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THE ROLE OF RECOVERY COSTS IN OPTIMIZATION OF INFRASTRUCTURE RESILIENCE

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RESILIENCE AND CRITICAL INFRASTRUCTURE PROTECTION

Historically, U.S. government policy toward critical infrastructure protection (CIP) has focused on physical protection and asset hardening (for examples, see [1], [2], [3], and [4]). Recently, the federal government has realized "protection, in isolation, is a brittle strategy" [5] and not all disruptive events, natural or manmade, can be prevented. Critical infrastructure resilience (CIR) is the concept concerned with how critical infrastructures absorb, adapt, and recover from the effects of a disruptive event to ensure delivery of critical infrastructure services, and CIR has become a top priority for the Department of Homeland Security (DHS).

Successful integration of CIR concepts into CIP policies requires the development of formal metrics and analysis methods In a recent U. S. Government Accountability Office (GAO) report [6], the GAO stated that DHS should develop and adopt resilience metrics so that "DHS could be able to demonstrate its effectiveness in promoting resiliency among the asset owners and operators it works with and would have a basis for analyzing performance gaps. Regarding the latter, DHS managers would have a valuable tool to help them assess where problems might be occurring or alternatively provide insights into the tools used to assess vulnerability and risk and whether they were focusing on the correct elements of resiliency at individual facilities or groups of facilities." Additionally, these metrics should be broadly applicable across all 18 of DHS's critical infrastructure systems to enable comparison of different sectors and infrastructure systems.

Vugrin et al. [7] have proposed a new framework that expands upon previous efforts by 1) being generally applicable to all infrastructure types and 2) explicitly accounting for the costs of recovery processes in resilience cost calculations. This paper describes how optimal feedback control design can be integrated into Vugrin et al.'s framework in order to identify optimal recovery strategies that minimize resilience costs.

MEASUREMENT OF RESILIENCE COSTS

Though the concept of resilience is relatively new to the critical infrastructure protection community, it has been studied in many different academic disciplines for more than three decades. Several different resilience definitions have been developed and proposed, but few quantitative methods have been developed for analysis of infrastructures and economic systems. Some of these methods (for example, see [8]) rely on subjective evaluations of a system's resilience enhancement features, such as redundancy and adaptivity, to provide a resilience metric. These approaches are difficult to impose consistently across different infrastructure systems due to the subjective nature and variability of the evaluations, so these approaches are typically most informative when combined with a objective or simulation-based approach. Objective assessment methods, such as those developed by Bruneau et al. [9] and Rose [10], typically evaluate the difference between disrupted and undisrupted performance levels to estimate resilience. That difference is an important factor affecting resilience, but, in general these approaches have a common limitation: they do not explicitly consider the important role that recovery processes and costs have in determining system resilience. Resource allocation can be a critical concern during crisis events, and emergency responders must decide how limited resources should be directed to minimize deleterious impacts and maximize response efficiencies.

To address this limitation, Vugrin et al. have proposed a new approach to measuring resilience costs. They define system resilience as follows:

Given the occurrence of a particular disruptive event (or set of events), the resilience of a system to that event (or events) is the ability to reduce efficiently both the magnitude and duration of the deviation from targeted system performance levels.

This definition implies that calculation of resilience costs requires two quantities:

- Systemic Impact (SI), shown in Figure 1a, is the cumulative impact that a disruption has on system performance. SI is measured using the difference between targeted system performance levels and actual system performance levels. This quantity is analogous to Bruneau et al.'s and Rose's resilience measurements.
- Total Recovery Effort (TRE), shown in Figure 1b, is the cumulative amount of resources expended during the recovery effort. This term provides the efficiency measure for the system recovery and is unique to Sandia's resilience assessment framework. The inclusion of this term enables the consideration of recovery resource constraints and comparison of recovery costs.

Rather than directly calculating resilience, Sandia's resilience assessment framework indicates how to calculate two types of resilience costs. The first type of cost is termed Recovery-dependent Resilience (RDR) costs, which are the total disruption costs to the system under a specified recovery strategy. That is, if α is a weighting term that assigns the relative importance of *SI* and *TRE* and $|TSP|$ represents the magnitude of the targeted system performance levels, the *RDR* costs for a specific recovery strategy (*RS*) are calculated as

$$RDR(RS) = (SI + \alpha \times TRE) / |TSP| . \quad (1)$$

The denominator is a normalizing term that allows comparison of systems of differing magnitudes. Sandia's framework also includes the concept of Optimal Resilience (OR) costs. OR costs are the resilience costs when the optimal recovery strategy that minimizes Eq. (1) (i.e., the linear combination of SI and TRE) is employed.

