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Test Slide

Fault tolerant quantum computing with o

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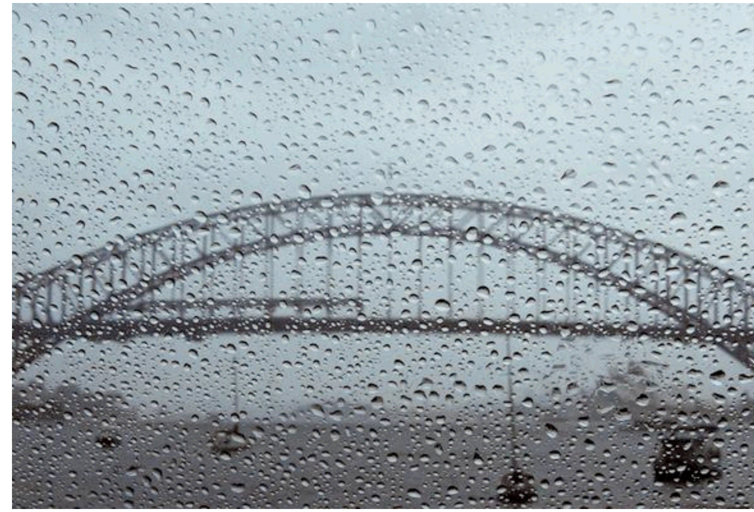
This work was supported by United States National Science Foundation and the Laboratory Directed Research and Development program at Sandia National Laboratories. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin company, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Fault-tolerant quantum computing

It's all about simulation.



Ideal

Circuit C

Size L

Faulty

Circuit C'

Size $O(\varepsilon^{-1} L \log^{c} L)$

Theorem: Simulation possible when elementary ops fail with error probability below accuracy threshold p_c.

Proof: By some explicit construction having a particular c, p_c and big-O constant.

Assumption junction

- Nonincreasing error rate
- Parallel operation
- Refreshable qubits

Necessary

- Reliable classical computation
- Fast classical computation
- No qubit leakage
- Uncorrelated noise

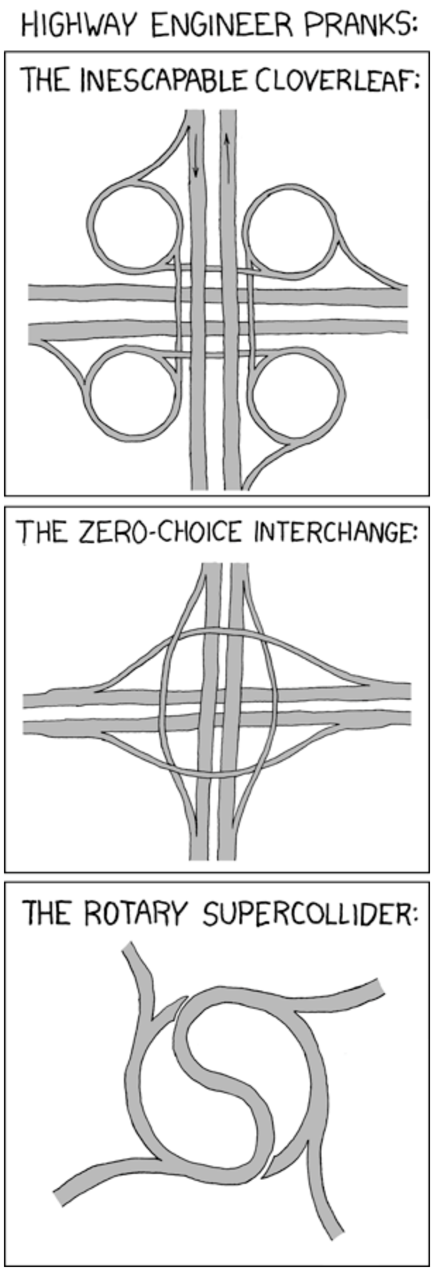
Helpful

- Standard gate basis
- Equal-time gates
- Uniformly faulty gates

Convenient

- 2D layout
- Local quantum processing

Realistic



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Quintessential fault tolerance

1. Code Family
   - Color codes (triangular, 4.8.8)
2. Syndrome extraction protocol
   - CNOT schedule into an ancilla
3. (Syndrome) ancilla distillation protocol
   - None (N/A).
4. (Classical) decoding algorithm
   - Most-likely error (MLE) decoding
5. Encoded universal gate basis
   - Clifford group by transversal ops + code deformation
   - *T* gate by magic state $|T\rangle$ (encoded or injected)

<u>Strategy:</u> Given an ε, dial up a large enough code.

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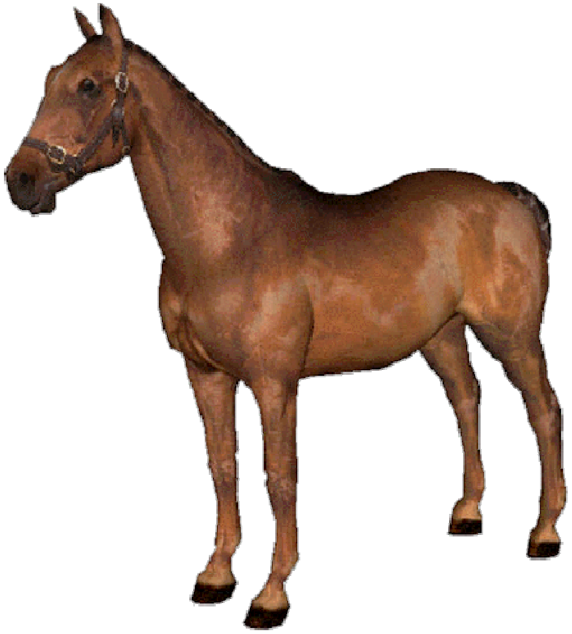
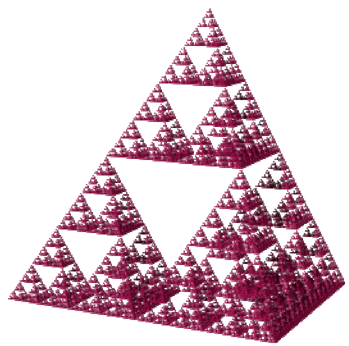
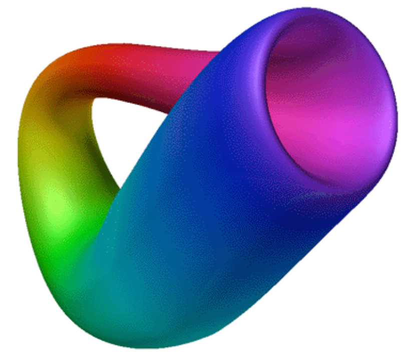
Results

Before defining color codes, the syndrome extraction protocol, the decoder, etc., let's review the accuracy threshold we estimated...



