

# Adiabatic quantum computing with dressed Rydberg atoms

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Krittika Goyal  
with Bob Keating, Ivan Deutsch,  
Yuan-Yu Jau\*, Grant Biedermann\* and Andrew Landahl\*

University of New Mexico  
\*Sandia National Laboratories

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DAMOP



LABORATORY DIRECTED RESEARCH & DEVELOPMENT



Krittika Goyal (CQuIC, UNM)

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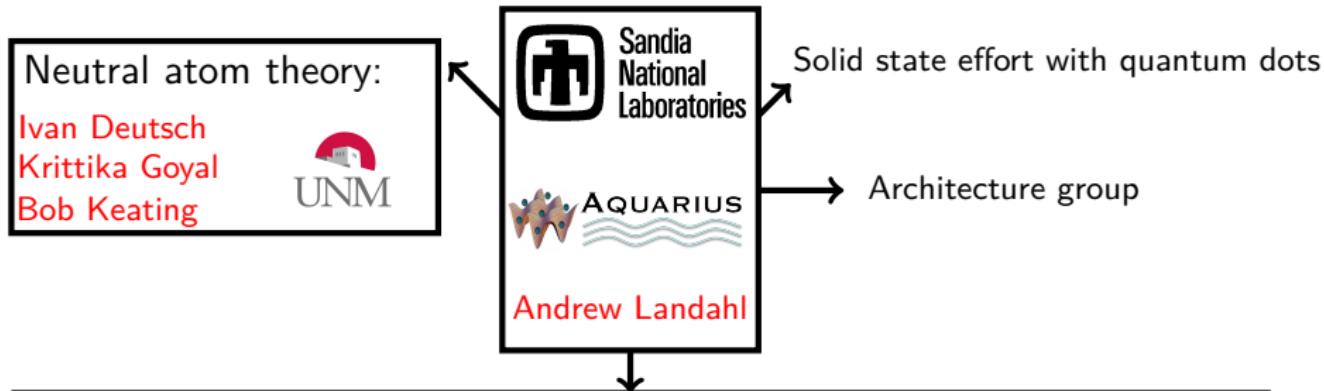


AQC with dressed Rydberg atoms



DAMOP 1 / 16

# Adiabatic QUantum ARchitectures In Ultracold Systems



## Neutral Atom Expt:



Greg Brady, Kevin Fortier, Tom Hamilton,  
Aaron Hankin, **Yuan-Yu Jau**, Cort Johnson,  
Shanalyn Kemme, Michael Mangan, Paul Parazzoli,  
Peter Schwindt and **Grant Biedermann**

# What is AQC?

- Goal: Solve an optimization problem using Adiabatic Quantum Computation (AQC)

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<sup>1</sup>E. Farhi and J. Goldstone, arXiv:quant-ph/0001106v1 (2000).

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# What is AQC?

- Goal: Solve an optimization problem using Adiabatic Quantum Computation (AQC)
- Map on to problem Hamiltonian  $H_P \Rightarrow$  *Ground state is the solution*
- Beginning Hamiltonian  $H_B \Rightarrow$  *Ground state is easy to prepare*
- $H(s) = (1 - s)H_B + sH_P$ <sup>1</sup>  
Change  $s$  *adiabatically (slowly)* so that system is always in the *ground state* of  $H(s)$

<sup>1</sup>E. Farhi and J. Goldstone, arXiv:quant-ph/0001106v1 (2000).

# Quadratic Unconstrained Binary Optimization (QUBO)

- The general QUBO problem is to find the  $\vec{z} = (z_1, z_2, \dots, z_n)$  that **minimizes** the function

$$f(\vec{z}) = \sum_{i=1}^n h_i z_i + \sum_{i,j=1}^n J_{ij} z_i z_j, \text{ where } z_i \in \{0, 1\}$$

- NP-hard because the number of possible values of  $\vec{z}$  scales as  $2^n$ .

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- NP-hard because the number of possible values of  $\vec{z}$  scales as  $2^n$ .
- This problem is equivalent to finding the ground state of the Hamiltonian

$$H_P = \sum_{i=1}^n h_i \left( \frac{\mathbb{I} - \sigma_z^{(i)}}{2} \right) + \sum_{i,j=1}^n J_{ij} \left( \frac{\mathbb{I} - \sigma_z^{(i)}}{2} \right) \otimes \left( \frac{\mathbb{I} - \sigma_z^{(j)}}{2} \right)$$

The  $z_i$  have been mapped onto  $\left( \frac{\mathbb{I} - \sigma_z^{(i)}}{2} \right)$

# Why dressed ground state Rydberg atoms?

- Ground state neutral atoms allow for precise and robust single qubit control.

