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A New Approach to Understanding the Fundamental Physical Phenomena Governing Tunnel Wall Stability under Shock Loading using Peridynamic Theory

Award Number HDTRA1-08-10-BRCWMD


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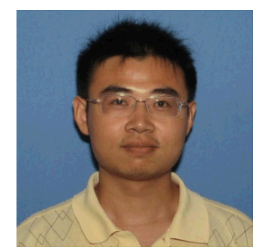
Mr. Hady Joumaa


Mr. Jun Li

Presentation for

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Outline of Presentation

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  - "Fractal Mechanics" (Mr. Joumaa)
  - "Wave Propagation in Peridynamic Materials" (Dr. Demmie)
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- Conclusions
- Future Directions

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A New Approach to Understanding the Fundamental Physical Phenomena Governing Tunnel Wall Stability under Shock Loading using Peridynamic Theory,

Dr. Paul N. Demmie, Sandia National Laboratories,

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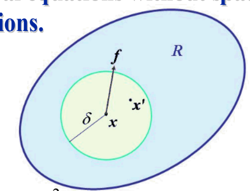
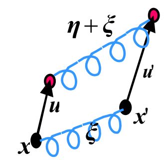
Objectives: The objectives are (1) to use peridynamic theory (PD) to identify key parameters and determine their relative importance for tunnel wall stability and (2) to develop advanced numerical models that relate ground motion parameters, rock strength properties, and tunnel geometry and reinforcement.

Relevance: The proposed research will provide (1) a greater knowledge and understanding of physical phenomena and observable facts pertinent to tunnel wall stability under shock loading and (2) a scientific basis to enhance our capabilities to defeat WMD.

Peridynamic theory **(PD) is a theory of continuum mechanics that uses differo-integral equations without spatial derivatives rather than partial differen equations.**

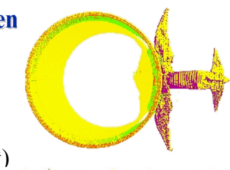


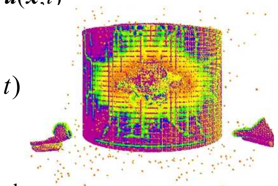


Configuration Variables

$\xi = x' - x$

$\eta = u(x',t) - u(x,t)$





$$\rho(x)\frac{d^2}{dt^2}u(x,t) = \iiint_R f(u(x',t)-u(x,t), x'-x)\,dV' + b(x,t)$$

where
- ρ is the density at x,
- t is the time,
- R is the computational domain,
- b is the body force.
- x is the position vector,
- u is the displacement vector,
- f is the pairwise force function, and

Approach: Our approach is based on the peridynamic theory of continuum mechanics with its non-local, pairwise interactions and failure determined at the bond level. The research investigates (1) the representation of rock as a random, peridynamic material, (2) the fundamental physics of shock propagation through complex, fractured media, and (3) the failure of such media at boundaries.

Personnel Support: The project will partially support Dr. Paul N. Demmie at Sandia National Laboratories and Dr. Martin Ostoja-Starzewski at the University of Illinois. An objective of the project is to train university students. Therefore, the project will support one graduate student full time and partially support another graduate student at the University of Illinois.

Tasks:

- **FY10**: Represent rock as a random PD material. Study shock propagation in PD.
- **FY11**: Represent rock as a random-fractal-PD material. Study shock propagation in such PD materials
- **FY12**: Investigate boundary effects and tunnel wall stability. Validate numerical models and identify key parameters and their relative importance.

Funding Profile:

FY10: $180K; **FY11:** $189K ; **FY12:** $199K

Principal Investigator (PI) Contact Information:

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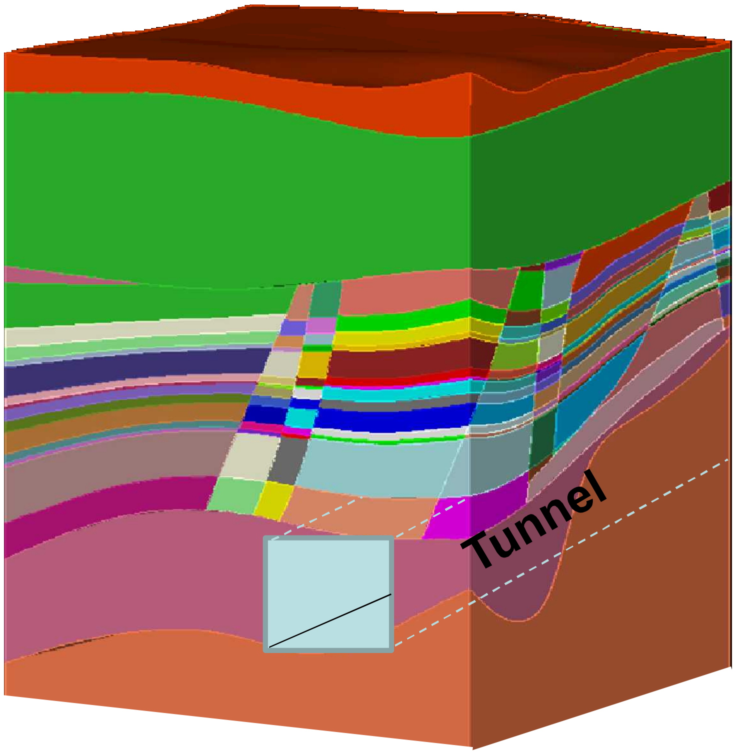
Project Objectives

1. To obtain a better understanding of the fundamental physical phenomena governing tunnel wall stability under shock loading including identification of key parameters and their relative importance for tunnel wall stability during shock loading.
2. To develop advanced numerical models that relate ground motion parameters, rock strength properties, tunnel geometry, features, and reinforcement such as rock bolts.

