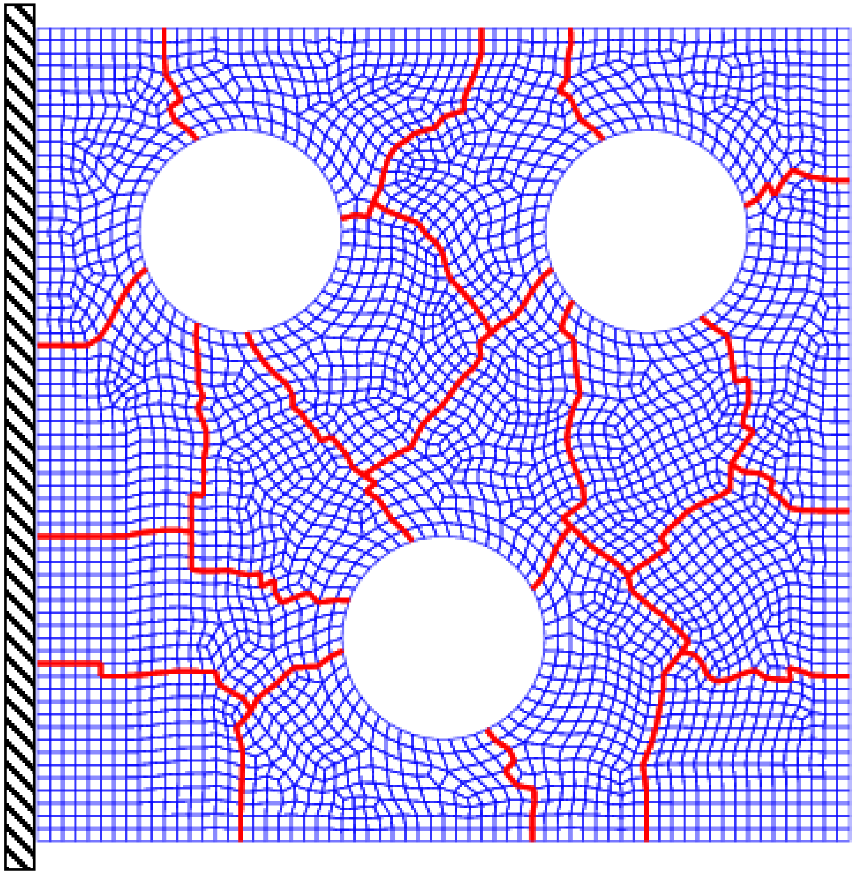
- **Helmholtz problem matrix:** $A = K + i\omega C - \omega^2 M$
  - A may be positive definite, indefinite, or singular
  - Solvers and theory for Helmholtz problems much less mature
  - Naïve application of non-Helmholtz solvers can be problematic
- **Solution options:**
  - Sparse direct solvers:
    - Impractical for larger 3D problems - O(n^2) operations, O($n^{4/3}$) memory
  - Multigrid:
    - Shifted Laplacian, …
  - Domain Decomposition:
    - Iterative substructuring (FETI-DPH, BDDC)
    - Optimized Schwarz
    - **Overlapping Schwarz**

Background

- **Solver Basics:** $Ax = b$
  - Preconditioner:
    - $M^{-1}Ax = M^{-1}b$ (left preconditioning)
    - $x = M^{-1}y \Rightarrow AM^{-1}y = b$ (right preconditioning)
    - Goals: preconditioned system easier to solve, but not too costly
  - Krylov Method:
    - Conjugate gradients, GMRES, BiCGSTAB, …
    - Used to solve preconditioned system
- **Domain Decomposition Basics:**
  - Partition domain Ω into smaller subdomains $\Omega_1, \cdots, \Omega_N$
  - Construct and solve global (coarse) problem(s)
  - Construct and solve local problems
  - Preconditioner combines local and global solutions

Background

- $M^{-1}r = \underbrace{\Phi_c(\Phi_c^H A \Phi_c)^{-1}\Phi_c^H r}_{\textbf{global}} + \underbrace{\sum_{i=1}^{N} R_i^T(R_i A R_i^T)^{-1}R_i r}_{\textbf{local}}$