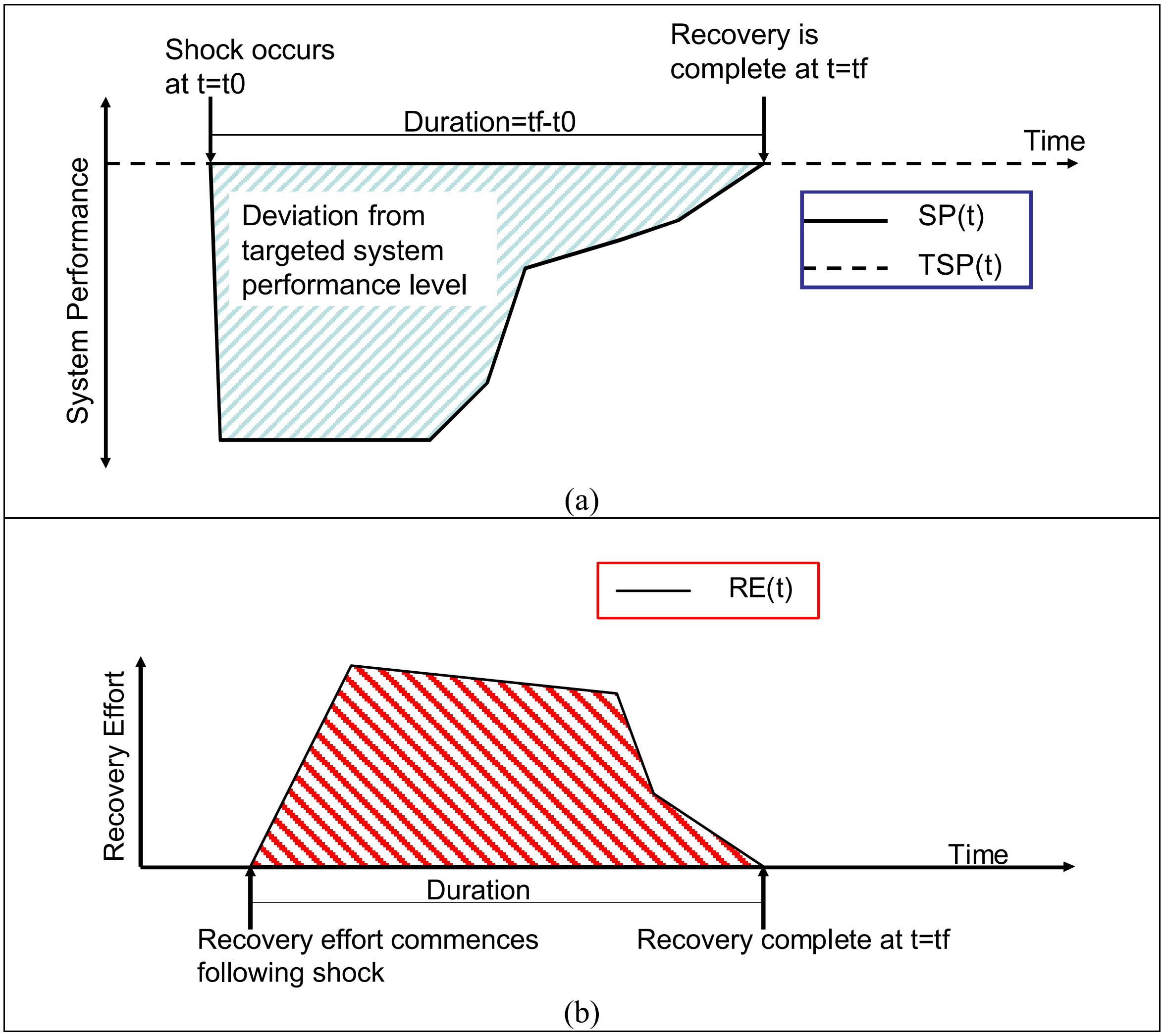


Figure 1: Systemic impact (a) and total recovery effort (b) are measured by calculating the shaded areas under the curves

Using this approach to measuring resilience costs, we find that decreasing *RDR* and *OR* values imply increasing resilience. Also, since the *RDR* and *OR* values are dimensionless quantities, they are most informative when used in a comparative manner. For example, they can be used to compare the resilience of different systems to the same disruption or to compare the resilience of the same system to different types of disruptions. Moreover, they can be used to compare the resilience of a system to a disruption under different recovery strategies.

APPLICATION OF FEEDBACK CONTROL DESIGN

When only systemic impact (or a comparable quantity) is considered, the primary challenge is simulation of the system and analysis of the system performance impacts. The introduction of the total recovery effort term complexifies the mathematics of resilience assessment. In mathematical terms, the recovery term acts as a system "controller" that dynamically affects the system performance. Changes in system performance thus affect the selection of a recovery strategy, creating a closed feedback loop.

