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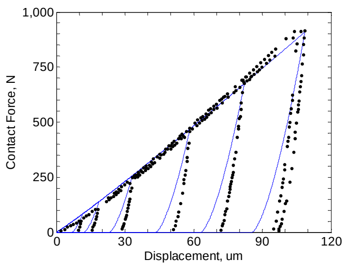

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An Analytical Elastic Plastic Contact Model with Strain Hardening

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November, 2013

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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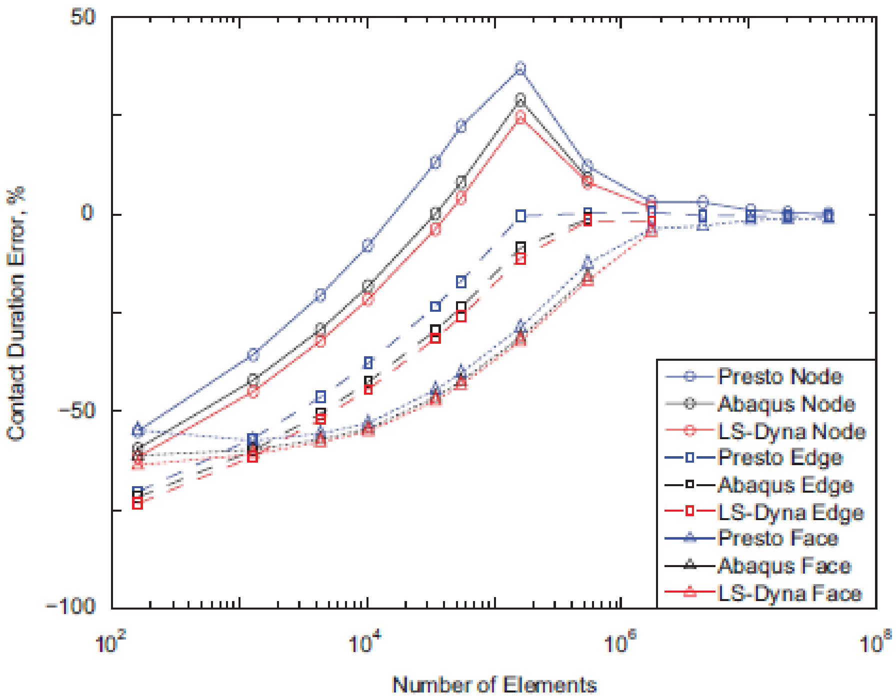
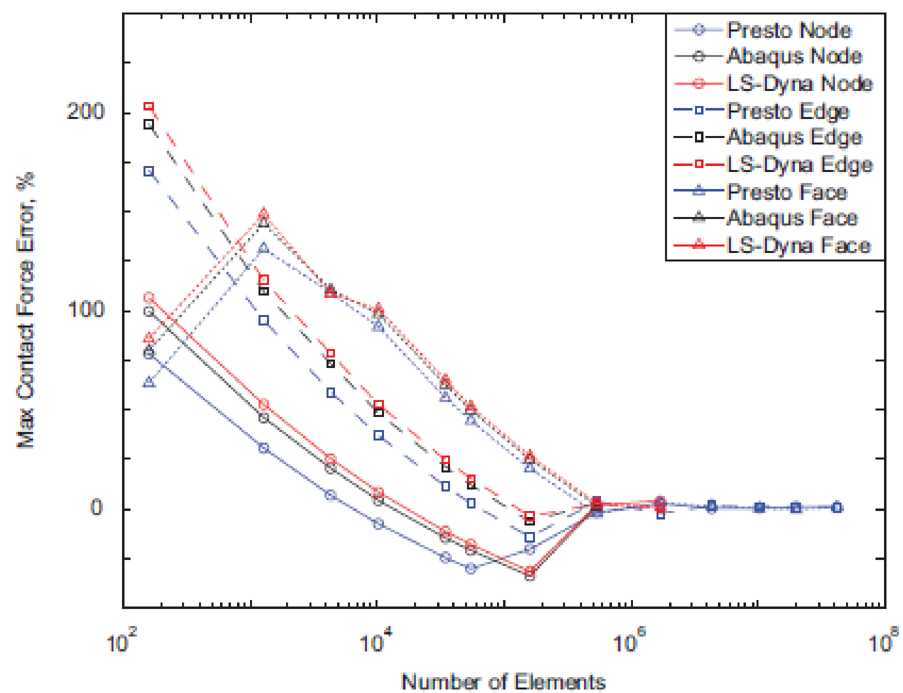
High Fidelity Numerical Modeling Is Impractical For General Problems

