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Towards Coupling Plasticity and Gradient Damage

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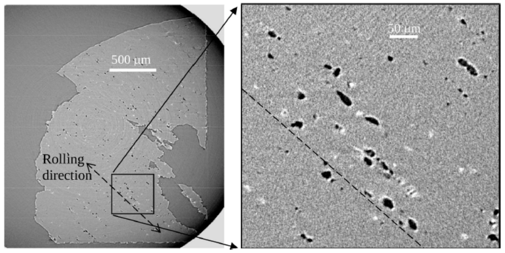
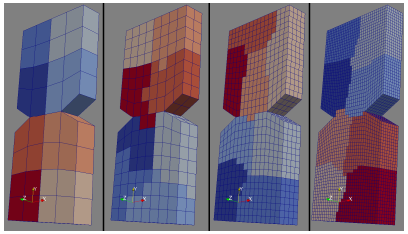
Outline

- **Motivation**
  - Our group and problems we are addressing
  - Pitfalls to avoid
- **Formulation of Gradient Damage**
  - Microforce Balance Concepts
  - Hyperelasticity and Damage
  - Plasticity and Damage
- **Segregated Example (Plasticity)**
- **Monolithic Example (Hyperelasticity)**
- **Current Work (in progress)**
- **Summary and Conclusions**

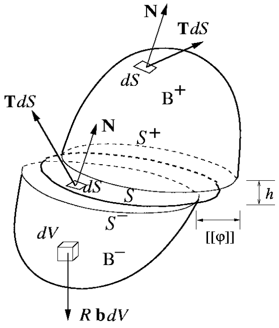
SNL - Mechanics of Materials Computational Mechanics

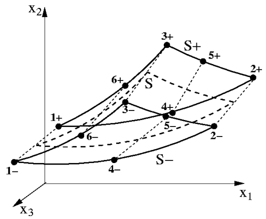
- **We are asked to provide verified, quantitative models and fracture/failure methods**
  - QMU ready (mesh-independent)
  - Applicable to engineering alloys (AL6061,AL7075,SS304L,PH17-4,PH13-8,etc…)
- **Our research is focused on constitutive models, failure models, and numerical methods**
- **Active projects**
  - Tomography
  - Variational Nonlocal Method (Mota)
  - Localization Elements (Foulk)
  - Gradient Methods, Damage
  - Generalized Bifurcation Criteria
  - Hydrogen Assisted Fracture
  - State Variable Re-mapping
  - Computational Mechanics Research Environment





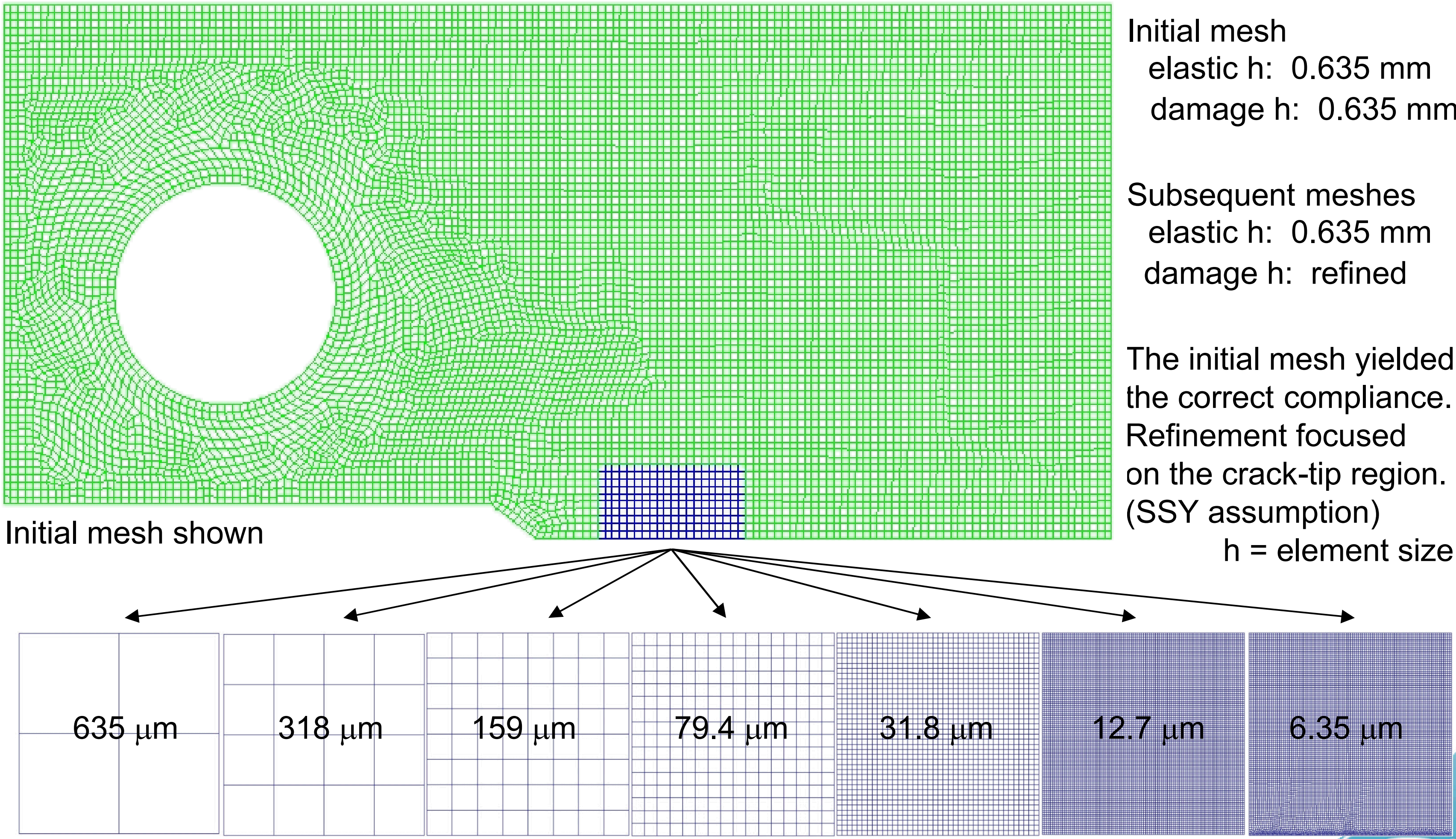
- **Future projects (FY12)**
  - Multi-Grid/Multi-Scale Methods
  - Low Triaxiality Failure Regimes
    - *Shear dominance*
    - *Thin walled structures*





This could be you…

- Finite elements are only used to solve a partial differential equation
- Any correlation of a finite element with a physical process can be misleading



