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Regularizing damage evolution for ductile fracture using localization elements

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Issues

MISSION

Need to predict fracture/failure of critical components loaded in abnormal environments – crack initiation and propagation

MATERIALS AND MECHANISMS

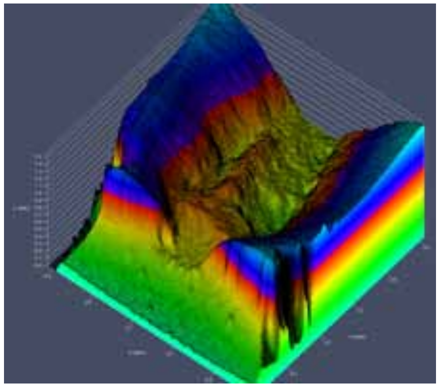
Aluminum and steel alloys which can exhibit distributed and localized processes evolving from void nucleation, growth, microinertia, and coalescence. Failure processes are rate and temperature dependent.

CURRENT MODELS

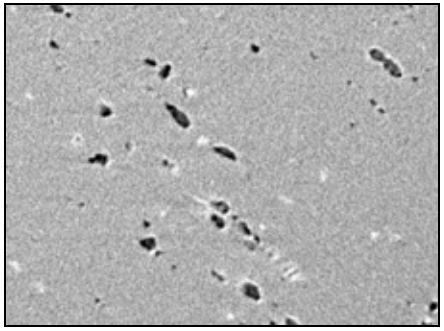
The community has developed plasticity/damage models to address rate dependence, temperature dependence, and void growth. Literature is rich with damage models for metals, polymers, and composites. Glitch – they corrupt the PDE.

REGULARIZATION

Our goal is to retain the micromechanics of local damage models and add regularization through multiple methods. *We need to work on the numerics and the physics simultaneously.*