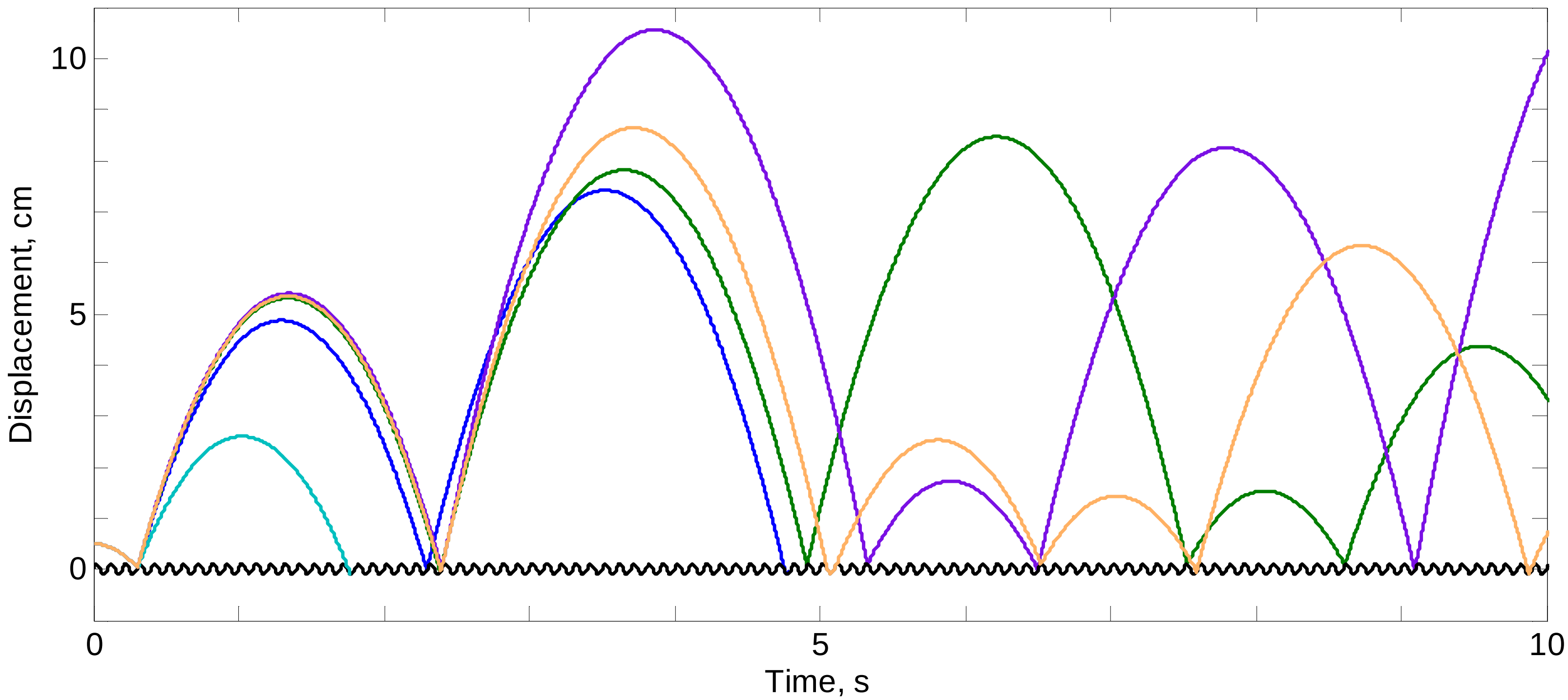




- Numerical solutions compared to analytical (Hertzian) solution for elastic contact
- An impractical number of elements is required to accurately model contact in just the elastic regime

Nonlinearities Can Lead to Significantly Different System Responses

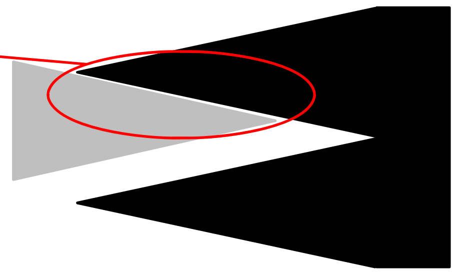
- Small changes in the input parameters can lead to large changes in the response
- High fidelity modeling can only take us so far…to probabilistically explore the design spaces of complex mechanisms, we need efficient models
- Imperative that we have accurate and efficient contact and dissipation laws



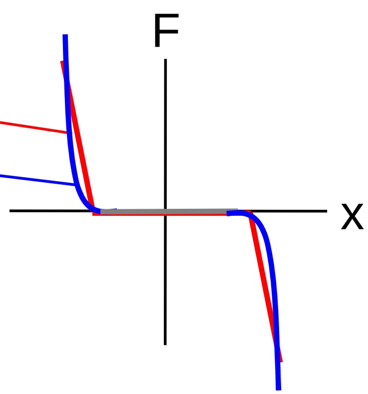
Shown: coefficient of restitution (green), Piecewise-Linear (purple), the present elastic-plastic (blue), a similar elastic plastic (orange), and a dissimilar elastic plastic (cyan)

Considerations for Constitutive Modeling

- Physical process
  - Plasticity
  - Localized yielding
  - Asperities
  - etc.

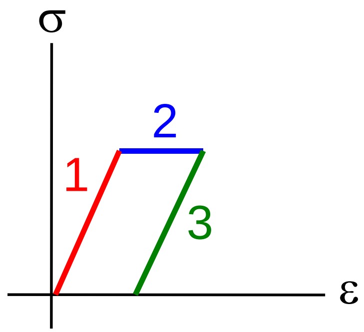


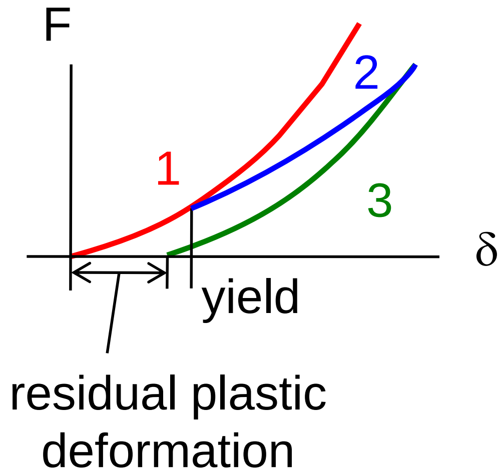
- Methods of simplified impact dynamics modeling
  - Iwan-type models (bolts, joints, frictional interfaces)
  - Penalty stiffness
  - Hertzian contact
  - Coefficient of restitution
  - Elastic-Plastic models