Initial mesh shown

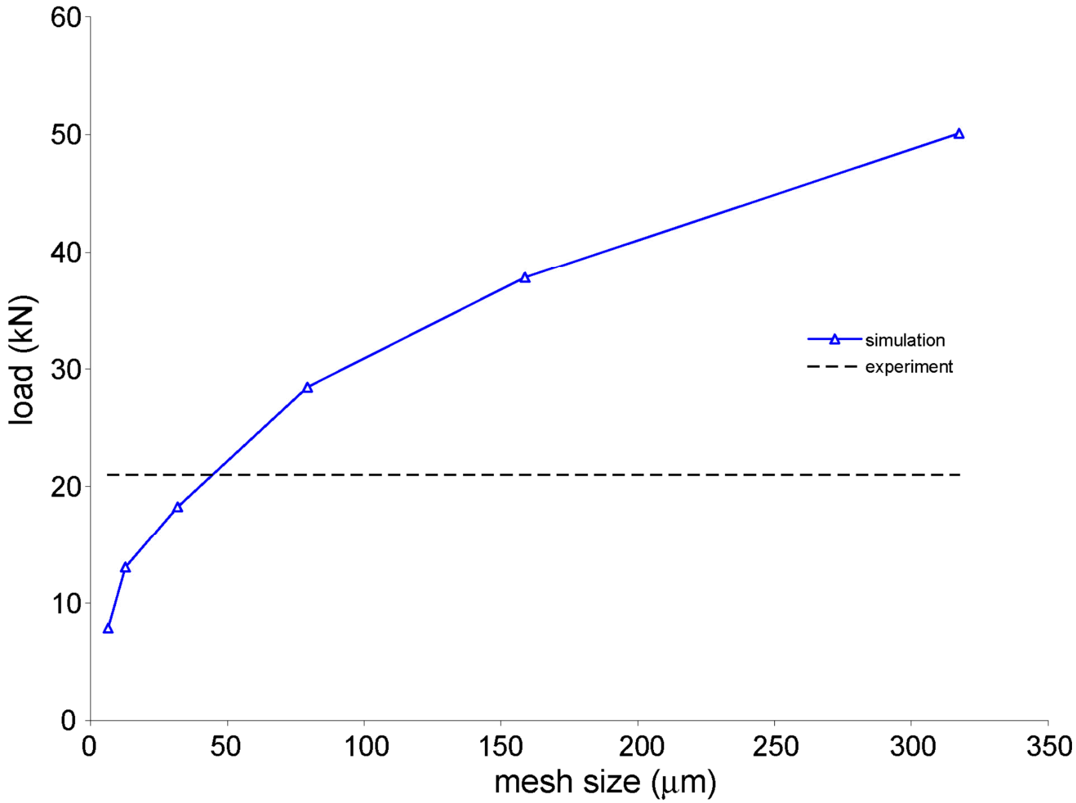
Initial mesh
elastic h: 0.635 mm
damage h: 0.635 mm

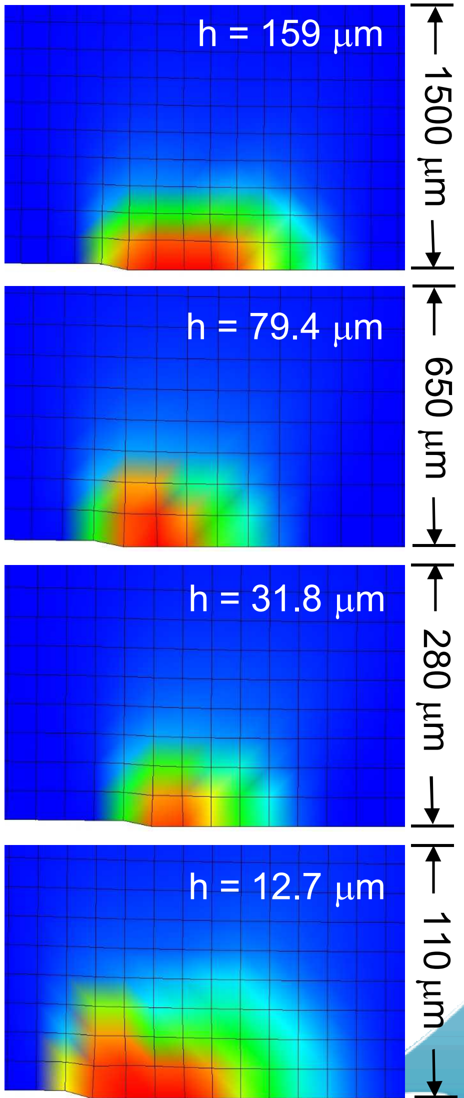
Subsequent meshes
elastic h: 0.635 mm
damage h: refined

The initial mesh yielded the correct compliance. Refinement focused on the crack-tip region. (SSY assumption)
h = element size

Seeking a length scale for damage

Snapshots of damage taken at propagation. The process zone scales with the mesh size and the predicted loads span the experimental finding.

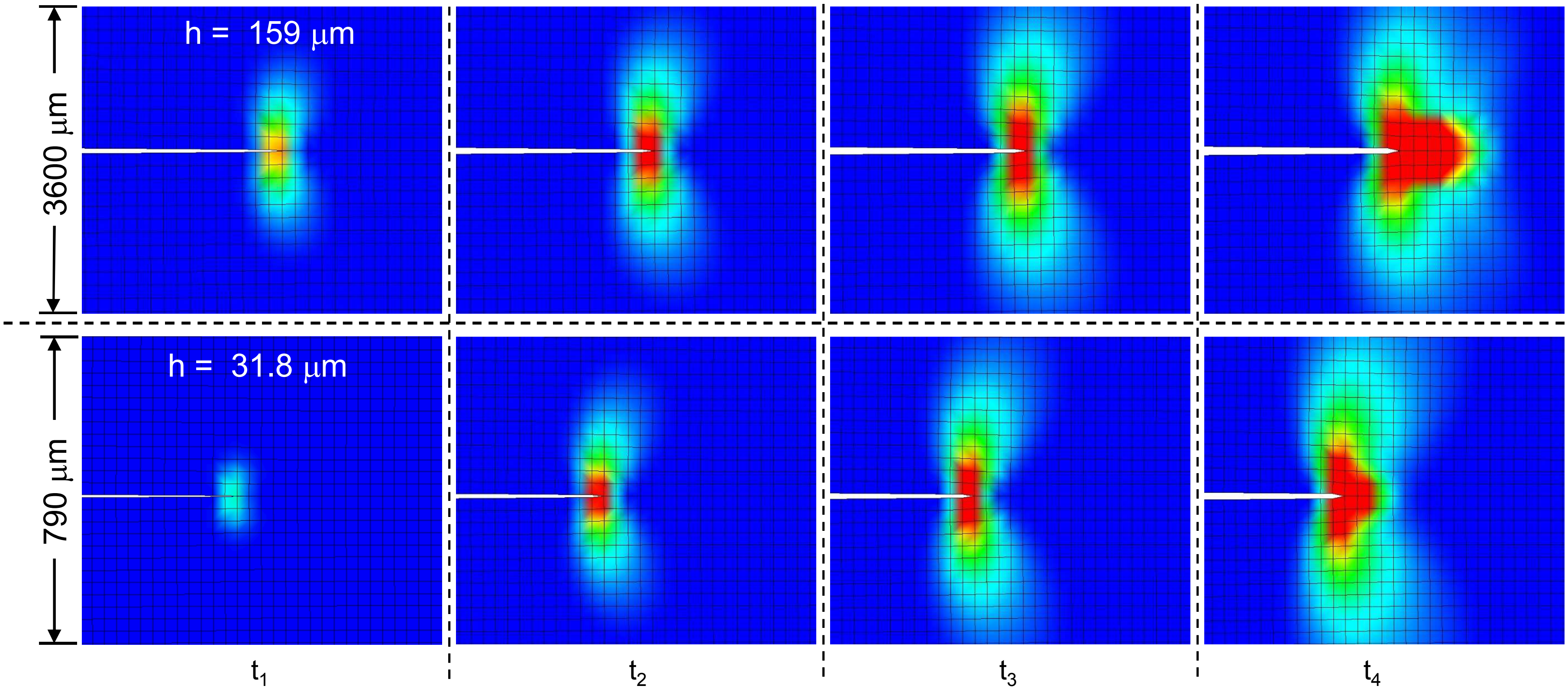




Note: Contours of porosity, 1.0E-4 to 1.0, log scale

Seeking a length scale for plasticity

Because the plastic zone size is tied to damage, plasticity is also mesh dependent

