SAND2011-0532 C

SAND2011-0532C

# Scaling of Photon-Phonon Coupling in Nano-Optomechanical Systems

**Peter T. Rakich[1] , Zheng Wang[2], Charles E. Reinke[1], Ryan Camacho[1], Paul Davids[1]**

[1] *Applied Photonic Microsystems, Sandia National Laboratories, Albuquerque NM, 87123, USA*
[2] *Physics, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139, USA*
*rakich@alum.mit.edu*

**Abstract:** Through systematic examination of material and topological degrees of freedom in nano-optomechanical systems, we identify the fundamental barriers and opportunities for the creation of large photon-phonon coupling.

Coherent phonon generation in dielectric waveguides and media was demonstrated some four decades ago via laser induced electrostrictive forces [1,2]. With the advent of nanophotonics, nano-enhanced radiation pressure from highly confined modes has been shown to produce efficient phonon generation at low powers with chip scale-devices [3]. Optomechanical photon-phonon coupling of this form has caught the attention of many since nanoscale light confinement produced remarkably large forces within miniscule volumes [3-5], and it is known to produce high frequency phonon transduction [3-5]. Furthermore, the unique range of length-scales and time-scales accessible with such nano-scale systems show the potential for benefit to numerous RF and signal processing applications, fuelling the investigation of such physical mechanism for use in high frequency signal transduction. However, without a unified framework through which optical forces and optomechanical parametric processes can be understood, it is difficult to determine whether technologically relevant data rates can be achieved through use of such systems.

In this paper, we develop a unified framework through which optomechanical transduction can be understood in virtually all optomechanical systems, elucidating the bandwidth and efficiency limitations of such technologies. Through examination of material and geometric degrees of freedom, we develop scaling laws which describe the magnitude of optical forces produced by electrostriction and radiation pressure in any optomechanical system [4,5]. Using these scaling laws, we explore the practical upper-bounds of light-induced forces, and identify materials systems with favourable characteristics for optomechanical transduction. Through generalized treatment of photon-phonon coupling, a fundamental scaling law that governs the efficiency and bandwidth limitations *of all radiation pressure driven optomechanical devices* can be derived, enabling the comparison of all optomechanical systems in a unified framework. With this theory, we show that the maximum transduction bandwidth and phononic power output (i.e. photon-phonon coupling) of any optomechanical device is determined by: (1) the maximum possible magnitude of radiation pressure, (2) the device dimension, and (3) the effective mechanical impedance of the system.

Simply stated, the challenges associated with broadband stimulated phonon emission in optomechanical systems arise from: (1) *limited optical forces*, and (2) the high *mechanical impedance of naturally occurring media*. One can show the maximum driving force produced by light within any optomechanical system is fundamentally limited by the optical power and energy density limitations of optical materials. Consequently, radiation pressure is typically restricted to a maximum value of $10^4 (\mathrm{N}/\mathrm{m}^2)$ within materials such as silicon. We term this limit, the "Radiation-Pressure Limit''. With this practical limitation in mind, one can prove that the maximum transduction bandwidth and phononic power output (i.e. photon-phonon coupling) of any optomechanical device is determined by: (1) the maximum possible magnitude of radiation pressure, (2) the device dimension, and (3) the effective mechanical impedance.

Forces in media generally arise from, (1) radiation pressure and (2) electrostrictively induced forces, both of which have historically played a very important role in the understanding of transduction with light. In contrast to radiation pressure, which originates from the momentum transfer from scattered photons at dielectric boundaries, electrostriction is derived from the strain-dependence of dielectric permittivity and such forces are present in all dielectric media. The maximum driving force produced by light within any optomechanical system is fundamentally limited by the optical power and energy density limitations of optical materials. This limitation of optical driving forces becomes apparent

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

once it is understood that the *exact form of the radiation pressure induced optical force density within any dielectric medium* can be expressed as [5]:

$$F_j^{rp} = \partial_i T_{ij} = \tfrac{1}{2}\varepsilon_o \,|\,\mathbf{E}(\mathbf{r})|^2\, \partial_j \varepsilon(\mathbf{r}). \qquad (1)$$