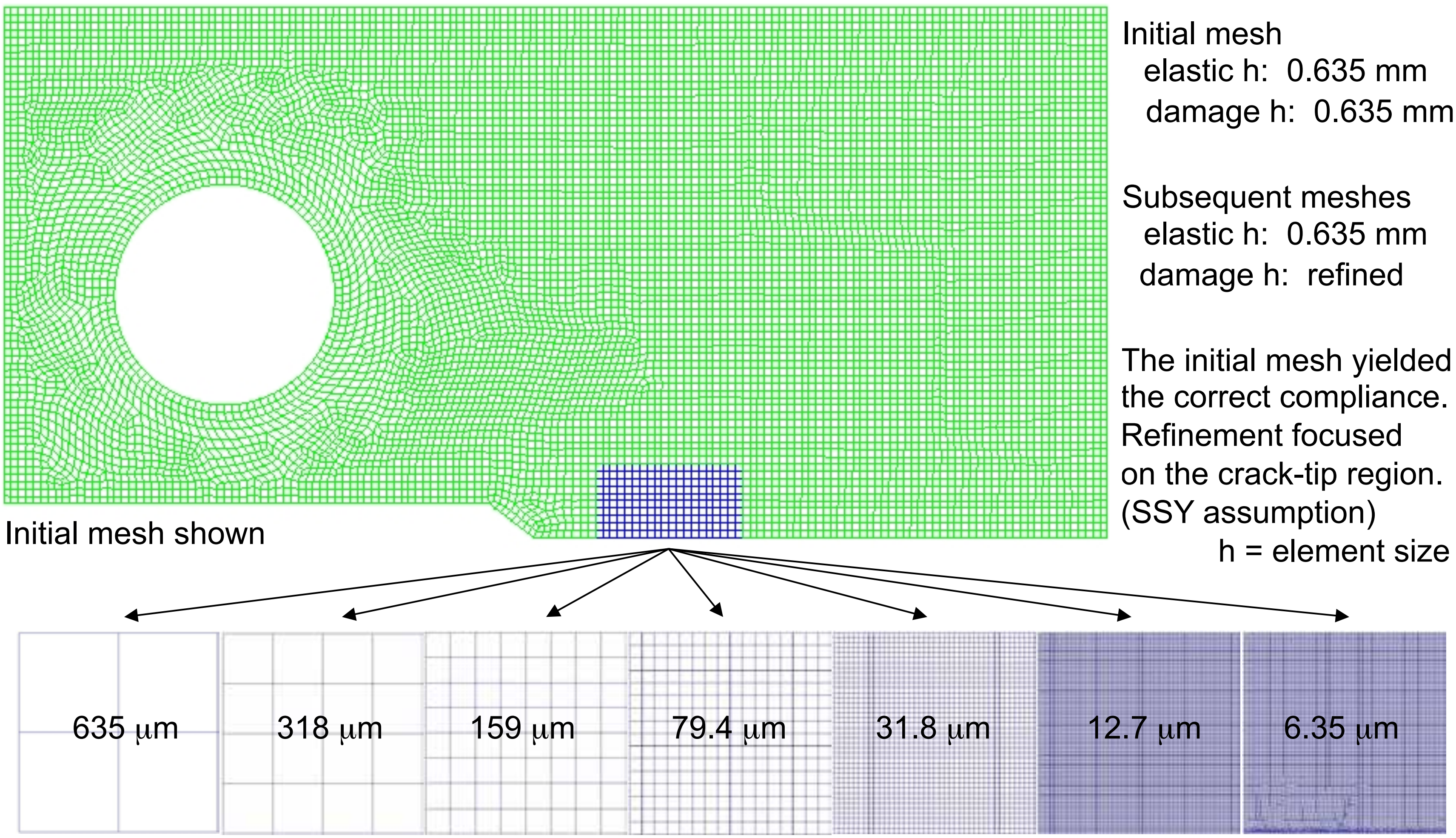
Boyce, PH13-8, H950



ALS, 7075-T7351

This could be you...

- Finite elements are only used to solve a partial differential equation
- Any correlation of a finite element with a physical process can be misleading



Initial mesh
  elastic h: 0.635 mm
  damage h: 0.635 mm

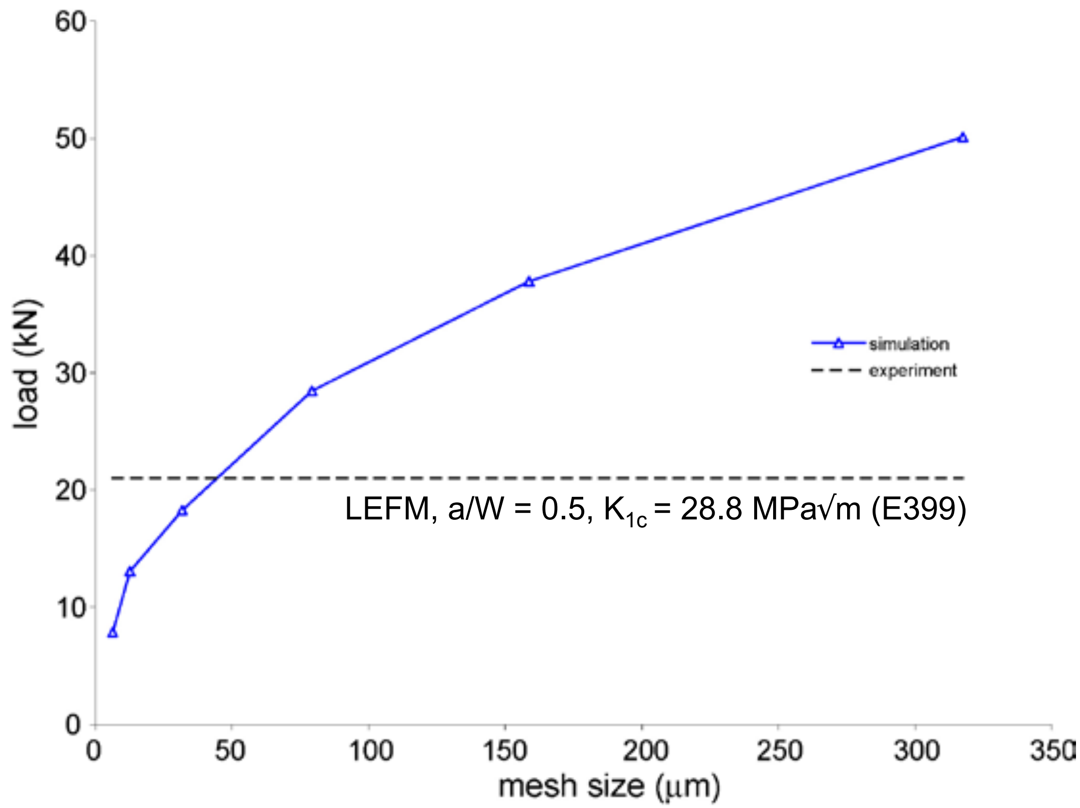
Subsequent meshes
  elastic h: 0.635 mm
  damage h: refined