D. Schrader PRL **93**, 150501 (2004)

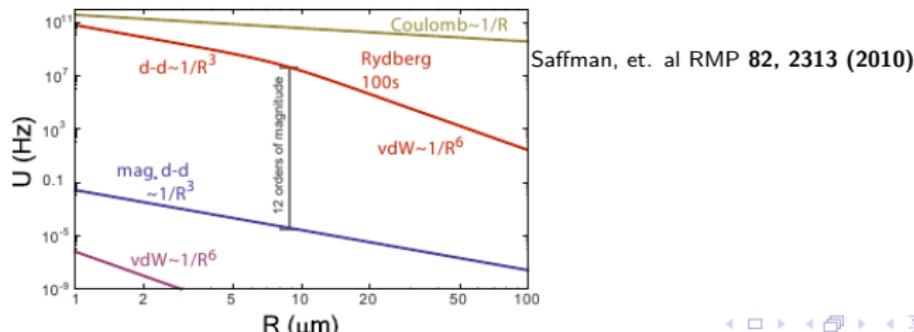
C. Zhang PRA **74**, 042316 (2006)

W. Rakreungdet PRA **79**, 022316 (2009)

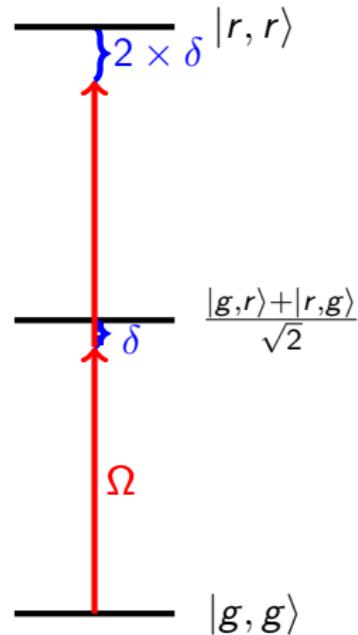
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# Why dressed ground state Rydberg atoms?

- Ground state neutral atoms allow for precise and robust single qubit control.
- Interactions between ground state neutral atoms are very weak, making it hard to implement two qubit gates, without contact interaction
- Dipole-dipole interactions between Rydberg atoms are several orders of magnitude stronger than ground state neutral atoms.

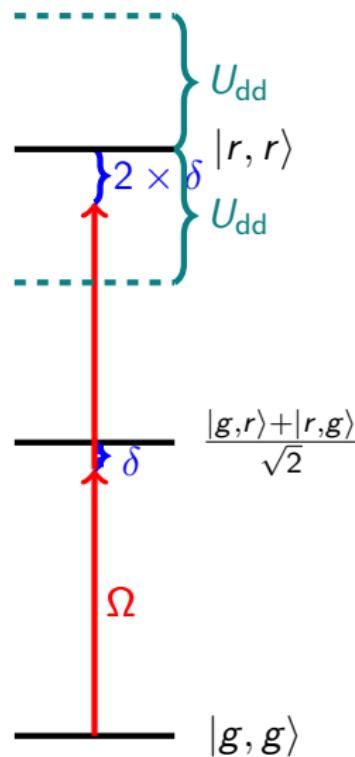


# The Rydberg blockade



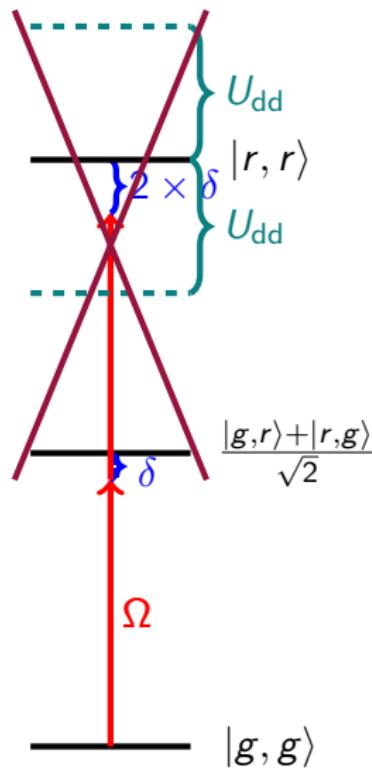
- $U_{dd}$  is the interaction energy between two Rydberg atoms.
- Blockade interaction results in entangled state  $\frac{|g, r\rangle + |r, g\rangle}{\sqrt{2}}$
- References: Y. Miroshnychenko et al. PRA **82**, 013405 (2010).  
E. Urban et al. Nature Physics **5**, 110. (2009).