Simplified Elastic Plastic Impact Modeling

- Deflection divided into three phases:
  - Elastic loading (1)
    - Hertzian force-deflection relationship
    - Spans from the initial contact until the onset of yielding
  - Mixed Elastic-Plastic loading (2)
    - Elastic forces hypothesized to decrease smoothly
    - Plastic forces hypothesized to increase smoothly
  - Elastic unloading (3)
    - Hertzian, but with a different contact radius than for loading
    - A portion of the plastic deflection is unrecoverable

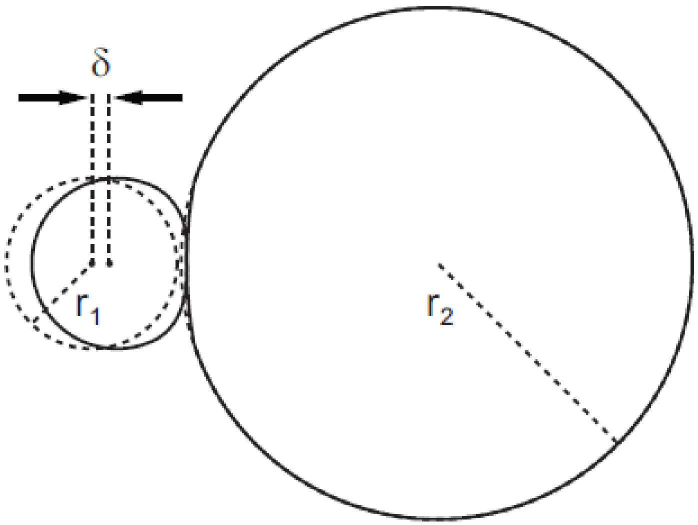




Elastic Regime

- Hertzian Loading

$$F = \frac{4}{3} E \sqrt{r} \delta^{3/2} \qquad a = \sqrt{r\delta}$$

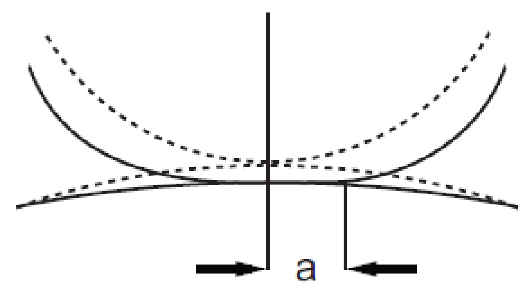


- With effective properties

$$E = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{-1}$$

$$H = \left(\frac{2}{H_1} + \frac{2}{H_2} \right)^{-1}$$

$$r = \left(\frac{1}{r_1} + \frac{1}{r_2} \right)^{-1}$$



- Inception of yield based on stress field and the von Mises criterion

$$f(\nu) = \max_{u \in \mathbb{R}} \left(-(1+\nu) \left(1 - u \, \mathrm{atan} \left(\frac{1}{u} \right) \right) + \frac{3}{2} \frac{1}{1+u^2} \right)^2$$

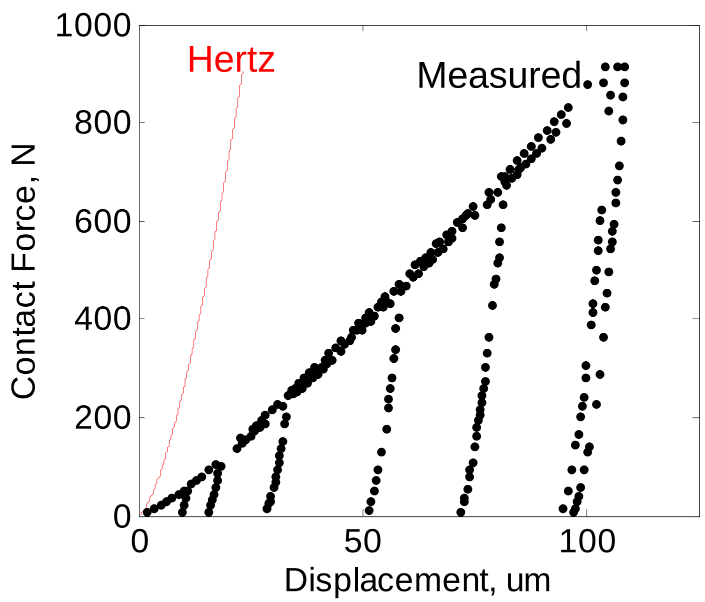
$$\delta_y = \frac{r}{f(\nu)} \left(\frac{\pi \sigma_y}{2E} \right)^2$$