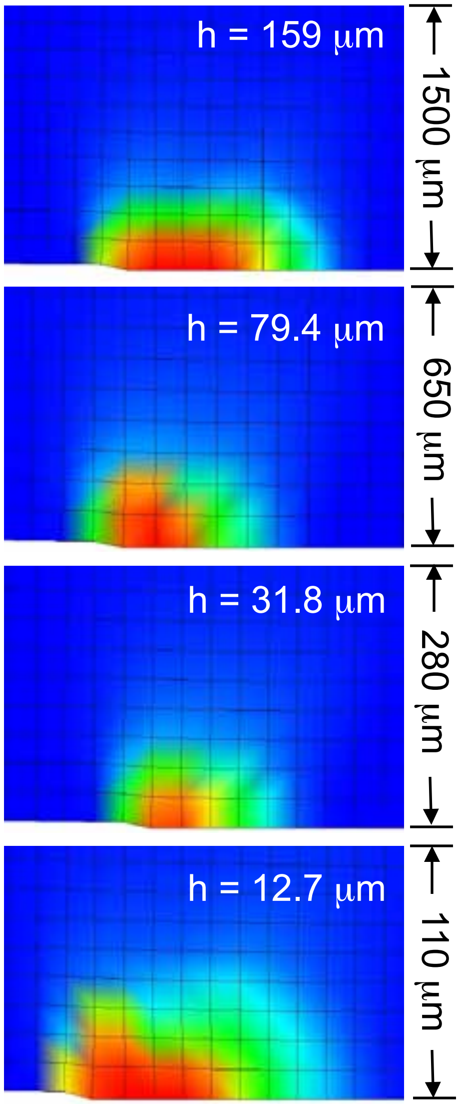
The initial mesh yielded the correct compliance. Refinement focused on the crack-tip region. (SSY assumption)

h = element size

Seeking a length scale for damage

Snapshots of damage taken at propagation. The process zone scales with the mesh size and the predicted loads span the experimental finding.





Note: Contours of porosity, 1.0E-4 to 1.0, log scale

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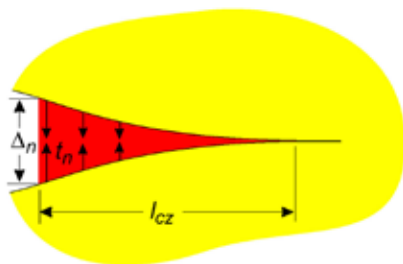
Exploring multiple methods for regularization