- elements partitioned into subdomains
- one or more subdomains assigned to each processor
- multilevel extensions straightforward
- local or global problems may be solved approximately
- multiplicative (Gauss-Seidel) variants
- hybrid overlapping Schwarz/iterative substructuring can be very effective*

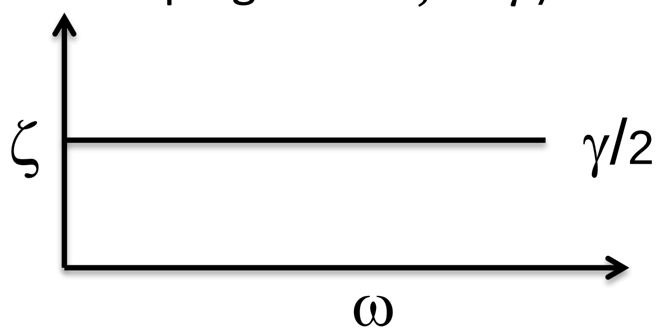
* Int. J. Numer. Meth. Engng (2010) 82, 157-183

Background

- **Preconditioner Challenges:** $A = K + i\omega C - \omega^2 M$
  - Local or global problems may be singular or nearly singular (resonance)
  - Potentially very slow convergence of iterative methods
- **Artificial Damping:**
  - Several options exist: absorbing BCs, shifted Laplacian, PML layers, …
  - Our approach: introduce structural damping*
  - Construct preconditioner for damped linear system

$$\big((1 + i\gamma)K + i\omega C - \omega^2 M\big)x = b$$

  - Damping factor $\zeta = \gamma/2$



Connection to shifted Laplacian for $C = 0$:

$$\left(K - \omega^2 \left(\frac{1}{1 + i\gamma}\right) M \right) x = b/(1 + i\gamma)$$

* Roy R. Craig, Jr., Structural Dynamics, Wiley (1981) 101-103

Fixed Frequency Problems

- **Problem:** solve $Ax_k = b_k$ for $k = 1, \cdots, \mathrm{M}$
- **Example:** source identification at fixed frequency
- **Approach:** reuse Krylov subspaces to accelerate convergence*
- **Starting Point:** subspaces stored in Φ

$$min_q \|A\Phi q - r\|_2,\ A\Phi = QR \ \Rightarrow Rq = Q^H r$$

- **Preconditioner:** $z_1 = M^{-1}r,\ r_1 = r - Az_1,\ q = R^{-1}Q^H r_1,$

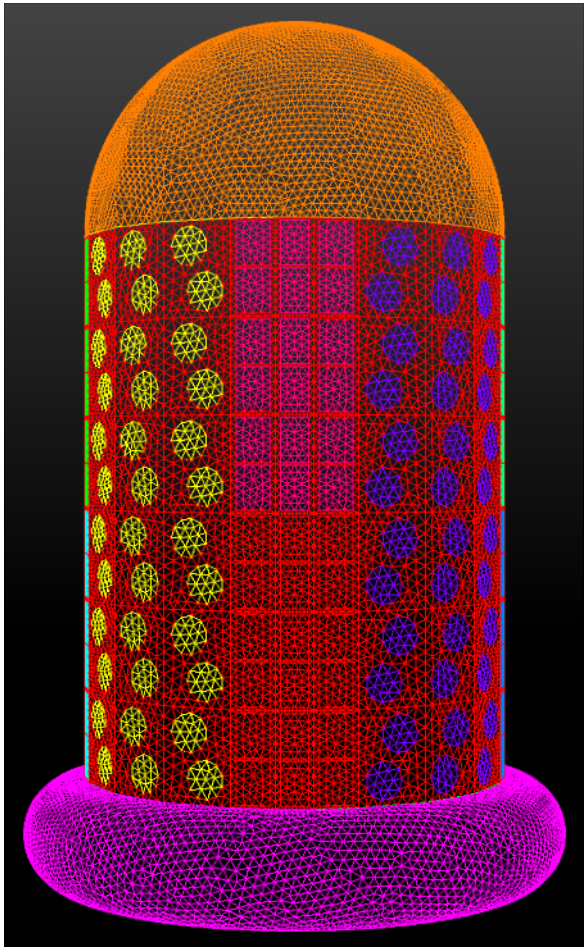
$$z_2 = \Phi q, z = z_1 + z_2 = \widehat{M}^{-1}r$$

