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Code verification for the eXtended Finite Element Method (XFEM): the compound cohesionless impact problem

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Sandia National Laboratories

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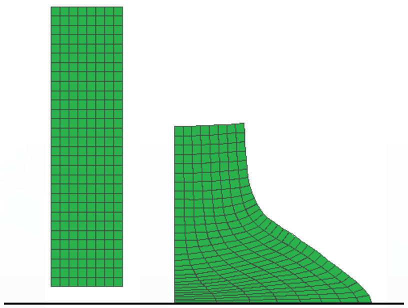
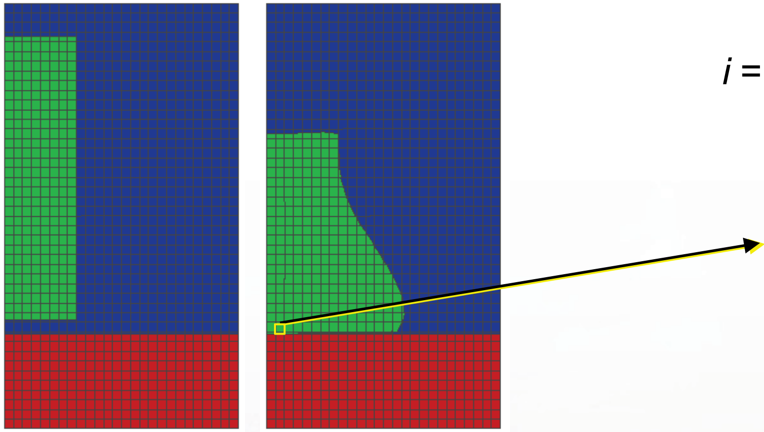
Interface treatment in Eulerian-frame solid-dynamics simulations is a quandary

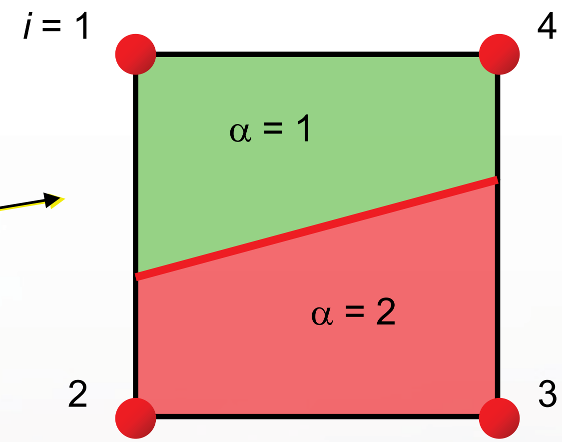
Eulerian frame: material interfaces do not correspond to element boundaries. Thus, mixed-material elements containing interfaces are present.



Lagrangian frame:
mesh fixed to material



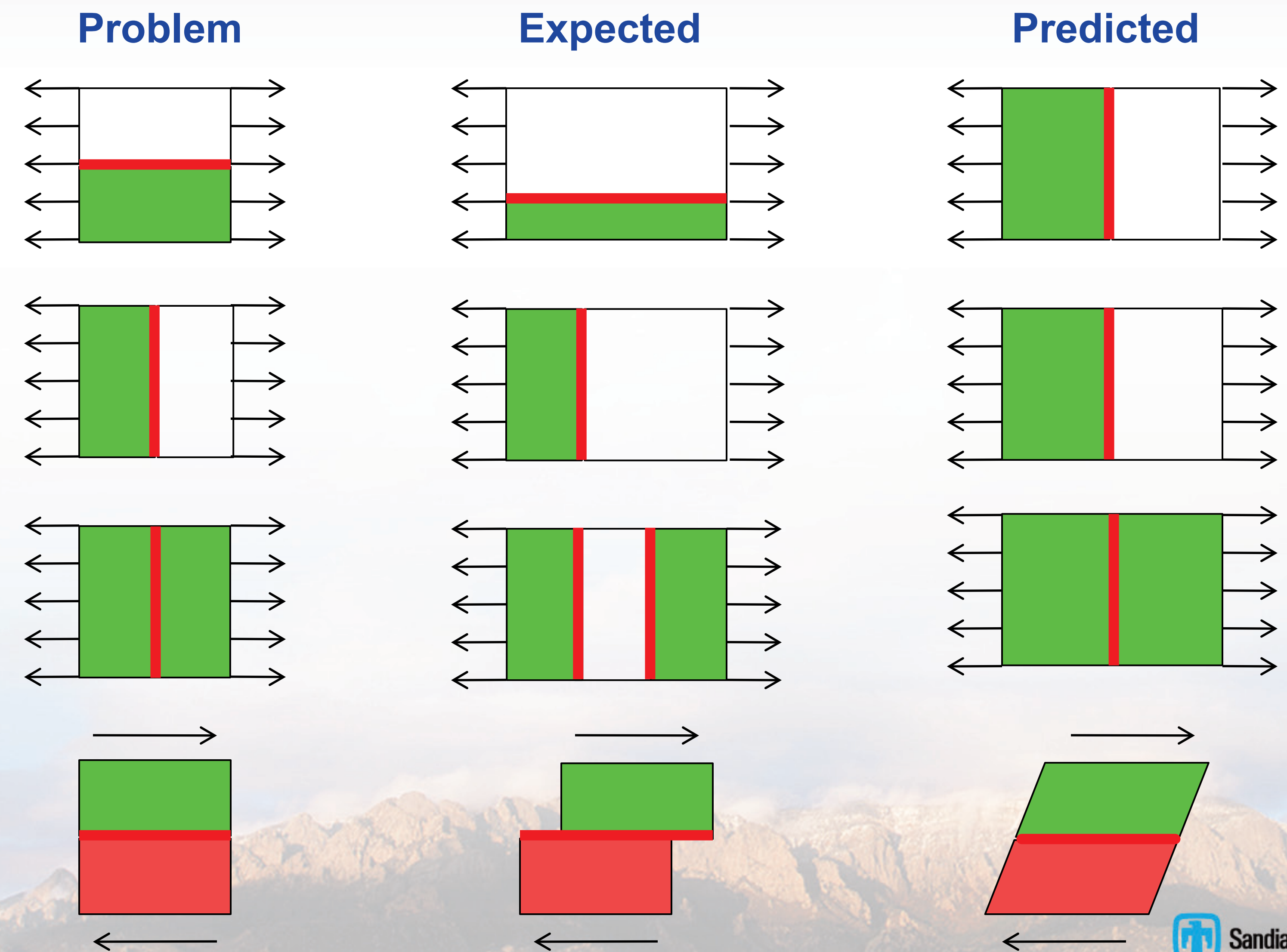
Eulerian frame:
material moves across mesh



Quantities necessary for computing motion and deformation cannot be multiply defined within a single mixed-material element: stress, strain, strain rate, etc. Consequences:

- No relative motion or void-opening between materials is possible
- Shearing and tensile motion can induce erronous material states
- Excessive adhesion and erroneous dissipative effects