Plastic Forces

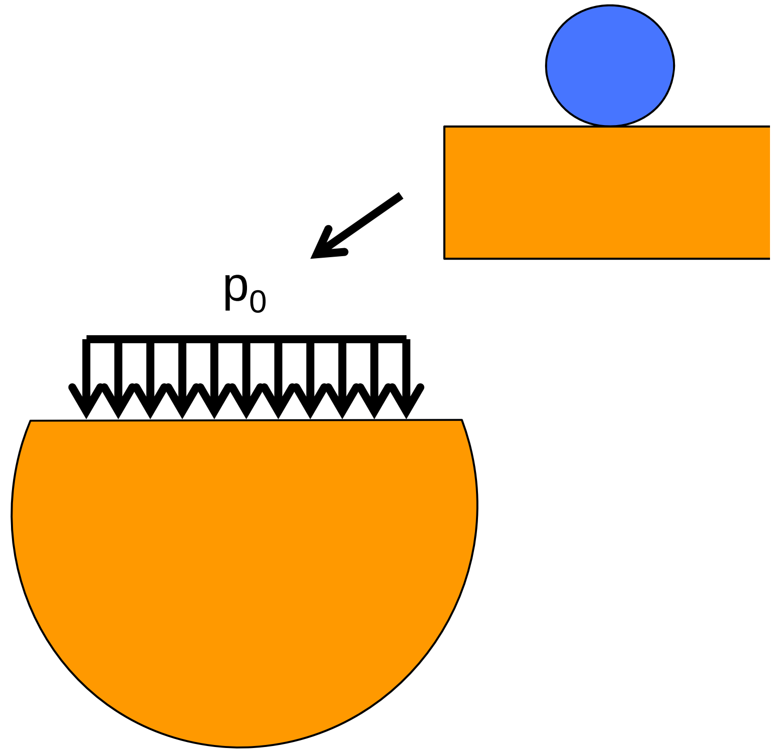
- Assumes uniform pressure distribution and conservation of volume

$$F = p_0 \pi \frac{a^n}{a_p^{n-2}} \qquad a^2 = 2r\delta \qquad p_0 = Hg10^6$$

- Strain hardening incorporated with Meyer's hardness exponent *n*.



Stainless Steel measurements from Bartier, Hernot, and Mauvoisin, "Theoretical and Experimental Analysis of Contact Radius for Spherical Indentation," Mechanics of Materials, 42 (2010), 640-656



Elastic Plastic Transitionary Behavior

$$F_{EP} = \phi_1(\delta)F_E(\delta) + \phi_2(\delta)F_P(\delta)$$

1. The transitionary functions must recover the elastic force at the onset of yield

$$\phi_1(\delta_y) = 1 \quad \text{and} \quad \phi_2(\delta_y) = 0.$$

2. The contact force must always increase with displacement

$$\frac{\mathrm{d}F_{EP}}{\mathrm{d}\delta} > 0 \quad \text{for} \quad \delta \geq \delta_y.$$

3. The rate at which the contact force increases with displacement cannot decrease

$$\frac{\mathrm{d}^2F_{EP}}{\mathrm{d}\delta^2} \geq 0 \quad \text{for} \quad \delta \geq \delta_y.$$

4. The Hertzian contact force (F_E) is greater than the elastic plastic contact force after yield occurs

$$F_E \geq F_{EP} \quad \text{for} \quad \delta \geq \delta_y.$$

5. The elastic plastic contact force is greater than the plastic contact force due to the contribution of the elastic contact force

$$F_{EP} \geq F_P \quad \text{for} \quad \delta \geq \delta_y.$$

6. The elastic plastic contact force approaches the value of the plastic contact force for large displacements

$$\lim_{\delta\to\infty} \phi_1(\delta) \to 0 \quad \text{and} \quad \lim_{\delta\to\infty} \phi_2(\delta) \to 1.$$

7. The contact force must be smooth across the inception of yield

$$\frac{\mathrm{d}\phi_1(\delta_y)}{\mathrm{d}\delta} = 0 \quad \text{and} \quad \frac{\mathrm{d}\phi_2(\delta_y)}{\mathrm{d}\delta} = 0.$$

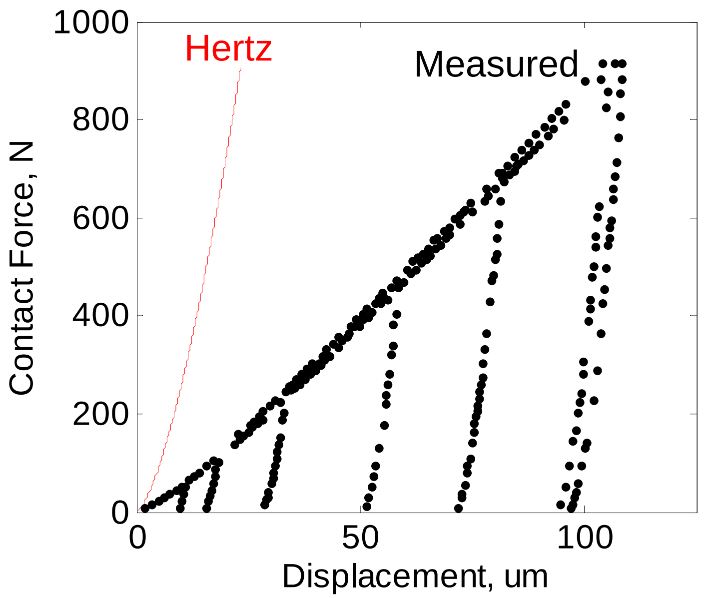
8. The transitionary functions are bounded

$$0 \leq \phi_1(\delta) \leq 1 \quad \text{and} \quad 0 \leq \phi_2(\delta) \leq 1 \quad \text{for} \quad \delta \geq \delta_y.$$

9. The derivative of the contact force must be smooth across the inception of yield

$$\frac{\mathrm{d}F_E(\delta_y)}{\partial\delta} = \frac{\mathrm{d}F_{EP}(\delta_y)}{\partial\delta}.$$