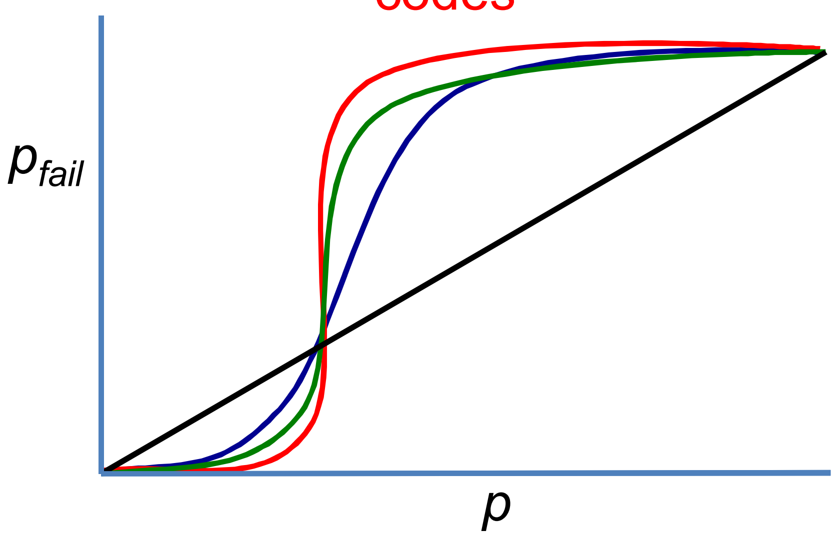
Qualitative threshold estimate

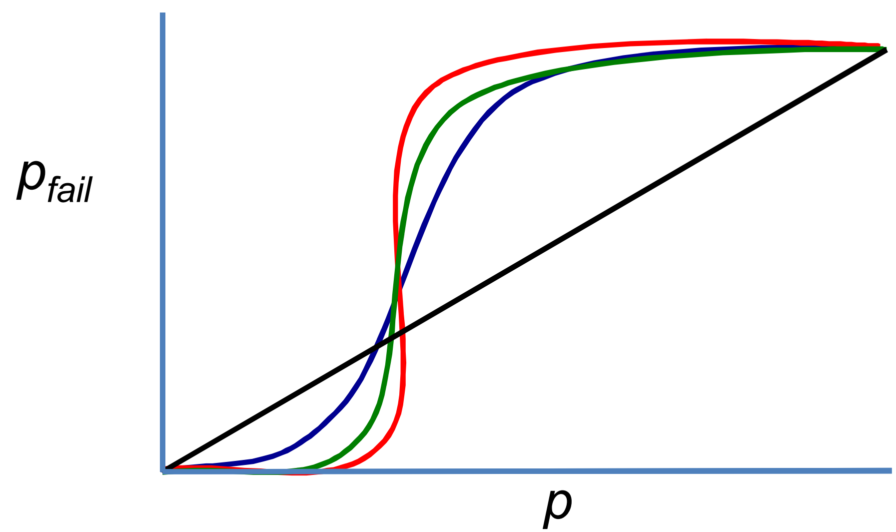




Concatenated codes

Topological codes (so far)





Computing the threshold

- Repeat for increasing code distances *d* (until it becomes intractable)
  - Repeat for multiple fault rates per operation *p* near the th p_c
    - Repeat *N* times, where *N* is on the order of $100/(p_c)^2$
      1. Throw down errors
      2. Decode and record success/failure (subtle)

$$p_{fail} \approx \frac{N_{fail}}{N} \pm \sqrt{\frac{p_{fail}(1-p_{fail})}{N}}$$

    -

$$p_{fail} = A + B(p-p_c)d^{-\nu}$$

- Fit data using method of differential corrections* to a form such as

$$p_{fail} = A + B(p-p_c)d^{-\nu} + C(p-p_c)^2 d^{-\nu}$$

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*statpages.org/nonlin.html

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Three noise models

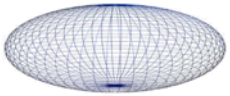
1. Bit-flip channel followed by phase-flip channel on data qubits only. ("Code capacity")

2. Same, but also bit-flip channel on syndrome qubits. ("Phenomenological noise model")

3. Bit-flip channel followed by phase-flip channel on data qubits and on each gate in the syndrome extraction circuit. ("Circuit-based noise model")

