SAND2011-1235C

Optimal antenna beamwidth for stripmap SAR

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ABSTRACT

The classical rule-of-thumb for Synthetic Aperture Radar (SAR) is that a uniformly illuminated antenna aperture may allow continuous stripmap imaging to a resolution of half its azimuth dimension. This is applied to classical line-by-line processing as well as mosaicked image patches, that is, a stripmap formed from mosaicked spotlight images; often the more efficient technique often used in real-time systems. However, as with all rules-of-thumb, a close inspection reveals some flaws. In particular, with mosaicked patches there is significant Signal to Noise ratio (SNR) degradation at the edges of the patches due to antenna beam roll-off. We present in this paper a calculation for the optimum antenna beamwidth as a function of resolution that maximizes SNR at patch edges. This leads to a wider desired beamwidth than the classical calculation.

Keywords: SAR, antenna, stripmap

1 INTRODUCTION

The question is "For a Synthetic Aperture Radar (SAR) stripmap made of mosaicked spotlight images, what is the optimal antenna azimuth beamwidth to maximize the minimum Signal-to-Noise Ratio (SNR) anywhere in a patch for an image at a particular resolution?" The minimum SNR is assumed to be at the patch edge.

The answer is "It depends on the actual antenna beam roll-off characteristics, but may approach 50% wider than the classical limits suggest."

Below is the justification for this.

2 DISCUSSION

It is well-known that the aspect angle subtended by the synthetic aperture of a SAR needs to meet

$$\theta_{ap} = \frac{a_w \lambda}{2\rho_a}, \tag{1}$$

where

a_w = nominal bandwidth factor due to window functions during processing,
λ = nominal wavelength of radar, and
ρ_a = desired azimuth resolution after processing. (2)

For stripmap SAR using mosaicked patch processing, the image patch width needs to be at least as wide as the synthetic aperture is long (assuming broadside imaging). In fact, to allow time for antenna slewing between adjacent synthetic

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apertures, the patch width needs to be slightly wider than the synthetic aperture length. In addition, antenna pointing tolerance should also be accommodated. Consequently, we define the patch azimuth width to minimally meet

$$\theta_{patch} = (1+\eta_{ap})\theta_{ap} , \tag{3}$$

where

$$\eta_{ap} = \text{factor to account for slew time, tolerance, etc. } (\eta_{ap} > 0) \tag{4}$$

We note that η_{ap} will actually be somewhat velocity dependent.

The antenna azimuth beam pattern needs to sufficiently illuminate the entire patch of interest. We now define

$$\theta_{az} = \text{the } -3 \text{ dB azimuth beamwidth of the antenna.} \tag{5}$$

While embodied in the common rule-of-thumb (where achievable resolution equals half the aperture size of a uniformly illuminated aperture[1]), it is nevertheless rather simplistic to presume that all we need is the –3 dB antenna beamwidth to span the imaged patch, that is $\theta_{az} = \theta_{patch}$. Were this the case, then this equality yields a 6 dB round-trip SNR reduction at the patch edge. It is not at all clear that this is optimum for strip mapping. In fact it is not. We will show that by widening the antenna beamwidth slightly, even though the gain at the beam center diminishes, the gain at the patch edge will increase somewhat before falling off again. This is illustrated in Figure 1.