These assumptions are to enforce smoothness in the compliance curve…



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Elastic Plastic Transitionary Behavior

- From the previous assumptions, the transitionary behavior is derived

$$a_p = \left(\frac{3p_0}{4E} 2^{n/2} \pi r^{n/2-1/2} \delta_y^{n/2-3/2}\right)^{1/(n-2)}$$

$$\phi_1(\delta) = \text{sech}\left((1+\xi)\frac{\delta - \delta_y}{\delta_p - \delta_y}\right)$$

$$\xi = n - 2$$

$$\phi_2(\delta) = 1 - \text{sech}\left((1-\xi)\frac{\delta - \delta_y}{\delta_p - \delta_y}\right)$$

$$F(\delta) = \text{sech}\left((1+\xi)\frac{\delta - \delta_y}{\delta_p - \delta_y}\right)\frac{4}{3}E\sqrt{r}\delta^{3/2} + \text{sech}\left((1-\xi)\frac{\delta - \delta_y}{\delta_p - \delta_y}\right) p_0 \pi \frac{a^n}{a_p^{n-2}}$$

$$a = \sqrt{r\delta\left(2 - \text{sech}\left(\xi\frac{\delta - \delta_y}{\delta_p - \delta_y}\right)\right)}$$

Restitution Model

- Unloading is elastic

$$F = \frac{4}{3}E\sqrt{\bar{r}}\left(\delta - \bar{\delta}\right)^{3/2}$$

- Assumption: the residual plastic deformation is proportional to the elastic plastic force/equivalent elastic force

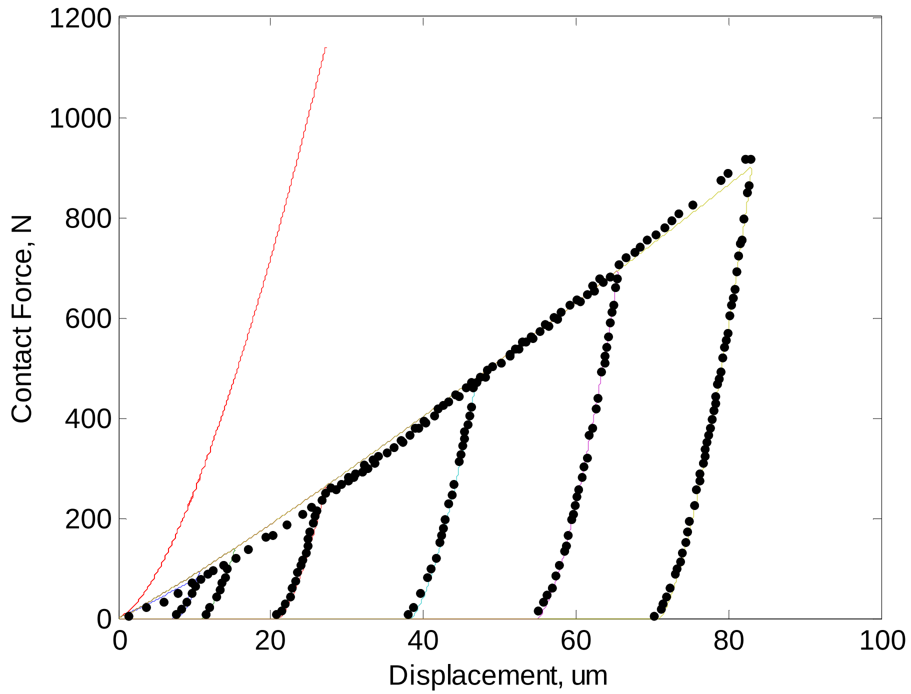
$$\bar{\delta} = \delta_m \left(1 - \frac{F_m}{4/3E\sqrt{\bar{r}}\delta_m^{3/2}}\right)$$

- Modified contact radius based off of compatibility

$$\bar{r} = \frac{F_m^2}{(4/3E)^2\left(\delta_m - \bar{\delta}\right)^3}$$

- Fully prescribes contact model in terms of material properties
- No tuning or calibration parameters

Direct Validation



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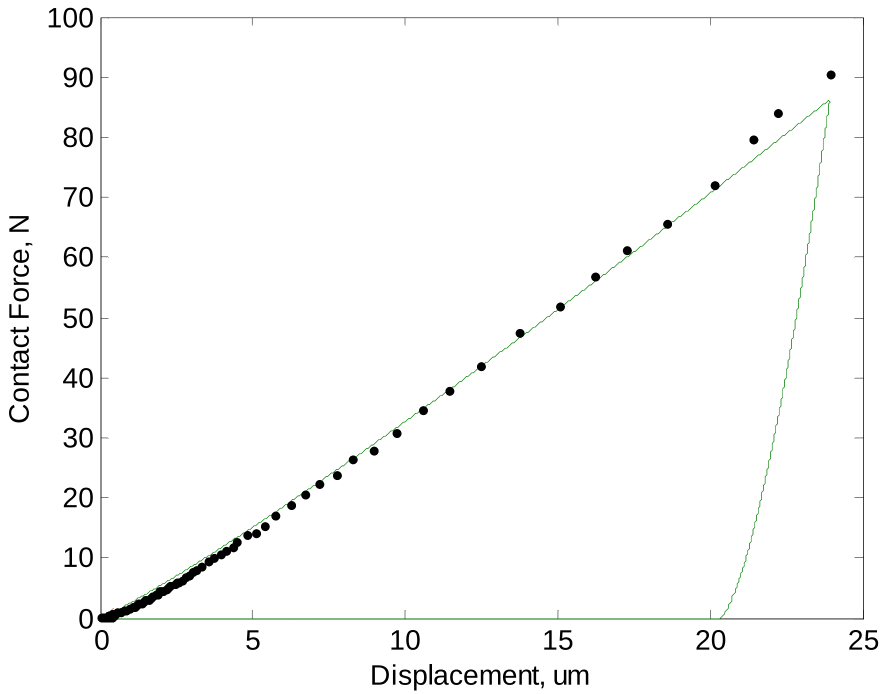