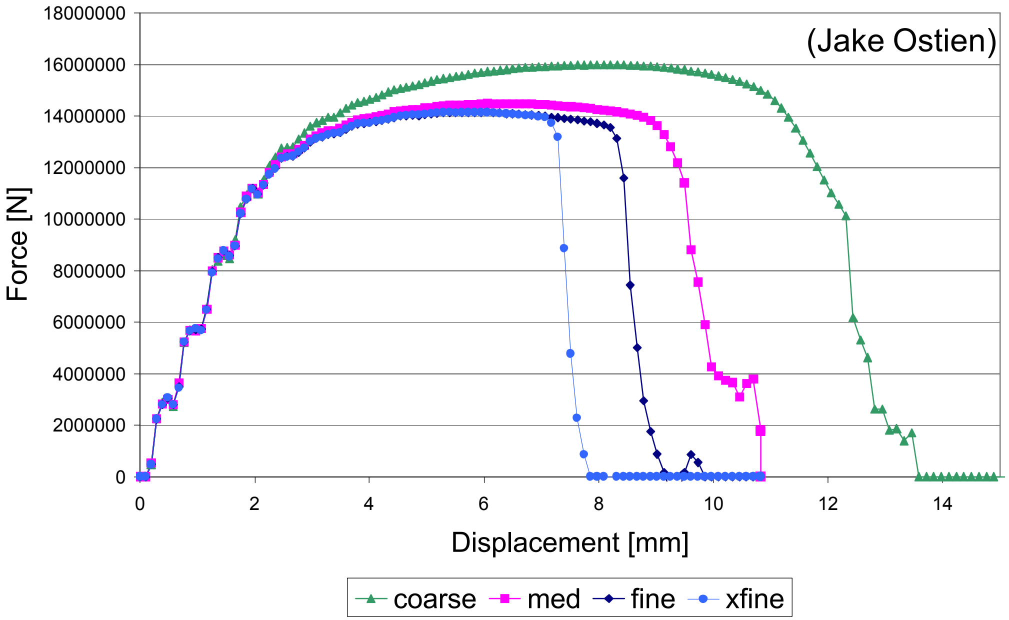


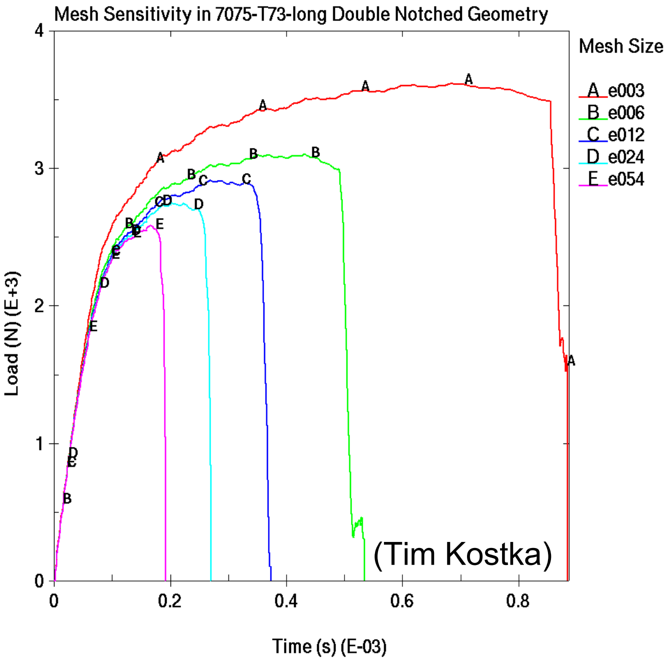
$$r_p = \frac{K^2}{4\pi\sigma_{ys}^2}\left[\frac{3}{2}+(1-2\nu)^2\right]$$

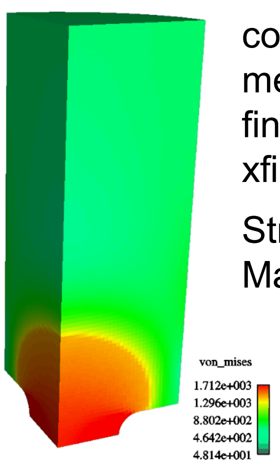
K_Q = 28.8 MPa√m (D. Dawson) r_p = 1390 μm (major axis)

Note: Contours of equivalent plastic strain, 0.0 to 2.0%. Time t_4 taken at propagation.

Mesh dependence under notched tension







coarse – 752 elements
med – 6016 elements
fine – 48128 elements
xfine – 385024 elements

Strain rate = 50/s
Material = A286

- Specimens of various notched radii for "fitting" model
- The results depend on the mesh size
- The fitted damage parameters are convoluted
- Goodness of the model is not known
- The issue stems from the governing PDEs

Finite elements is just a method for solving a partial differential equation. Numerics should be transparent and not add physics. The real issue is that we are using a local model without a length scale to solve a localization problem. It corrupts the axiom for QMU.

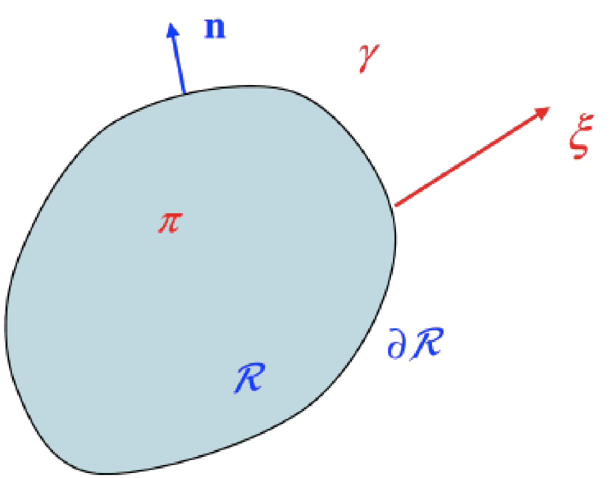
Regularization

- **Mesh dependence shown in motivational slides can be remedied by adding a length scale to the failure process**

- **In this work, we will accomplish that using a gradient methodology**

Microforce Balance

- **Following Gurtin (1996) and Solanki & Bammann (2009)**
- **Propose additional degrees of freedom related to damage**
- **Additional fields obey a balance law: *Microforce Balance***
- **Evolution equation for additional fields derived via Coleman & Noll thermodynamical arguments**



Micro-forces:

γ -external body force

π -internal body force

ξ -stress vector

- Local *microforce* balance
- $\nabla \cdot \xi + \pi + \gamma = 0$

Hyperelasticity and Gradient Damage

- **For a hyperelastic material with scalar damage**

$$\psi_0 = \frac{1}{2}\left(\frac{1}{2}(J^2 - 1) - \ln(J)\right) + \frac{1}{2}\mu(\mathrm{tr}\,\bar{C} - 3)$$

$$\psi = (1 - \phi)\psi_0 ...$$

- **Quasi-static Balance of Linear Momentum, no Body Forces**

$$\nabla \cdot \sigma = 0$$

- **Assume a Microforce Balance, arrive at a transport like equation**

$$\beta\dot{\phi} = G_\phi + l^2\nabla^2\phi$$

- **Others have used Helmholtz Equation (phase field) De Borst, Peerlings, Miehe, Bordin**

$$\phi - l^2\nabla^2\phi = 0$$

Plasticity and Gradient Damage

- **For a multiplicative decomposition**

$$\psi_0 = \frac{1}{2}\left(\frac{1}{2}({J^e}^2 - 1) - \ln(J^e)\right) + \frac{1}{2}\mu(\mathrm{tr}\,\bar{C}^e - 3)$$

