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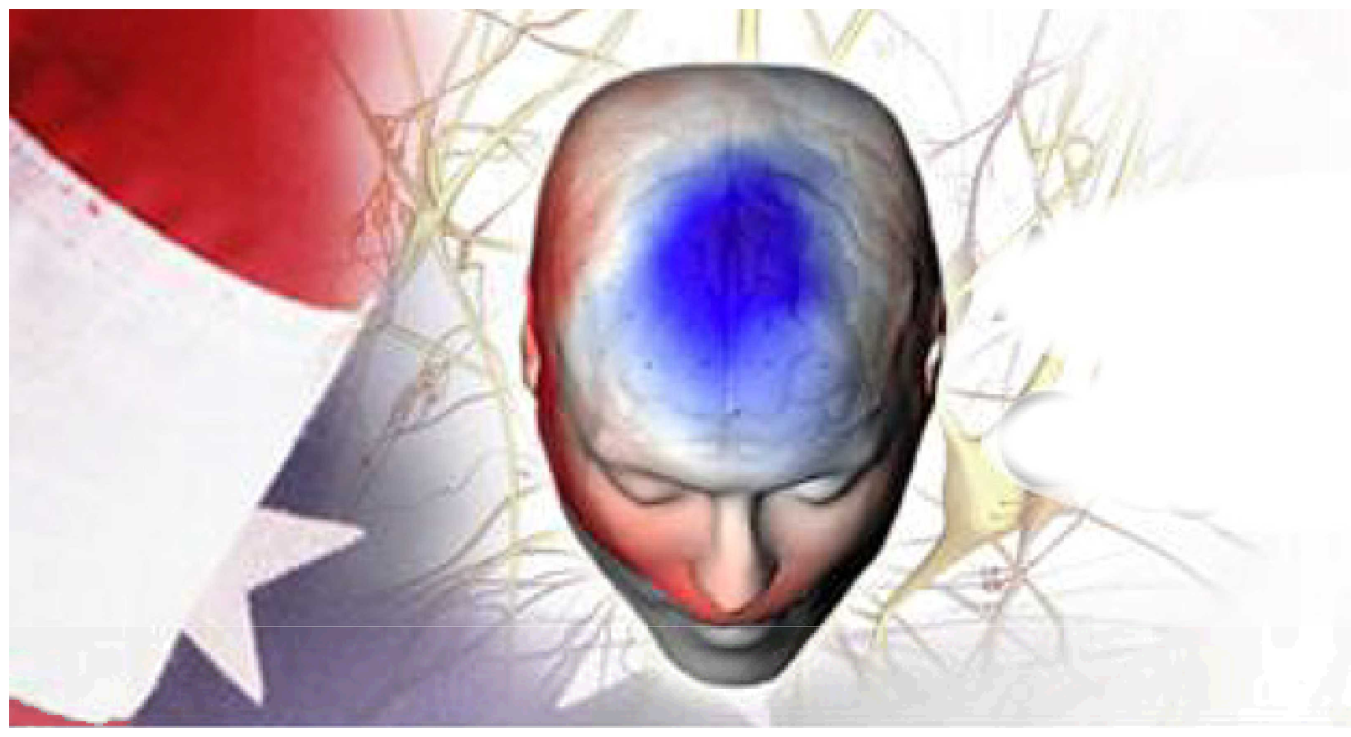


Parallel design simulation for neurologically inspired systems

Richard Schiek[1], Christy Warrender[2], Heidi Thornquist[1], Ting Mei[1], Eric Keiter[1], Tom Russo[1]
[1]Electrical Systems Modeling [2]Cognitive Modeling
Sandia National Laboratories, Albuquerque, NM USA

Abstract
While neurological systems carry information as electrical signals, the typical neurological circuit topology is very different from a circuit topology. For example, neurons within the brain connect to on the order of 10^4 neurons, and not always the nearest neighbors.[Gordon20 04] Additionally, the underlying electrical signal is analog in nature and has many forms of sub-critical, bias like behavior, single spikes to spike trains or packets. The resulting functional unit is capable of apparent computational efficiency at very low power. [Brown & Bashir, 2002]
The goal of this work is to develop computational and statistical tools to enable efficient parallel simulation of neurologically inspired systems. Specifically, we're extending traditional Spice style circuit simulation to accurately model individual neuron behavior in a highly connected circuit. An important additional aspect to this work is uncertainty quantification of neural model parameters so that one can gauge stability of a neural based system. We have implemented common neurological ion-channel models (e.g. Hodgkin-Huxley, Connor-Stevens) in a dynamic cable-equation format within a circuit simulator, Xyce (xyce.sandia.gov); see simulation outline below. This allows one to use a netlist style syntax to describe a collection of neurons for simulation. As with any circuit simulation, the model parameters for the circuit components are critical in determining the circuit's performance.