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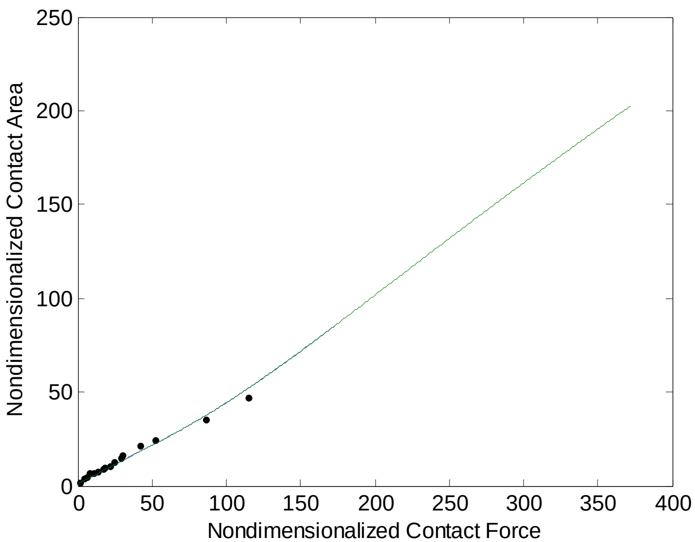

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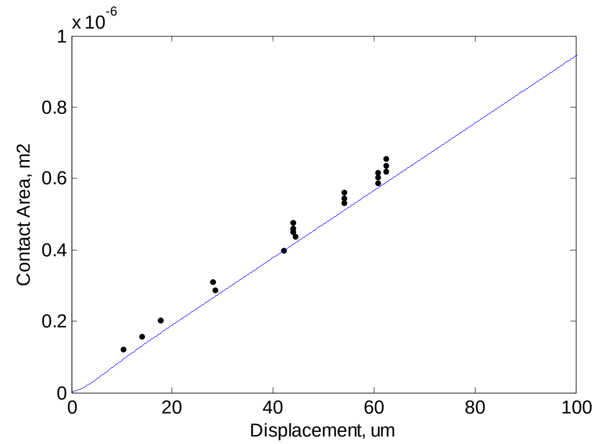
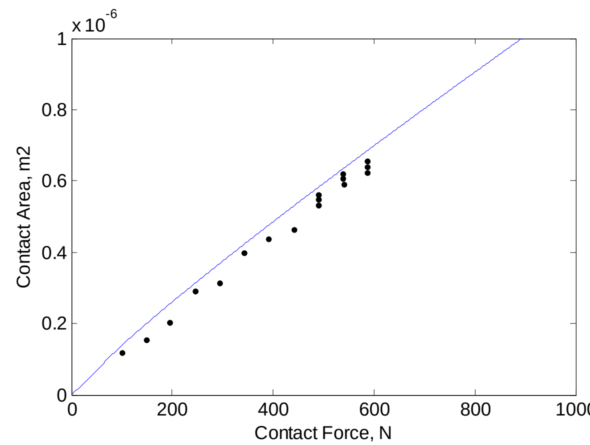
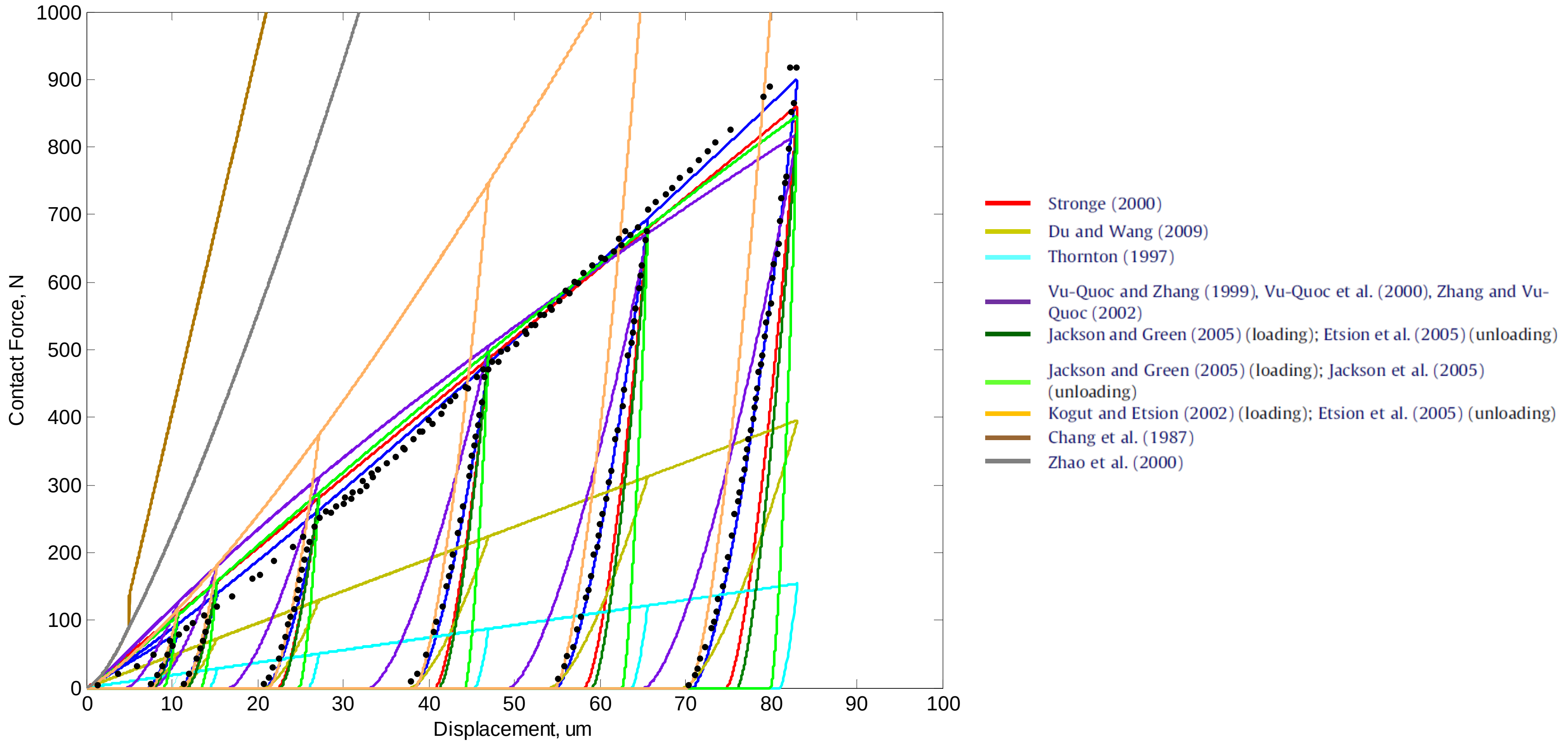