- **Note:** Initial correction made so $Q^H b_k = 0$. Further, we have after each iteration $(A\Phi)^H r = 0$

* Comput. Methods Appl. Mech. Engrg. (1994) 117, 195-209

Fixed Frequency Problems

- **Example:** acoustic source identification (ka ≈ 15)



	Basic	Reuse vectors
Total iterations	2399	147
Total solve time	327 sec	49 sec
Time/iteration	0.13 sec	0.27 sec
Analysis time	5:35 min	1:01 min

Note: Related direct field acoustic test (DFAT) model with multiple element types (tets, hexes, shells, beams), non-conforming structural-acoustic interface, and 32K+ constraint equations solved on 80 processors.

Variable Frequency Problems

- **Problem:** solve $A(\omega_k)x_k = b$ for $k = 1, \cdots, \mathrm{M}$
- **Example:** frequency sweep
- **Approach:** reuse previous solutions for initial guess*
- **Starting Point:**

$$min_q \|A\Psi q - r\|_2, \; A\Psi = QR \; \Rightarrow Rq = Q^H r \Rightarrow x_{init} = \Psi R^{-1} Q^H r$$

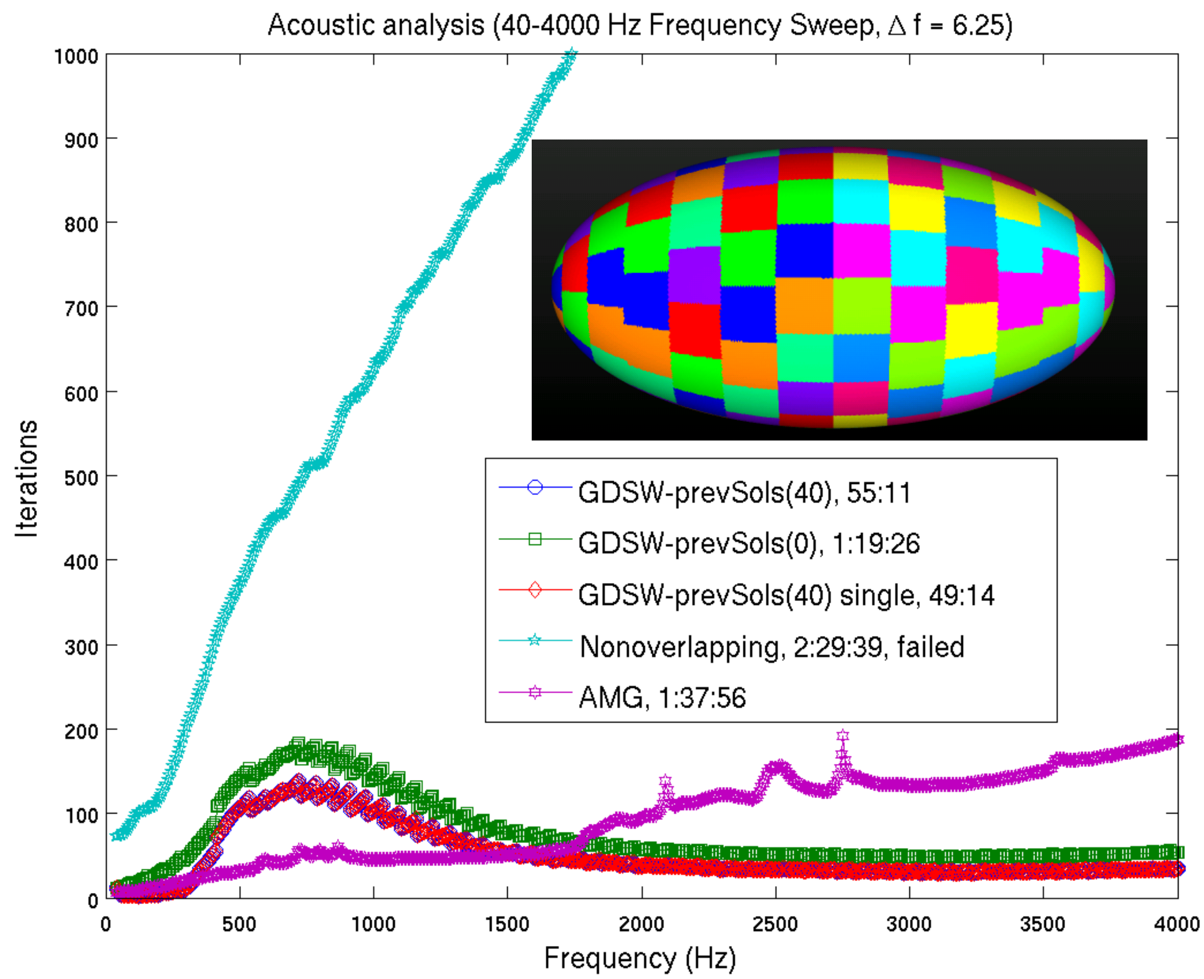
- **Preconditioner:** Standard M^{-1}, but can be combined with reuse of Krylov subspaces