Incorrect dynamics are predicted on intra-element interfaces in standard FEM



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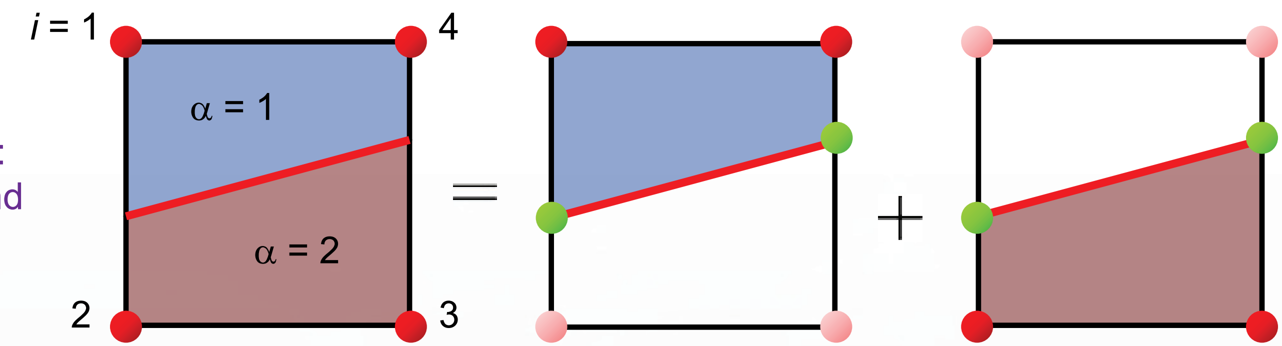
The eXtended Finite Element Method allows interfaces to be modeled correctly

XFEM is a scheme for explicitly capturing intra-element interface physics in FEM simulations: decompose displacements u^h within element using Heaviside function.

ALEGRA developers: Tom Voth and J. Mosso



"enriched" elements

i = node index
α = material index

$$u^h(\vec{x}) = \sum_i u_i^0 N_i(\vec{x}) + \sum_\alpha \sum_i u_i^\alpha N_i(\vec{x}) H_\alpha(\vec{x})$$

XFEM, with the necessary contact, interface reconstruction, and remap algorithms, is being implemented in **ALEGRA**: a finite-element code available on most DoD HPC platforms for simulating high-deformation multimaterial shock hydrodynamics, solid dynamics, and MHD.

ALEGRA-related talks at UGC:

- R. Doney, Tuesday morning, 6/20 – Parallel scaling exercises
- C. Nicely, Wednesday afternoon, 6/22 – Material model evaluation

This implementation of XFEM in ALEGRA is revolutionary and needs verification

	XFEM implementations to-date	XFEM implementation in ALEGRA
Context	Lagrangian-frame structural mechanics	Eulerian-frame multimaterial shock hydrodynamics
Physics focus	Crack and/or void growth and propagation	Dynamics of materials interfaces

To characterize accuracy and correctness of XFEM implementation in ALEGRA for shock problems, a shock verification problem is needed.

Crack propagation
Hansbo and Hansbo, *CMAME* 2004

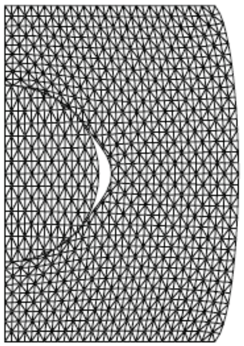


Fig. 4. Displacements for the contact problem.

Circular void expansion
Dolbow *et al*, *CMAME* 2008

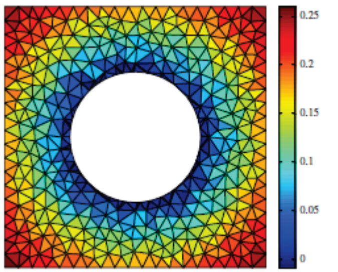
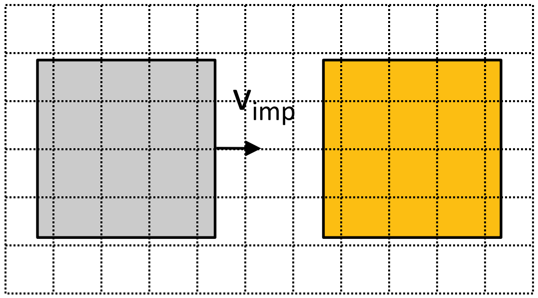


Fig. 8. Numerical approximation u^h to the solution u of the one-sided problem.

1D high-velocity impact and shock
Present work



See Carpenter *et al.*, *IJNME* 1991

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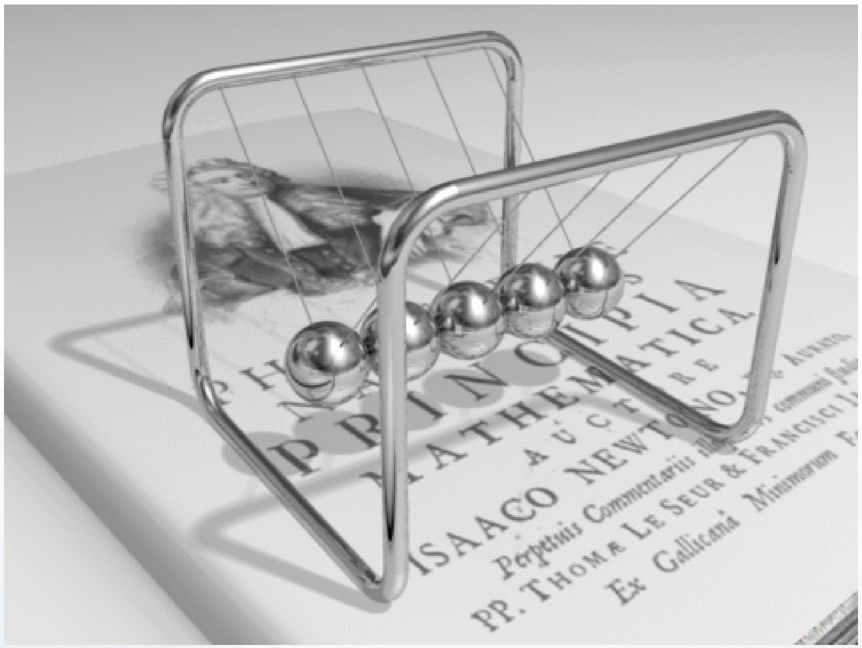
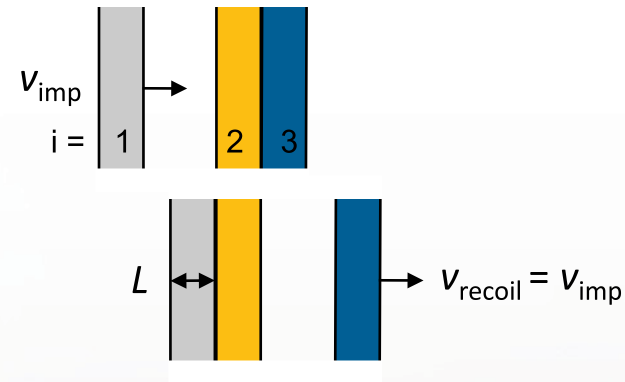
The Newton's cradle problem is useful for XFEM verification

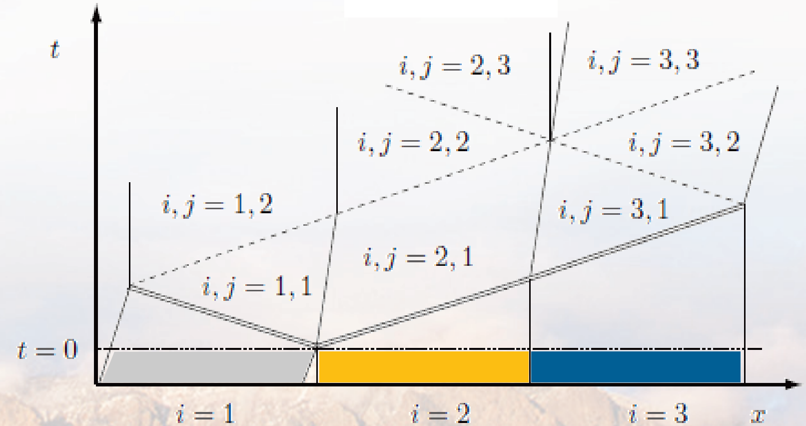
We made the 1D impact verification problem more difficult and compelling by transforming it into a familiar and seemingly simple problem: "Newton's cradle."