Copper measurements from Jamari and Schipper, “Experimental Investigation of Fully Plastic Contact of a Sphere Against a Hard Flat,” ASME Journal of Tribology, 128 (2006), 230-235

Comparison to Other Models



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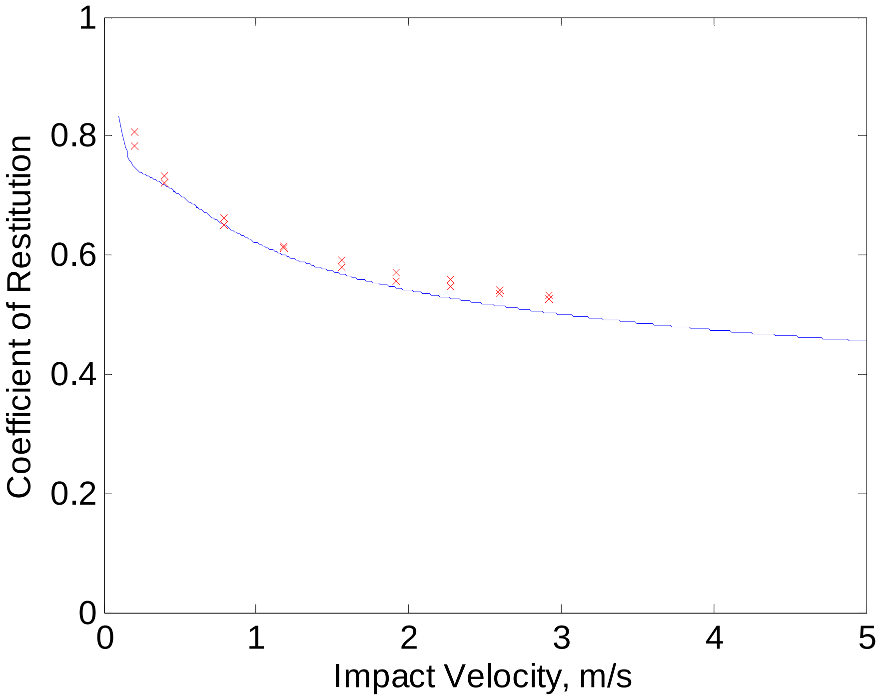
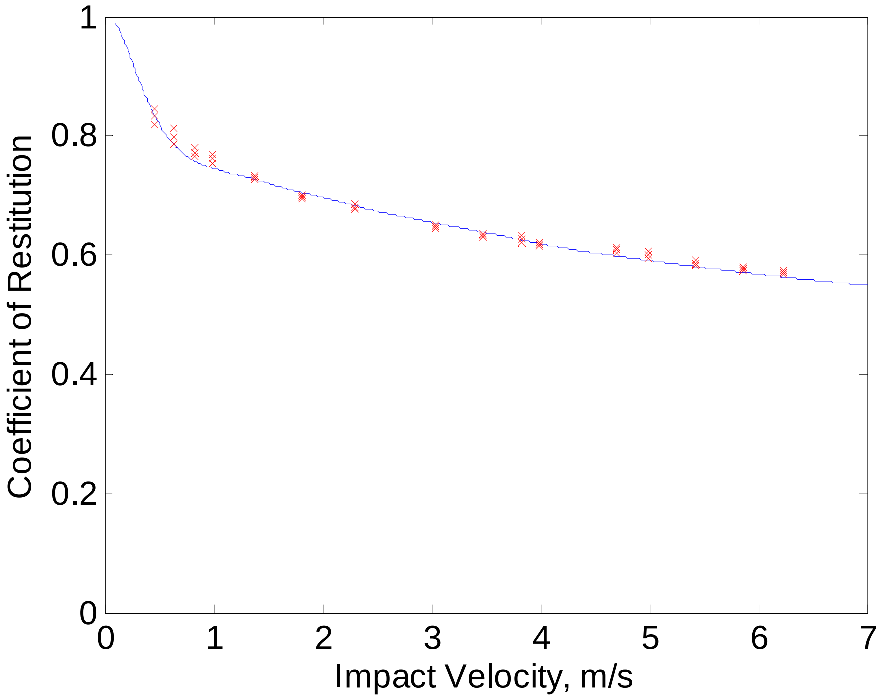
Indirect Validation