cohesive zone, crack band

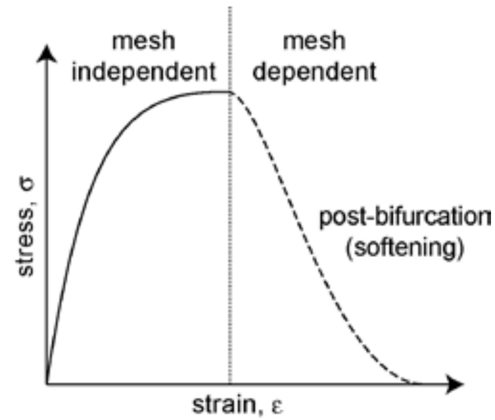
PROBLEMATIC, BUT RICH

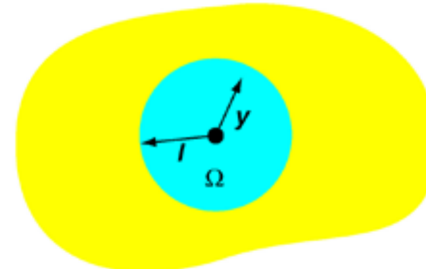
nonlocal



$$t_n = t_n(\sigma_{max}, \phi, \Delta_n/\ell)$$

Adaptive insertion before bifurcation
Bulk + surface model, mesh topology

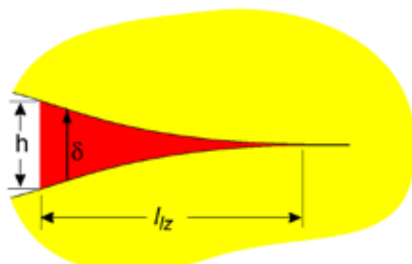




$$\bar{\dot{\phi}}(x) = \frac{1}{\Psi(x)} \int_\Omega \psi(y,x)\dot{\phi}(y)\, d\Omega(x)$$

Integrate over a ball
Rescaling at boundaries

localization elements



$$\boldsymbol{F} = \boldsymbol{F}^{\|}\boldsymbol{F}^{\perp} \quad \boldsymbol{F}^{\perp} = \boldsymbol{I} + \frac{\phi}{h} \otimes \boldsymbol{N}$$

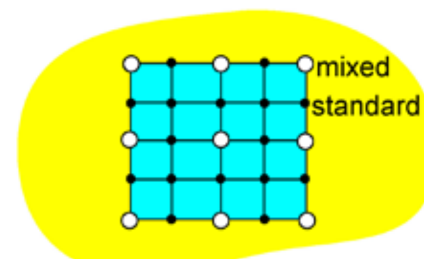
Adaptive insertion before bifurcation
Restricted path via mesh topology

gradients

$$\dot{\phi} = f(\sigma, \phi) + \ell \frac{\partial \phi}{\partial x} + + \ell^2 \frac{\partial^2 \phi}{\partial x^2}$$

Solve additional pde(s)
Specify boundary conditions

variational nonlocal



$$\bar{\boldsymbol{Z}} = \frac{1}{\mathrm{vol}(D)} \int_D \boldsymbol{Z}\, dV$$

Volume averaging on coarse domain

Mesh adaption needed ← → *independent of mesh topology*

NOTE: Artificial viscosity can be used but it convolutes the energetics of failure.