Animation by Dominique Toussaint (www.edumedia-share.com, GNU)

1D cohesionless, compound impact:





Analytic solution: $v_{3,3} = v_{1,0} = v_{imp}$

An attractive problem for verification:

- Shock and release play important roles.
- Analytic solution is easily available,…

…if dissipation is absent.

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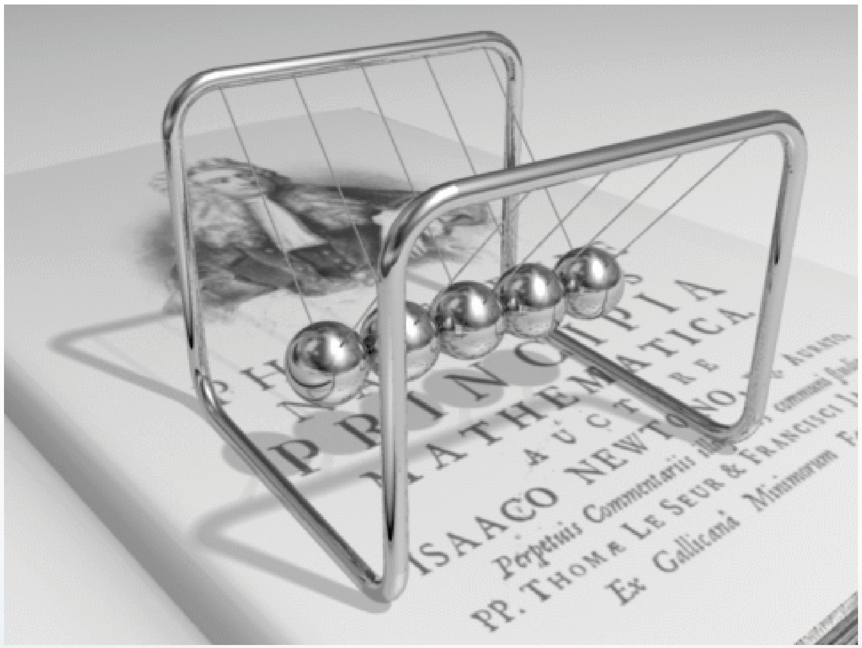

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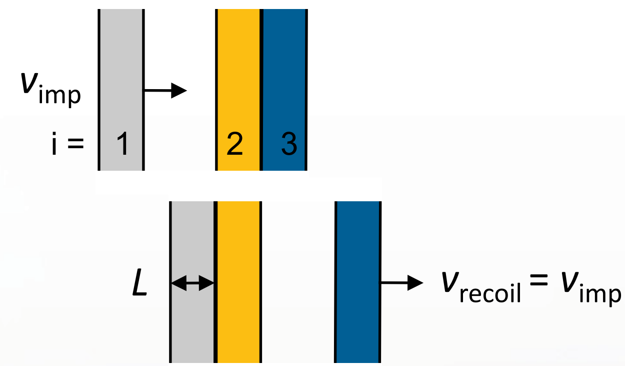
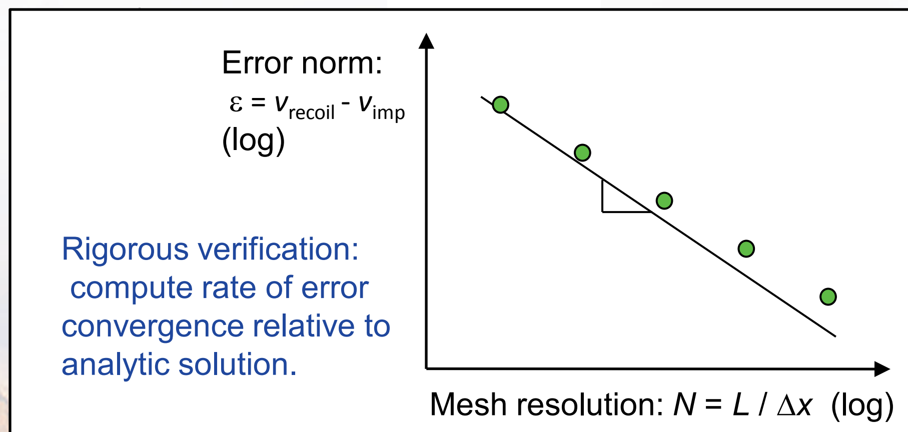
1D cohesionless, compound impact:





Analytic solution: $v_{3,3} = v_{1,0} = v_{imp}$

An attractive problem for verification:

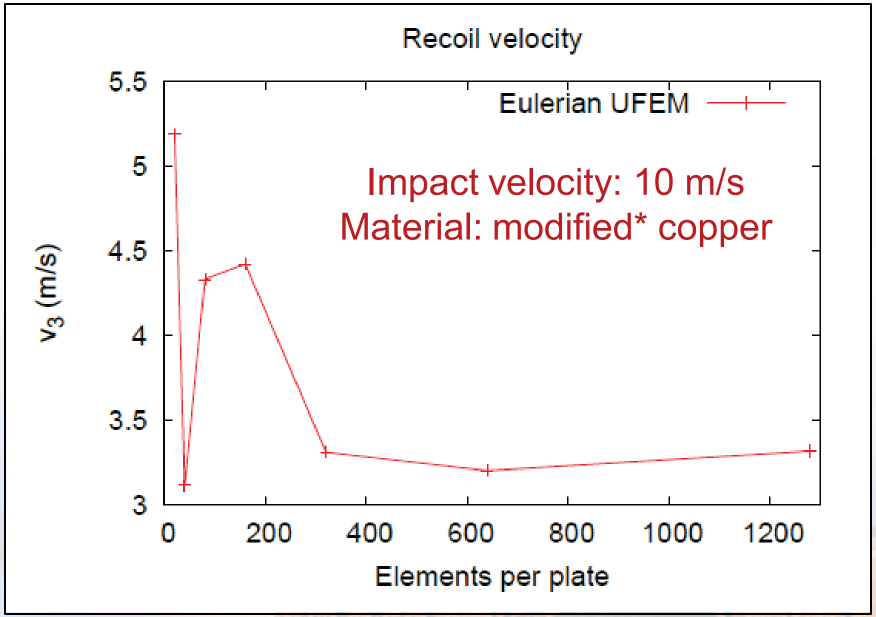
- Shock and release play important roles.
- Analytic solution is easily available,…