Here, $T_{ij}$ is the Maxwell stress, $\varepsilon_o$ is the free space permeability, $\mathbf{E}(\mathbf{r})$ is the electric field distribution, and $\partial_j\varepsilon(\mathbf{r})$ is the gradient of the dielectric distribution. Clearly, large dielectric gradients (i.e. high index-contrast) benefit the production of large optical forces, and optical forces occur only at dielectric surfaces in step-index structures. More importantly, optical force density is fundamentally limited by the achievable electromagnetic energy density $\frac{1}{2}\varepsilon\,|\,\mathbf{E}|^2$. Energy density is typically bound to $10^4$ J/m$^3$ in high-index materials, such as silicon, before the onset of appreciable two-photon absorption and the associated heating which limit practically achievable powers. Consequently radiation pressure is fundamentally restricted to a maximum value of ~$10^4$ N/m$^2$, which we term the "Radiation Pressure Limit", in all optomechanical devices including those utilizing high-Q electromagnetic resonances. Bear in mind that the "Radiation Pressure Limit " is only attainable by *optimally confined modes* which interact *very* strongly with the boundaries of the system [4,5].

In its most basic form, optomechanically stimulated phonon emission is a third-order nonlinear process (illustrated as an energy-level diagram in Fig. III.A.1) through which the interference between optical waves of two different frequencies $(\omega_p, \omega_s)$ produces a time-harmonically modulated optical driving force of frequency, $\Omega = (\omega_p - \omega_s)$. In describing stimulated phonon generation, the optical powers (particle fluxes) $P_p$ ($\Phi_p$), $P_s$ ($\Phi_s$), corresponding to an optical pump ($\omega_p$) and a Stokes waves ($\omega_s$) are coupled by way of acoustic phonons of frequency $\Omega = (\omega_p - \omega_s)$ and power (particle flux) $P_\Omega$ ($\Phi_\Omega$). From the time-varying optical force distributions, the generated elastic wave power can be computed the parametric conversion for both photons and phonons.

Through generalized treatment, we show that a fundamental scaling law that governs the efficiency and bandwidth limitations *of all radiation pressure driven optomechanical devices* can be derived, enabling the comparison of all optomechanical systems in a unified framework. As discussed in Section II.A.5, a unique maximum of optical force from radiation pressure can be established within an optical medium, regardless of device topology. This limit, which is defined by the maximum sustainable electromagnetic energy density $(u_{em}^{max})$ of an optical material, we term the "Radiation-Pressure Limit". Based on this fundamental limit, our scaling law reveals that the maximum quantum efficiency obtained via a radiation pressure mediated process within a unit length $(\Delta z)$ of guided wave optomechanical interaction is of the form,

$$\eta^{\max} = \frac{d\Phi_\Omega}{dz}\frac{\Delta z}{\Phi_p} \cong \alpha \cdot \frac{u_{em}^{max}}{c}\frac{\omega_s}{\Omega} \cdot \frac{(n_g - n_g)^2}{n_g \cdot Z(\Omega)}\Delta z \qquad (2)$$

Here $\Phi_p$ $(\Phi_\Omega)$ is the incident (generated) photon (phonon) flux, $Z(\Omega)$ is the frequency dependent mechanical impedance of the body into which the phonon is being transduced, and $\alpha$ is a factor that depends weakly on geometry. Within nanoscale waveguides and cavities, $u_{em}^{max}$ can be easily obtained at miliwatt laser powers. Thus, for energy densities corresponding to $u_{em}^{max}$, or the "Radiation Pressure Limit," the only means by which transduction efficiency can be increased is through: (1) an increase in modal dispersion $(n_g - n_p)$, (2) a decrease in mechanical impedance $Z(\Omega)$, or (3) an increase in interaction length, $\Delta z$. The role of dispersion $(n_g - n_p)$, is derived from its fundamental connection to radiation pressure [5]. While Eq. 2 describes a fundamental limit associated with radiation pressure induced parametric processes, it should be noted that *this limit can be exceeded through designs which efficiently utilize electrostrictive forces*, which have been widely neglected in the context of nano-optomechanics.

## References

[1] R. Chiao, C. Townes, and B. Stoicheff, Phys. Rev. Lett., vol. 12, no. 21, pp. 592–595, 1964.
[2] E. Ippen and R. Stolen, Appl. Phys. Lett., vol. 21, no. 11, pp. 539–541, 1972.
[3] M. Eichenfield, R. Camacho, J. Chan, K. J. Vahala, and O. Painter, Nature, vol. 459, pp. 550–U79, MAY 28 2009.
[4] P. Rakich, P. Davids, and Z. Wang, Opt. Express, vol. 18, no. 14, pp. 14439–14453, 2010.
[5] P. Rakich, Z. Wang, and P. Davids, Opt. Lett., vol. 36, no. 2, pp. 217–219, 2011