Vugrin et al.'s resilience cost measurement approach lends itself nicely to mathematical formulations utilized for the development of optimal feedback control laws. When applied to a system, feedback controllers utilize measured system outputs to regulate system behaviors to target conditions while simultaneously providing a measure of the cost in doing so. Feedback control has been successfully used in a wide variety of settings and applications, from simple household temperature thermostats to more complicated applications such as the mitigation of aero-acoustic noise in supersonic jets.

The application of optimal control feedback laws for resilience analysis allow one to answer the following question: given a disruption to an infrastructure system, what it the optimal recovery strategy that minimizes resilience costs as calculated by Eq. (1). The following section presents a set of representative infrastructure systems that differ according to variety of factors, such as redundancy, presence of emergency inventory reserves, etc., that affect resilience. We model these systems with systems dynamics conventions and apply optimal feedback control laws to demonstrate the following:

1) we can quantitatively assess the resilience of these systems to a particular disruption;
2) we can identify optimal recovery strategies in a predictive manner; and
3) we can confirm that Vugrin et al.'s resilience assessment approach recognizes the system that logic indicates is most resilient.

REPRESENTATIVE INFRASTRUCTURE SYSTEM MODELS

For this paper we assess the resilience of four simple infrastructure models. We refer to the first and simplest system as base system (Figure 2). This model is an adaptation of a simple supply-consumption system dynamics model. In this system a stock manager tries to manage stock acquisition rates so that

1) The stock inventory tracks a desired stock profile; and
2) Stock consumption tracks an external demand signal.

A few key system characteristics should be noted:

1) The stock loss rate is proportional to the stock inventory level;
2) The stock acquisition rate is the sum of the manager's stock adjustment strategy and the average consumption.
3) To track the desired stock profile, the manager uses a stock adjustment strategy that is proportional to the difference between the desired stock inventory profile and actual stock inventory.
4) A simulated disruption to the system hinders the acquisition rate.

Specifically, we model the system with the following delay-differential equation:

$$\frac{dS}{dt} = \frac{\left[3 + .5\sin\left(4t\right)\right] - S}{8} - .5S + \left[C\left(t\right) - C\left(t-1\right)\right] + N(t)$$
$$\frac{dC}{dt} = .5S \qquad (2)$$

where S represents the stock, C denotes the rate of stock consumption, and N represents a potential disruption to the stock acquisition rate. For this analysis, the disruption decreases the stock acquisition rate by one unit from time 1 to 1.2, i.e.,

$$N(t) = \begin{cases} -1, & 1 \leq t \leq 1.2 \\ 0 & \end{cases} \qquad (3)$$

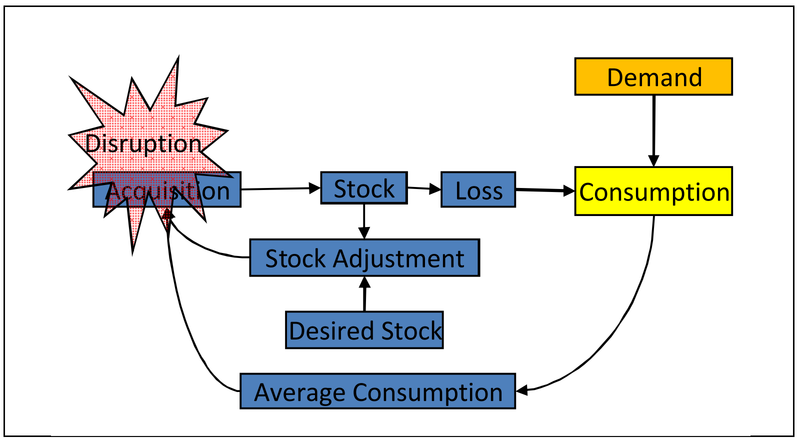
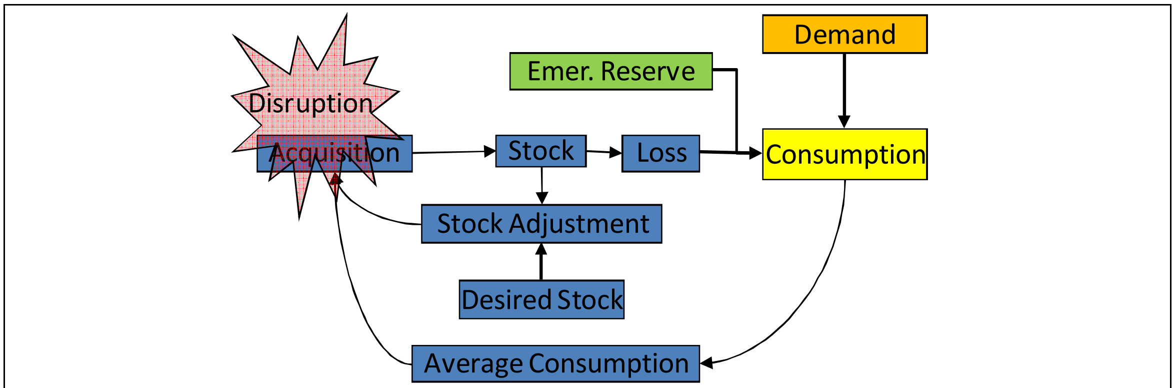