- **Note:** Initial correction made so $Q^H b_k = 0$. Further, we have after each iteration $(A\Psi)^H r = 0$

* Comput. Methods Appl. Mech. Engrg. (1998) 163, 193-204

Variable Frequency Problems

Example: acoustic source identification for aerospace testing



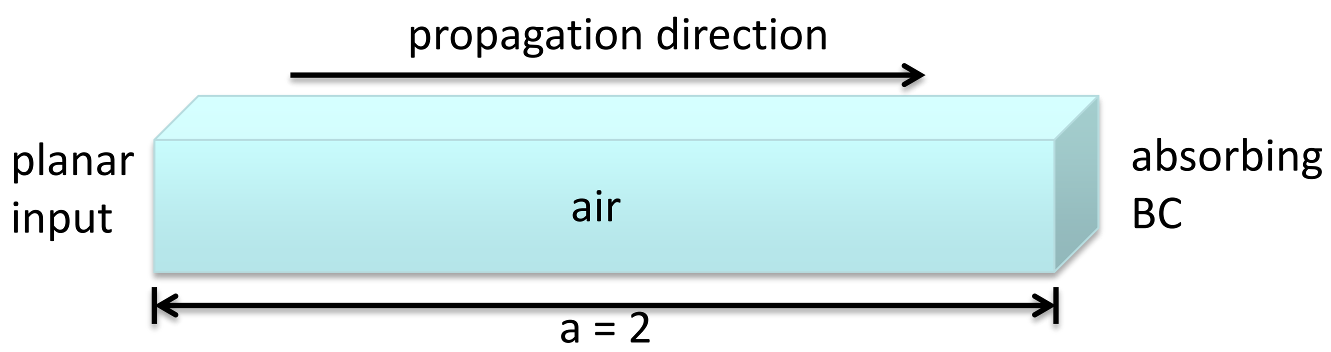
Higher order elements

- Element formulation:
  - H^1-conforming hierarchical p-FEM shape functions*
    - Integrated Legendre polynomials
  - Internal element variables statically condensed
    - vertex, edge and face unknowns remain
- Implementation:
  - based on hp3d code from UT Austin (Demkowicz et al.)
  - Other options possible, but very convenient
  - Hex8 or Tet4 mesh ⇒ internal edge-face-volume data structures ⇒ dial in polynomial degree on the fly
  - parallel assembly and solution
  - research code for now