# The Rydberg blockade



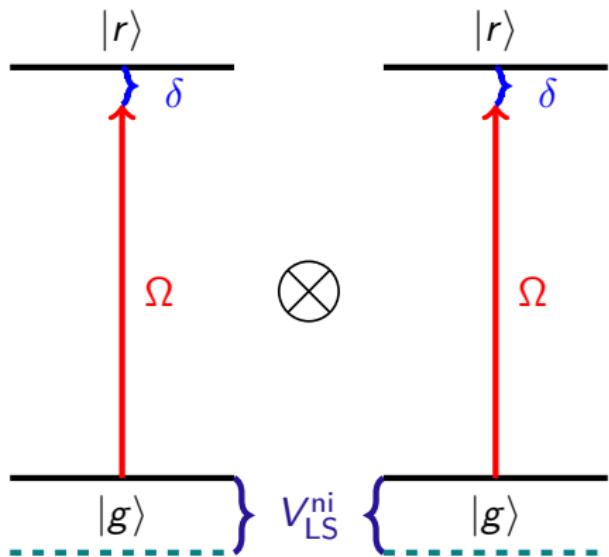
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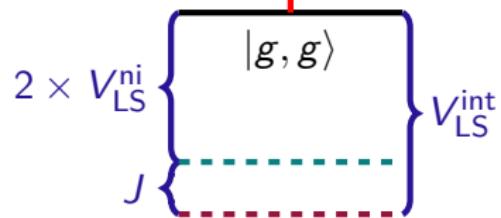
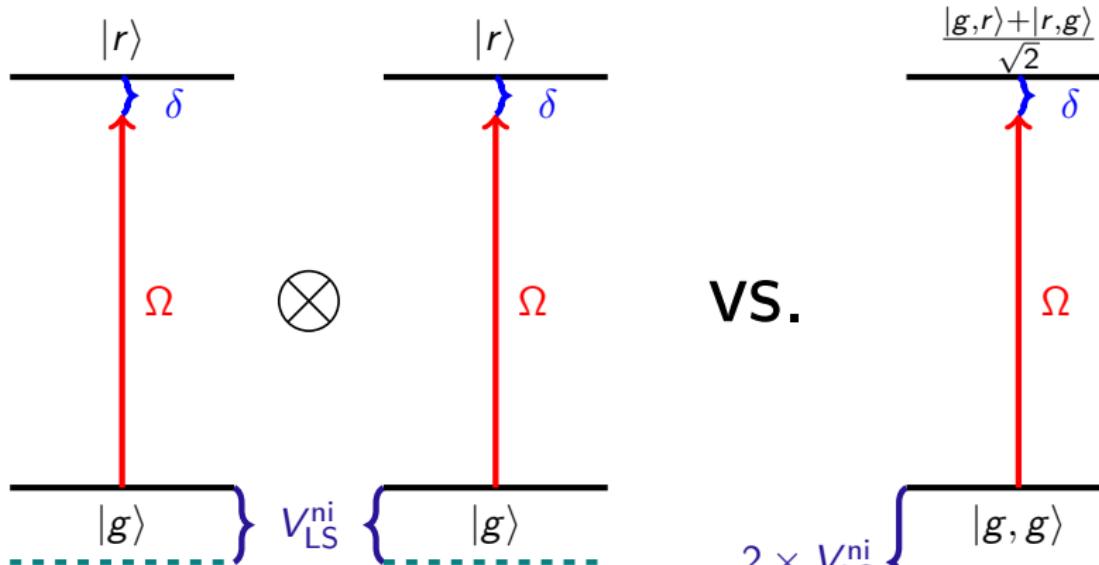


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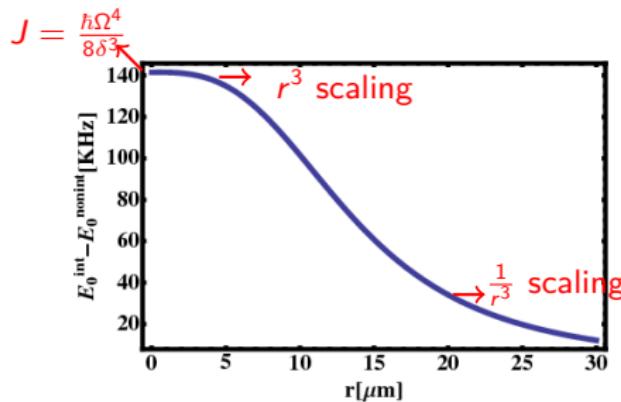
## Dressed ground state Rydberg atoms