Background and Significance

- **The stability of tunnels in hard rock geologies under ground shock loading is of direct consequence to studies of vulnerability or survivability of deeply-buried hard targets.**
- **Host media is characterized by highly jointed, faulted, and irregular 3D geology.**
- **The physical processes and parameters that determine stability include tunnel depth, geometry and dimensions, rock strength, character of joints and faults, wall reinforcement, and the magnitude and duration of ground shock.**
- **Ground shock propagation in hard rock geologies, and the basic interaction and response of tunnels to ground shock loading has been the subject of considerable research. However, there is insufficient understanding of the interactions of the dynamic loading near tunnels and at tunnel walls and the degree of such loading required to destabilize the walls and produce rock ejection, spallation, or collapse.**



- **Relationships between ground motion parameters, rock properties, tunnel characteristic and reinforcement, need to be understood, and advanced numerical models incorporating such relationships need to be developed.**
- ***To meet these challenges, peridynamic theory, the mechanics of random media, and the mechanics of fractal media will be combined.***

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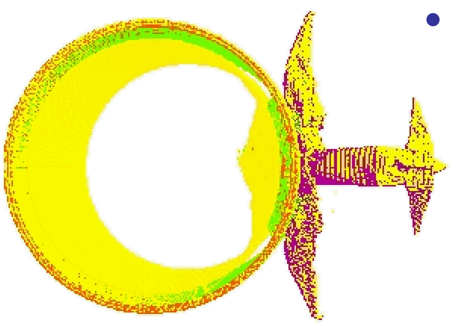
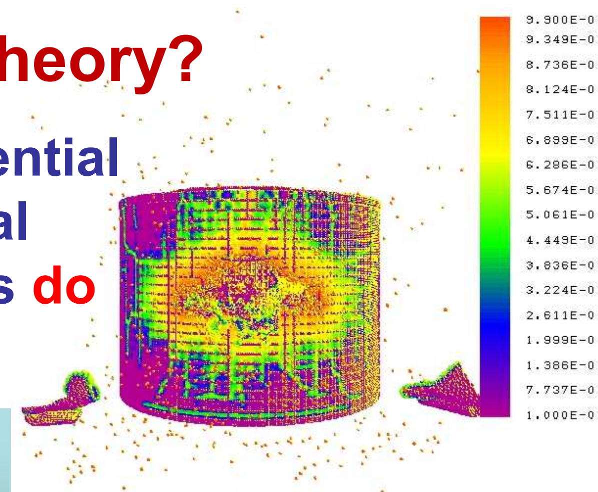
Computational Method: Peridynamic Theory

- ***Peridynamic theory*** **is a theory of continuum mechanics that uses integro-differential equations without spatial derivatives rather than partial differential equations.**
  - **Bond-Based Peridynamics[1]**
  - **State-Based Peridynamics[2]**
- **Peridynamic means "near force".**

Why do we use peridynamic theory?



- **The fundamental partial differential equations used in conventional finite element or particle codes do not apply at discontinuities.**



With peridynamics, cracks are part of the solution, not part of the problem.

[1]Silling, "Reformulation of elasticity theory for discontinuities and long-range forces", in *Journal of the Mechanics and Physics of Solids*, 48 (2000) , pp. 175-209. (Silling 2000)

[2] S.A. Silling et al. "Peridynamic States and Constitutive Modeling", in *J Elasticity*, 88 (2007), pp. 151–184. (Silling 2007)

Summary of Technical Project Plan

- The proposed research falls naturally into five top-level, basic-research tasks:
  - Task 1: Represent rock as a stochastic peridynamic material.
  - Task 2: Represent rock as a stochastic-fractal peridynamic material.
  - Task 3: Investigate shock propagation in peridynamic materials.
  - Task 4: Investigate boundary effects from shock loading and tunnel wall stability.
  - Task 5: Validate numerical models and identify key parameters.
- It is the goal of this research to complete these tasks and achieve the objectives of this technical proposal in three-five years.
  - To attain this goal, we plan to acquire and utilize existing data for material characterization and failure processes and for validation of the numerical models.
- More details of tasks and subtasks are found in the technical project plan.

Results

- Results and discussion from team members are given in the following slides:
  - "Towards Stochastic Peridynamics" (Dr. Ostoja-Starzewski)
  - "Fractal Mechanics" (Mr. Joumaa)
  - "Wave Propagation in Peridynamic Materials" (Dr. Demmie)
- How did the results modify or confirm your approach?
  - The results thus far confirm our confidence that a stochastic peridynamic theory combining peridynamic theory with random-fractal material characterization is a viable approach to enhance the understanding of the fundamental physical phenomena governing tunnel wall stability in hard rock under dynamic shock loading conditions.
- Residual Risk/Milestone Status
  - There is technical risk in using any new approach like peridynamics. But, the biggest risk in meeting milestones is implementing and testing our numerical models since the codes available are export controlled and students are usually not citizens of the United States. We may need help from DTRA to mitigate this risk.
  - No milestones exist through August 2011. The milestone for Task 1 is at the end of December 2011.