$$\psi = (1-\phi)\psi_0 + f(\phi)...$$

- **Quasi-static Balance of Linear Momentum, no Body Forces**

$$\nabla \cdot \sigma = 0$$

- **Assume a Microforce Balance, arrive at a transport like equation**

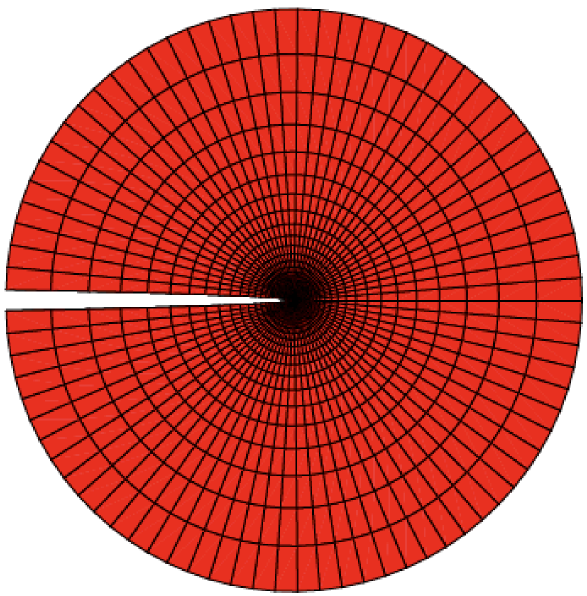
$$\beta\dot{\phi} = G_\phi + l^2\nabla^2\phi$$

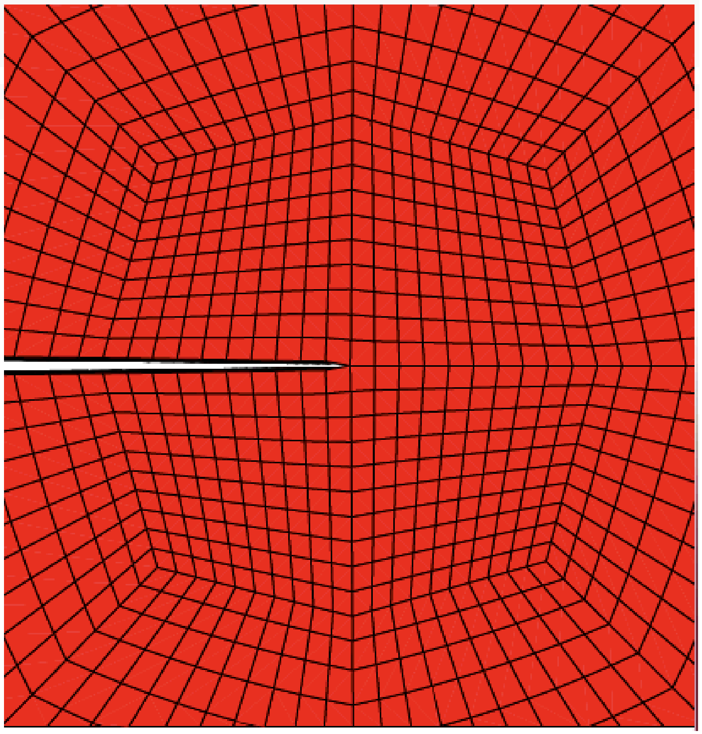
- **Cocks-Ashby Damage Evolution**

$$G_\phi = \left[\frac{1}{(1-\phi)^m} - (1-\phi)\right]\sinh\left[\frac{2(2m-1)}{m+1}\frac{\langle p\rangle}{\bar{\sigma}}\right]|d^p|$$