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# Dressed ground state Rydberg atoms



J. E. Johnson and S. L. Rolston,  
PRA **82**, 033412 (2010)

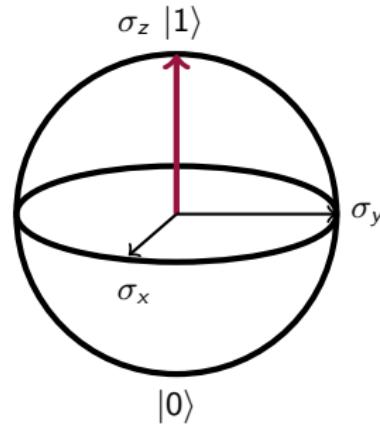
- The interaction is robust to fluctuations in  $r$ .
- Dressed ground state Rydberg atoms are easy to trap.
- The interaction can stay on at all times which is required for AQC.

## Cs energy level diagram:

 $100S_{1/2} \rightarrow |r\rangle$ 

- Prepare atoms in  $6S_{1/2}F = 3 \rightarrow |1\rangle$  (eigenstate of  $\sigma_z$ )

- Turn on  $\pi/2$  microwave pulse  $\rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$  (ground state of  $\sigma_x$ )

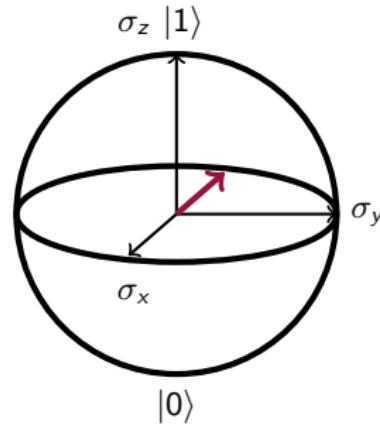
 $7P_{1/2}$  $6P_{1/2}$  $6S_{1/2}F = 4 \rightarrow |0\rangle$  $6S_{1/2}F = 3 \rightarrow |1\rangle$ 

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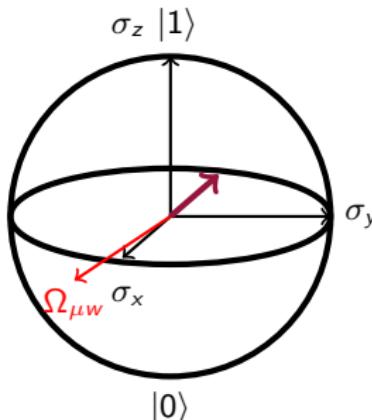
- Turn on  $\pi/2$  microwave pulse  $\rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$  (ground state of  $\sigma_x$ )

—  $7P_{1/2}$

- Turn on the microwaves.

$$H_B = \frac{\hbar\Omega_{\mu w}(t)}{2} \left( \sigma_x^{(1)} + \sigma_x^{(2)} \right)$$

—  $6P_{1/2}$



9.2GHz  
 $\Omega_{\mu w}$

—  $6S_{1/2}F = 4 \rightarrow |0\rangle$   
 { —  $6S_{1/2}F = 3 \rightarrow |1\rangle$

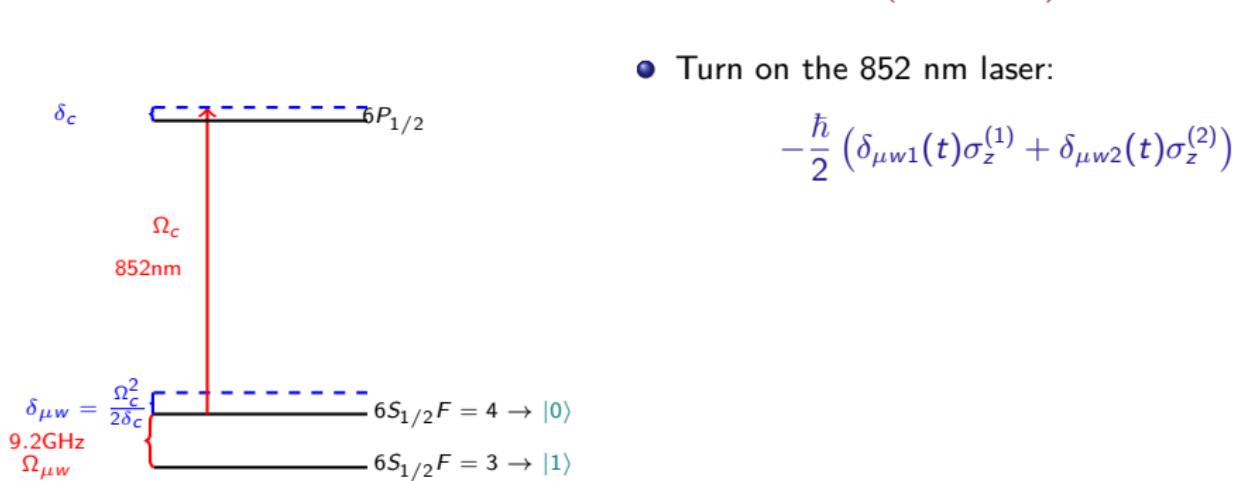
## Cs energy level diagram:

$$\text{---} 100S_{1/2} \rightarrow |r\rangle$$

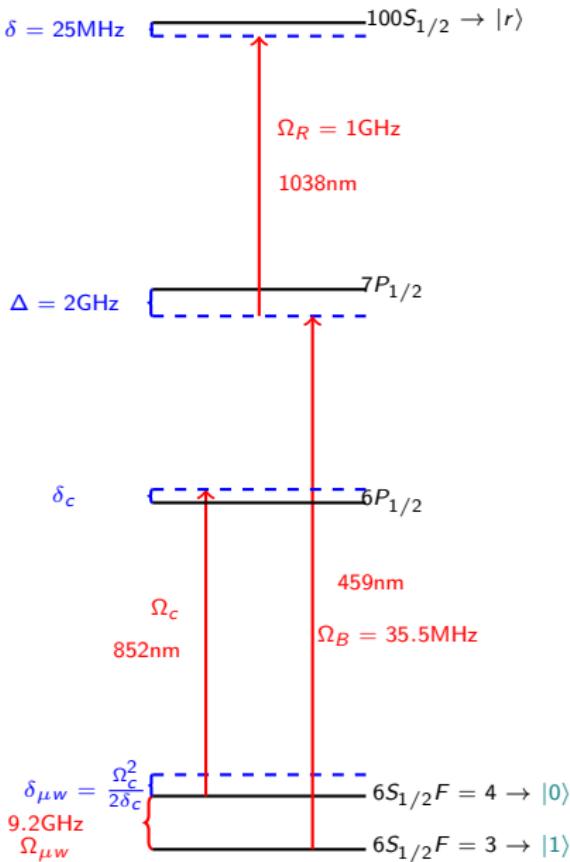
- Prepare atoms in  $6S_{1/2}F = 3 \rightarrow |1\rangle$  (eigenstate of  $\sigma_z$ )

$$\text{---} 7P_{1/2}$$

- Turn on  $\pi/2$  microwave pulse  $\rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$  (ground state of  $\sigma_x$ )



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- Turn on the microwaves.  

$$H_B = \frac{\hbar\Omega_{\mu w}(t)}{2} \left( \sigma_x^{(1)} + \sigma_x^{(2)} \right)$$
- Turn on the 852 nm laser:  

$$-\frac{\hbar}{2} \left( \delta_{\mu w 1}(t) \sigma_z^{(1)} + \delta_{\mu w 2}(t) \sigma_z^{(2)} \right)$$
- Turn on Rydberg lasers:

$$J(t) \left( \frac{\mathbb{I} - \sigma_z^{(1)}}{2} \right) \otimes \left( \frac{\mathbb{I} - \sigma_z^{(2)}}{2} \right)$$

# Evolution time: 2 qubit QUBO

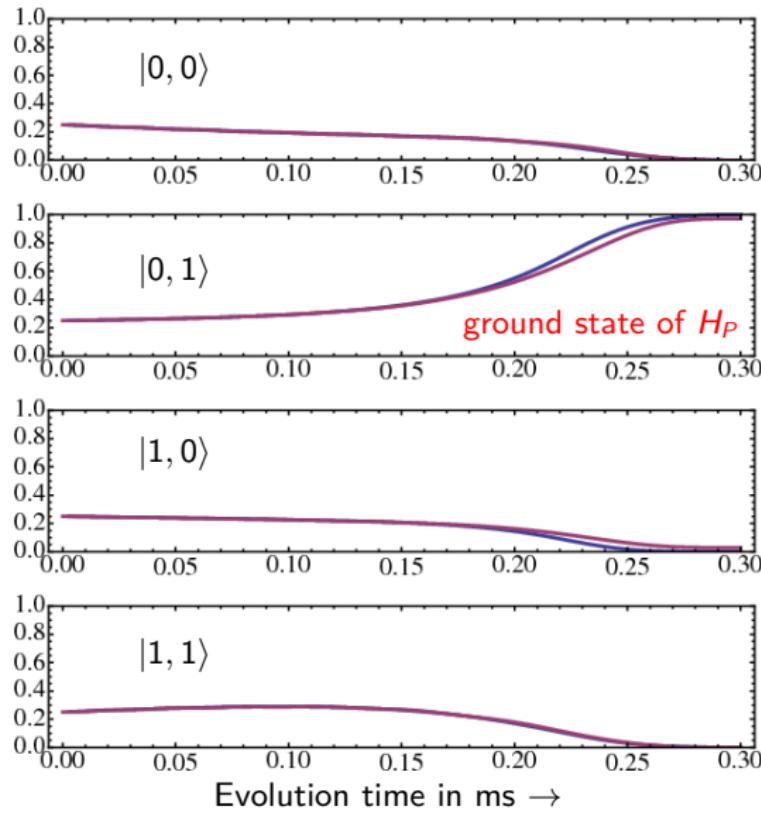
2 qubit QUBO Hamiltonian in KHz:

$$H(t) = 20 \left(1 - \frac{t}{t_{\max}}\right) \left(\sigma_x^{(1)} + \sigma_x^{(2)}\right) + \\ 5 \frac{t}{t_{\max}} \sigma_z^{(1)} + 10 \frac{t}{t_{\max}} \sigma_z^{(2)} + 10 \left(\frac{t}{t_{\max}}\right)^2 \left(\frac{\mathbb{I} - \sigma_z^{(1)}}{2}\right) \otimes \left(\frac{\mathbb{I} - \sigma_z^{(2)}}{2}\right)$$

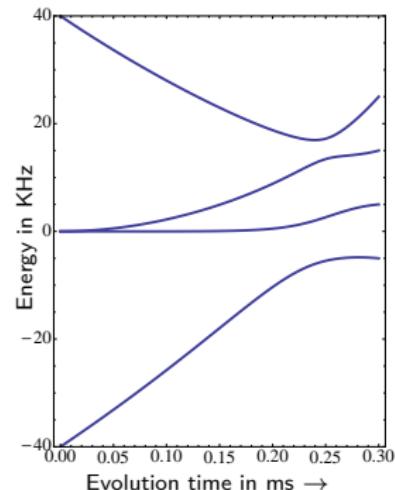
Ground state of  $H_P$  is  $|0, 1\rangle$