Towards Stochastic Peridynamics

<u>**three scales:**</u>

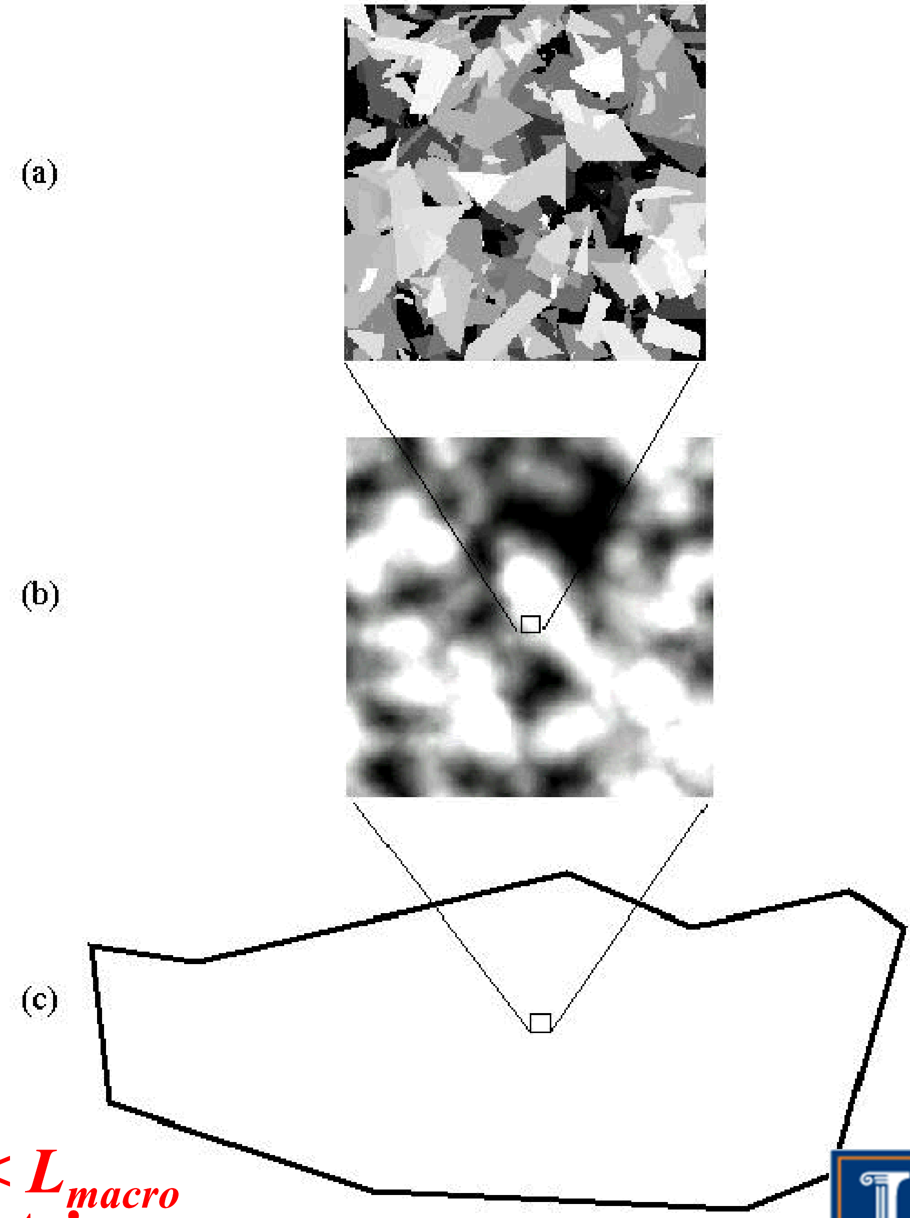
microscale*: average grain size *d
(microstructure)

mesoscale*: *L
if not RVE, then
inhomogeneous
continuum

$$\Rightarrow \qquad \delta = L / d$$

***macroscale*: L_{macro}**

separation of scales $d << L << L_{macro}$
does not hold on wavefronts!



Formulating stochastic peridynamics

Field operator acting on $\mathbf{u}$: $\mathbf{L}(\omega)\mathbf{u} = \mathbf{s} \qquad \omega \in \Omega$

Randomness enters through random field of material

properties, e.g. stiffness tensor field $\mathbf{C}(\omega,\delta,\mathbf{x}) = \langle \mathbf{C} \rangle + \mathbf{C}'(\omega,\delta,\mathbf{x})$

$\Rightarrow \qquad \nabla \cdot [\mathbf{C}(\omega,\delta,\mathbf{x}) : \varepsilon] = \mathbf{s}$

$\Rightarrow \qquad \mathbf{B} = \{B(\omega,\delta,\mathbf{x}); \omega \in \Omega, \mathbf{x} \in E^3\}$ *random medium*

set of deterministic realizations

Basic question of stochastic mechanics:

How different is the average response $\langle \mathbf{u} \rangle$ of the random medium governed by $\left\langle \mathbf{L}^{-1} \right\rangle^{-1} \langle \mathbf{u} \rangle = \mathbf{s}$

from the response of a directly averaged medium $\langle \mathbf{L} \rangle \mathbf{u}_{ave} = \mathbf{s}$?

Formulating stochastic peridynamics

field equation: $\rho(\mathbf{x},\omega)\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{B(\omega)} \mathbf{f}\left(\eta,\xi,\omega\right) d\mathbf{x}' + \mathbf{b}(\mathbf{x},t) \qquad \omega \in \Omega$

randomness enters through random fields $\rho(\mathbf{x},\omega), \mathbf{f}\left(\eta,\xi,\omega\right)$

$$\Rightarrow \qquad \mathbf{B} = \{B(\omega,\mathbf{x});\omega \in \Omega, \in E^3\} = \{\rho(\mathbf{x},\omega),\mathbf{f}\left(\eta,\xi,\omega\right)\}$$

random medium

Focus on microelastic material with pairwise force function (PFF):

$$\rho(\mathbf{x},\omega)\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{B(\omega)} H\left(p,\xi,\omega\right)\left(\eta+\xi\right) d\mathbf{x}' + \mathbf{b}(\mathbf{x},t,\omega)$$

with

$$f\left(\eta,\xi,\omega\right) = H\left(p,\xi,\omega\right)\left(\eta+\xi\right) \qquad p = \left|\eta+\xi\right| \qquad \omega \in \Omega$$

Formulating stochastic peridynamics

Adopt a scalar-valued function $H\left(p,\xi,\omega\right)$ as a random field.

To specify that random field, use 1st and 2nd order statistics,

(It is not simply a white-noise random field.)

work with wide-sense stationary (WSS) random fields

use *correlation function*

$$\rho_H(\mathbf{x}_1,\mathbf{x}_2)=\frac{\langle[H(\mathbf{x}_1)-\langle H(\mathbf{x}_1)\rangle][H(\mathbf{x}_2)-\langle H(\mathbf{x}_2)\rangle]\rangle}{\sigma_H(\mathbf{x}_1)\sigma_H(\mathbf{x}_2)}$$

$$\rho_H(\mathbf{x}_1,\mathbf{x}_2)=\rho_H(r)\qquad r=\left|\mathbf{x}_1-\mathbf{x}_2\right|$$

Formulating stochastic peridynamics

<u>**Given our interest in multiscale, stochastic field phenomena**</u>**, employ correlation function with decoupling of the *fractal geometric effect* from the *Hurst effect***

Fractality is typically present on smaller length scales, while Hurst effect arises on large scales (persistence).