* Finite Elements in Analysis and Design (2010) 474-486

Higher order elements

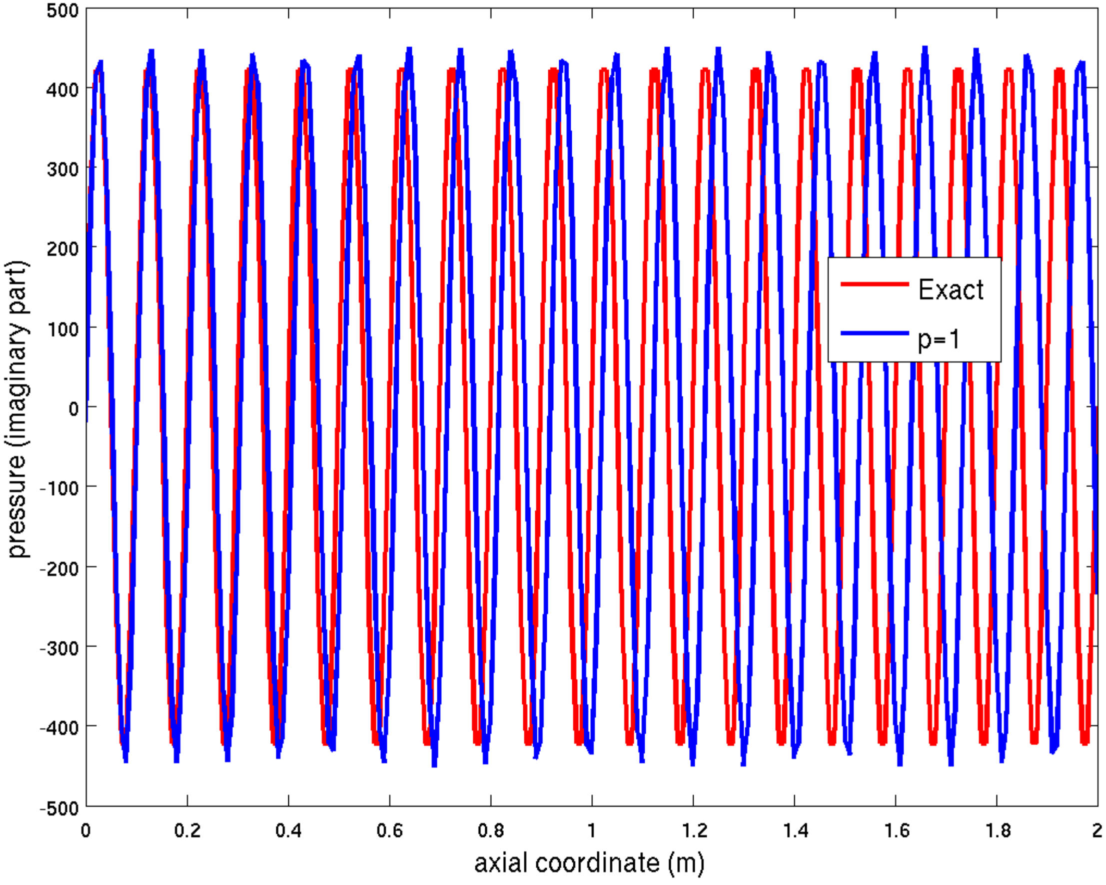
- Waveguide example:
  - sound speed = 343, f = 3430 Hz, λ= 0.1, ka = 2π(20)