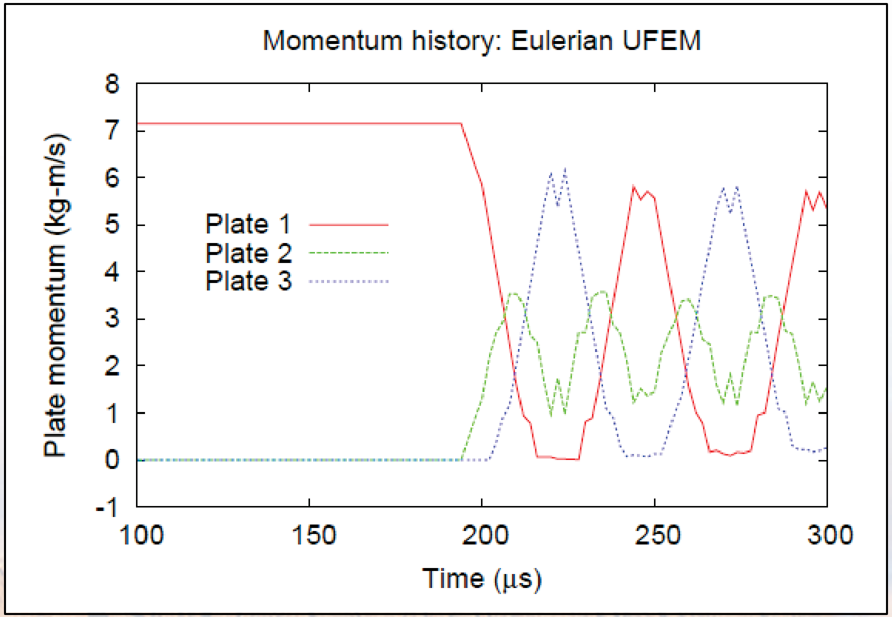
…if dissipation is absent.

Standard "unenriched" Eulerian FEM fails to capture plate separation

This problem is trivial for Lagrangian methods. However, ALEGRA's conventional Eulerian "unenriched" FEM ("UFEM") has no natural capacity for
(1) Void closure during impact, or
(2) Void opening during recoil

For v_{imp} = 10 m/s, it therefore fails to recover the analytic recoil velocity:





* "Modified": s = 0.0, Γ = 0.0

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Standard "unenriched" Eulerian FEM fails to capture plate separation

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(1) Void closure during impact, or
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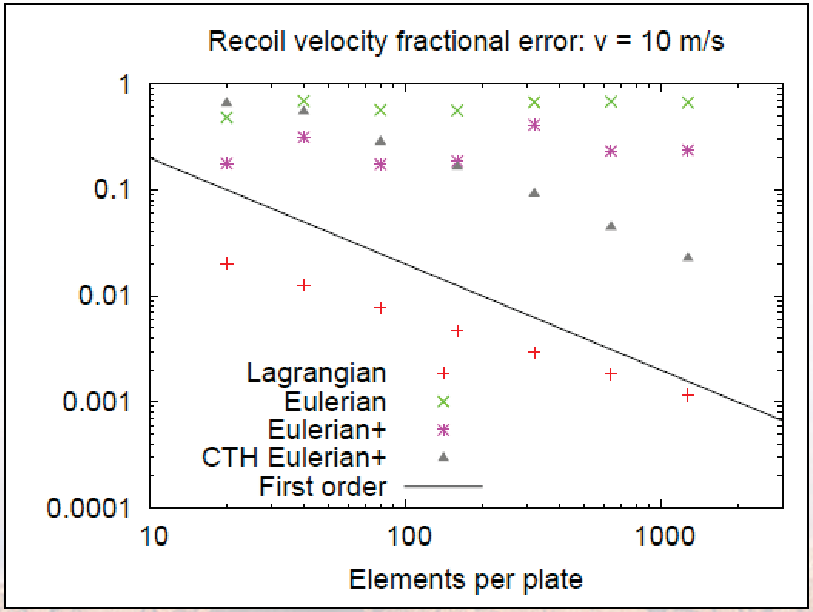
This is reflected unambiguously in the error convergence plots :

Fractional error:
$\varepsilon = (v_{recoil} - v_{imp}) / v_{recoil}$



- Lagrangian: standard unenriched ALEGRA, no remap.
- Eulerian: standard unenriched ALEGRA with remap.
- Eulerian+: activate internal *ad hoc* interface physics modifications.
- CTH Eulerian+: run identical simulation in CTH with ad hoc interface physics modifications (Marlin Kipp).

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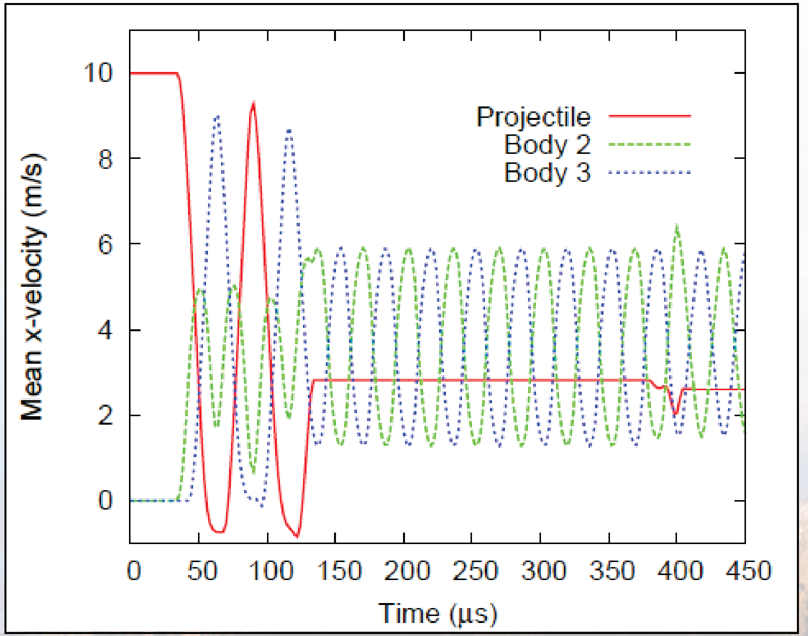
With XFEM, the void closure and opening is captured naturally, even in Eulerian frame

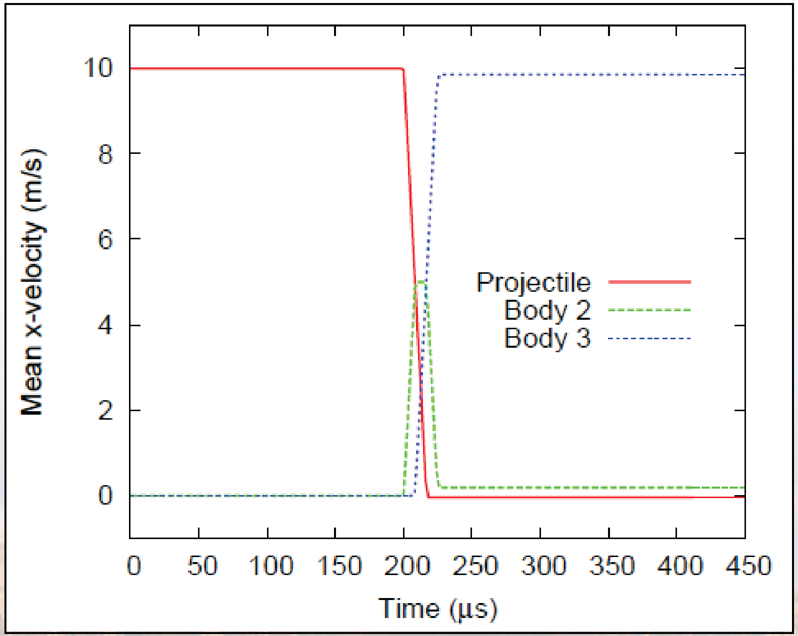
ALEGRA simulations for 1D cohesionless compound impact with UFEM and XFEM:

- Copper projectile incident on compound copper+copper target
- Plates have same material properties, different material ID's
- Impact velocity: v_{imp} = 10 m/s
- Mesh resolution: L = 40 elements per plate

Standard Eulerian UFEM



Eulerian XFEM

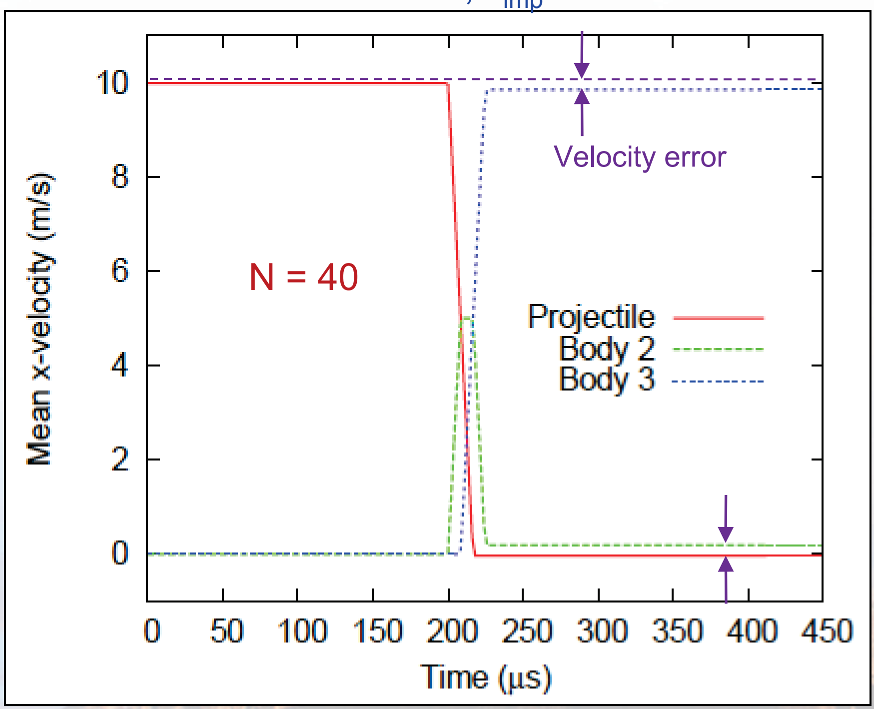


Eulerian XFEM captures plate separation at high and low impact velocity

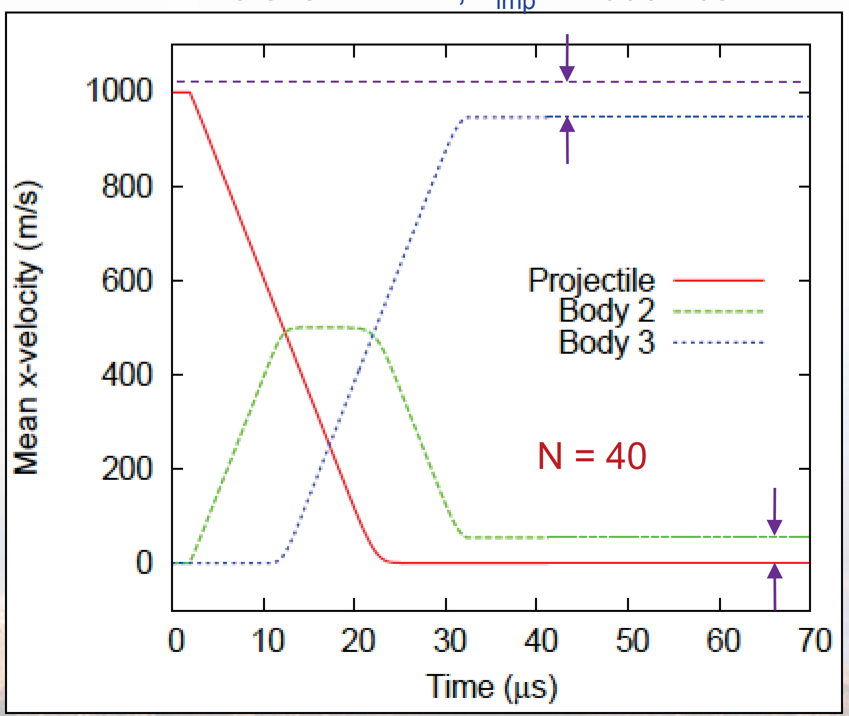
ALEGRA simulations for 1D cohesionless compound impact with XFEM:

- Copper projectile incident on compound copper+copper target
- Plates have same material properties, different material ID's
- Impact velocities: v_{imp} = 10 m/s and 1000 m/s
- Mesh resolution: N = 40 elements per plate

Eulerian XFEM, v_{imp} = 10 m/s



Eulerian XFEM, v_{imp} = 1000 m/s



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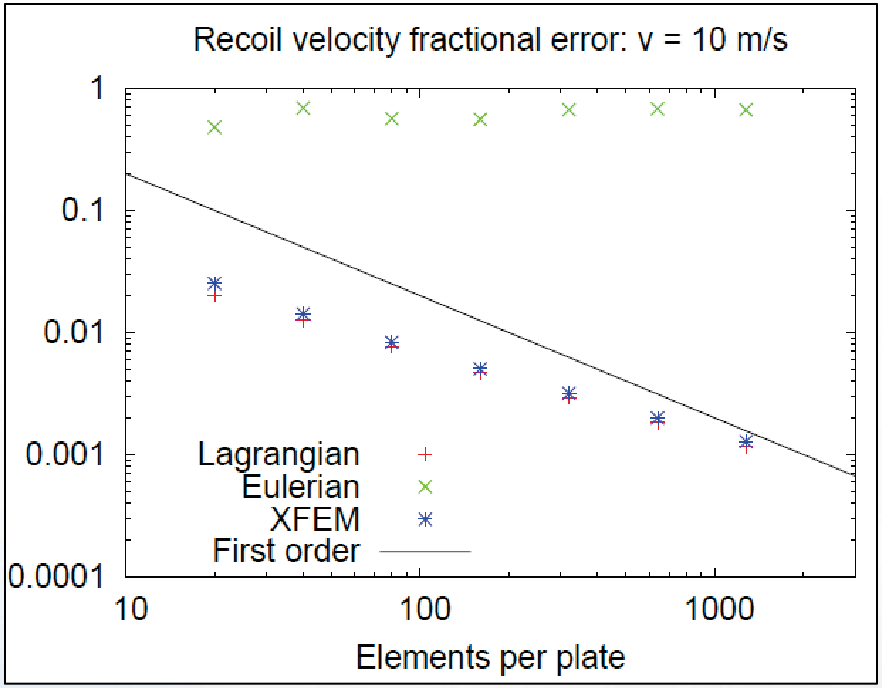
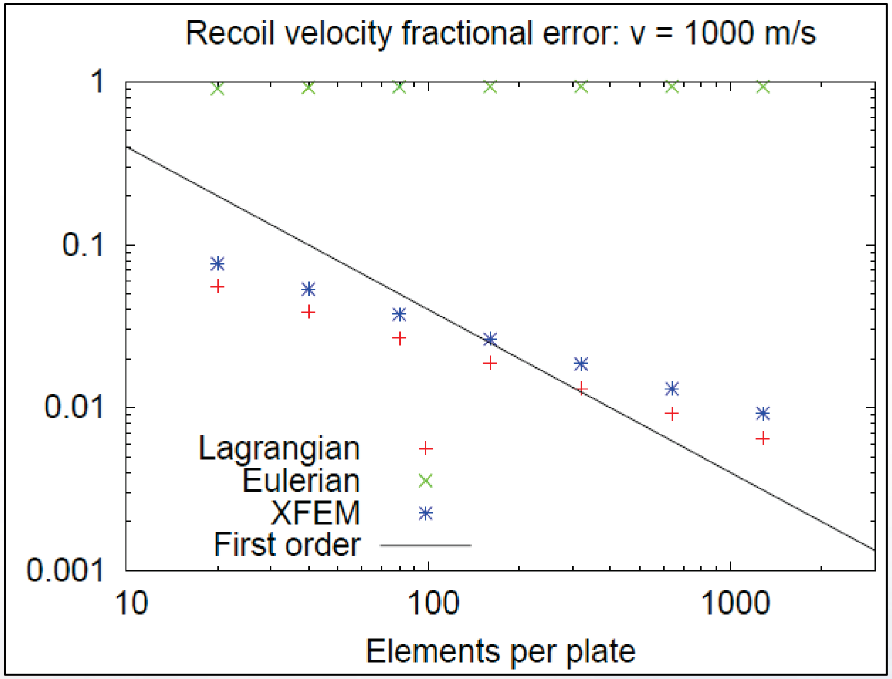
Error convergence for recoil velocity is only observed with XFEM