- Cauchy correlation function:

$$\rho(x) = 1 - \left(1 + x^{\theta}\right)^{-\eta/\theta}, \quad 0 < \theta \leq 2, \quad \eta > 0$$

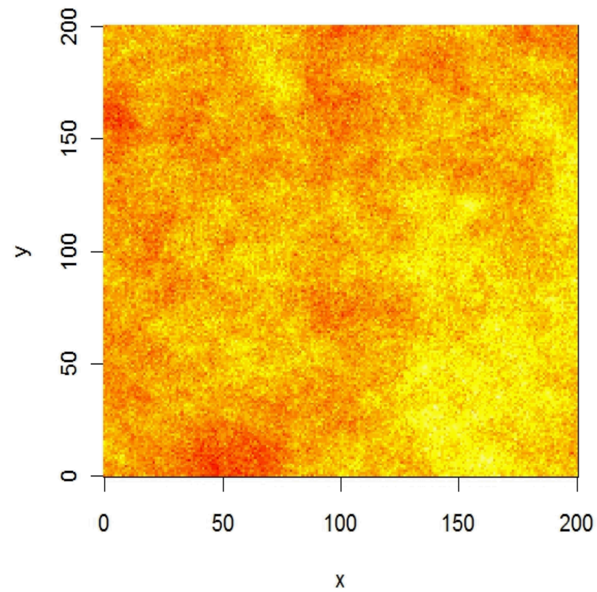
- Dagum correlation function:

$$\rho(x) = 1 - \left(\frac{x^{\beta}}{1 + x^{\beta}}\right)^{\gamma}, \quad \beta > 0, \quad \gamma > 0$$

Formulating stochastic peridynamics

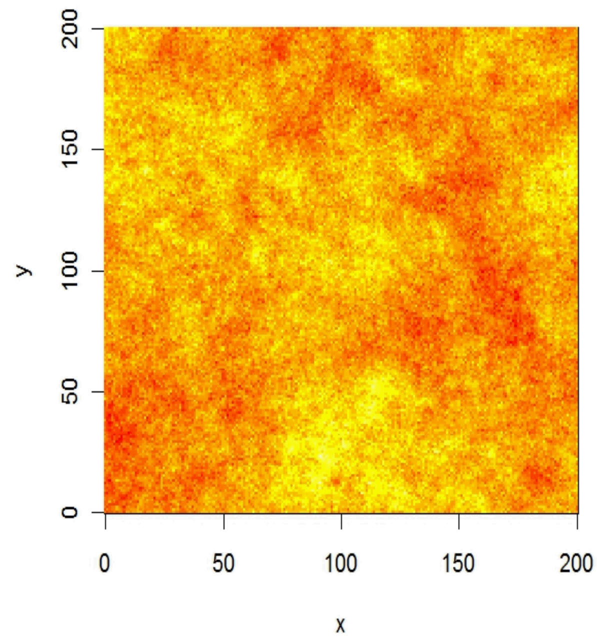
- **Cauchy correlation function:**

  one realization *B*(ω)

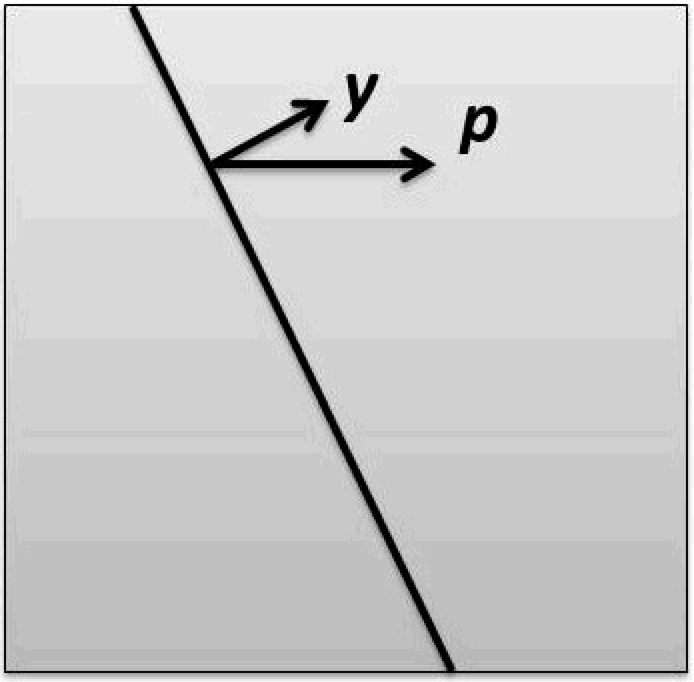


- **Dagum correlation function:**

  one realization *B*(ω)

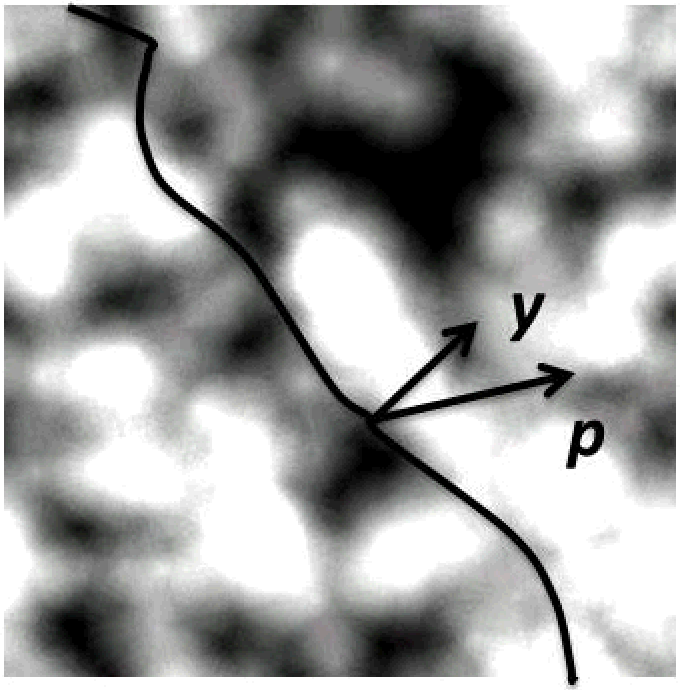


Formulating stochastic peridynamics



(a)

Wavefront in a homogeneous anisotropic medium, propagating in direction ***p***, locally along a ray of direction ***y***.