K field BVP – Example Segregated Scheme

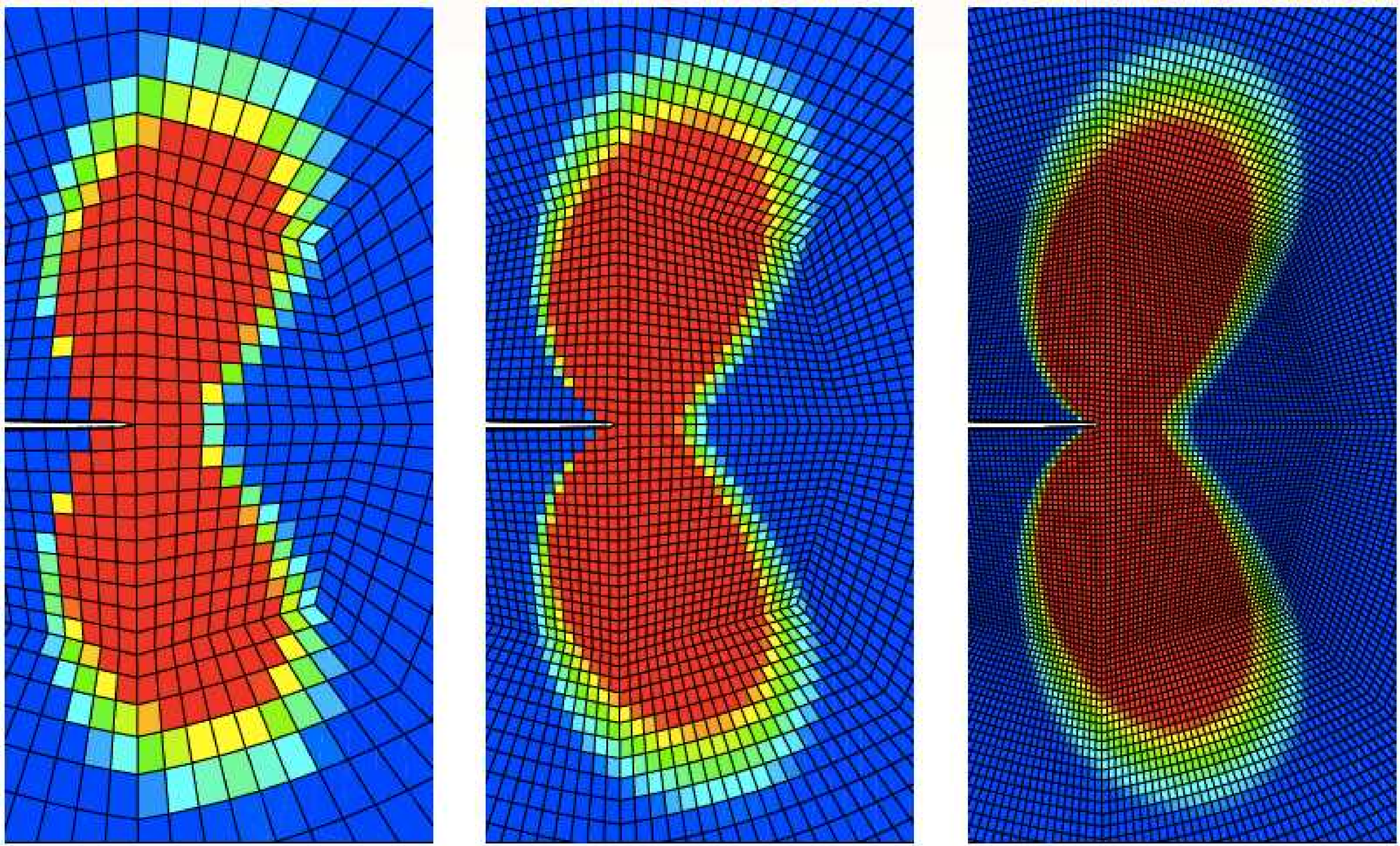
- Radius 150 mm, h = 60 mm





Mesh Independent Plasticity Field

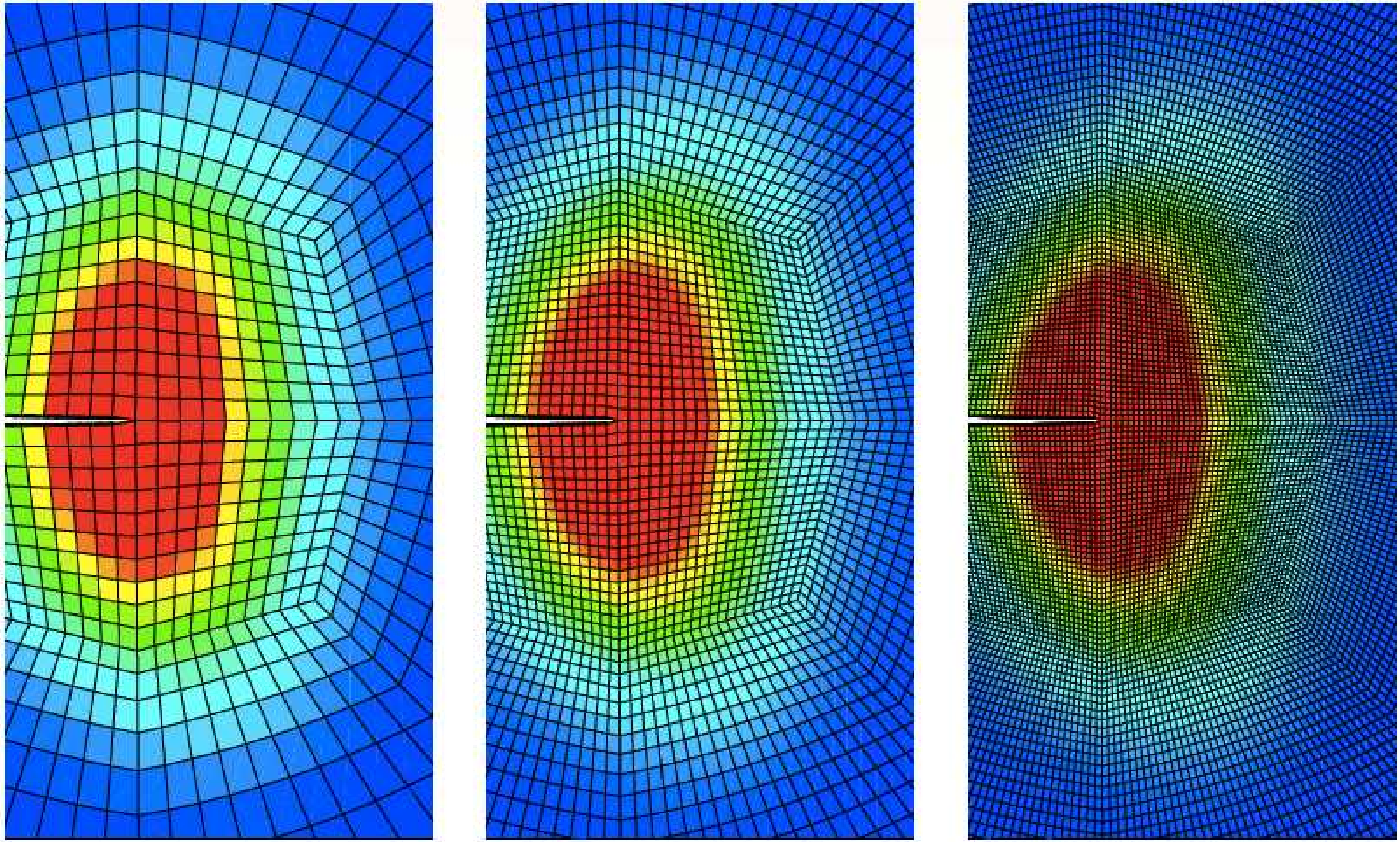
SierraSM/SierraTF



Contours of equivalent plastic strain, (0,.001), mesh sizes 60μm, 30μm, 15μm

Mesh Independent Damage Field

SierraSM/SierraTF



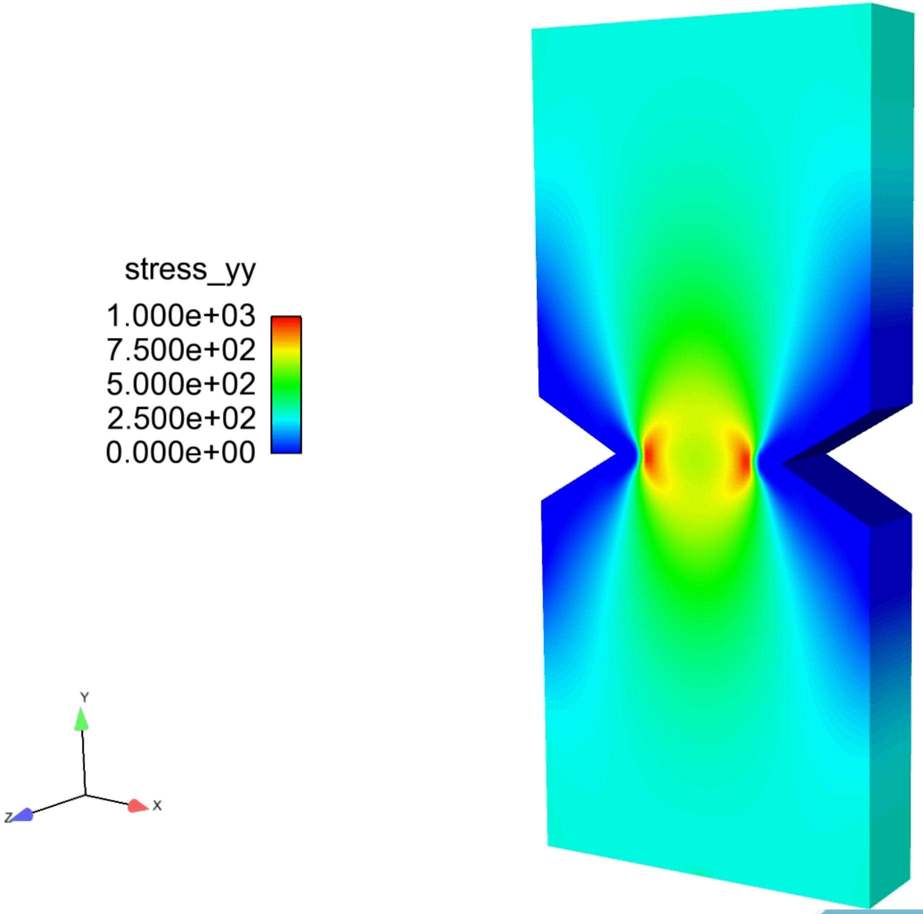
Contours of damage, (1E-3,1.5E-3), mesh sizes 60μm, 30μm, 15μm

Fully Coupled Systems

- **How about a monolithic system of multiple PDEs**
- **Revisit the Hyperelastic Damage model**
  - **Use nonlinear elasticity as a basis – NeoHookean**
  - **Implement a Damage source term consistent with Holzapfel's model**