Error convergence under spatial refinement shows XFEM provides accuracy near that of the Lagrangian formulation. The standard Eulerian formulation fails to converge.





Mean convergence rate

	v_{imp} = 10 m/s	v_{imp} = 1000 m/s
Lagrangian	0.688	0.518
Eulerian	-0.079	0.007
XFEM	**0.718**	**0.509**

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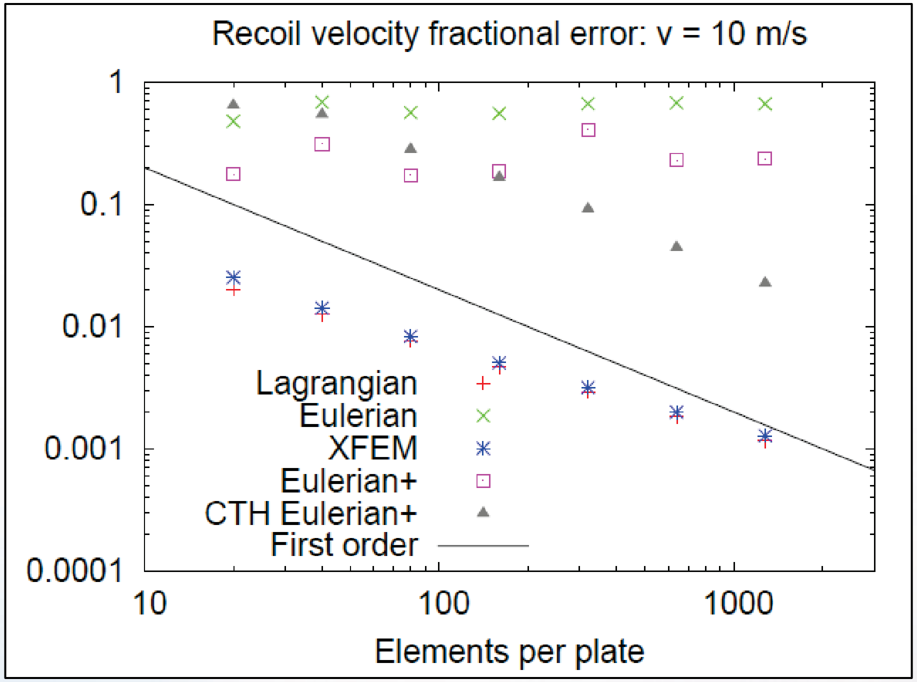
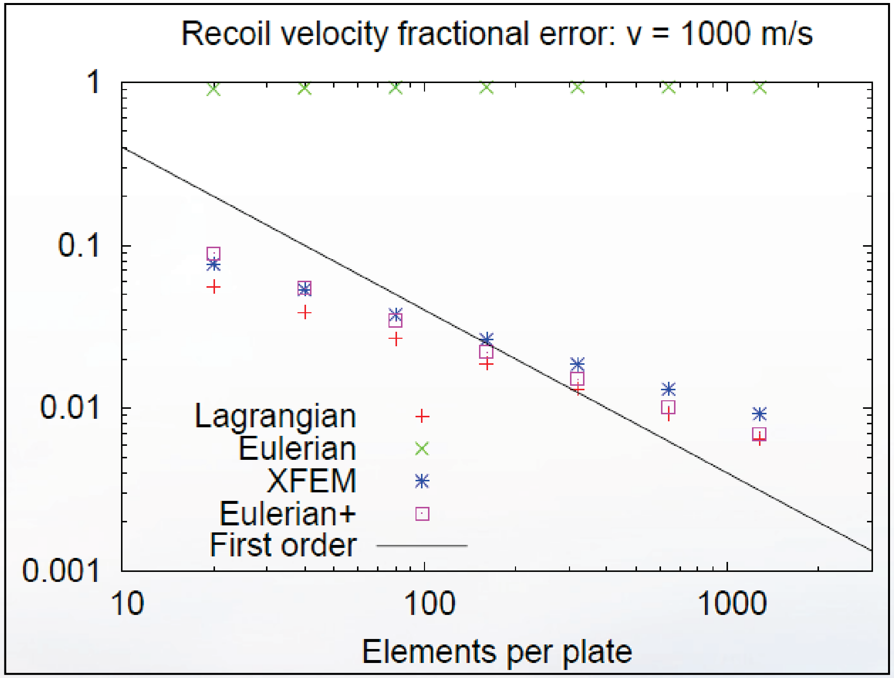
Ad hoc modifications to standard Eulerian FEM give mixed results

Error convergence under spatial refinement shows XFEM provides accuracy near that of the Lagrangian formulation. The standard Eulerian formulation fails to converge.





Mean convergence rate

	v_{imp} = 10 m/s	v_{imp} = 1000 m/s
Lagrangian	0.688	0.518
Eulerian+	-0.070	0.588
XFEM	**0.718**	**0.509**
CTH Eulerian+	0.807	---

Eulerian+: Eulerian with "intermaterial fracture" (ad hoc physics)