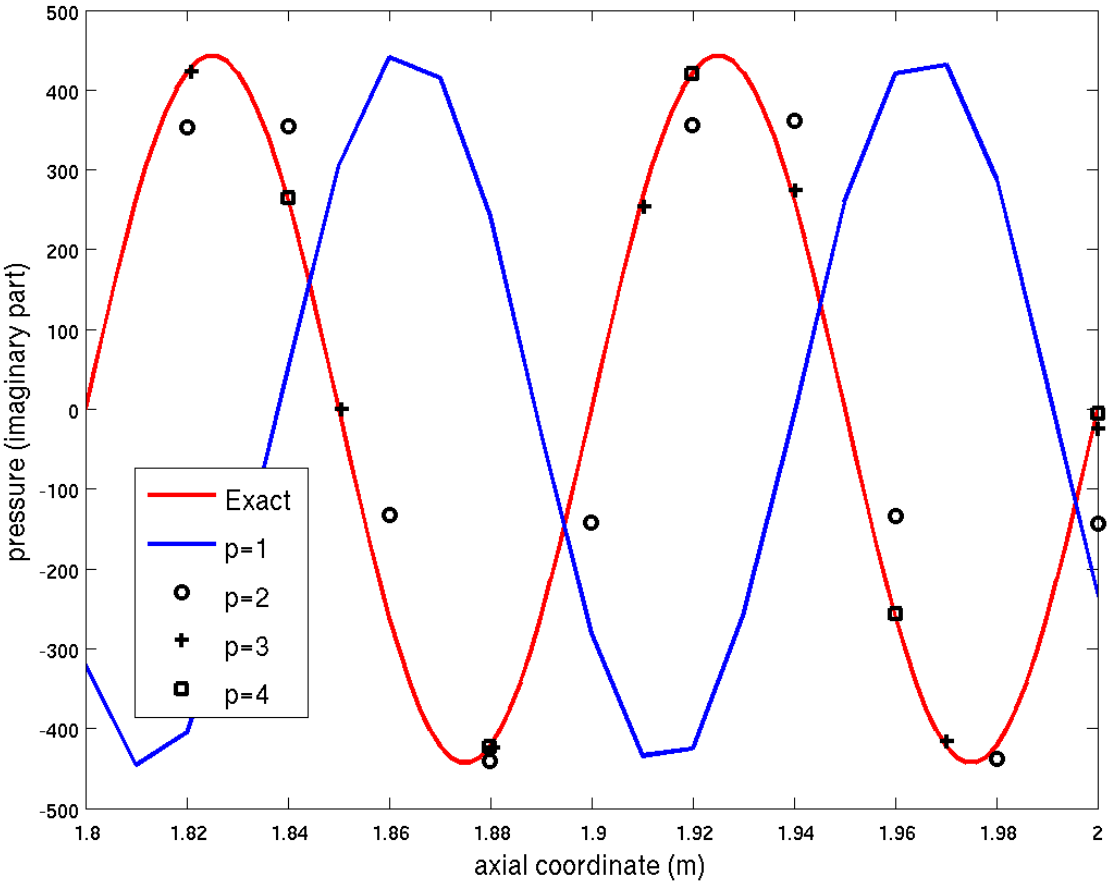
- FE meshes:
  - single element along transverse directions
  - propagation direction: λ/h = 10 for p = 1, λ/h = 5 for p = 2, …
  - mimics same total number of dofs in 3D meshes for different p
  - trapezoidal elements to model non-mesh-aligned wave propagation

Higher order elements

- Linear elements: significant dispersion (phase) & amplitude errors



Higher order elements

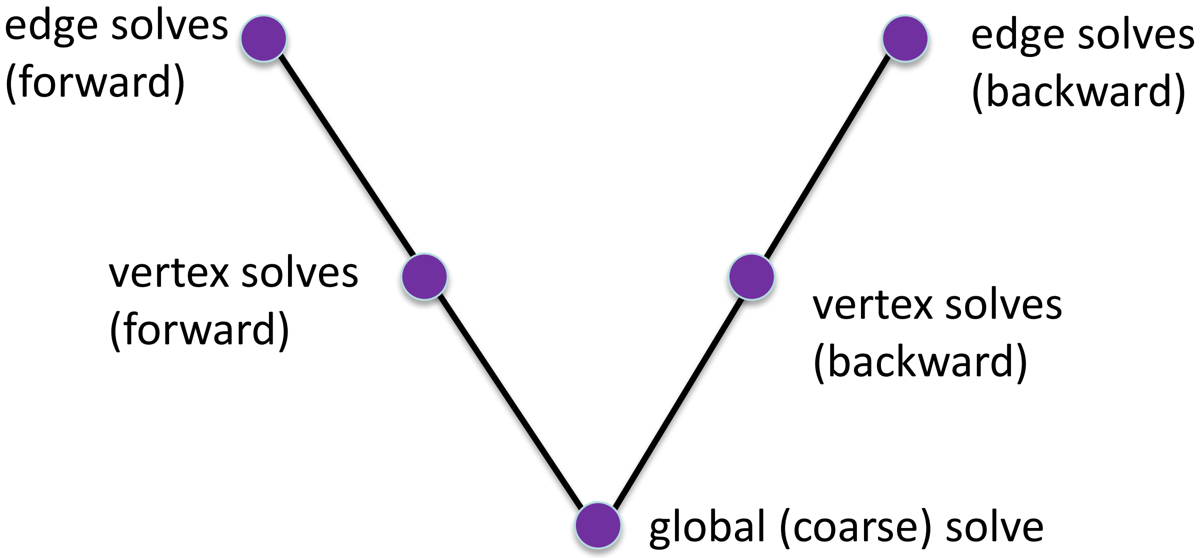


same dof density in propagation direction for all p $\Rightarrow$ fair comparison for different p.