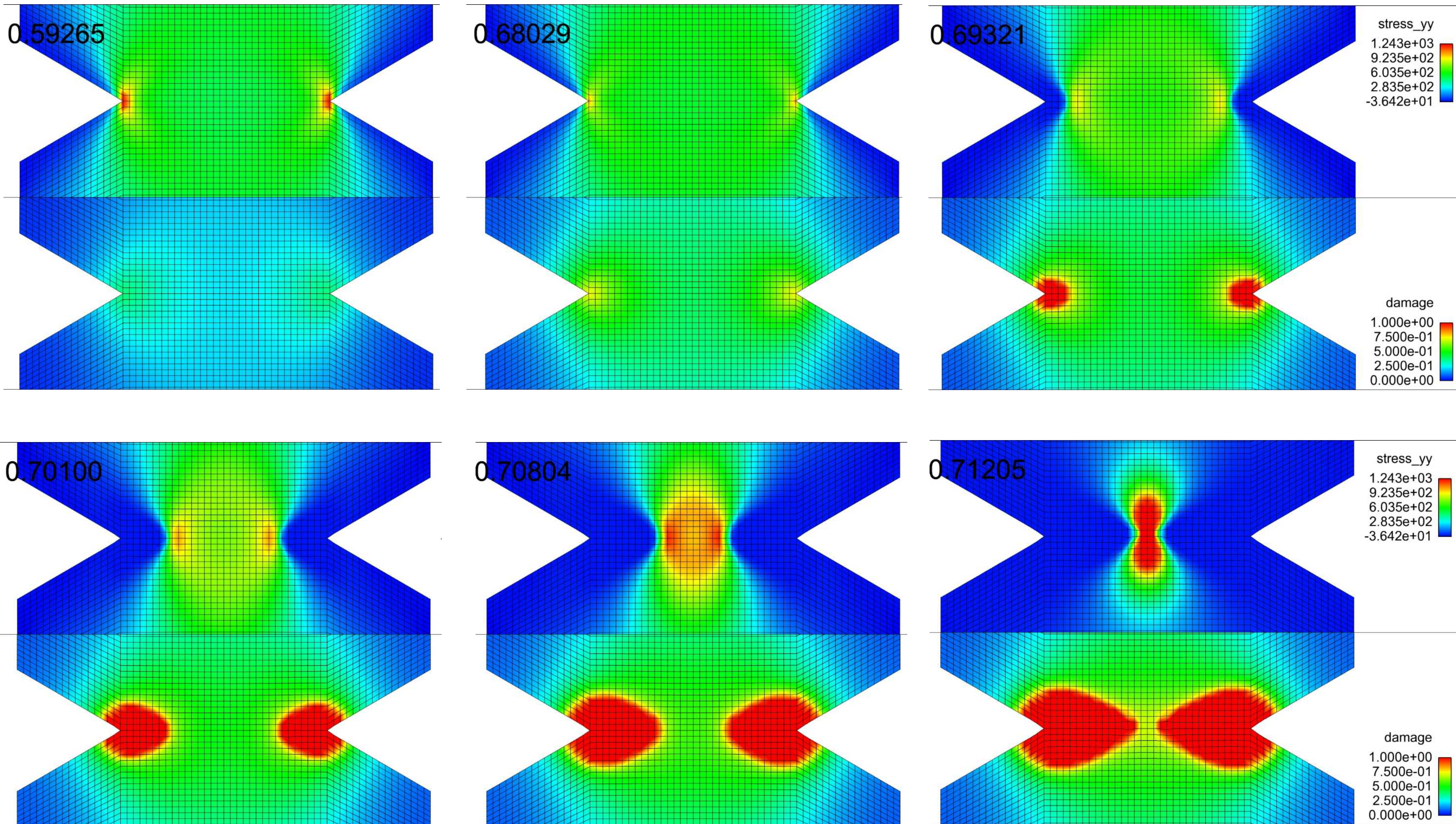
Double Notch BVP

- Hyperelastic Damage model, similar to Holzapfel's
- Damage evolution depends on Helmholtz free energy
- 9mm x 4mm x 1mm
- 30 degree notch angle
- Young's modulus = 200 GPA
- Poisson's ratio = 0.25