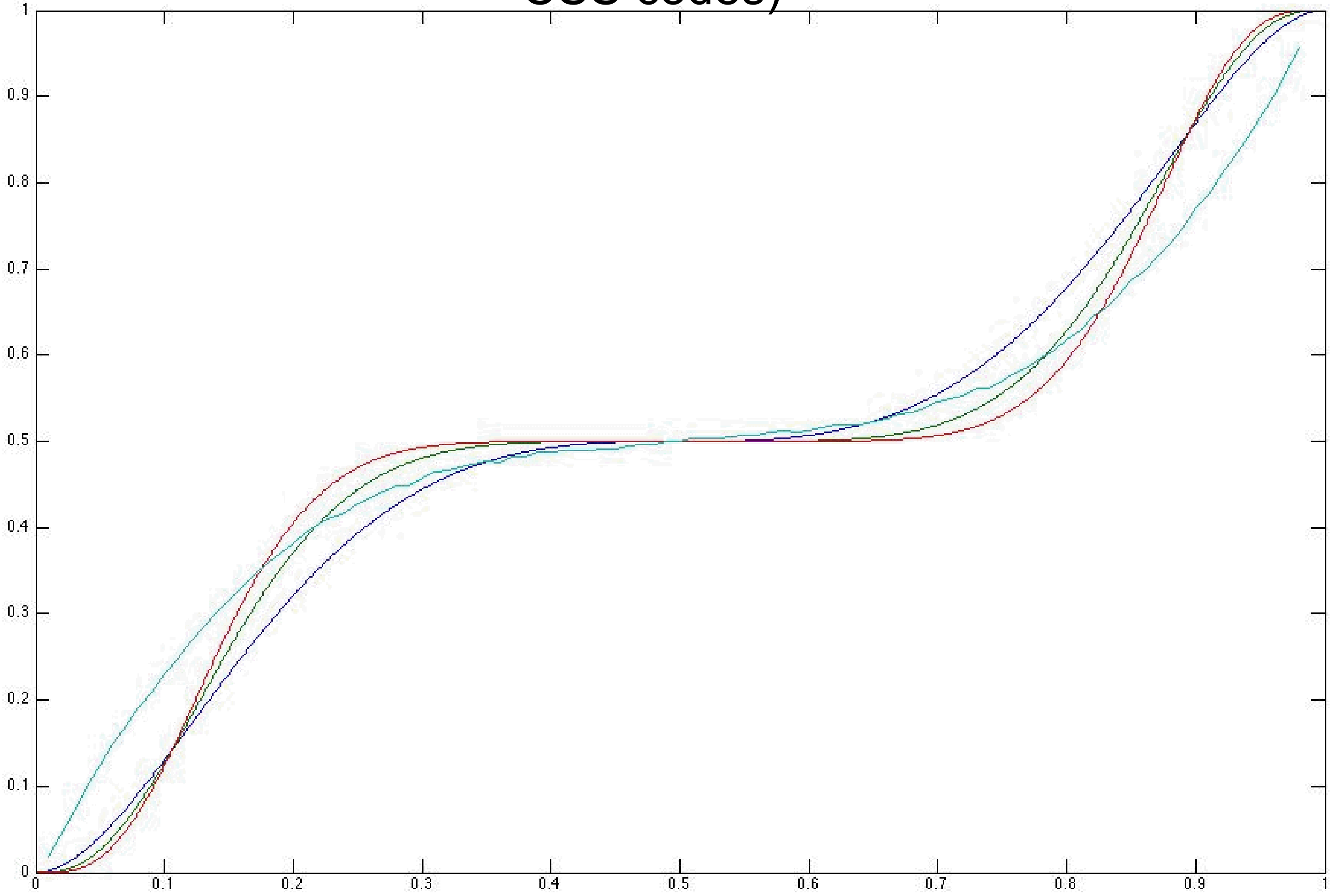
$$\rho \mapsto (1-p)\rho + pZ\rho Z$$

$$\rho \mapsto (1-p)\rho + pX\rho X$$



Code capacity

10.5% (cf. 10.3% for MLE decoding of toric code; 10.9% optimal decoding of toric, 4.8.8 color codes; 11% bound for CSS codes)



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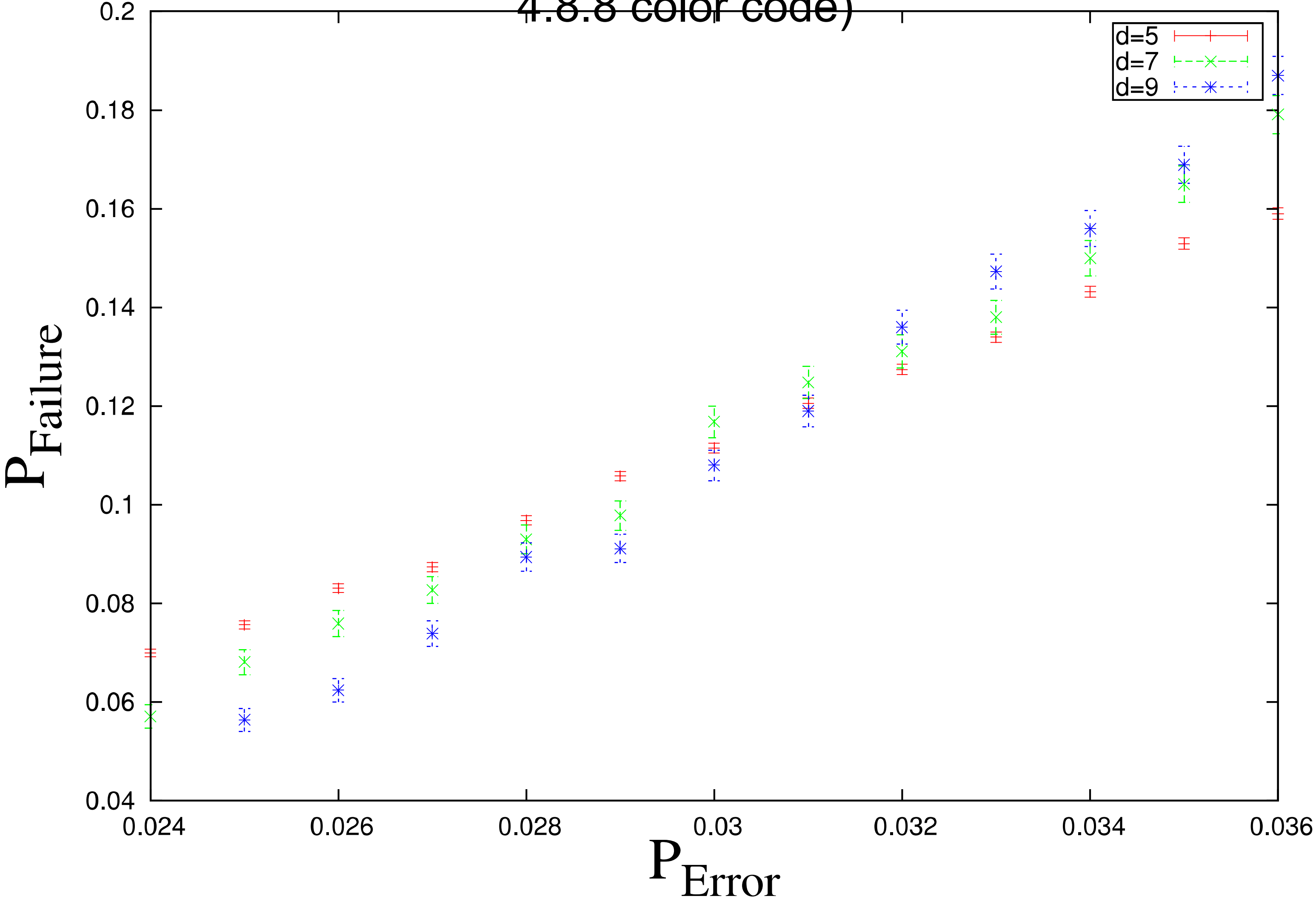
“Phenomenological” FTQC threshold