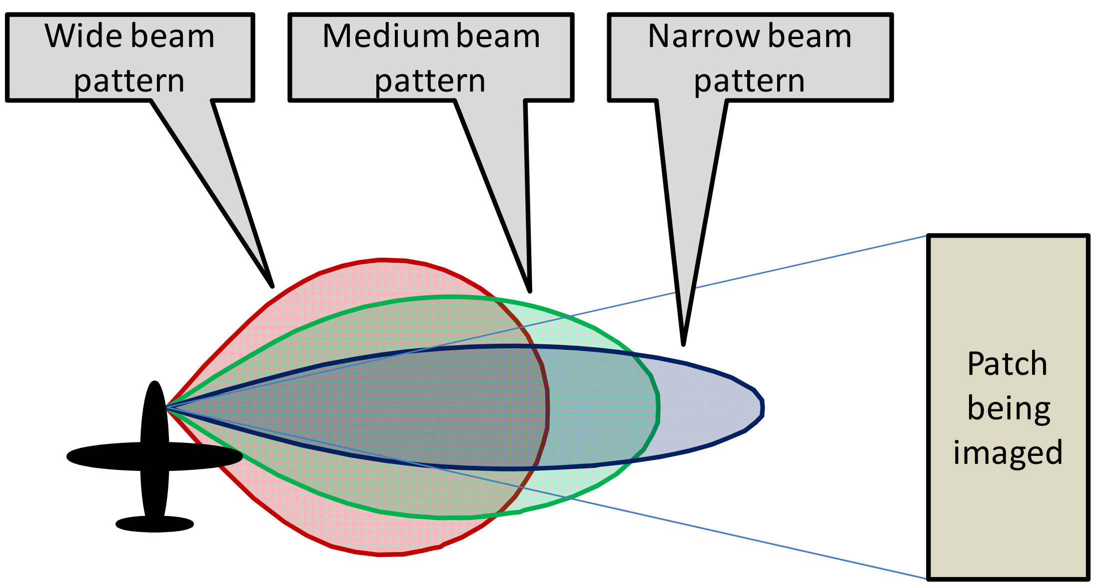

Figure 1. Of the antenna patterns depicted, the medium beam pattern yields the greatest gain in the directions of the edges of the patch being imaged. Consequently it offers the maximum of the minimum SNR across the image patch.

Consider an antenna beam pattern described by the function

$$g(\phi, \theta_{az}) = \text{antenna beam pattern in the power domain,} \tag{6}$$

where

$$\phi = \text{the azimuth angle variable off boresight of the antenna.} \tag{7}$$

We will assume a symmetric antenna pattern. Some properties of $g(\phi,\theta_{az})$ include

1. The definition of half power beamwidth

$$g\left(\frac{\theta_{az}}{2},\theta_{az}\right)=0.5\,, \tag{8}$$

2. The peak is at the center of the beam

$$g(0,\theta_{az})\geq g(\phi,\theta_{az})\,, \tag{9}$$

3. Gain is inversely proportional to beamwidth

$$g(0,a\theta_{az})=\frac{1}{|a|}g(0,\theta_{az})\,. \tag{10}$$

The task at hand is to find θ_{az} such that $g(\theta_{patch}/2,\theta_{az})$ is maximum.

Clearly, if θ_{az} is too narrow (small) then $\theta_{patch}/2$ is outside the main lobe of the antenna, suffering too much attenuation. However, if θ_{az} is too wide (large) then in spite of being in the mainlobe of the antenna, the mainlobe will have reduced gain, as will $\theta_{patch}/2$. Consequently, an optimum θ_{az} exists that is neither too wide nor too narrow for a particular $\theta_{patch}/2$.

We do note that the two-way antenna pattern is in fact

$$g^2(\phi,\theta_{az}) = \text{two-way antenna beam pattern in the power domain.} \tag{11}$$

Clearly this is maximized if $g(\phi,\theta_{az})$ is maximized.

<u>Case 1. Parabolic Mainlobe</u>

A very common characteristic at the peak of an antenna mainlobe is a paraboloid, consequently we will assume a quadratic function to the –3 dB points, namely

$$g(\phi,\theta_{az})=\frac{k}{\theta_{az}}\left(1-2\left(\frac{\phi}{\theta_{az}}\right)^2\right) \tag{12}$$

where

$$k = \text{a constant that embodies a gain.} \tag{13}$$

We find the maximum gain at the imaged patch edge by setting

$$\frac{d}{d\theta_{az}}g\left(\frac{\theta_{patch}}{2},\theta_{az}\right)=0 \tag{14}$$

and solving for θ_{az}. Performing the differentiation yields

$$\frac{d}{d\theta_{az}} g\left(\frac{\theta_{patch}}{2}, \theta_{az}\right) = k\left(\frac{3}{2}\theta_{patch}^{2} - \theta_{az}^{2}\right) \Big/ \theta_{az}^{4} \ . \tag{15}$$

Consequently, with some algebraic manipulation, the maximum SNR at the patch edge is achieved when

$$\theta_{az} = \sqrt{3/2}\ \theta_{patch} \, . \tag{16}$$

This in turn implies that for the paraboloid antenna model, we calculate the optimum −3 dB width as a function of resolution (and other parameters) as

$$\theta_{az} = \sqrt{3/2}\ (1 + \eta_{ap}) \frac{a_w \lambda}{2\rho_a} \ . \tag{17}$$

Example

Consider the following SAR stripmap imaging parameters

$$\begin{aligned} \lambda &= 0.018 \text{ m (Ku-band)}, \\ \rho_a &= 0.3 \text{ m}, \\ a_w &= 1.2, \\ \eta_{ap} &= 0.1 \end{aligned} \tag{18}$$

From these we calculate

$$\begin{aligned} \theta_{ap} &= 2.06 \text{ degrees}, \\ \theta_{patch} &= 2.27 \text{ degrees}, \\ \theta_{az} &= 2.78 \text{ degrees}. \end{aligned} \tag{19}$$

Note that this calculates the optimum antenna beamwidth to be 34% wider than the classical limit.