Figure 2: Base system

We consider three variations of this system:

1) The Emergency Inventory (EI) system: This system is identical to the base system except that it includes an emergency inventory of stock. The emergency supply is activated when the consumption cannot meet the external demand signal. In this event the emergency supply rate can supply that commodity directly to the consumer. The emergency supply enhances the restorative capacity of the system. Thus, it is expected that resilience costs for this system are expected to decrease, relative to the base system, indicating greater resilience.
2) The Redundant Stock (RS) system: This system is identical to the base system except that it has four locations for stock production instead of one location. The inherent redundancy enhances the absorptive capacity of the system. Thus, it is expected that resilience costs for this system are expected to decrease, relative to the base system, indicating greater resilience.
3) The Redundant Stock, Emergency Inventory (RSEI) system: this system has four stock production locations and an emergency supply rate. The inherent redundancy and emergency supply enhance the absorptive and restorative capacities of the system. Thus, it is expected that resilience costs for this system are the least for all the systems, indicating this system is the most resilient.


Demand
Emer. Reserve
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Acquisition
Stock
Loss
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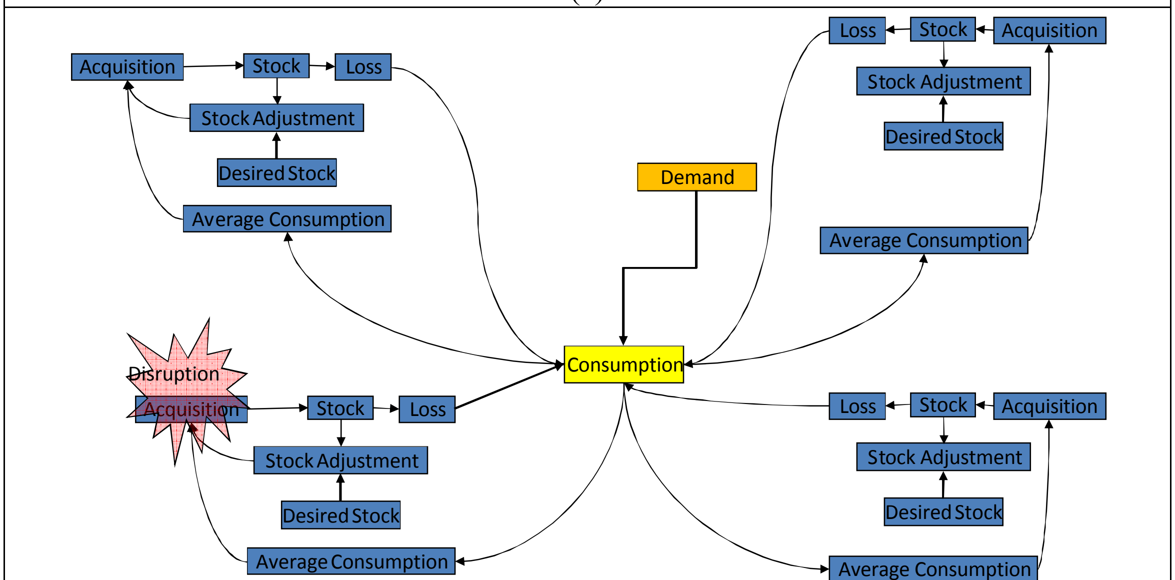
Stock Adjustment
Desired Stock
Average Consumption
(a)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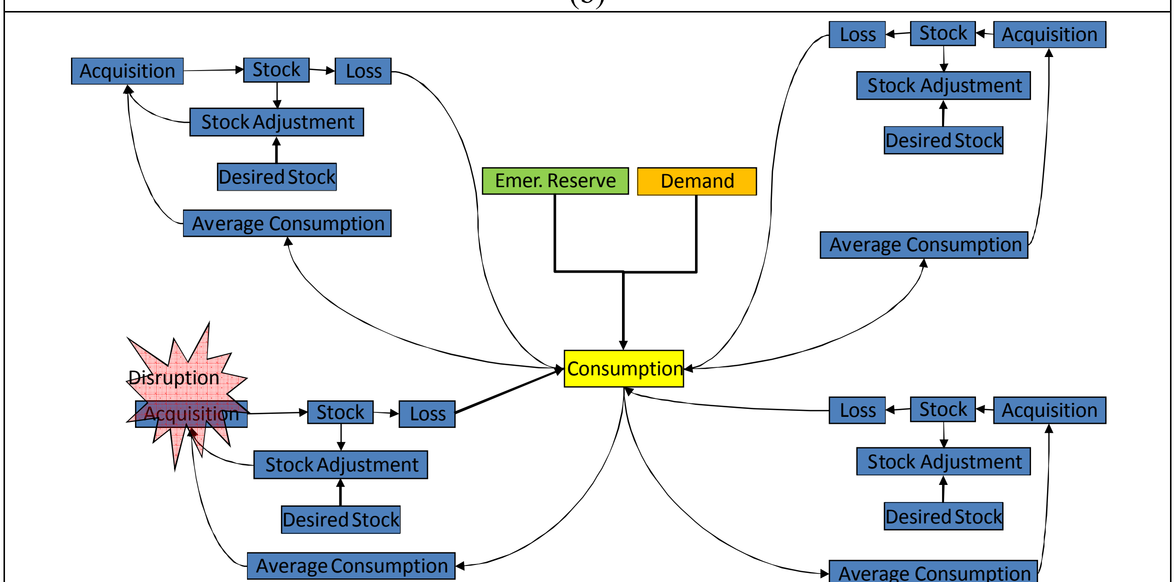
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Emer. Reserve
Demand
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Disruption
(c)