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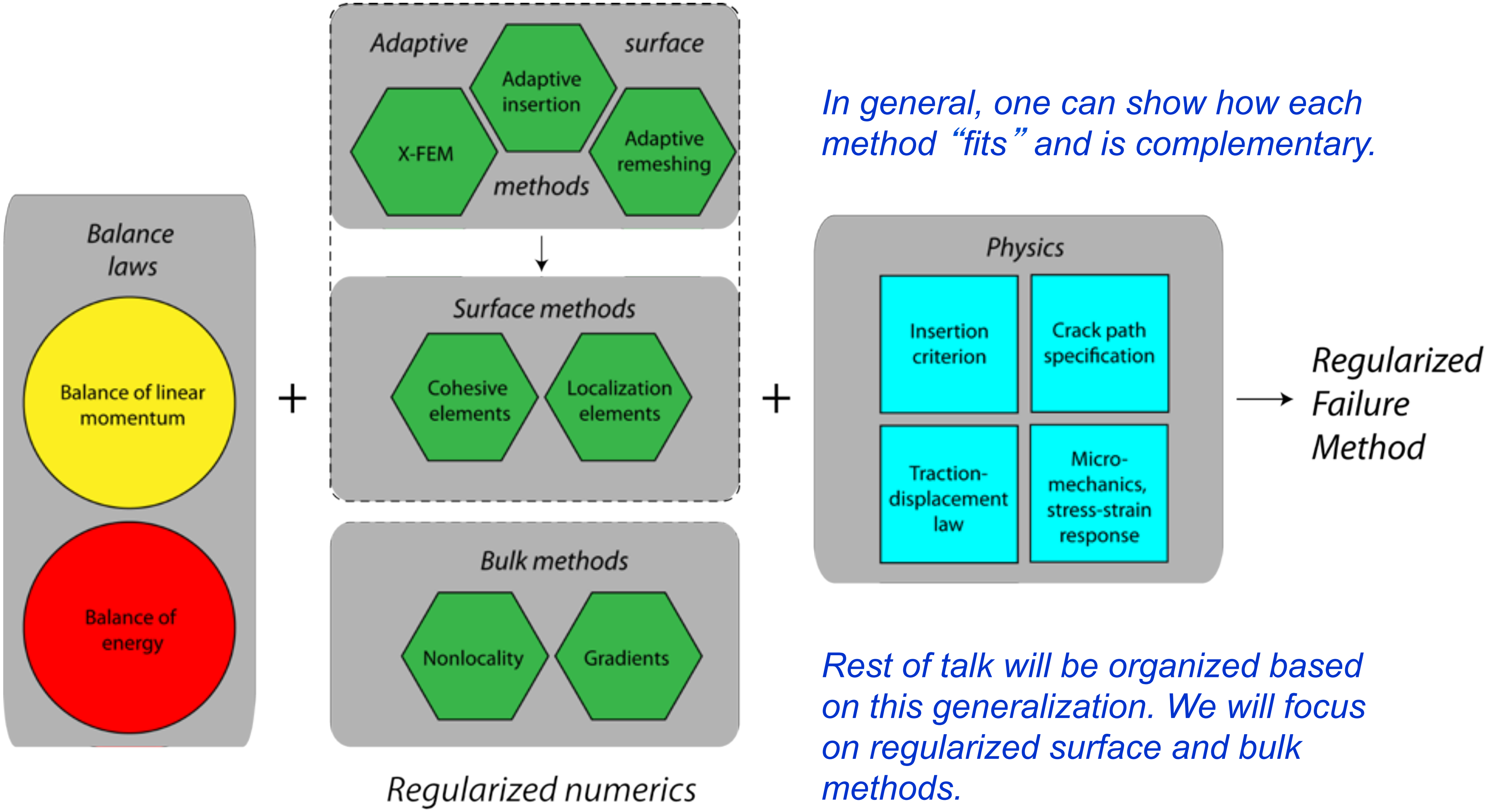
Generalizing methods