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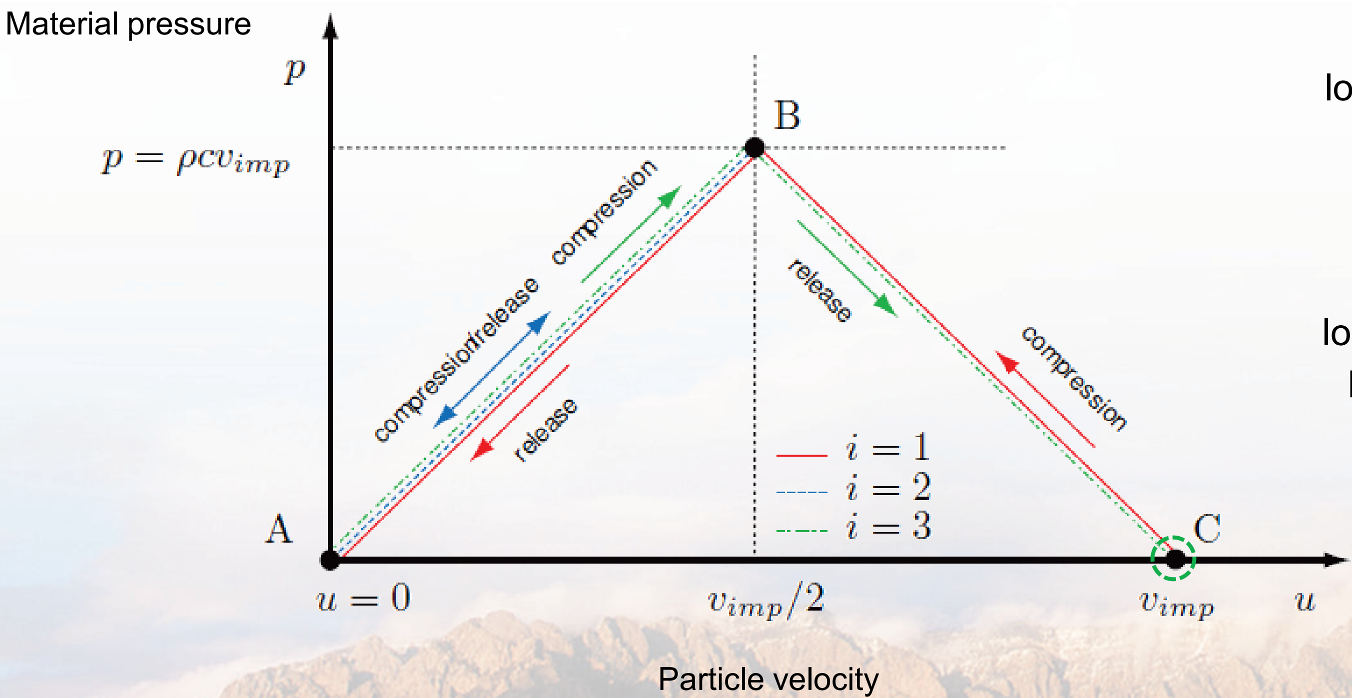
The exact solution can be obtained using a graphical analysis

The analytic solution for the case of symmetric, dissipationless impact can be obtained by a simple graphical analysis. (Full mathematical analysis is discussed in the extra slides.)

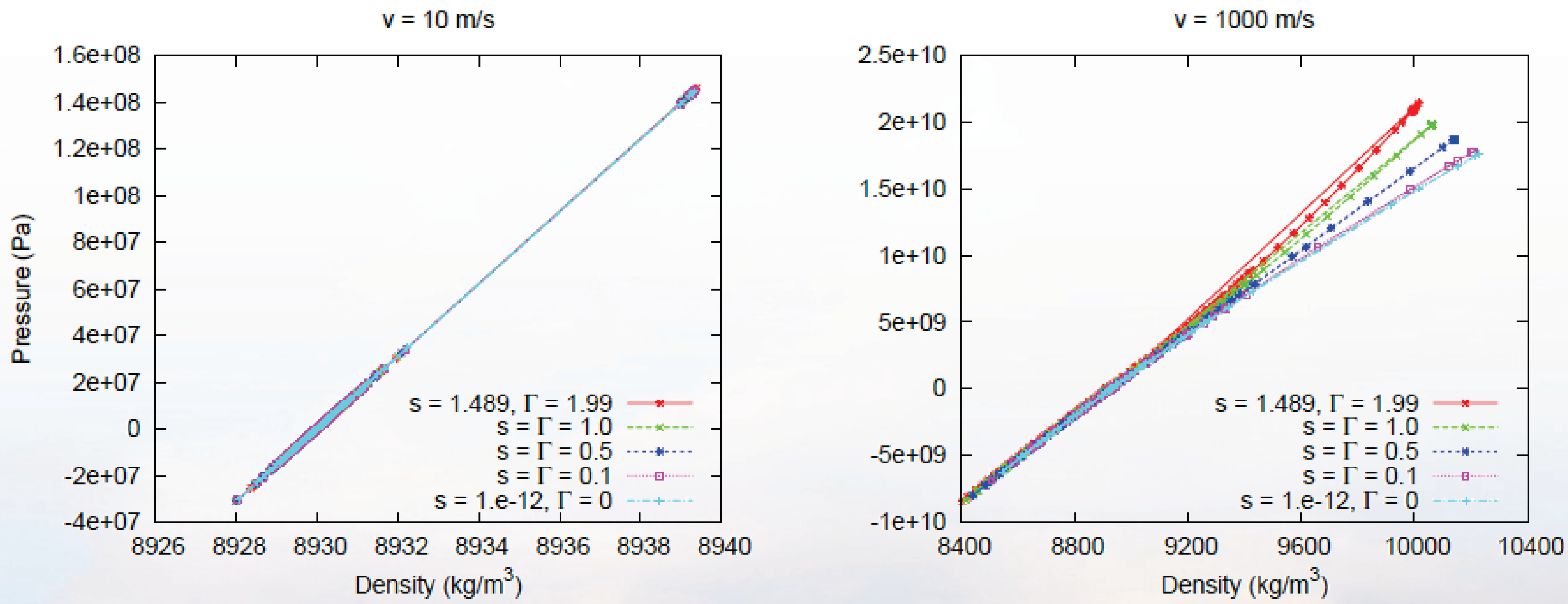


Symmetric: loading paths all have the same slope magnitude.

Dissipationless: loading and unloading paths are identical.

Material response can introduce dissipation, leading to analysis complications

At high impact velocity, the pressure-density response is not linear. Further, the response is also not exactly irreversible at high velocity, even with s = 0 and Γ = 0.



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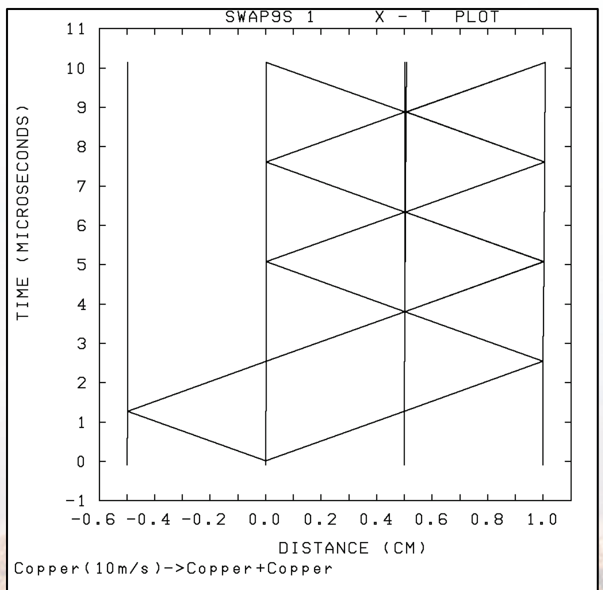
Due to ambiguities in reference solution, verification study is incomplete

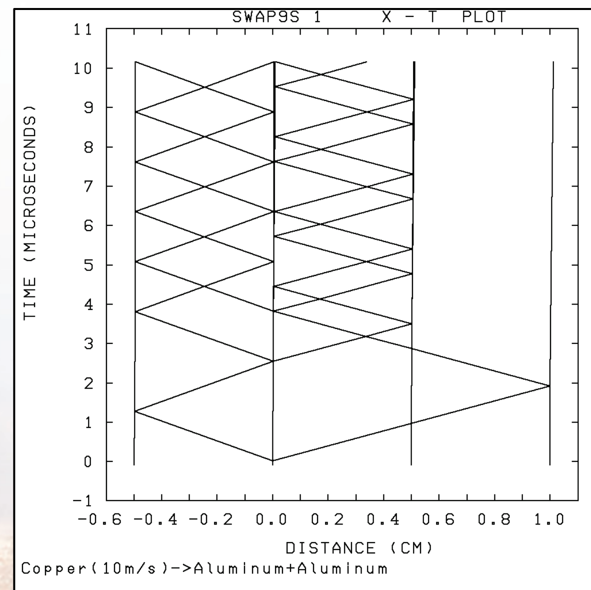
Analytic result for dissipationless, symmetric case is interesting and useful, but not realistic, particularly at high impact velocities.