Figure 3: Systems with resilience enhancements: (a) emergency inventory (EI) only, (b) redundant system (RS) only, and (c) Emergency inventory and redundant system (RSEI)

Eq. (2) can be generalized to model the additional systems. (For complete details, see [11]). To assess the resilience costs, we define

$$SI = \int [D - C]^2 \, dt$$
$$TRE = \int .01 [ESR]^2 \, dt \qquad (4)$$

where D represents the external demand (shown in Figure ##) and targeted system performance and $0.01xESR^2$ represents the cost of the emergency inventory used in the recovery process. To calculate the optimal resilience costs, we formulate the linear quadratic regulator problem that enables us to identify the identify the optimal ESR recovery strategy that minimizes Eq. (1) when systemic impact and total recovery effort are defined as in Eq. (4) (see [11] for mathematical details).

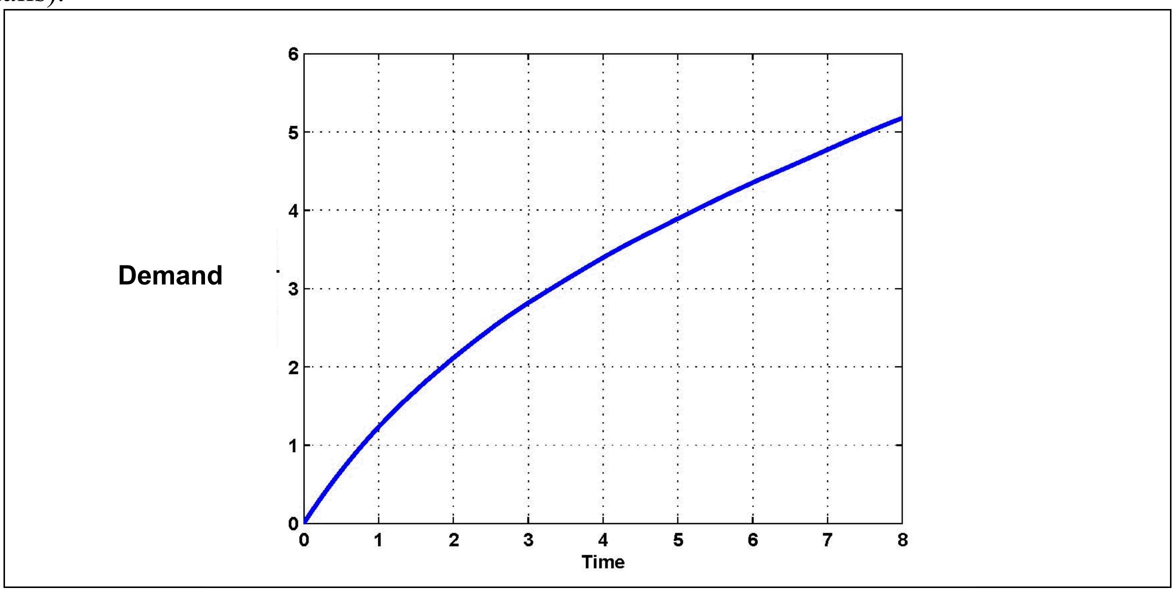


Figure 4: Demand

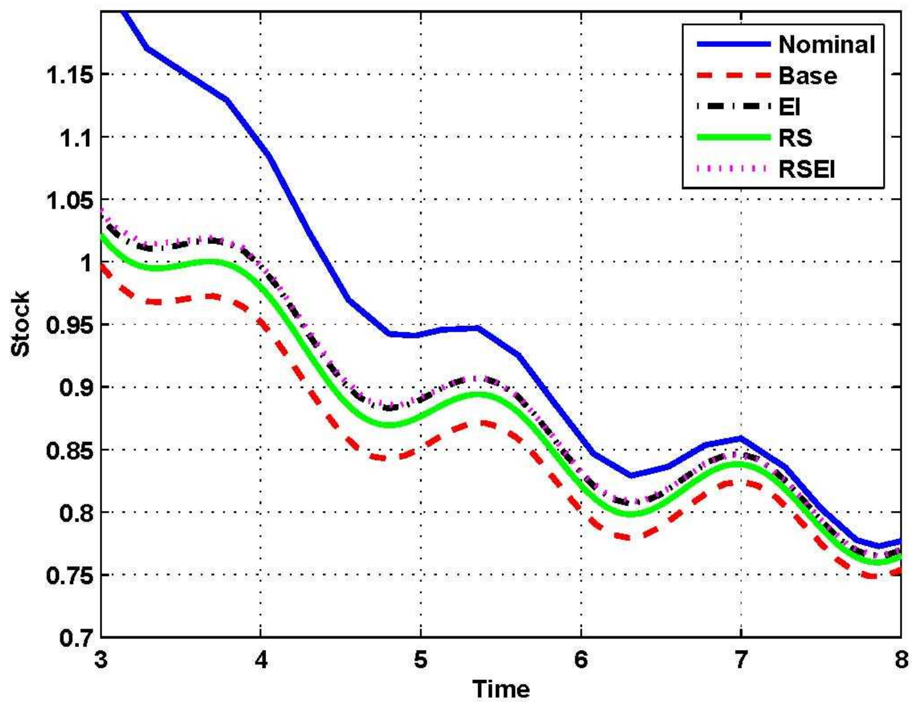
ANALYSIS

Figure 5a shows the inventory levels for the stocks whose acquisition rates are disrupted, and Figure 5b shows inventory levels for the other 3 stocks that are not directly affected in the RS and RSEI systems. The stocks for systems with emergency supply