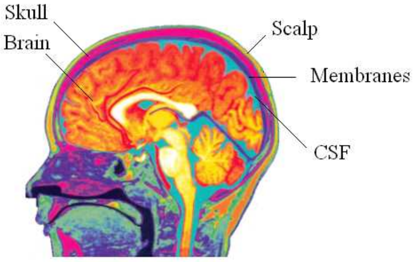


(b)

Wavefront in a realization *B(ω)* of a randomly inhomogeneous anisotropic medium.

Fractal Mechanics: Approach & Methods

- Development of governing formulation of mechanical behavior of fractal materials applying basic fractional calculus
- Application of homogenization techniques to "regularize" the fractional integrals
- Understand the wave motion in two different models, analytically and computationally
  - Isotropic: fluid
  - Anisotropic: solid



Pine tree leaves & human brain are examples of random (natural) fractals

Fractal Fluid Model

- **Formulation of wave propagation equation by application of dimensional regularization**

$$\frac{\partial^2 p}{\partial t^2} = \left(c \frac{2^{D-3}\Gamma\left(D/2\right)}{\Gamma\left(3/2\right)} |R|^{2-D} \right)^2 \left[(3-D)\vec{R}\cdot\vec{\nabla}p + |R|^2 \nabla^2 p \right]$$

- **Derived in [V. Tarasov, *Ann. Phys*. (2005), Fractional hydrodynamic equations for fractal media]**
- **Solved analytically and numerically in [H. Joumaa & M. Ostoja-Starzewski, *ZAMP* (2011), On the wave propagation in isotropic fractal media]**

Analytical Solution

- **Modal decomposition**

$$H'' + m^2 H = 0$$

$$G'' + \frac{G'}{\tan\theta} + G\left[n(n+1) - \frac{m^2}{\sin^2\theta}\right] = 0$$

$$r^2 F'' + (5-D) rF' + k^2\lambda^2 r^{2D-4} F - n(n+1) F = 0$$

- **Decoupled solution**

fractal radial harmonic functions of first and second kind

$$H(\phi) = h_m e^{\pm im\phi}$$

$$G(\theta) = g_{mn} L_n^m(\cos\theta)$$

$$F(r) = f_1 F_\nu^{(1)}(r) + f_2 F_\nu^{(2)}(r)$$

$$F_\nu^{(2)}(r,k,D) = r^{\frac{D-4}{2}} Y_\nu\left(\frac{k\lambda}{D-2} r^{D-2}\right)$$

$$F_\nu^{(1)}(r,k,D) = r^{\frac{D-4}{2}} J_\nu\left(\frac{k\lambda}{D-2} r^{D-2}\right)$$

$$\nu = \frac{\sqrt{4n(n+1) + (D-4)^2}}{2(D-2)} > 0$$

Numerical Solution

- **Finite element formulation**

$$\left(\frac{\lambda}{c}\right)^2 \int_V \frac{\partial^2 p}{\partial t^2}\hat{p}\,dV = \int_V (3-D)\,\hat{p}\,|R|^{4-2D}\,\vec{R}\cdot\vec{\nabla}p\,dV + \int_V \hat{p}\,|R|^{6-2D}\,\nabla^2 p\,dV$$

$$\left(\frac{\lambda}{c}\right)^2 \sum_{e=1}^{N_e}\int_{V_e} \frac{\partial^2 p}{\partial t^2}\hat{p}\,dV_e + (3-D)\sum_{e=1}^{N_e}\int_{V_e} \hat{p}\,|R|^{4-2D}\,\vec{R}\cdot\vec{\nabla}p\,dV_e + \sum_{e=1}^{N_e}\int_{V_e} |R|^{6-2D}\,\vec{\nabla}\hat{p}\cdot\vec{\nabla}p\,dV_e = 0$$

inertia term — fractal elastic term — regular elastic term

- **Elastodynamic approach**
  - Elemental mass matrix $M_e = \left(\frac{\lambda}{c}\right)^2 \int_{V_e} H^T H dV_e$
  - Elemental elastic matrices

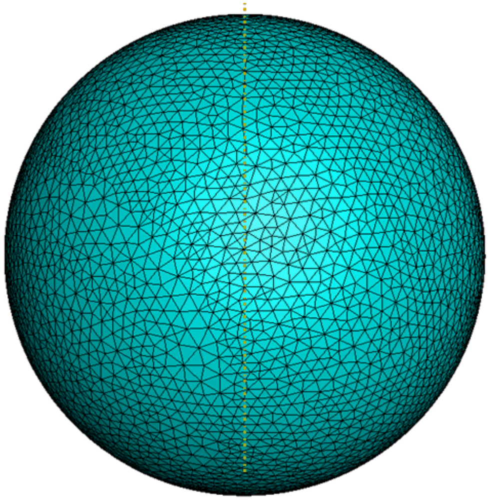
$$L_e = (3-D)\int_{V_e} |R|^{4-2D}\, H^T \left(\vec{R}\cdot\vec{\nabla}H\right) dV_e \qquad K_e = \int_{V_e} |R|^{6-2D}\,\vec{\nabla}H^T\cdot\vec{\nabla}H dV_e$$