Alternative: use method of characteristics to compute motion for:

- Multiple materials with disparate thicknesses (non-symmetric impact)
- Materials with real Hugoniot and EOS (s ≠ 0, $\Gamma \neq 0$, hence dissipation allowed)

Sandia MOC code: Stress Wave Analyzing Program (SWAP)*

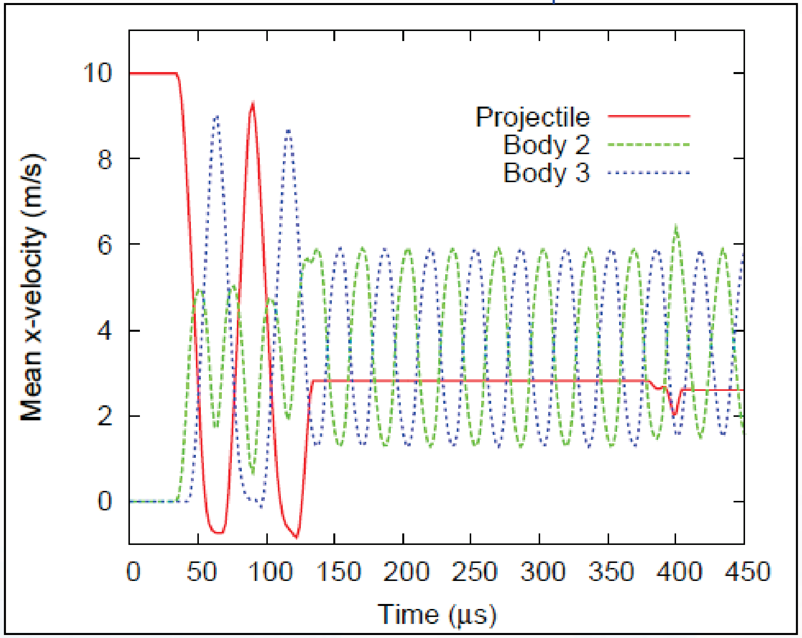




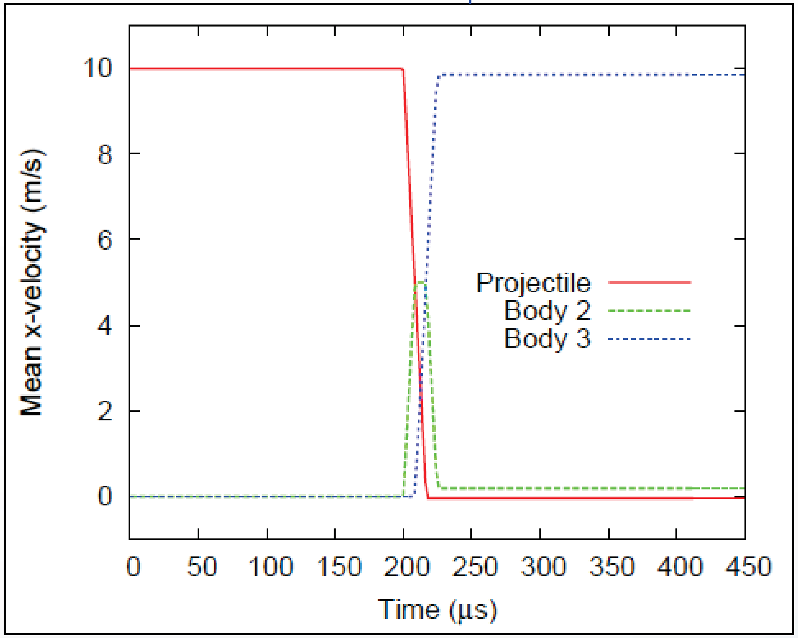
* Barker and Young, Sandia technical report SLA-74-0009, 1969.

Verification demonstrates XFEM has great potential for simulating shock and impact

Standard Eulerian UFEM, v_{imp} = 10 m/s



Eulerian XFEM, v_{imp} = 10 m/s



Summary and conclusion:

- Despite ambiguities, study has made clear demonstration of XFEM capability.
- Future verification work may extend to non-symmetric impact with real Hugoniot.
- Current investment by ALEGRA development team in XFEM is likely to yield enormous benefit in accuracy for multimaterial shock problems.

EXTRA SLIDES

An exact solution for Newton's cradle can also be obtained analytically

Three assumptions are necessary to make the problem dissipationless:

1. Deformation is uniaxial → stress tensor σ is spherical and p = tr(σ)/3
2. Material response is elastic → plasticity and fracture play no role
3. Thermal contribution to pressure much smaller than volumetric contribution

Pressure in the initial impact event is then determined solely by the Hugoniot:

$$\begin{aligned} p_{1,1} &= p_{1,0} + \rho_{1,0}(U_{1,1} - u_{1,0})(u_{1,1} - u_{1,0}) \\ p_{2,1} &= p_{2,0} + \rho_{2,0}(U_{2,1} - u_{2,0})(u_{2,1} - u_{2,0}) \end{aligned}$$

An additional assumption is required: shock wave speed = sound wave speed: fixed.

$$\begin{aligned} U_{1,1} &= u_{1,1} - c_1 \\ U_{2,1} &= u_{2,1} + c_2 \end{aligned}$$

Setting impact pressures and material velocities to be equal and solving, we obtain the shock state (j = 1) in impact:

$$u_{2,1} = \frac{u_{1,0}}{2} = \frac{v_{imp}}{2}$$

$$\rho_{1,1} = \rho_{1,0}\frac{U_{1,1} - u_{1,0}}{U_{1,1} - u_{1,1}}$$

$$\rho_{2,1} = \rho_{2,0}\frac{U_{2,1} - u_{2,0}}{U_{2,1} - u_{2,1}}.$$

An exact solution for Newton's cradle can also be obtained analytically

Pure transmission across the plate-2/plate-3 interface implies plate 3 has the same shock state as plate 2.

Our assumptions imply the release state also lies on the Hugoniot. Same expressions and process then yield plate-3 release state (j = 2), when pressure is set equal to zero:

$$p_{3,2} = p_{3,1} + \rho_{3,1}(U_{3,2} - u_{3,1})(u_{3,2} - u_{3,1}) = 0.$$

Substituting Hugoniot pressure, density, and particle speed for the shock state in plate 3 into this expression, and solving for plate-3 release particle speed yields:

$$u_{3,2} = u_{3,1} + u_{2,1} = 2u_{2,1}$$

$$u_{3,2} = u_{1,0} = v_{imp}.$$