rates, EI and RSEI, are most similar to the nominal inventories, followed by stocks for the RS and then the base system. Undisrupted stock levels for the RSEI system exceed their counterparts for the RS system. This relative increase is caused by the use of the emergency supply inventory. When the emergency supply initiates at t = 1, consumption levels for RSEI system exceed those of the EI system. Since the stock acquisition rates are a function of the average consumption rate, RSEI stock acquisition rates increase relative to the RS rates, leading to increased RSEI stock levels.

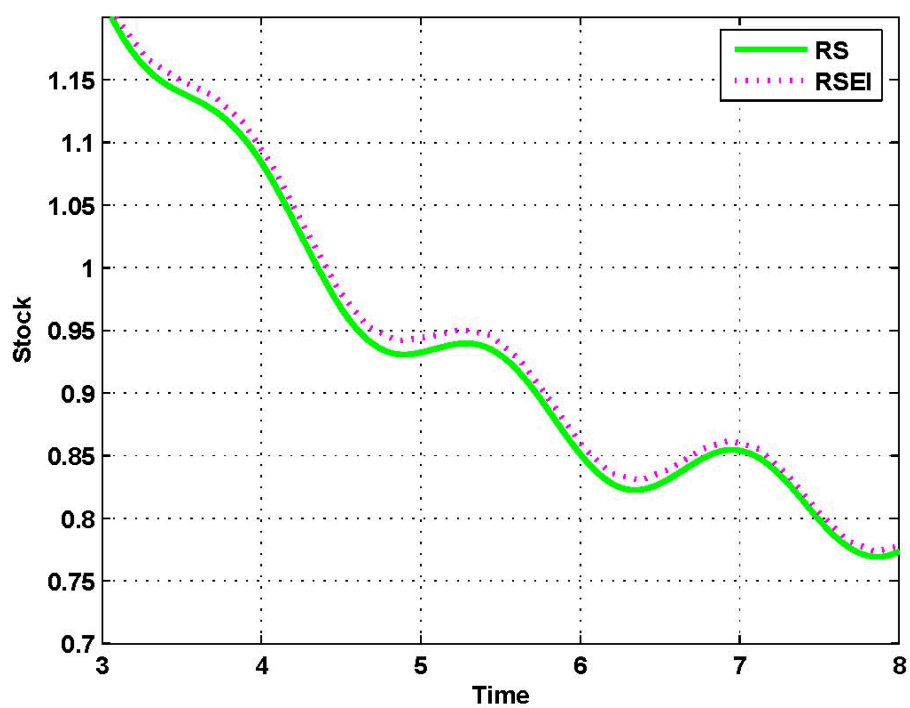
Systemic impact is measured in terms of the difference between the consumption term and the demand function, and the differences between these terms can be seen for all systems in Figure 6. Again, the systems with emergency supply inventories, EI and RSEI, track the target demand most closely, followed by the RS and base systems. Since the base and RS systems do not have emergency inventories, they have no means for overcoming the initial disruption. The RS system is not as severely affected as the base system since only 25 per cent of the stocks are affected, instead of 100 percent in the base system.