## Evolution time: 2 qubit QUBO

Population

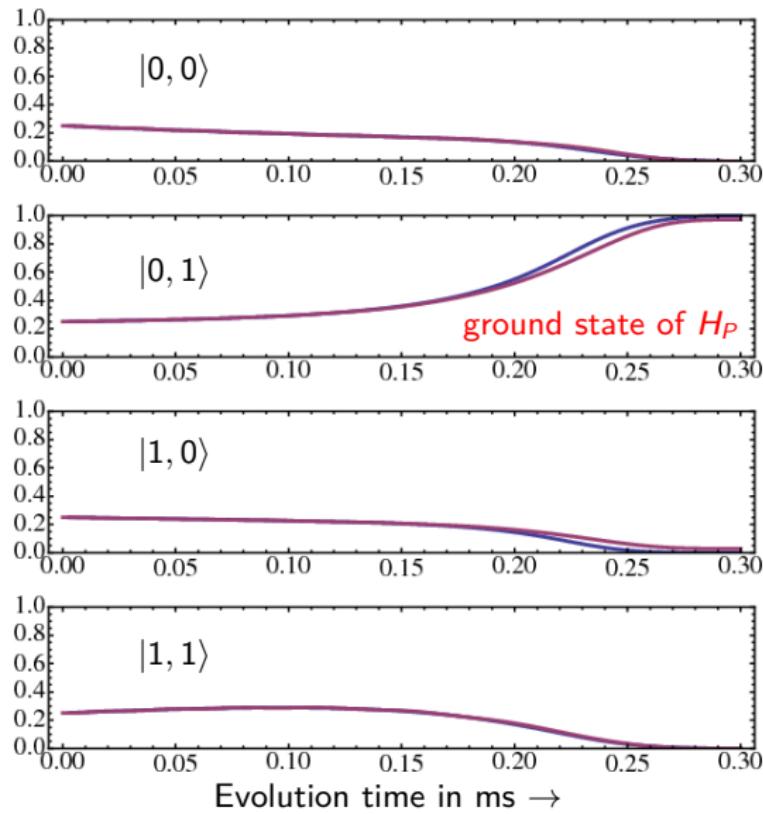


— Adiabatic evolution  
 — Time dependent Schrödinger eqn

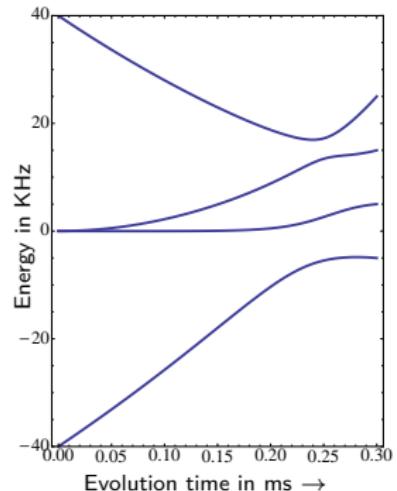


# Evolution time: 2 qubit QUBO

Population

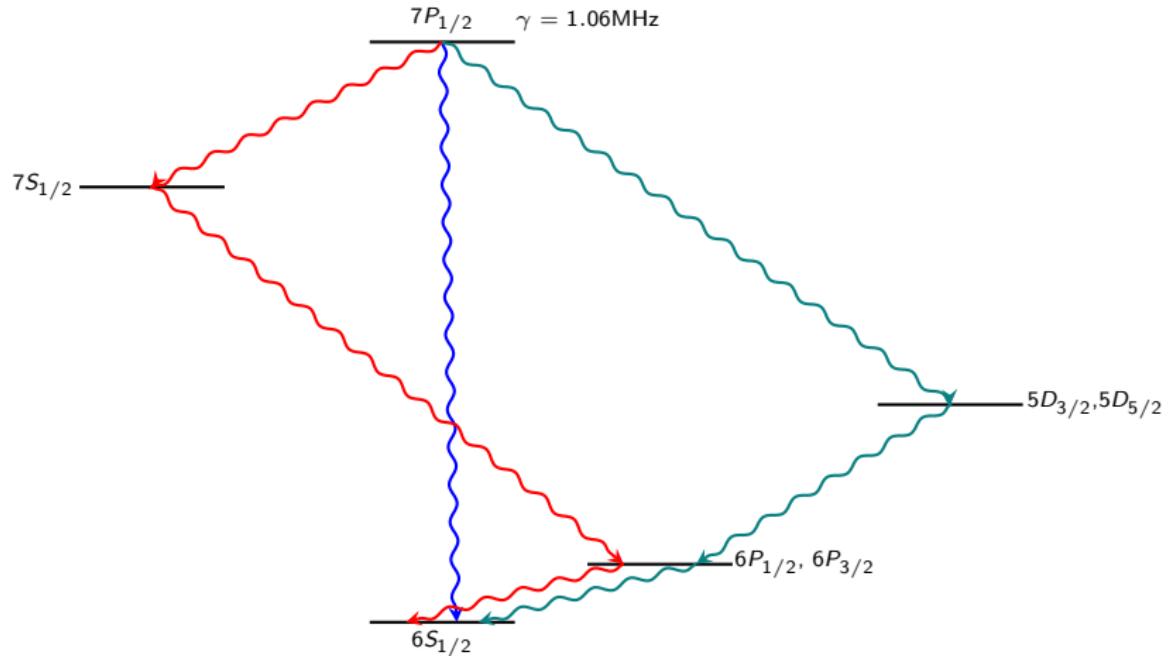


Adiabatic evolution  
Time dependent  
Schrödinger eqn



What about photon  
scattering?