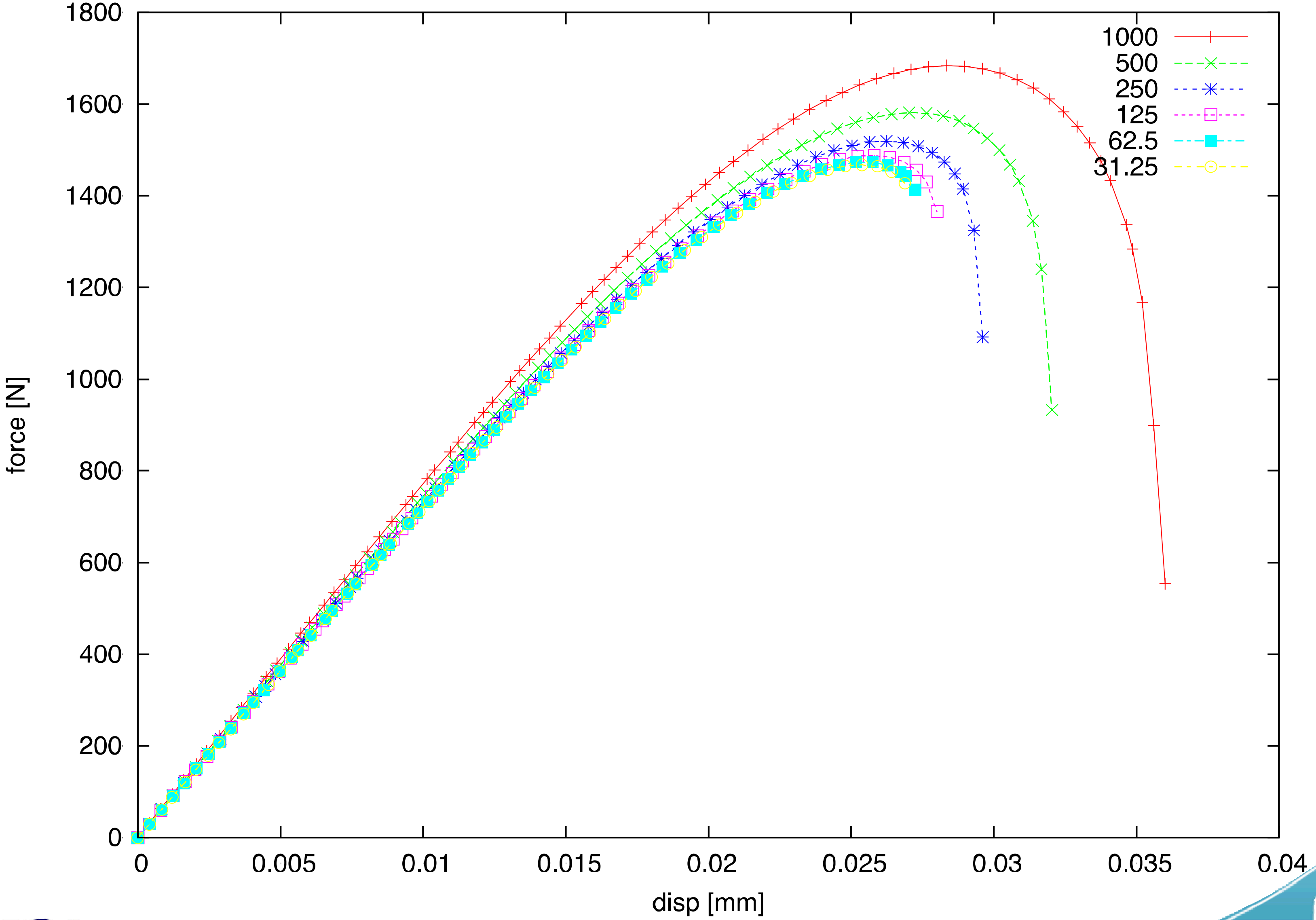


Stress and Damage Fields

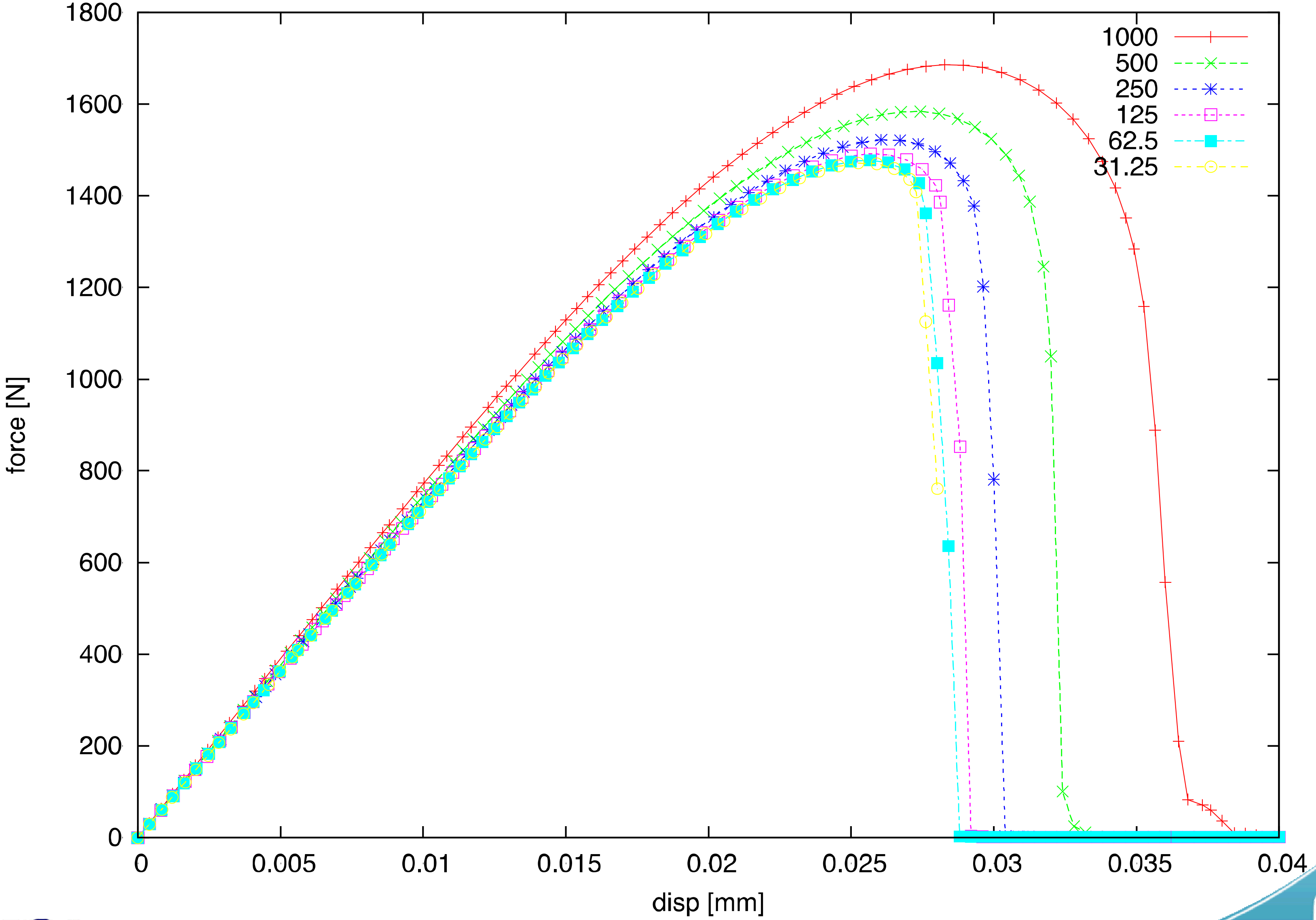
SierraTF



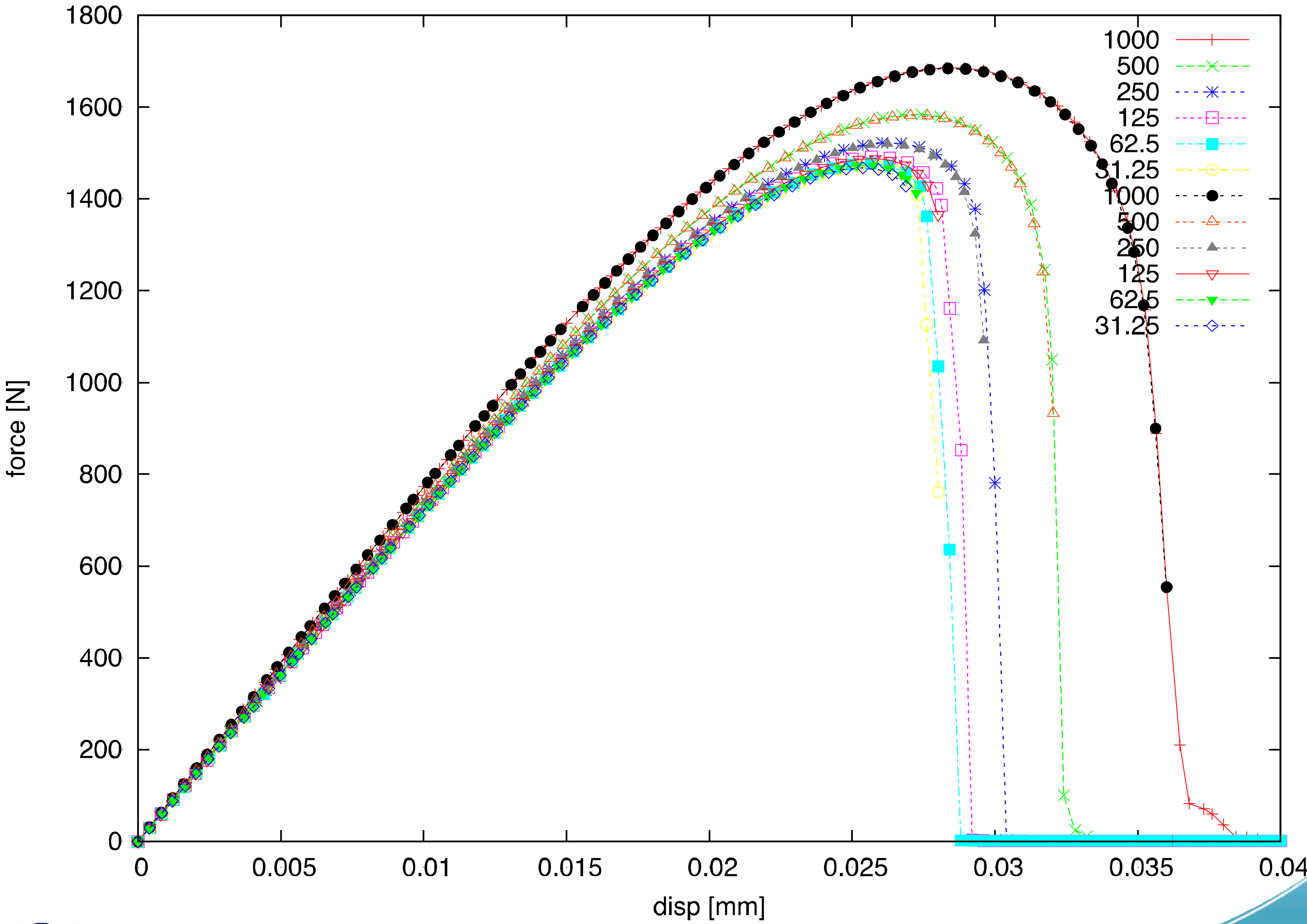
Global Response



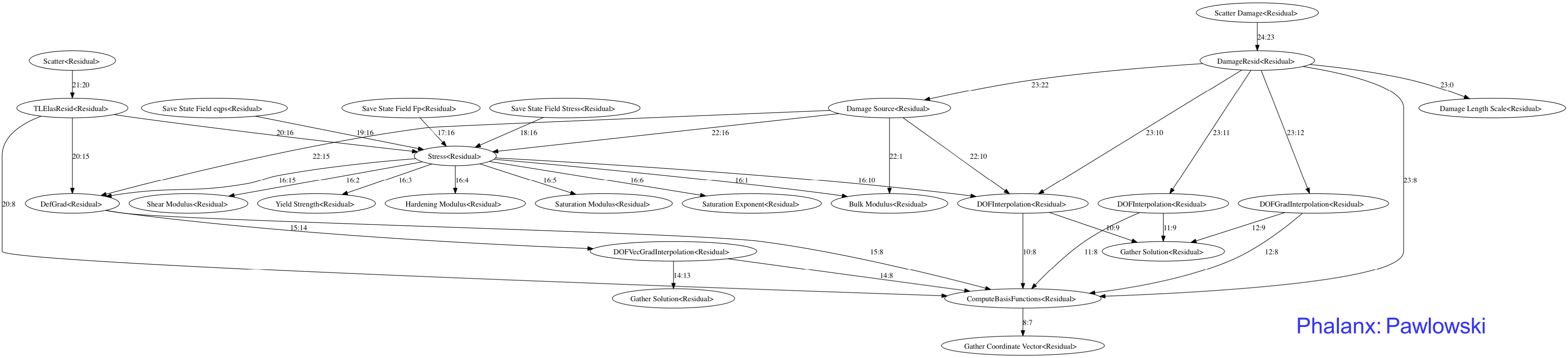
Global Response – Segregated Solve



Global Response



Gradient Damage and Plasticity

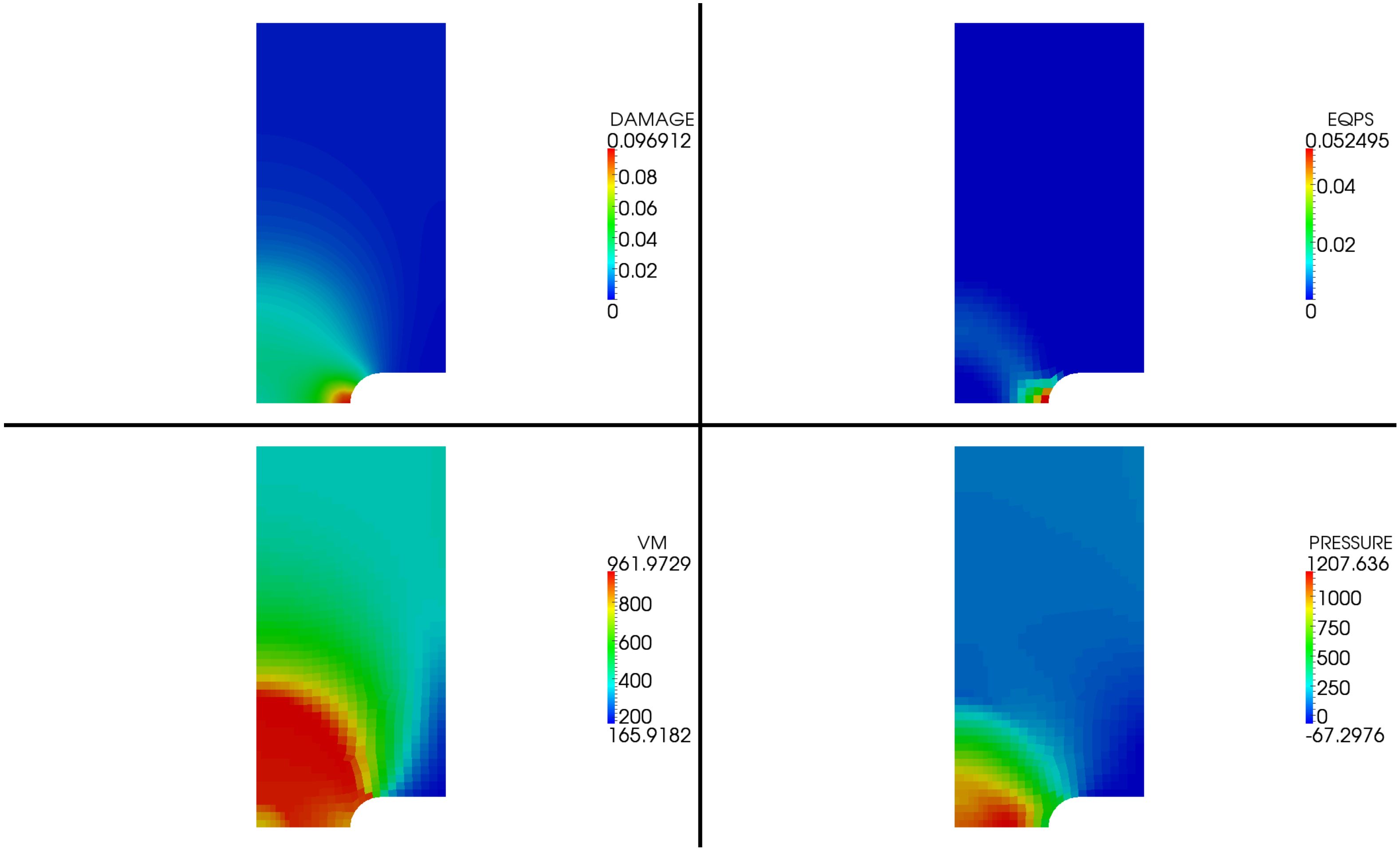


Phalanx: Pawlowski

- **Objective: Develop an environment capable of studying a range of damage models and coupling strategies**
  - Damage transport equations
  - Damage Helmholtz equations
  - Nonlocal methodologies
  - Monolithic/Segregated schemes
- **Status: Monolithic equations are implemented for select damage models**
- **Still to do: Segregated schemes (subset of the monolithic equations), Finish the nonlocal implementation**

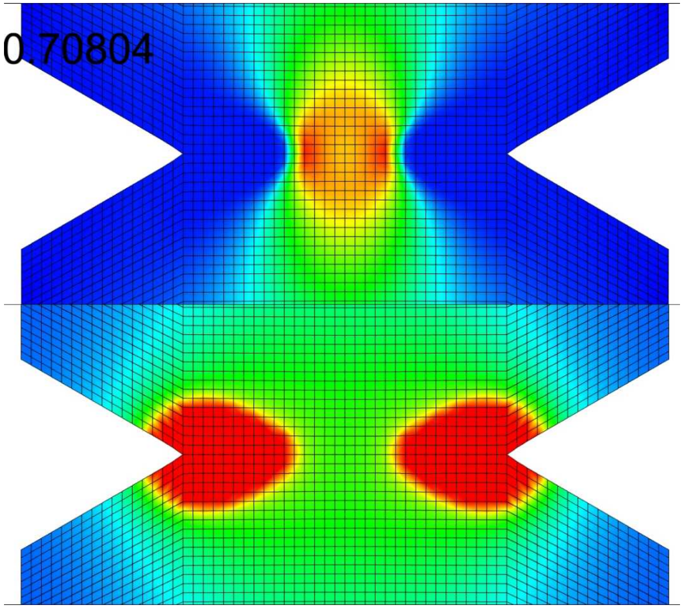
Monolithic Gradient Damage and Plasticity

Albany



Summary

- Damage evolution equations arise from fundamental, thermodynamic considerations – what the equations resemble depends on your assumptions
- Damage approaches are found, in some capacity, in multiple strength models, but here is adapted to simple hyperelastic nonlinear elasticity and plasticity
- Solving the coupled system of PDEs seems to regularize the problem, i.e. produces mesh independent solutions
- We are attempting to create a general framework to study the different damage models, for applicability to the problems we are trying to solve



Conclusions

- *Validation* is required of all the new methodologies we are developing
  - Requires close ties with Experimental Mechanics staff
  - Focus has been on *Verification*
  - These methods need to be used in production calculations
- *Validation* is crucial to identifying the physical multi-scale mechanisms, in this case we are looking at the plastic zone size versus the damage process zone, this work is creating the environment to enable that investigation
- There is some robustness to be had using segregated solution schemes (often observed), more work to do here

Thank You