3.05% (cf. 2.94% for MLE decoding of toric code; 3.30% optimal decoding of toric code; 4.5% optimal decoding of 4.8.8 color code)



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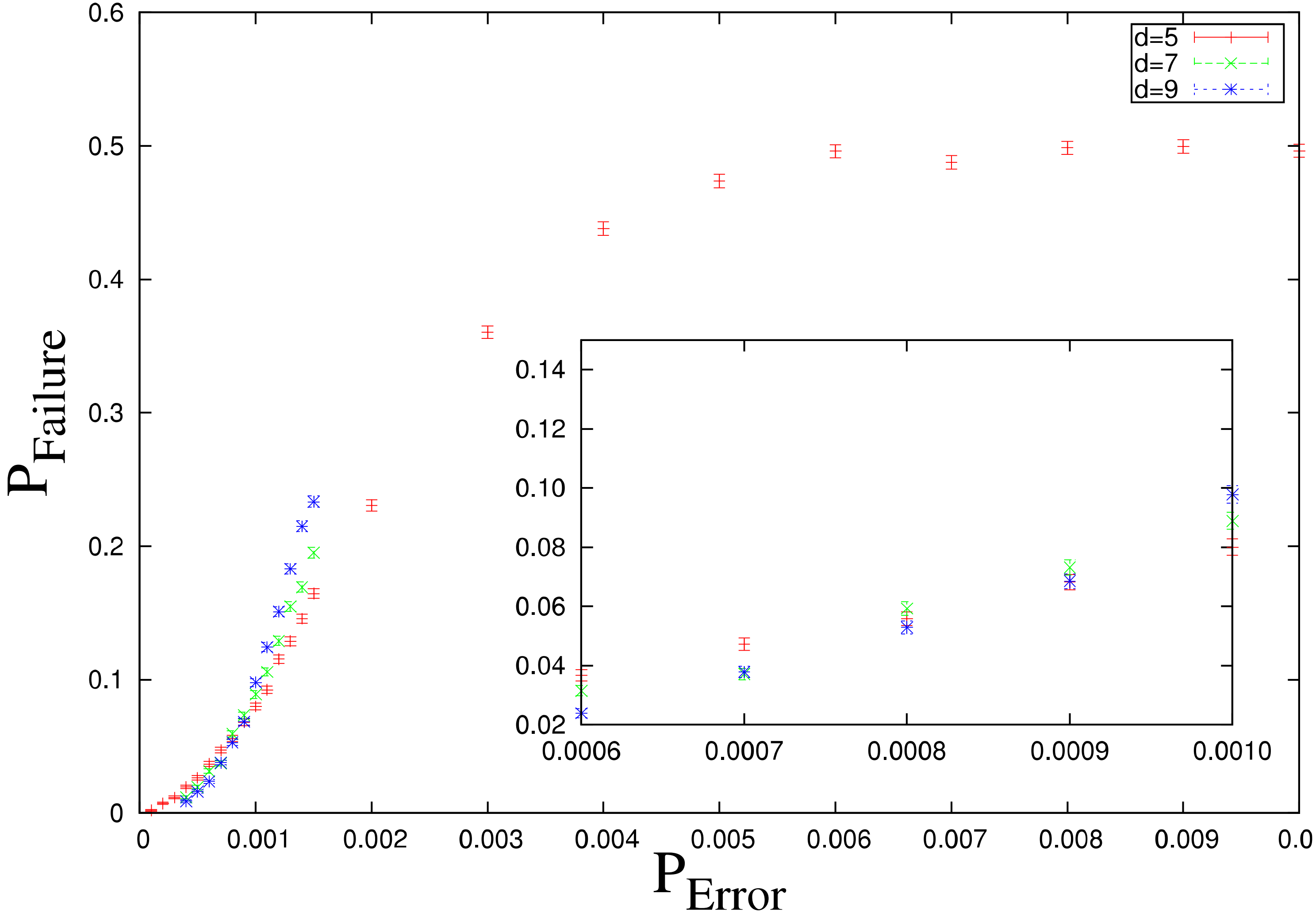
Circuit-based FTQC threshold

0.085% (cf. 0.75% to 1.1% for MLE decoding of toric code)

3.05×10^{-5} rigorous lower bound by combinatorial counting (cf. 1.7×10^{-4} rigorous lower bound by combinatorial counting for toric code)



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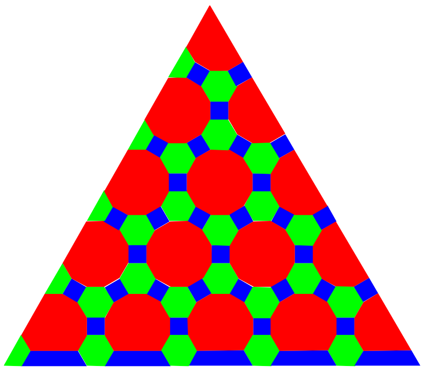
The take-home massage

The FTQC accuracy threshold for color codes is about 1/10 that of toric codes

But...

1. The threshold may increase for other FTQC schemes.

2. Color codes need far fewer magic state resources and use half the qubits for a comparable toric code.



Background

Let's define color codes, the syndrome extraction protocol, the decoder, etc., that we used, some of which we developed ourselves.

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How most people depict a toric code