Method
Neurons are highly branched cells that maintain a potential difference between the cell's interior and exterior. The branching and topological complexity is exemplified by the imaging from Caja shown at Figure 1.

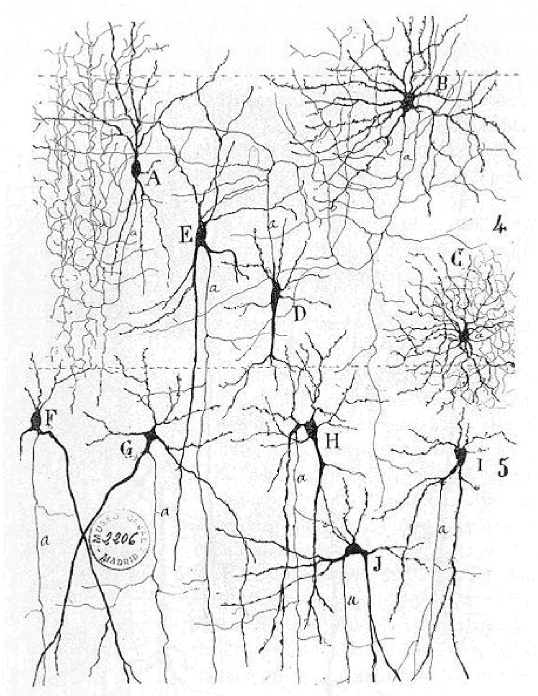


Figure 1 Texture of the Nervous System of Man and the Vertebrates by Santiago Ramon y Caja

Given a section of neuron membrane, the voltage potential across the membrane (i.e. from inside to outside) is described by Koch and Dayan [2001]

$$C_m \frac{dV}{dt} = i_m + \frac{I_e}{A}$$

where C_m is the membrane capacitance, V is the voltage difference, t is time, i_m is the current through the membrane while I_e represents any externally applied current into the cell and A is the surface area of the membrane.

The membrane current is dependent on the type of ion channels within the membrane. In this work we the Hodgkin-Huxley model where the current is described by:

$$i_m = i_{leak} + i_{Na} + i_K$$

where i_{leak} is the current that naturally leaks through the membrane, i_{Na} is the current associated with sodium ion channels and i_K is the current associated with potassium ion channels. This descriptive equation can be refined with algebraic expressions for the individual currents as:

$$i_m = \bar{g}_L (V - E_L) + \bar{g}_{Na} m^3 h (V - E_{Na}) + \bar{g}_K n^4 (V - E_K)$$

Parameters in this equation are: $\bar{g}_L$ is the maximal membrane conductance, E_L is the membrane reversing potential, $\bar{g}_{Na}$ is the sodium ion channel conductance, E_{Na} is the sodium channel reversing potential while $\bar{g}_K$ and E_K are the potassium channel maximal conductance and reversing potential respectively. The variables, m, h and n are voltage dependent gating variables that model the relative availability of the sodium and potassium channels. Gating variables, m, h and n are described by the ordinary differential equations:

$$\frac{dm}{dt} = \alpha_m(V) \cdot (1 - m) - \beta_m(V) \cdot m$$
$$\frac{dh}{dt} = \alpha_h(V) \cdot (1 - h) - \beta_h(V) \cdot h$$
$$\frac{dn}{dt} = \alpha_n(V) \cdot (1 - n) - \beta_n(V) \cdot n$$

The voltage dependent coefficients, $\alpha(V)$ and $\beta(V)$ are:

$$\alpha_m(V) = \frac{0.1(V + 40)}{1 - e^{-0.1(V+40)}}$$
$$\beta_m(V) = 4e^{-0.0556(V+65)}$$
$$\alpha_h(V) = 0.07e^{-0.05(V+65)}$$
$$\beta_h(V) = \frac{1}{1 + e^{-0.1(V+35)}}$$
$$\alpha_n(V) = \frac{0.1(V + 55)}{1 - e^{-0.1(V+55)}}$$
$$\beta_n(V) = 0.125e^{-0.0125(V+65)}$$

Note, in the equations for m, h and n, the voltage is given in units of milli-volts and time in milli-seconds.