Case 2. Sinc Function Mainlobe

For a uniformly illuminated antenna aperture, we identify for small ϕ/θ_{az}

$$g(\phi, \theta_{az}) = \frac{k}{\theta_{az}} \operatorname{sinc}^2\left(0.88 \frac{\phi}{\theta_{az}}\right). \tag{20}$$

We may again find the maximum gain at the imaged patch edge by setting

$$\frac{d}{d\theta_{az}} g(\theta_{patch}/2, \theta_{az}) = 0 \ , \tag{21}$$

and solving for the desired −3 dB beamwidth θ_{az} .

Alternately, we may simply plot $g(\theta_{patch}/2, \theta_{az})$ for parameters of interest and find the maximum value.

Example

For the following parameters (same as before)

k = 1,
λ = 0.018 m (Ku-band),
ρ_a = 0.3 m,
a_w = 1.2,
η_{ap} = 0.1, (22)

we plot the gain function $g\left(\theta_{patch}/2, \theta_{az}\right)$ as a function of nominal beamwidth θ_{az} in Figure 2. This yields a peak value at about θ_{az} = 2.69 degrees. A close-up of the peak is given in Figure 3.

Note that this is the –3 dB beamwidth. The classical number associated with a sinc() function is the distance from peak to first null, which is approximately 3.06 degrees.

Note also that for this example the 2-way attenuation at the patch edge is 4.1 dB with respect to the patch center.

Case 3. Sinc Function Mainlobe with Effects of a Presummer

A presummer is a Doppler filter that modifies SNR in a similar manner as the antenna beam pattern. Its effects will affect the optimum antenna beamwidth calculations above.

The two-way attenuation function in the power domain due to both the antenna pattern and the presummer is given by

$$g^2(\phi, \theta_{az})\, h(\phi, \theta_{az}) = \text{two-way combined pattern in the power domain} \tag{23}$$

where

$$h(\phi, \theta_{az}) = \operatorname{sinc}^2\left(0.88 \frac{\phi}{k_a \theta_{az}}\right) \tag{24}$$

and

$$k_a = \text{antenna beam Doppler oversampling factor after presuming.} \tag{25}$$

The presummer will tend to roll off the signal even more so, requiring compensation with a slightly wider antenna beamwidth.

Example

Figure 4 plots $g(\phi, \theta_{az})\sqrt{h(\phi, \theta_{az})}$ for $k_a = 1.5$ to be consistent with the scaling of Figure 2, and for the same parameters as in Figure 2.

This plot exhibits a peak at θ_{az} = 2.93 degrees. A smaller k_a will cause the optimum beamwidth to widen. For example $k_a = 1.4$ would increase the optimum beamwidth to grow to θ_{az} = 2.97 degrees.

Note that this calculates the optimum antenna beamwidth to be 44% wider than the classical limit.

Note also that an azimuth prefilter (a more sophisticated presummer with better filter stopband characteristics), while more complex, would have a lesser effect on increasing the optimum beamwidth.

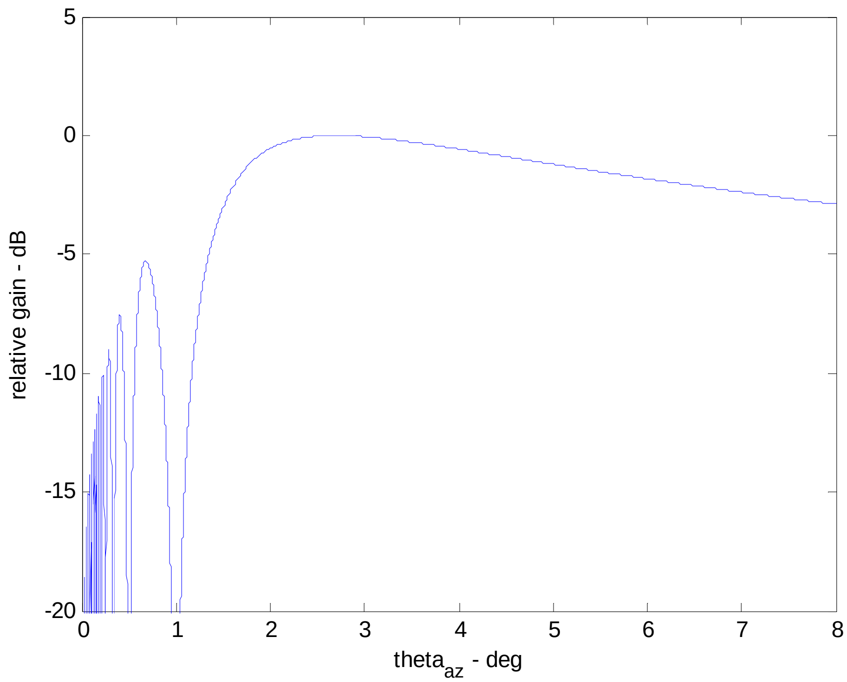

Figure 2. Relative gain versus beamwidth for patch width required for 0.3 m resolution stripmap SAR.