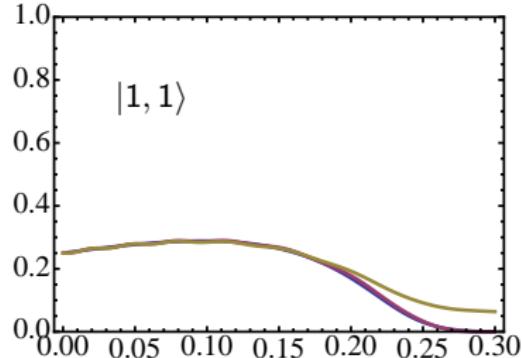
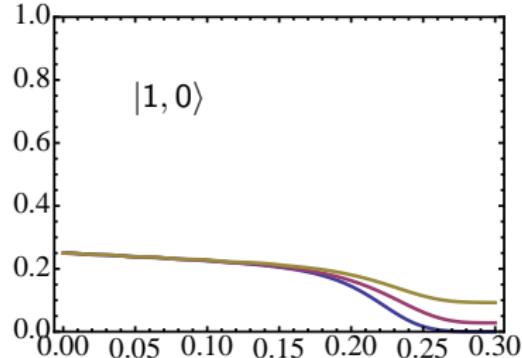
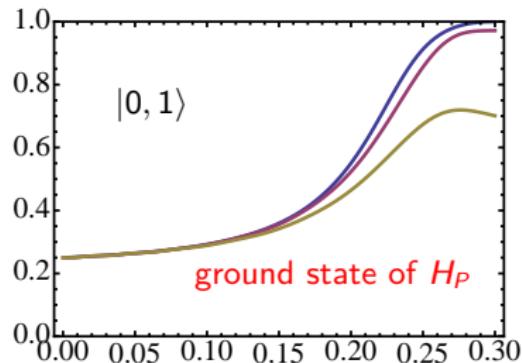
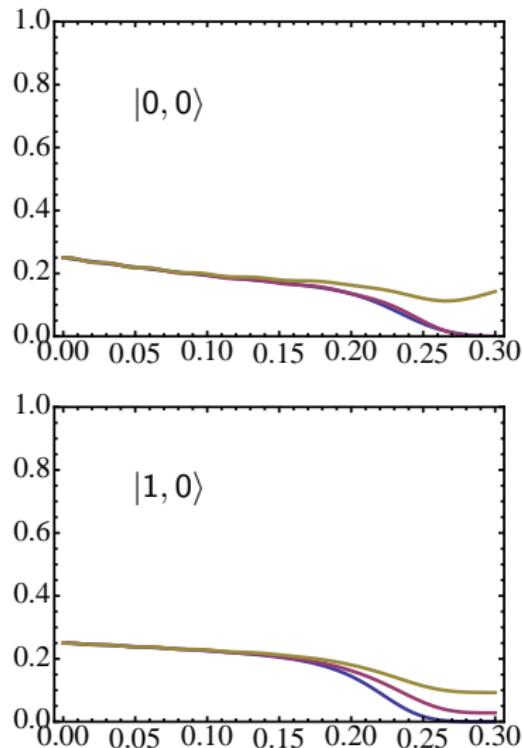
# Photon scattering



Photon scattering randomizes qubit, i. e. returns qubit with approximately equal probability to  $F = 3$  and  $F = 4$  states.

# Effect of dissipation

Population



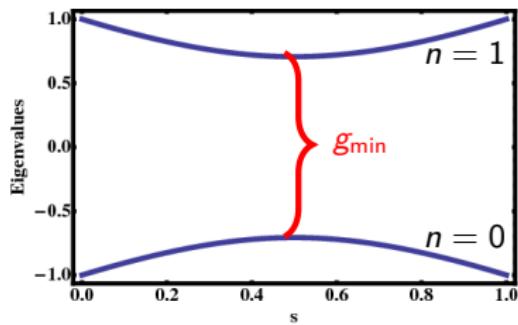
Evolution time in ms →

# Summary And Outlook

- Experimental scheme for two qubit QUBO problem.
- We need to combat the effects of dissipation.
- Optimize ramp to minimize time required for adiabaticity

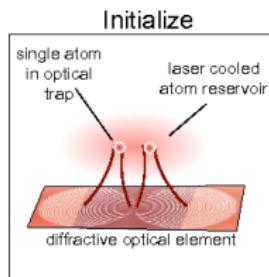
# How Slowly?

$$H(s) = (1 - s)\sigma_x + s\sigma_z$$



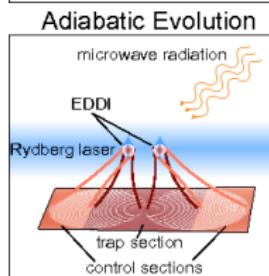
- By the adiabatic theorem the timescale for evolution must satisfy  $T \gg \frac{\varepsilon}{g_{\min}^2}$ <sup>a</sup>, where  $\varepsilon = \max \left| \langle n = 1; s | \frac{dH(s)}{ds} | n = 0; s \rangle \right|_{0 < s < 1} \approx 1$
- The feasibility of AQC depends on the scaling of  $g_{\min}$  with number of qubits.

<sup>a</sup>E. Farhi and J. Goldstone, arXiv:quant-ph/0001106v1 (2000).



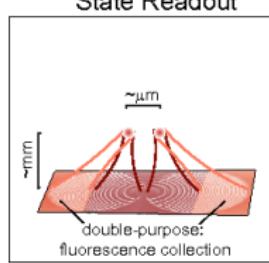
Prepare atoms in  
 $6S_{1/2}F = 3 \rightarrow |1\rangle$

Turn on  $\pi/2$  microwave  
 pulse  $\rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$



Turn on and adiabatically  
 ramp down  $\Omega_{\mu w}$

Adiabatically ramp up  
 intensities of all lasers.



# Evolution time

Population