The optimal recovery functions for the EI and RSEI systems are shown in Figure 7. The recovery function for the EI system exceeds its RSEI counterpart at all times after the disruption initiation; that is, the EI system requires a "greater" recovery effort than does the RSEI system. This occurs since only 25 per cent of the stocks are affected in the RSEI system, instead of 100 percent in the EI system.

The systemic impact is greatest for the base system, and systemic impact is ten and sixty times larger for the base system than for the RS and EI/RSEI systems, respectively (Table 1). Since the base and RS systems do not have a recovery mechanism (the emergency inventory), the total recovery efforts for those systems are 0. The total recovery effort terms for the EI and RSEI systems are both relatively small compared to the systemic impacts. However, the total recovery effort for the EI system is approximately 8.5 per cent larger for the EI system; hence, we are able to quantitatively assess the benefit that the redundancy in the RSEI system has towards the resilience costs. Since the RSEI system has the smallest resilience costs, that system is the most resilient system, followed by the EI system, then the RS system, and then the base system. This ranking matches the expected relative order of resilience that we expected.



(a)



(b)

Figure 5: Directly (a) and indirectly (b) affected stock inventory levels

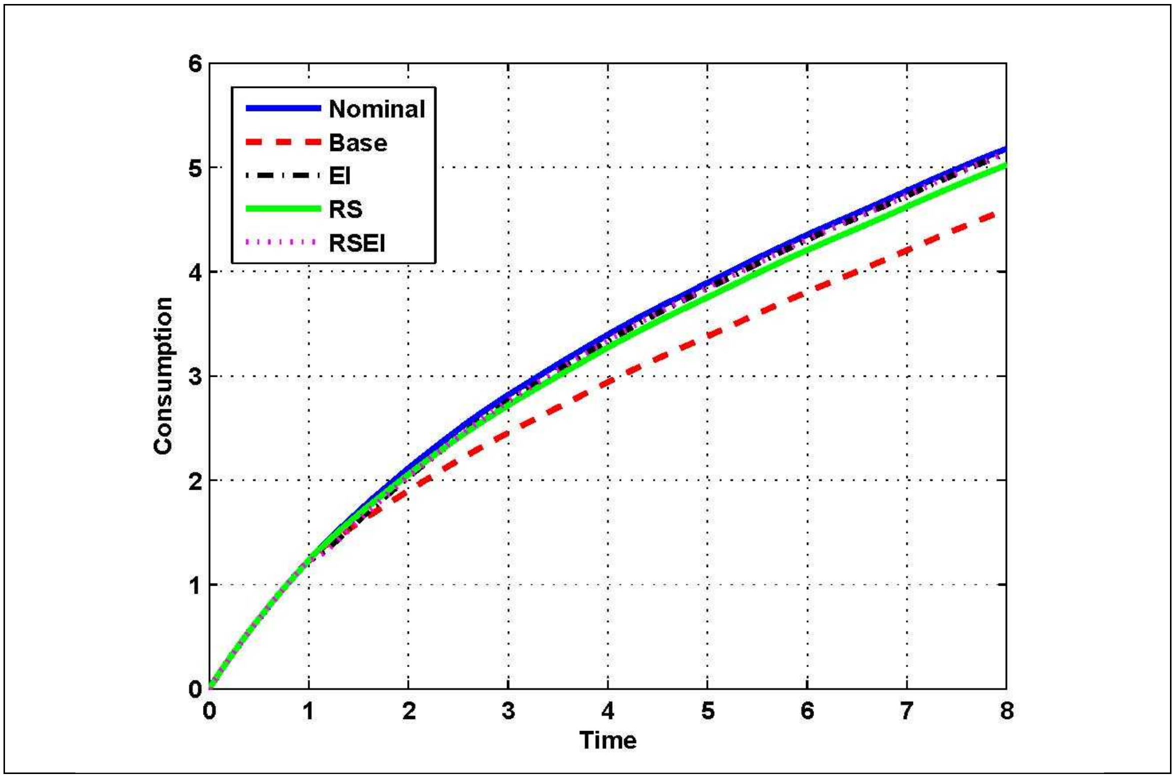


Figure 6: Consumption levels

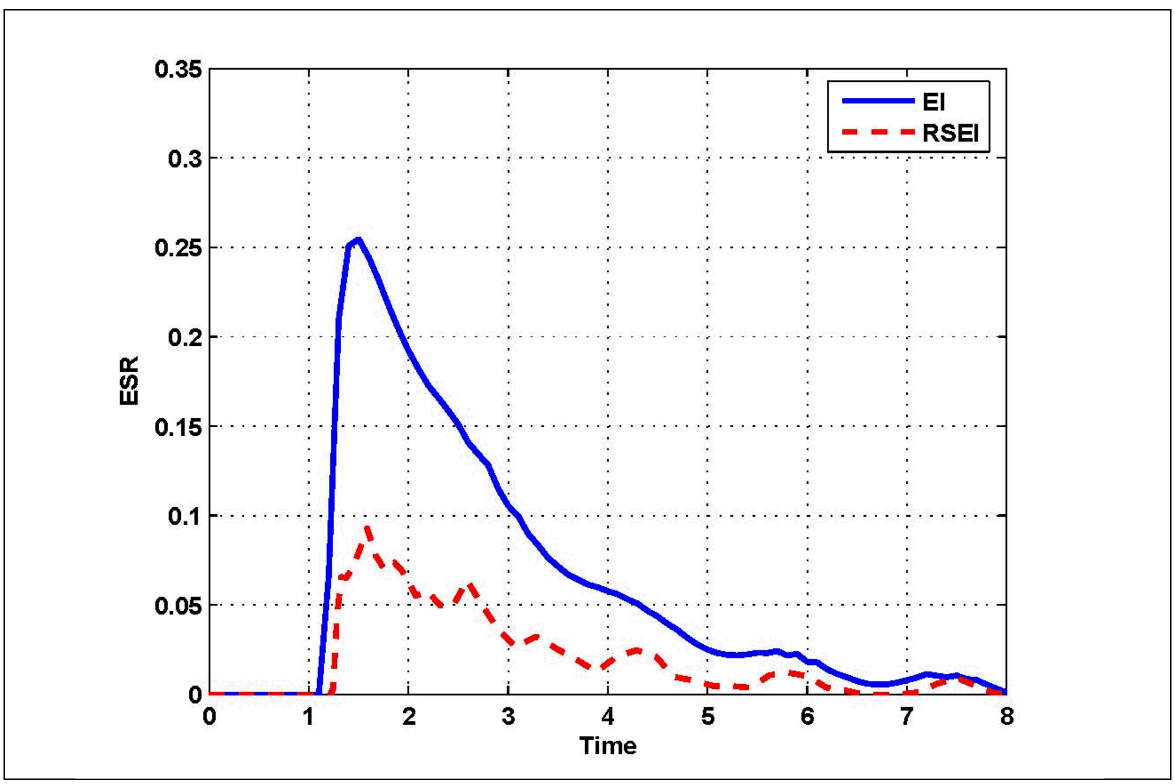


Figure 7: Recovery Effort

Table 1: Resilience Cost Calculations

	Base	Emer. Inventory Only	Redundancy Only	Emer. Inv. + Redundancy
Systemic Impact	1.4	.023	.11	.023
Total Recovery Effort	0	.0077	0	.0071
Optimal Resilience Costs[1]	.015	.00025	.0013.	.00024