  - Final assembly

$$\mathbf{M}\ddot{p} + (\mathbf{L}+\mathbf{K})\,p = 0$$

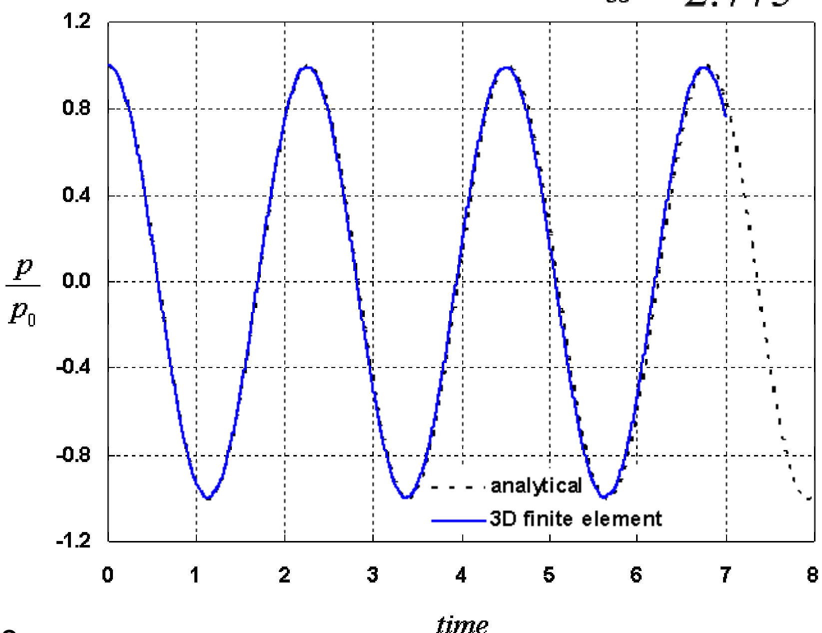
Simulation Results

- **Modal excitations on a spherical shell problem**
- **Newmark method (trapezoidal) for time march transient solution**

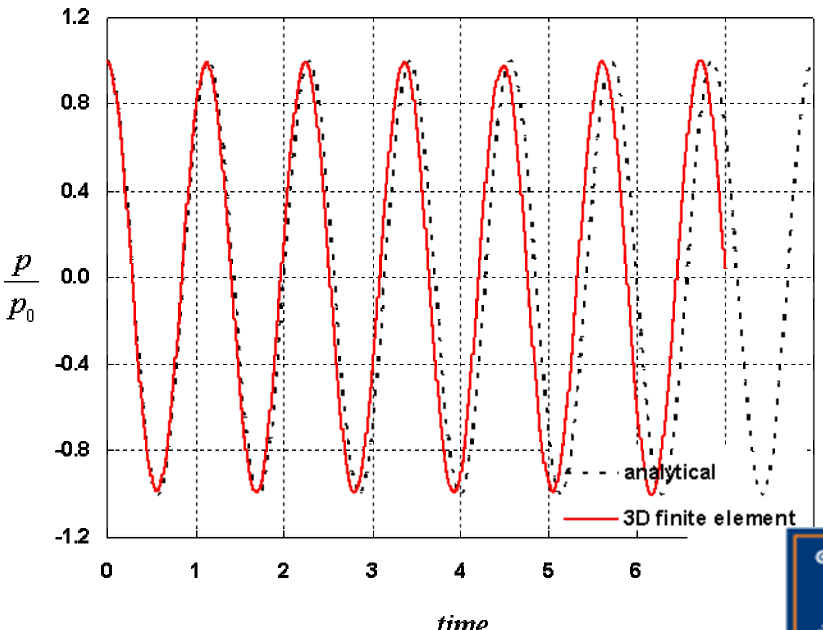


spherical shell meshed with tetrahedral elements

first mode $\omega = 2.775$



second mode $\omega = 5.501$



Solid Model

- **Formulation of general wave equation by [Li and Starzewski, *Proc. R. Soc.* (2009), Fractal solids, product measures and fractional wave equations]**

$$\rho \ddot{u}_i = \frac{\lambda + \mu}{g_3}\left(\frac{g_3 u_{j,i}}{g_i g_j}\right)_{,j} + \frac{\mu}{g_3}\left(\frac{g_3 u_{i,j}}{g_j g_j}\right)_{,j}$$

- **Analytical solution** $u_i \equiv u_i\left(x_i, t\right)$
  - **Limited to special problems**
  - **Modal decomposition in Cartesian system**

$$(L-x)^2 f''(x) + (D-1)(L-x) f'(x) + k^2 D^2 (L-x)^{2D} f(x) = 0$$

  - **Two independent homogeneous solutions (*fractal harmonic functions*)**

$$f_1(x) \equiv \cos\left[k(L-x)^D\right] \qquad f_2(x) \equiv \sin\left[k(L-x)^D\right]$$

Numerical Solution

- **Finite element formulation**

$$\int_{\Omega} \rho g_3 \ddot{u}_i w_i d\Omega = \int_{\Omega} (\lambda+\mu)\left[\left(\frac{g_3 u_{j,i} w_i}{g_i g_j}\right)_{,j} - \frac{g_3 u_{j,i} w_{i,j}}{g_i g_j}\right] d\Omega + \int_{\Omega} \mu \left[\left(\frac{g_3 u_{i,j} w_i}{g_j g_j}\right)_{,j} - \frac{g_3 u_{i,j} w_{i,j}}{g_j g_j}\right] d\Omega$$

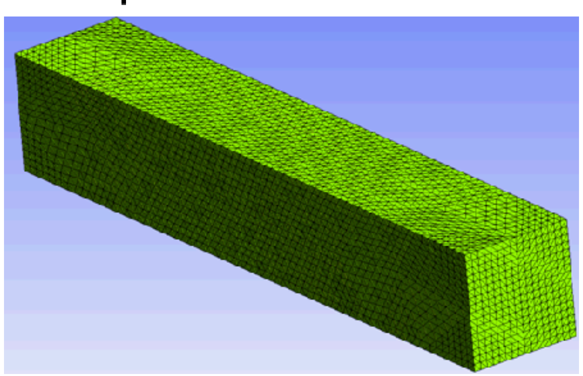
$$\int_{\Omega} \rho \ddot{u}_i w_i d\Omega + (\lambda+\mu)\int_{\Omega} \frac{u_{j,i} w_{i,j}}{g_i g_j} d\Omega + \mu \int_{\Omega} \frac{u_{i,j} w_{i,j}}{g_j g_j} d\Omega = 0$$