$$\frac{|\phi - \phi_{hp}|}{|\phi_{hp}|} \leq A(p) \left[\left(\frac{hk}{2p} \right)^{p} + Ck \left(\frac{hk}{2p} \right)^{2p} \right]$$

Higher order elements

- Preconditioning Strategy
  - goal: reduce memory and computations
  - local solves associated with edges and vertices
  - global solve for p = 1 sub-block (readily available)
  - Closely related strategy for Poisson by Schoberl et al.*
  - Symmetric Gauss-Seidel implementation



* IMA Journal of Numerical Analysis (2008) 28, 1-24

Higher order elements

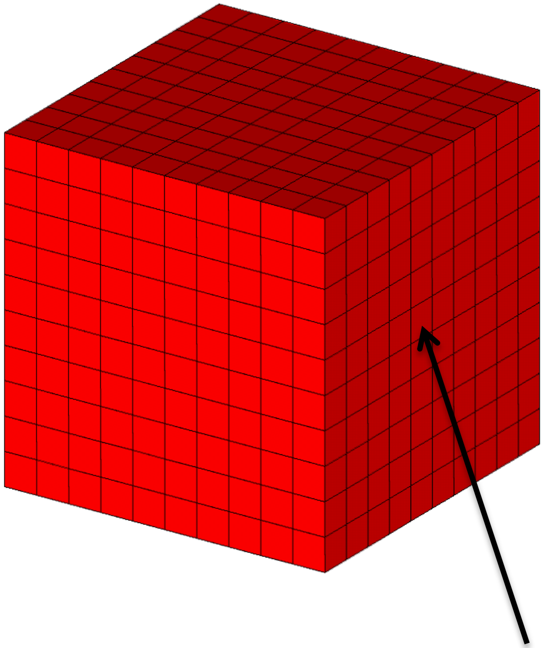
- Laplacian, 3x3x3 hex cube model results:

p	iterations	cond #	memory ratio
2	6	1.09	0.22
3	5	1.06	0.42
4	6	1.09	0.54
5	6	1.10	0.62
6	7	1.13	0.67

Notes: solver tolerance = 10^{-8}, memory ratio = ratio of local factorization memory to matrix storage memory

Higher order elements

Direct Solver, f = 343 Hz, $\lambda/h = 10$				
p	**iterations**	**init time**	**solve time**	**memory ratio**
2	1	16.3	0.04	3.19
4	1	55.7	0.52	3.21
6	1	528	1.5	3.21
Iterative Solver, f = 343 Hz				
2	7	1.4	0.32	0.66
4	7	8.5	5.9	1.19
6	7	79.9	34.3	1.55
Iterative Solver, f = 686, $\lambda/h = 5$				
2	21	1.4	1.7	0.66
4	20	10.4	17.0	1.19
6	19	81.9	92.7	1.55



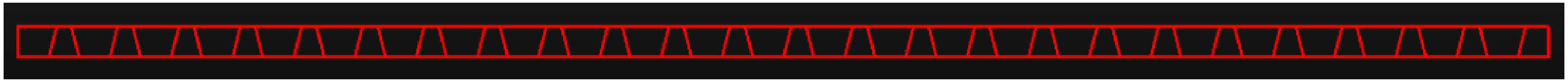
Absorbing BC, Neumann other 5 faces, center point source, solver tolerance = 10^{-8}

Closing Remarks

- **Domain decomposition solver promising:**
  - Artificial structural damping to address indefiniteness issues
  - Reuse of Krylov subspaces and previous solutions can noticeably accelerate convergence
  - Inexact solvers (e.g. use of single precision) can reduce both solution times and memory
  - Accommodates complex models with multiple element types & constraints, but not end of story
- **Solver for Higher Order Elements:**
  - Numerical results practically independent of polynomial degree
  - Very competitive with direct solvers for λ/h not too small
  - Investigate performance as inexact subdomain solver
  - Investigate enriched coarse spaces for smaller λ/h
  - Additional memory savings also possible