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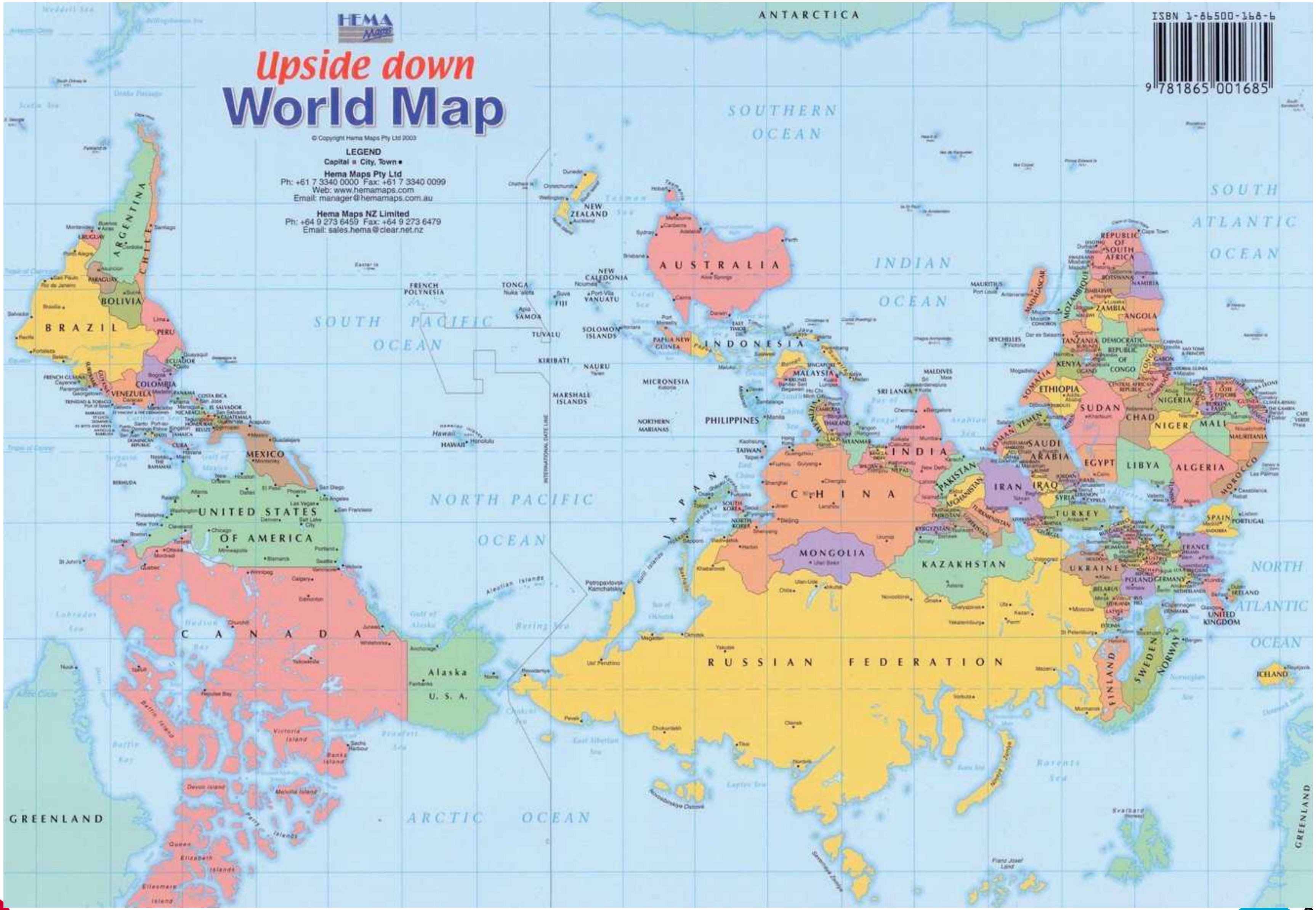
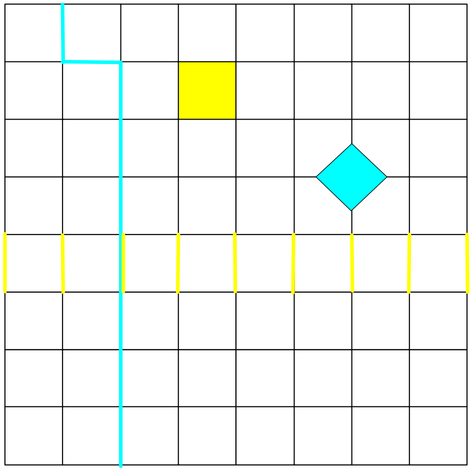
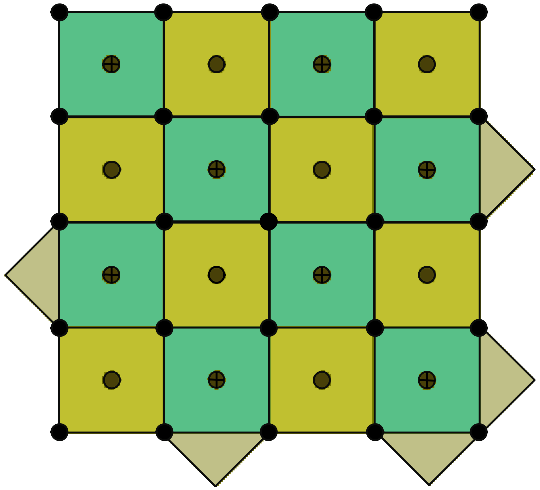
The right way to depict a toric code

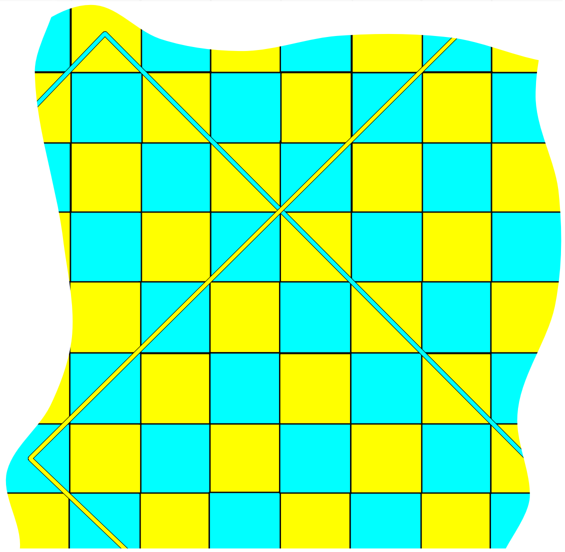


Bombinize!





Medial graph: combines a graph and its dual



Theorem: All medial graphs are 4-valent and face-2-colorable.