inertia term; first elastic term; second elastic term

- **Resulting equation**

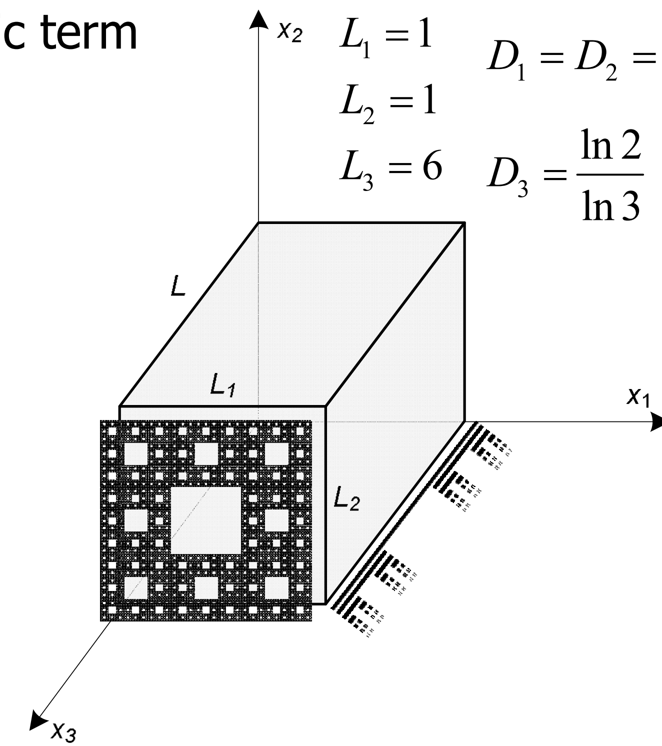
$$\mathbf{M} \cdot \ddot{\mathrm{U}} + (\mathbf{K} + \mathbf{H}) \cdot \mathrm{U} = 0$$

Carpinteri column FE mesh



$L_1 = 1$, $L_2 = 1$, $L_3 = 6$

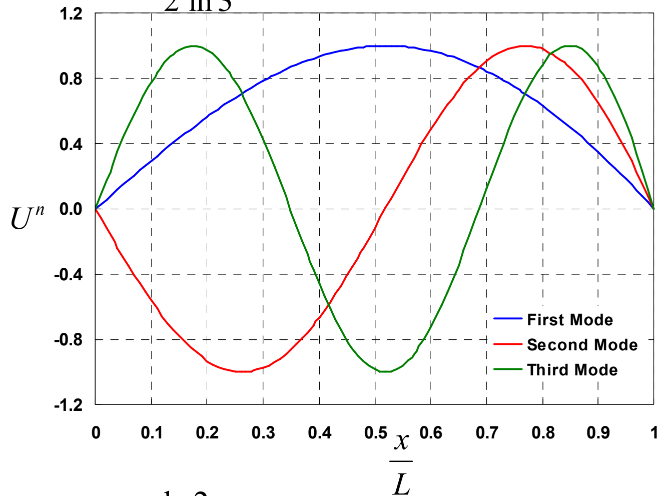
$$D_1 = D_2 = \frac{1}{2}\frac{\ln 8}{\ln 3} \qquad D_3 = \frac{\ln 2}{\ln 3}$$