These equations describe the voltage behavior of a section of neuron membrane. To model a section of a neuron process, such as an axon, a *cable-equation* formulation is used. Here, the cable equation is specified as:

The following equation was used to model the cable properties of the neuron:

$$C_m \frac{dV_i}{dt} = -i_i^m + \frac{I_i^E}{A_i} + g_{i,i+1}(V_{i+1} - V_i) + g_{i,i-1}(V_{i-1} - V_i)$$

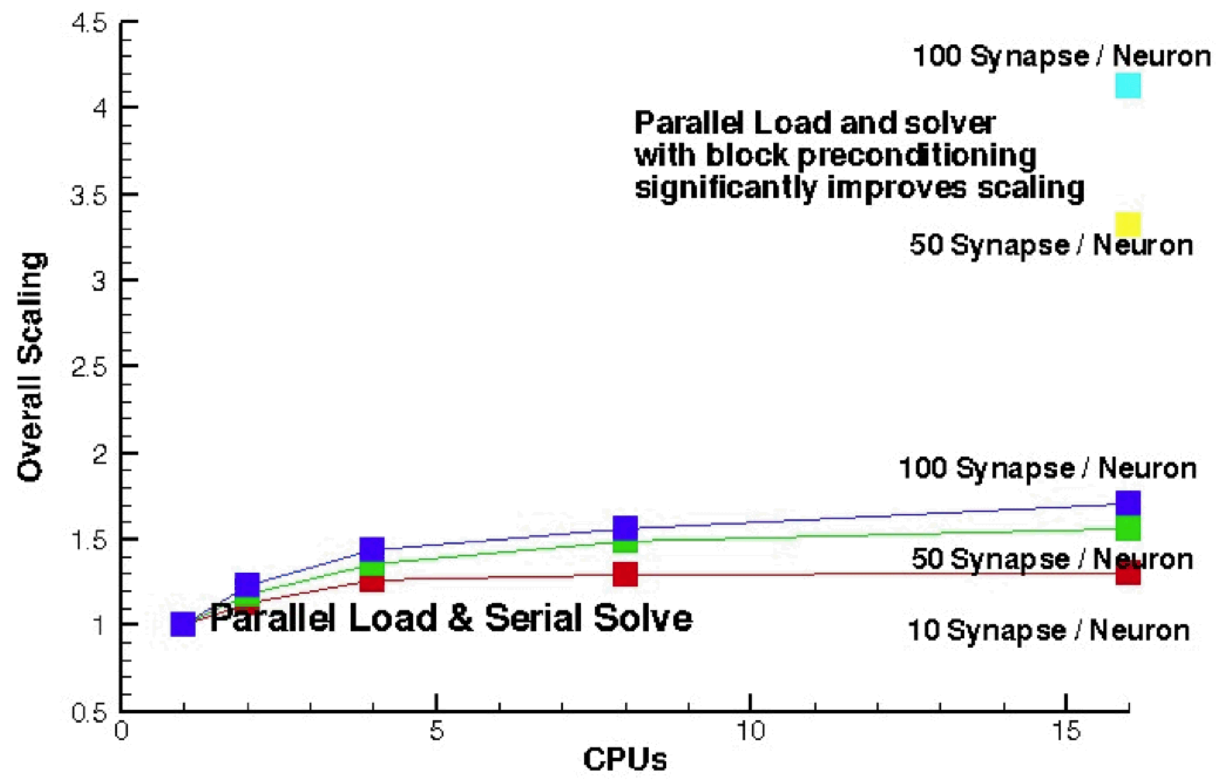
where C_m is the membrane capacitance, V_i is the voltage in compartment i relative to an external ground, t is time, i_i^m is the current through the membrane in compartment i, I_i^E represents any externally applied current into the cell and A_i is the surface area of the membrane in compartment i. The final two terms represent current flow into the adjoining compartments, $i-1$, for the previous compartment and $i+1$ for the next compartment. Conductance between the compartments, $g_{i,i+1}$ and $g_{i,i-1}$ can be calculated by:

$$g_{i,j} = \frac{a_i a_j^2}{r_{Long} L_i \left(L_i a_j^2 + L_j a_i^2\right)}$$

where a_i is the radius of compartment i, L_i is the length of compartment i and r_{Long} is the longitudinal intracellular resistance.

A benefit of the cable equation formulation is that a complex neuron can be represented as a collection of attached cable segments. These segments can then be divided and partitioned when simulating the system in parallel.