Resilience Ranking	4	2	3	1

Table 2 shows the resilience loss and resilience calculations that one gets when using Bruneau et al.'s and Rose approaches. Using those two approaches, one concludes that the additional redundancy in the RSEI system does nothing to enhance its resilience relative to the EI system that only has emergency inventory available to it. This conclusion is counterintuitive. However, the addition of the total recovery effort term in resilience costs results the conclusion that the redundancy combined with the emergency inventory further enhances the system's resilience, relative to the emergency inventory only system. This simple example demonstrates the importance of including recovery costs in resilience assessments.

Table 2: Alternative resilience approaches

	Base	Emer. Inventory Only	Redundancy Only	Emer. Inv. + Redundancy
Resilience Loss (Bruneau et al.) [9]	.015	.0024	.0013	.0024
Rank	4	**1**	3	**1**
Static Resilience (Rose [10])	0			
Rank	4	**1**	3	**1**
Dynamic Resilience (Rose [10])				
Ranking	4	**1**	3	**1**

SUMMARY

The development of objective, quantitative resilience assessment methods is an important step in the process of institutionalizing resilience within critical infrastructure protection. The authors assert that the recovery cost and resource utilization must be considered in these resilience assessment approaches. To demonstrate that importance, this paper presents a simple example that demonstrates when recovery costs are considered, one has an increased ability to discern between varying levels of resilience.

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[1] The quantity $|TSP|$ is the integral of the demand term and is calculated to be 95.

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