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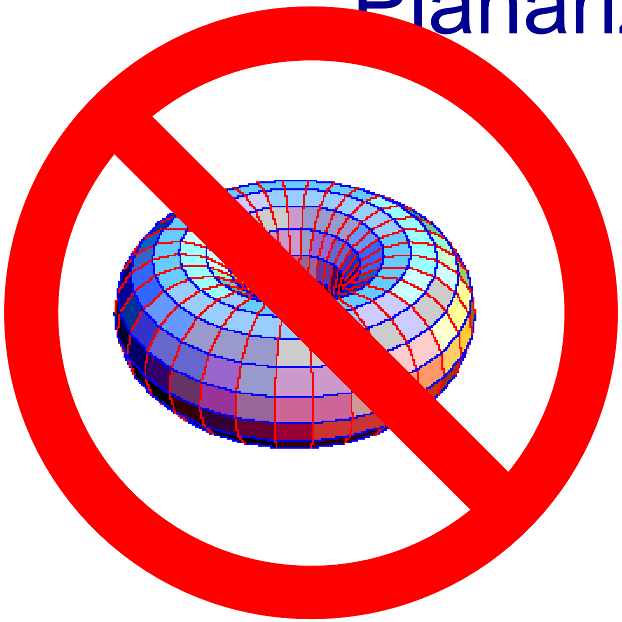
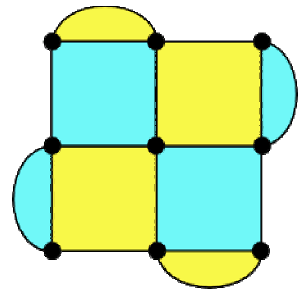
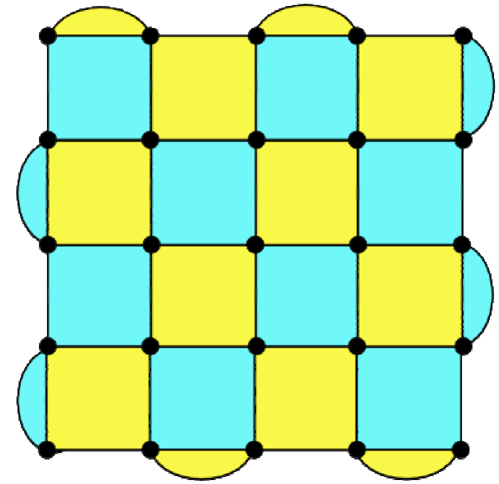
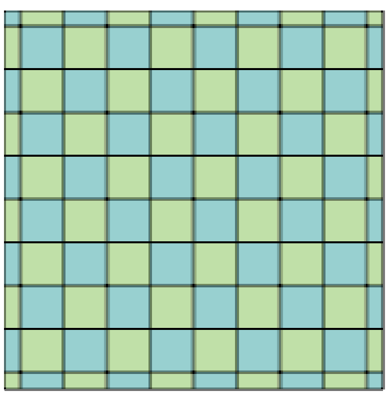
Planarize, planarize, planarize!





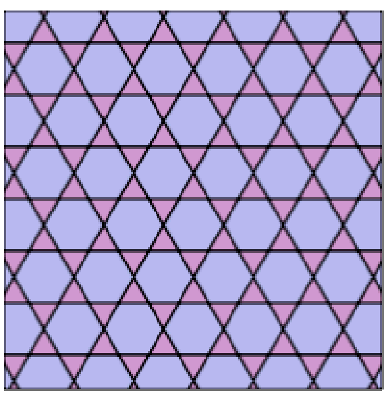


Vertex-transitive 4-valent face-2-colorable lattices:

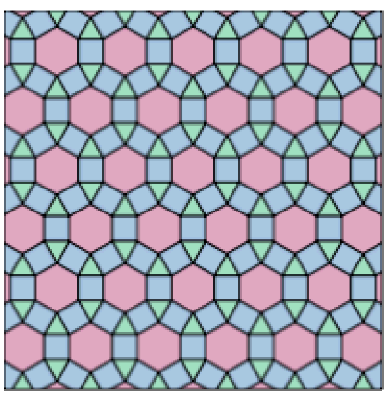


4.4.4.4

(Square)



3.6.3.6 (Kagome)



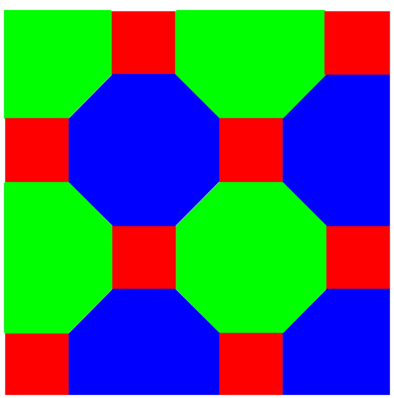
3.4.6.4

Color codes

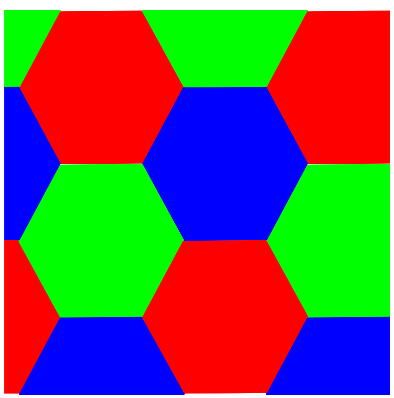
Defined for 3-valent face-3-colorable graphs

Each face in such a graph has an even number of vertices and edges

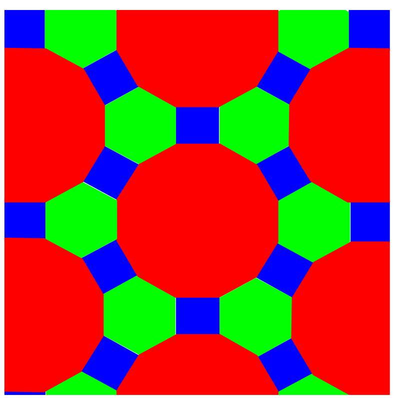
Vertex-transitive 3-valent face-3-colorable lattices:



4.8.8



6.6.6 (Hex)



4.6.12

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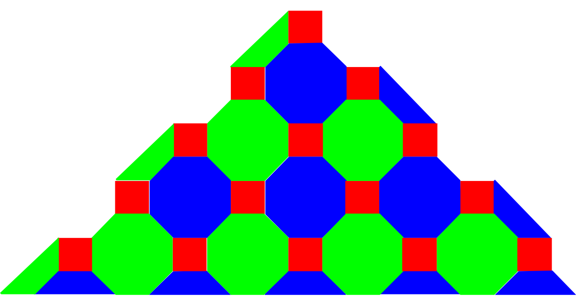
Planar color codes