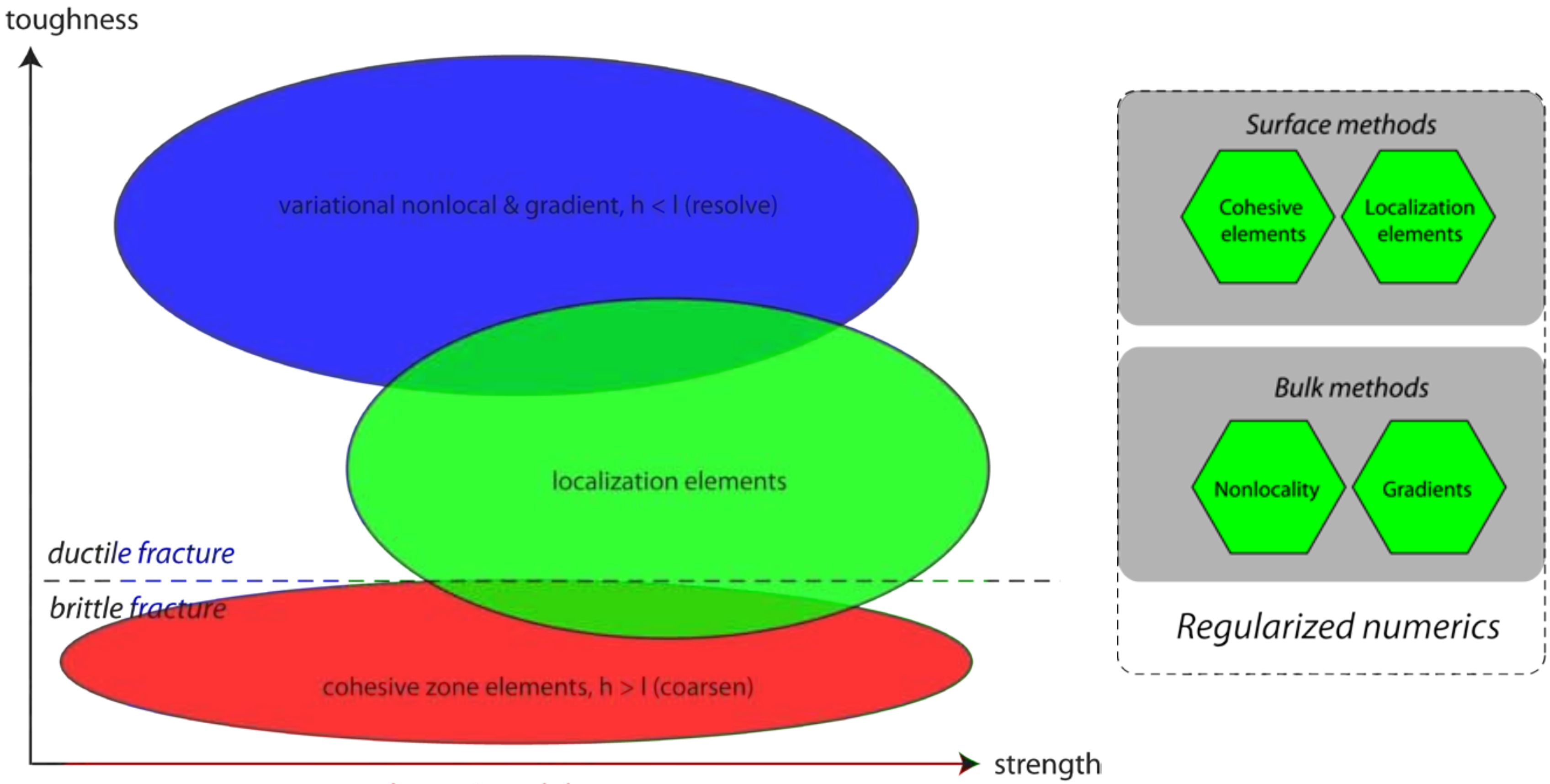
In general, one can show how each method "fits" and is complementary.

Rest of talk will be organized based on this generalization. We will focus on regularized surface and bulk methods.

NOTE: We will initially focus on deterministic studies at a single scale

Multiple approaches needed for components

- Each method has a region of applicability defined by strength and toughness
- Analysts need a toolbox to span aluminum to stainless steel alloys



l = length scale$_{mode\ I}$ = $\frac{\text{toughness} * \text{modulus}}{\text{strength}^2}$ h = element size

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Localization elements

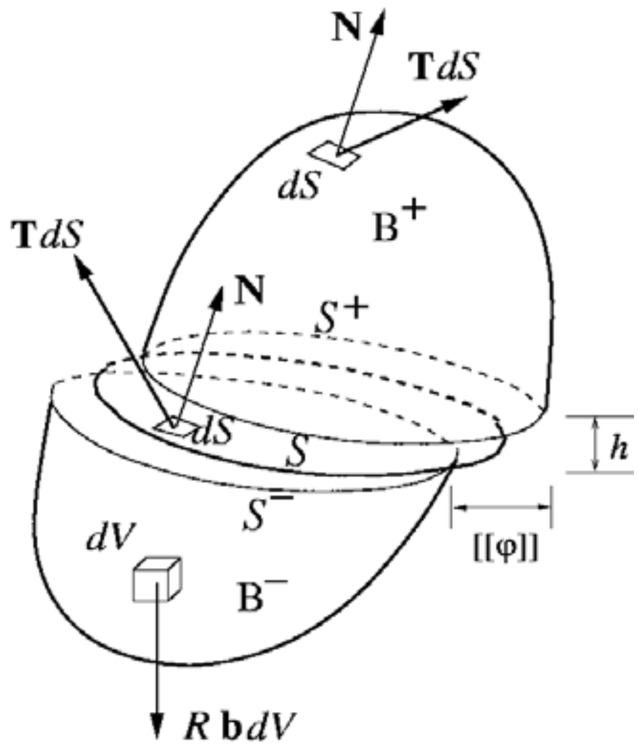
- Kinematic assumptions
- Improving/simplifying the formulation
- Test case
  - Fitting parameters (smooth tension, K-field)
  - X-prize geometry
- Addressing membrane forces
- Part of a family of methods for regularization