Extra Slides

p = 4 mesh



Variable Frequency Problems

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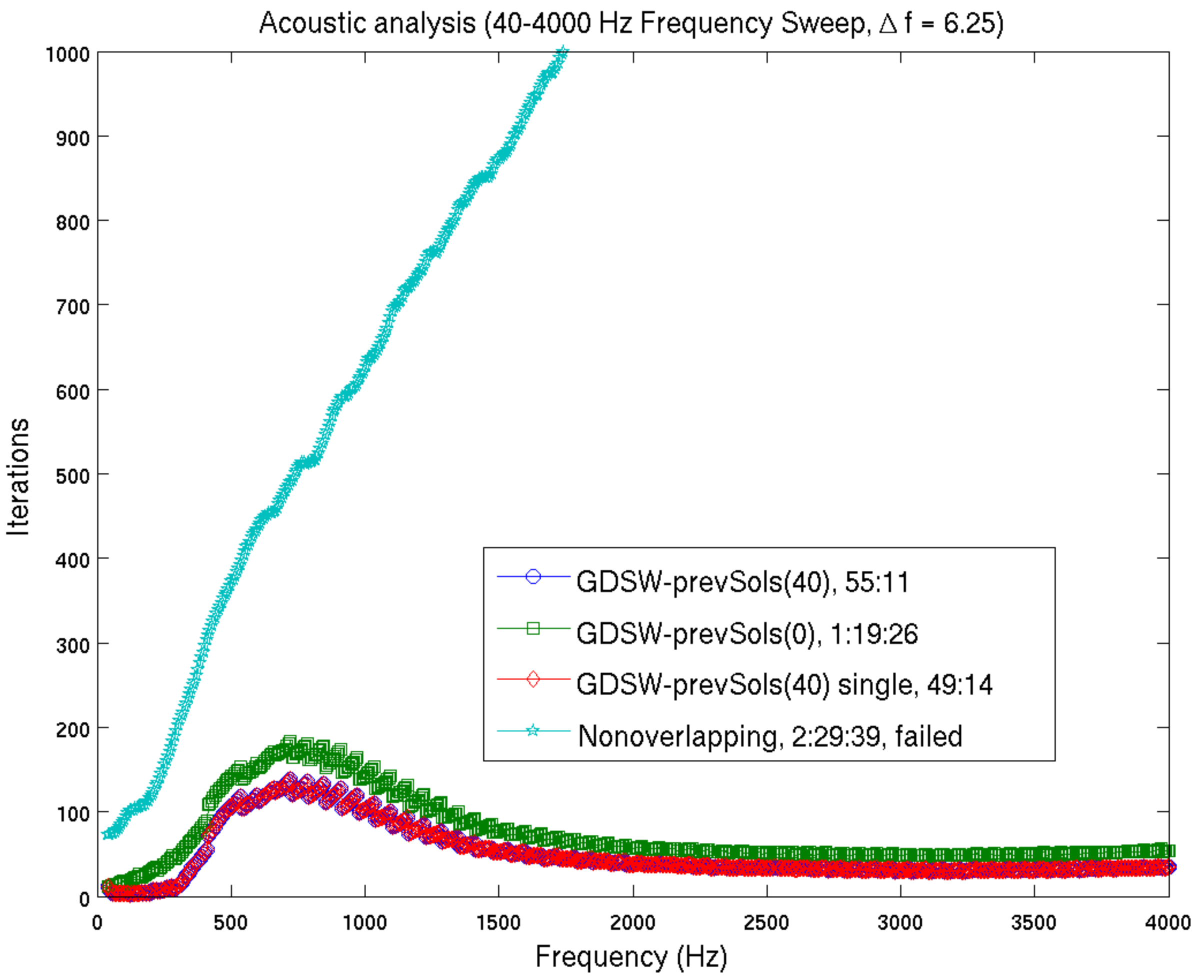

Helmholtz Solver Overview

- **Background:**
  - In final year of research project begun in FY12
  - New solver in code base and tested nightly (beta release)
  - Additional development and testing ongoing
- **Some Details:**
  - Frequency domain analysis:
    - acoustic and coupled structural-acoustic problems
    - models with wide variety of element types
    - models with large numbers of constraint equations
  - Initial Applications:
    - acoustic inverse problems (source identification)
    - direct field acoustic test analysis for structural-acoustic model