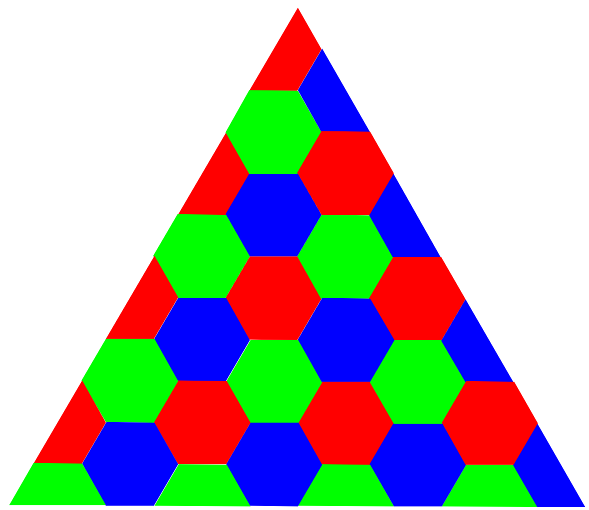
Check operators (stabilizer generators) $X_{\text{face}} = X^{\otimes i \in \text{face}}$ $Z_{\text{face}} = Z^{\otimes i \in \text{face}}$

Logical operator generators $\overline{X} = X^{\otimes i \in \text{side}}$ $\overline{Z} = Z^{\otimes i \in \text{side}}$



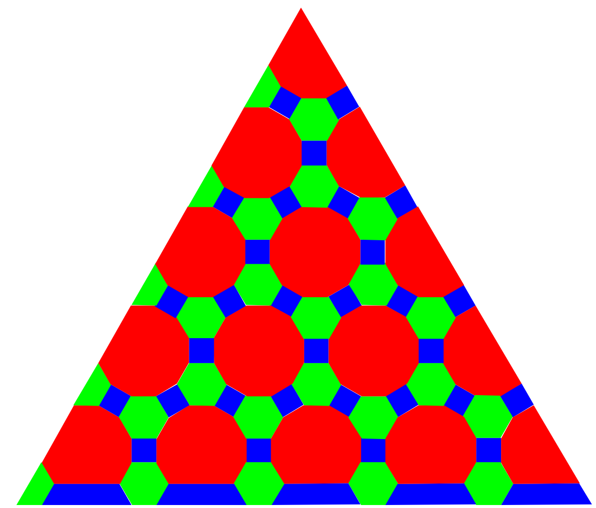
4.8.8

$n = \frac{1}{2}d^2 + d - \frac{1}{2}$



6.6.6 (Hex)

$n = \frac{3}{4}d^2 + \frac{1}{4}$



4.6.12

$n = \frac{3}{2}d^2 - 3d + \frac{5}{2}$

The 4.8.8 code uses half the qubits of the Kitaev surface code asymptotically!

All these codes have the [[7, 1, 3]] Steane code as their base.

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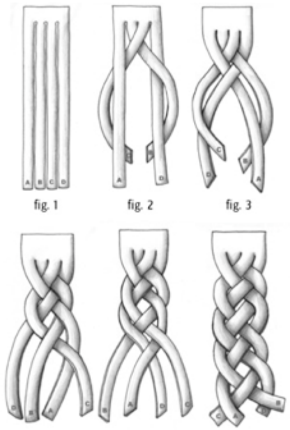
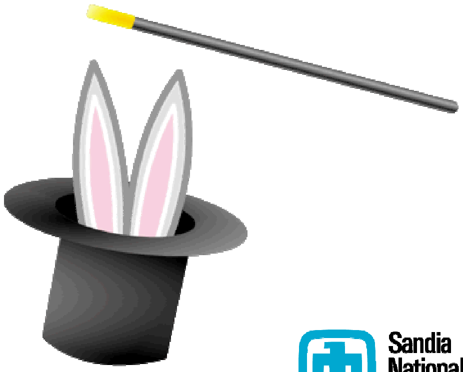
Encoded computations

Can show that encoded *H*, *S*, M_Z, *CNOT* (encoded Clifford circuits) are transversal

Can perform *CNOT* by braiding, just as for toric codes (and only between *Z*-type and *X*-type defects, as with toric codes).

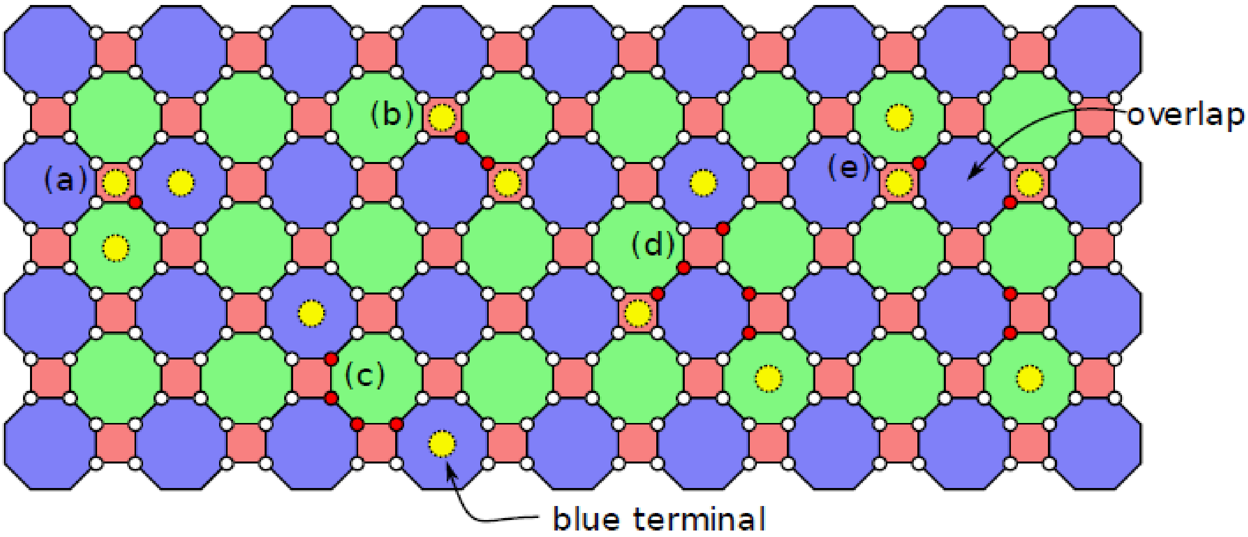


Need a magic state to get outside Clifford group. Use *T* state, *e.g.*



Most-likely error (MLE) decoding

Wang *et al*.: Heuristic decoder using perfect matching.