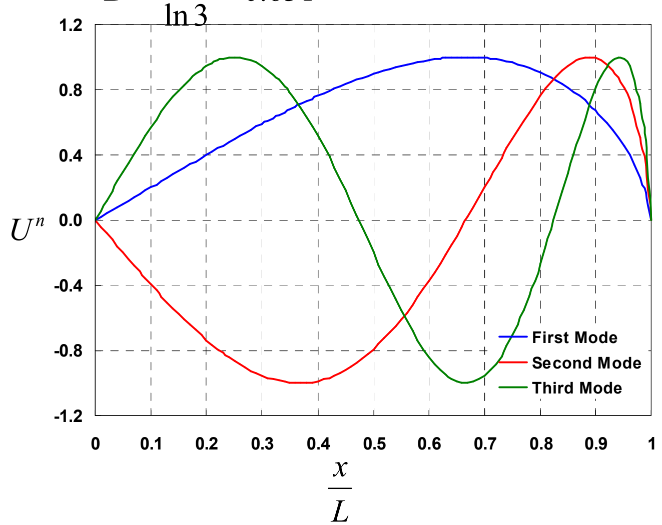
Simulation Results

- Modal shapes

$$D = \frac{1}{2}\frac{\ln 8}{\ln 3} = 0.946$$

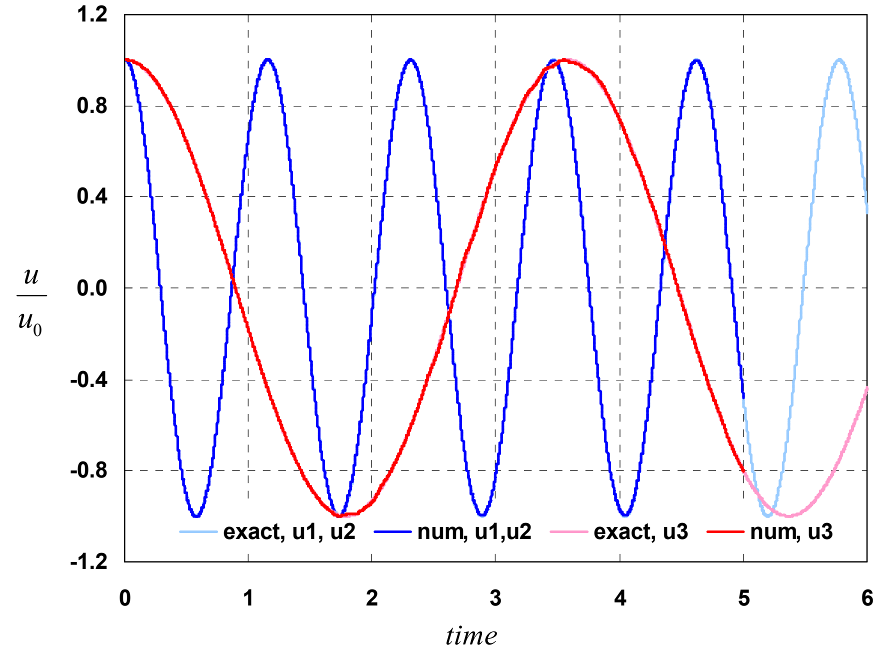


$$D = \frac{\ln 2}{\ln 3} = 0.631$$



- Modal excitation
- Transient response

first mode excitation in all directions



material properties

$\lambda = 1$

$\mu = 1$

$\rho = 1$

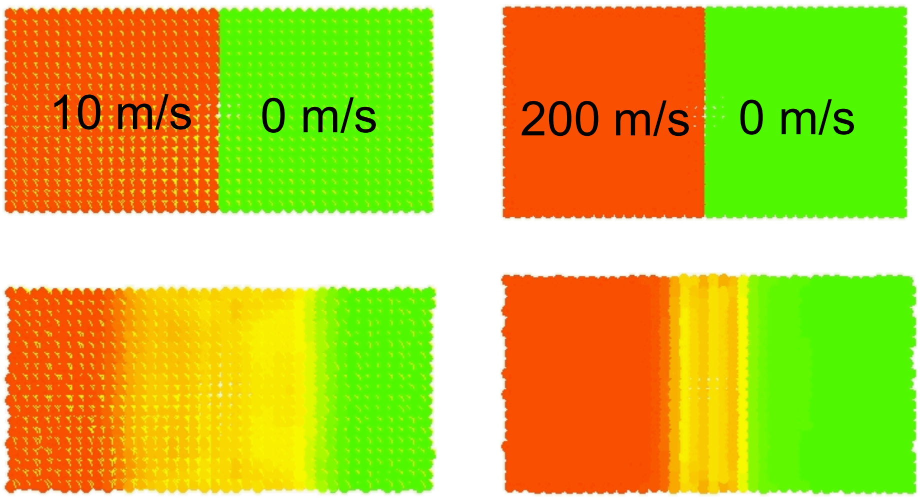
modal frequencies

$\omega_1 = \omega_2 = 5.441$

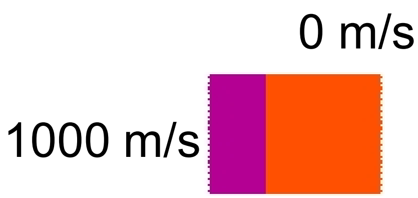
$\omega_3 = 1.757$

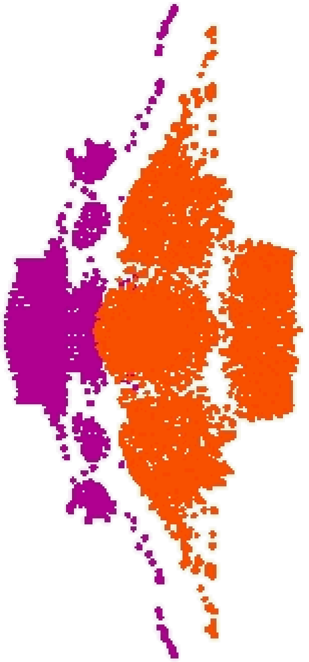
Wave Propagation in Peridynamic Materials

- **We performed impact simulations to study wave propagation and spall in Cu from impacts. The following are some examples:**



Velocity Magnitude





Materials

Accomplishments
(since August 2010)

- **Publications**
  - P.N. Demmie and M. Ostoja-Starzewski, "Waves in fractal media," *Journal of Elasticity*, *D.E. Carlson special issue*, online, 2011. DOI 10.1007/s10659-011-9333-6
  - H. Joumaa and M. Ostoja-Starzewski, "On the wave propagation in isotropic fractal media," *ZAMP*, in press, 2011. DOI: 10.1007/s00033-011-0135-2
  - J. Li and M. Ostoja-Starzewski, "Micropolar continuum mechanics of fractal media," *Int. J. Eng. Sci*. (*A.C. Eringen special issue*) online, 2011. doi: 10.1016/jijengsci.2011.03.010
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- **Presentations**
  - Demmie, "Peridynamic Theory: An Approach to Computational Mechanics without Spatial Derivatives", *Engineering Mechanics Institute Conference, Northeastern University, Boston, MA, June 2-4, 2011*
- **Graduate students**
  - Jun Li, PhD expected in 2012.
  - Hady Joumaa, PhD expected in 2012.

Coordination/Collaboration and Transition

- Coordination/Collaboration
  - Numerical models will be implemented and verified in the Kraken computer code whose development was funded by the Joint DoD/DOE Munitions Program (JMP).
  - Work on shocks and spall are of interest to JMP and are partially funded by JMP.
- Transition Plan
  - We plan to publish and present our work so that it is available to incorporate our numerical models in advanced computer codes.
  - Our students will be in a position to apply the methodology developed, continue to advance the state of the art, or train students.
- Other Funding Sources at University of Illinois
  - "Mechanics of Fractal Materials," NSF, $200,000, 2010-2013
  - "Shock Waves in Random Heterogeneous Materials," SAIC - Army - ARDEC, $70,000, 2011-2012
  - "Development of Fracture Model for High Explosive PBX9502," Los Alamos Natl. Lab, $100,000, 2011-2013

Conclusions

- We are confident that a stochastic peridynamic theory combining peridynamic theory with random-fractal material characterization has the ability to enhance understand the fundamental physical phenomena governing tunnel wall stability in hard rock under dynamic shock loading conditions.
- We made progress in developing stochastic peridynamics.
- We made progress in developing fractal mechanics and are in a position to apply it to peridynamics.
- We are publishing our work and training students.
- We continue to be excited about the opportunity to perform the research for this project.

Future Directions

- Continue developing stochastic peridynamic theory. (Task 1)
  - Implement and verify numerical models.
- Develop peridynamic theory of fractal media. (Task 2)
  - Implement and verify numerical models.
- Continue studies of wave and shock propagation in isotropic and random peridynamic media. (Task 3)
- Investigate boundary effects from shock loading and tunnel-wall stability. (Task 4)
- Validate numerical models and identify key parameters. (Task 5)