To simulate systems of graded complexity, we generated random networks of 1000 branched neurons connected on average at 10, 50 and 100 synapses per neuron. These systems were simulated for 1.0 sec while applying random inputs to all the neurons in the system. Additionally, the simulations were conducted using 1 to 64 processors on a cluster computer. In serial, Xyce and Neuron took similar times to run the test problems with Xyce taking average time steps of 4.2e-4 and Neuron taking 4.2e-5. Thus the advanced algorithms allow for larger time steps. In parallel, the best improvement (factor 3.3) in overall runtime came from using graph based partitioning and parallel-solvers coupled with inexact preconditioning (see attached figure). Analysis of the runtime data show that improvements in the system loading scale linearly until the size of the system per processor is small (approximately 22,000 unknowns) but this will vary with computer cluster. Switching from serial to parallel solvers pays off when the system size is over around 250,000. Thus, this work demonstrates that significant improvements in simulation capacity can be gained by using advanced algorithms.



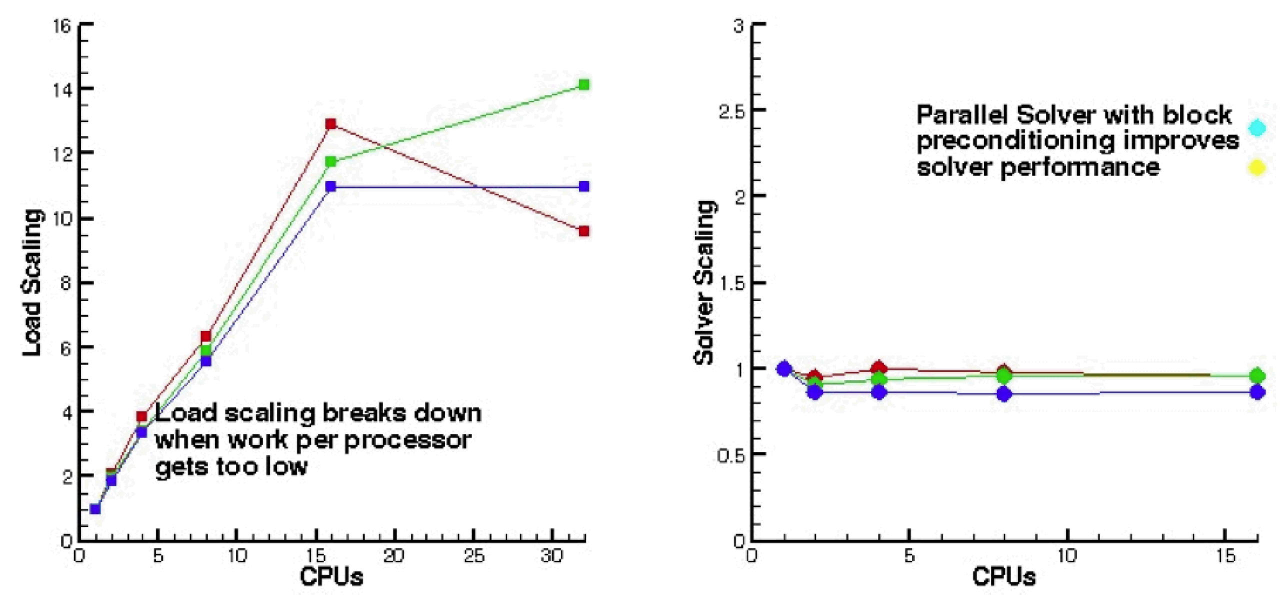


Figure 2: Paralel Scaling of Neural Circuits

Conclusions
Electrical circuit simulation tools can successfully address the greater topological complexity of neuron systems. The high degree of connectivity can be addressed by constructing the neuron unit from many devices. While synaptic densities have not yet approached that found in nature, hundreds to thousands of synapses per neuron is feasible.

References
Brown, M. W. & Bashir Z. I., "Evidence concerning how neurons of the perirhinal cortex may effect familiarity discrimination," Phil. Trans. R. Soc. Lond. B, 356, 10821095 (2002).
Dayan, Peter & Abbott L. F., "Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems" MIT Press, Cambridge Massachusetts 2001
Keiter E., Thornquist H., Hoekstra R., Russo T., Schiek R., Rankin E., "Parallel Transistor Level Circuit Simulation", Pp 1—21, *Advanced Simulation and Verification of Electrical and Biological System*, ed. Ping Li, Luis Miguel Silveira, Peter Feldmann, Springer 2010.
Koch, Christof, "Biophysics of Computation: Information Processing in Single Neurons," 1998
Shepherd, Gordon M., "The Synaptic Organization of the Brain," 5th edition, Oxford University Press, pg 7 & 458, Oxford University Press, 1998.

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