MLE decoder:

$$\min \sum_v x_v$$

$$\text{sto} \bigoplus_{v \in f} x_v = s_f \quad \forall f$$

$$x_v \in \mathbb{B} := \{0, 1\}$$

Canonical integer program (IP) form

$$\min \mathbf{1}^T \mathbf{x}$$

$$\text{sto}\, H\mathbf{x} = \mathbf{s}$$

$$\mathbf{x} \in \mathbb{B}^n$$

NP-hard in general.

For this class of codes, who knows?

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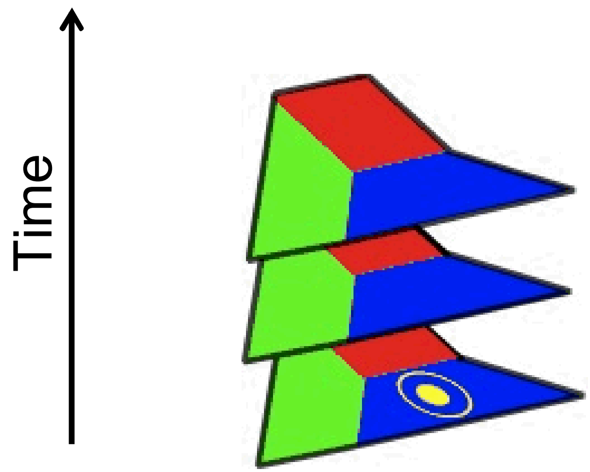
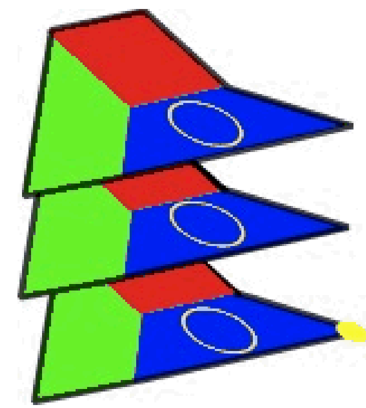
Fault-tolerant MLE decoding



Syndrome measurement error



Data error

Δs = *changes* in syndrome at each time step

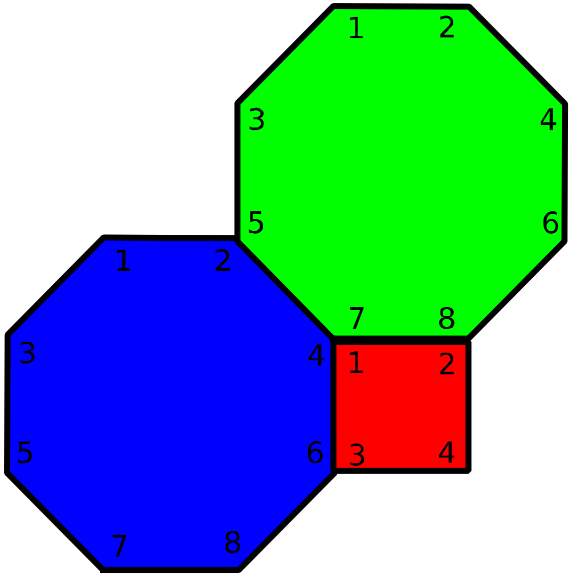
$$x = (\underbrace{d_{\text{round } 1} \cdots d_{\text{round } n}}_{\text{data qubits}} \; \underbrace{a_{\text{round } 1} \cdots a_{\text{round } n}}_{\text{ancilla qubits}})$$

Modified IP

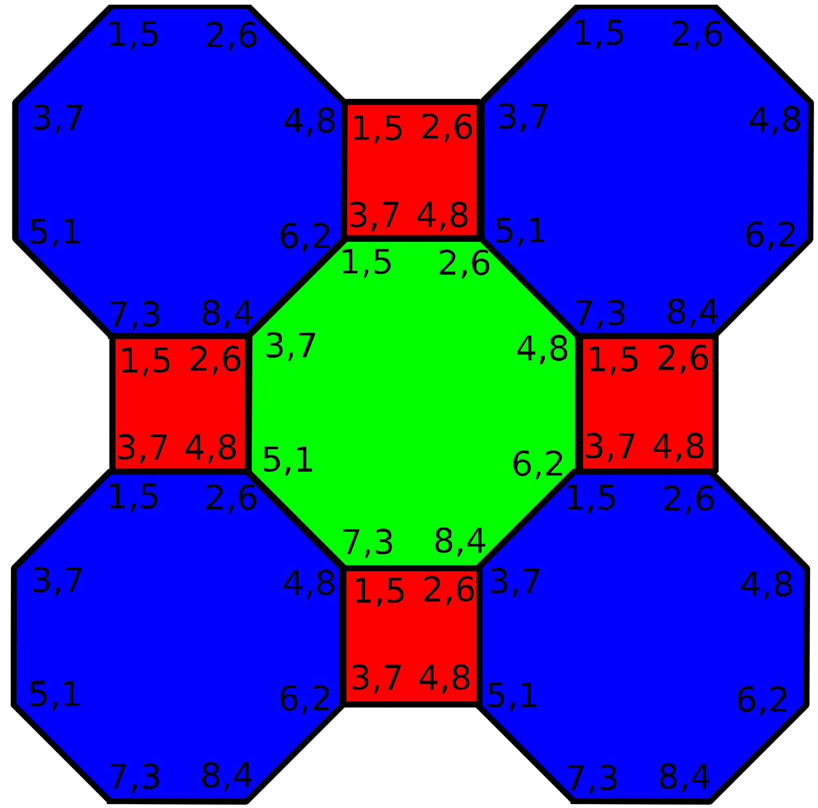
$$\min \mathbf{1}^T \mathbf{x}$$
$$\text{sto } A\mathbf{x} = \Delta \mathbf{s}$$
$$\mathbf{x} \in \mathbb{B}^{n^2+Fn}$$

Syndrome extraction schedule

16-step schedule



8-step schedule



Can show that these schedules propagate errors at most a bounded radius away in the syndrome extraction circuit

Quintessential fault tolerance

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Strategy: Given an ε, dial up a large enough code.

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Open questions

1. Linear program relaxation of IP?
2. Threshold against depolarizing noise?
3. Threshold for optimal decoding?
4. Threshold against leakage?
5. Cat state ancillas?
6. Better rigorous lower bounds?
7. 3D color codes? (Only magic state is logical $|+\rangle$).

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