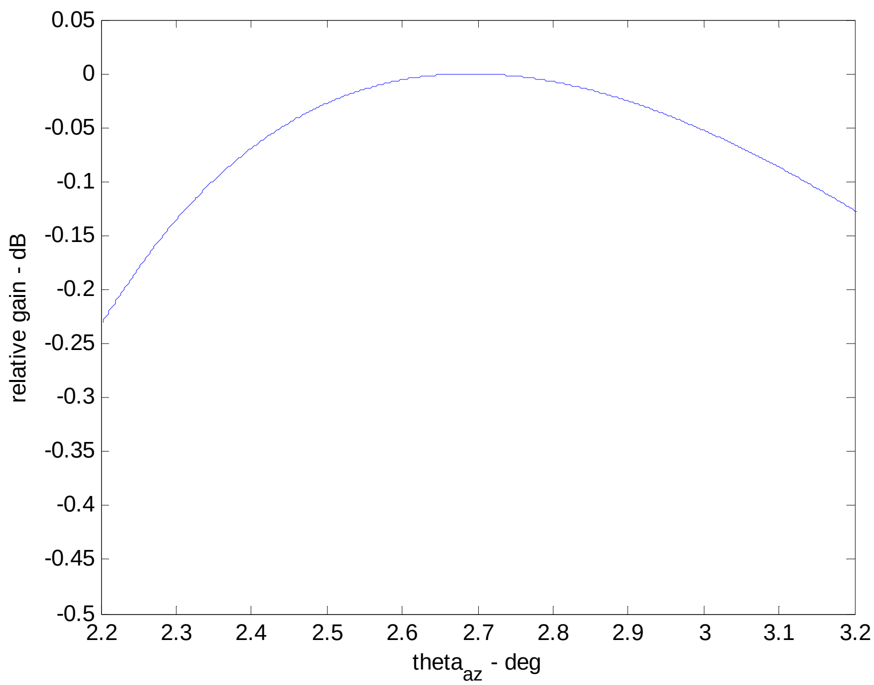

Figure 3. Close-up of peak response in Figure 2.

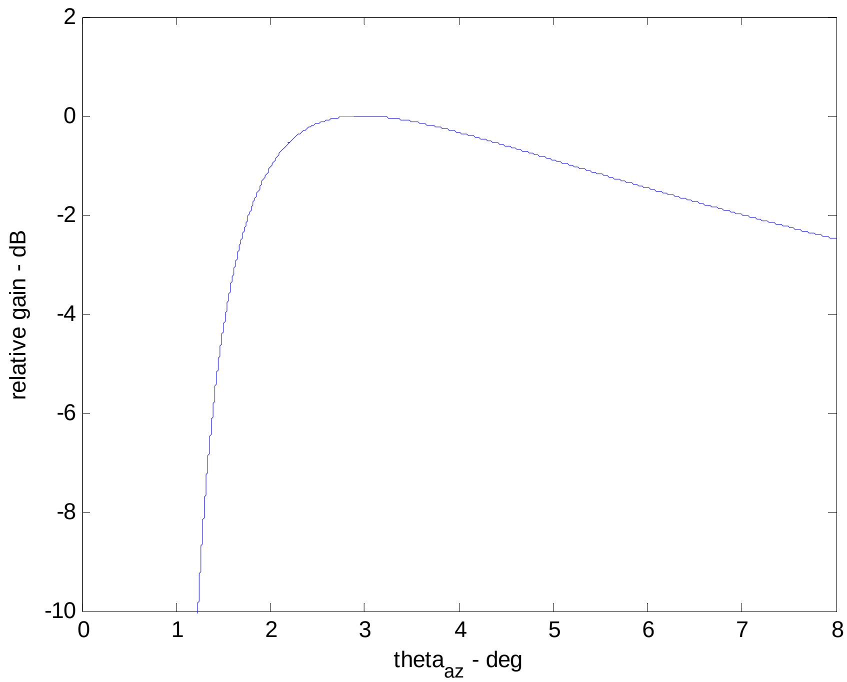

Figure 4. Same as Case 2 (Figure 2) except with effects of presummer.

3 CONCLUSIONS

Some key points to remember include:

- SAR imaging should consider the SNR over the whole image patch.
- There is a clear optimum value for beamwidth to achieve a maximum SNR at the image patch edge.
- A stripmap SAR is best served with an antenna beamwidth wider than the classical equations suggest.
- The exact optimal –3 dB beamwidth will depend on the specific roll-off characteristics of the antenna pattern.
- A reasonable first guess for the optimal beamwidth required for 0.3 m resolution stripmap at Ku-band using presumming is in the 3 degree range.

ACKNOWLEDGEMENTS

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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[1] Mark A. Richards, *Fundamentals of Radar Signal Processing*, ISBN 0-07-144474-2, McGraw Hill, 2005.