Stainless Steel measurements from Minamoto and Kawamura, “Effects of Material Strain Rate Sensitivity in Low Speed Impact Between Two Identical Spheres,” International Journal of Impact Engineering, 36 (2009), 680-686



Aluminum measurements from Kharaz and Gorham, “A Study of the Restitution Coefficient in Elastic-Plastic Impact,” Philosophical Magazine A – Physics of Condensed Matter Structure Defects and Mechanical Properties, 80 (2000), 549-559

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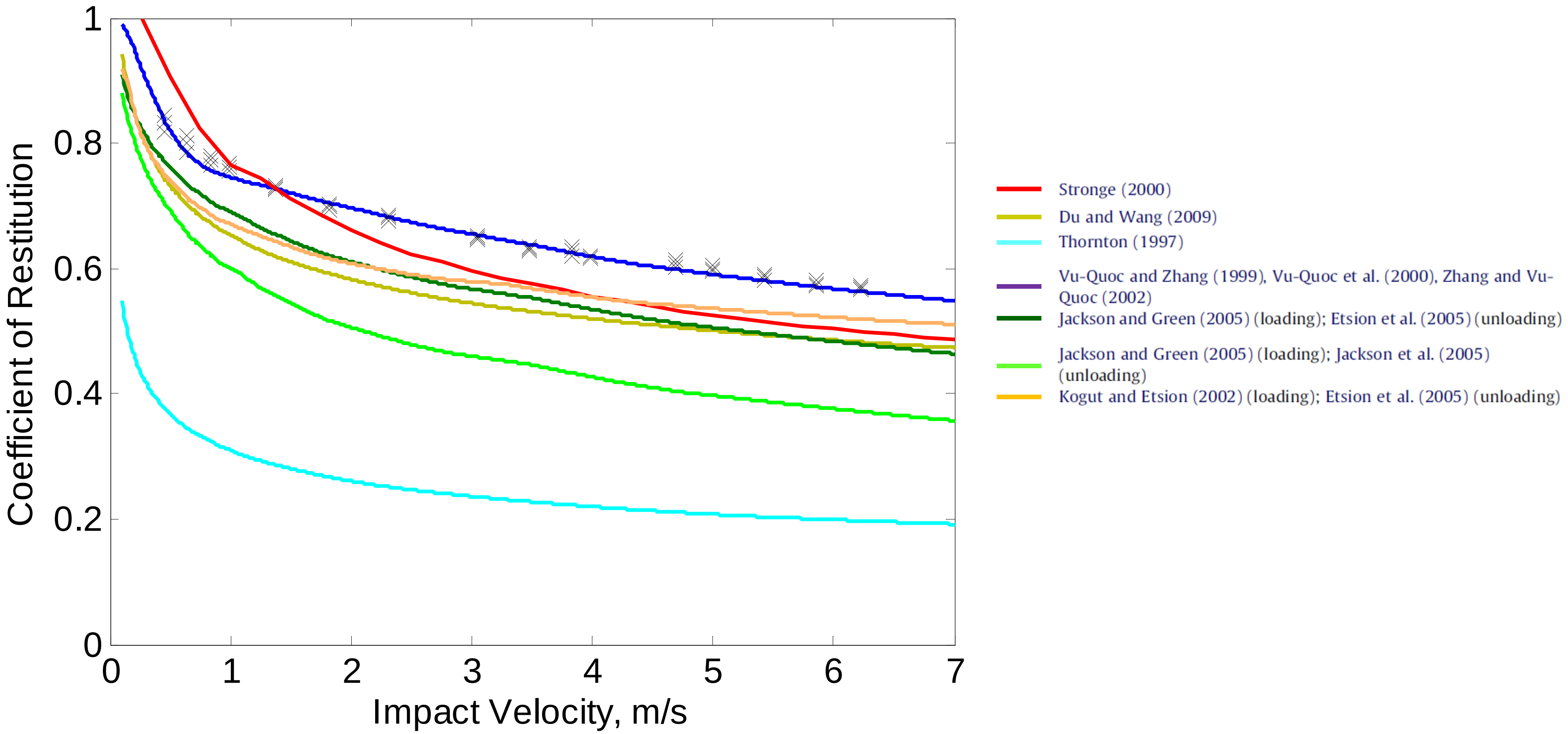

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Summary and Conclusions

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- A new analytical elastic plastic model that includes strain hardening has been developed
- No tuning or calibration parameters – entirely based on material properties, and several well supported assumptions
- Compliance after yield modeled as the contribution from elastic forces decreasing and the contribution from plastic forces increasing as the interference increases
- Very high agreement with available data; much more so than existing models in the literature