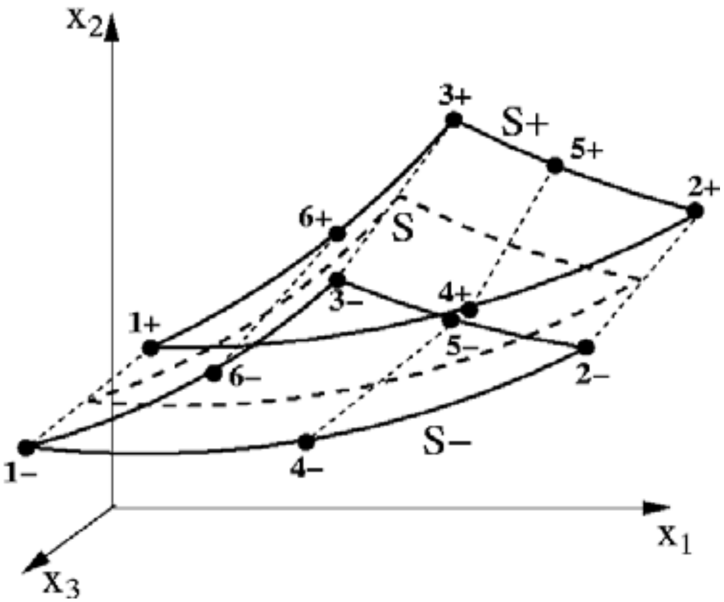
Kinematic assumptions

Surface methods

Cohesive elements

Localization elements





- Finite-deformation kinematics.
- Simulation of strain localization.
- No additional constitutive assumptions

$$\boldsymbol{F}^{\perp} = \boldsymbol{I} + \frac{[\![\varphi]\!]}{h} \otimes \boldsymbol{N} \qquad \boldsymbol{F}^{\parallel} = \boldsymbol{g}_i \otimes \boldsymbol{G}^i$$

$$\boldsymbol{F} = \boldsymbol{F}^{\parallel} \boldsymbol{F}^{\perp}$$

Yang, Mota and Ortiz, IJNME, 2005

Issues to remedy

$$\boldsymbol{F} = \boldsymbol{F}^{\parallel} + \frac{\boldsymbol{F}^{\parallel}[\![\boldsymbol{\varphi}]\!]}{h} \otimes \boldsymbol{N}$$

Configuration

$$P^D = \sum_{\pm} \int_{B_0^{\pm}} \boldsymbol{P} \cdot \dot{\boldsymbol{F}}\, dV + \int_{S_0} \boldsymbol{P} \cdot \dot{\boldsymbol{F}} h \, dS$$

$$= \sum_{\pm} \int_{B_0^{\pm}} \boldsymbol{P} \cdot \dot{\boldsymbol{F}}\, dV + \int_{S_0} \boldsymbol{P} \cdot \left[\dot{\boldsymbol{F}}^{\parallel} \boldsymbol{F}^{\perp} + \boldsymbol{F}^{\parallel} \dot{\boldsymbol{F}}^{\perp} \right] h \, dS$$

Conjugacy

$$= \sum_{\pm} \int_{B_0^{\pm}} \boldsymbol{P} \cdot \dot{\boldsymbol{F}}\, dV + \int_{S_0} \left[h \boldsymbol{P} (\boldsymbol{F}^{\perp})^T \cdot \dot{\boldsymbol{F}}^{\parallel} + (\boldsymbol{F}^{\parallel})^T \boldsymbol{P} \boldsymbol{N} \cdot [\![\dot{\boldsymbol{\varphi}}]\!] \right] dS$$

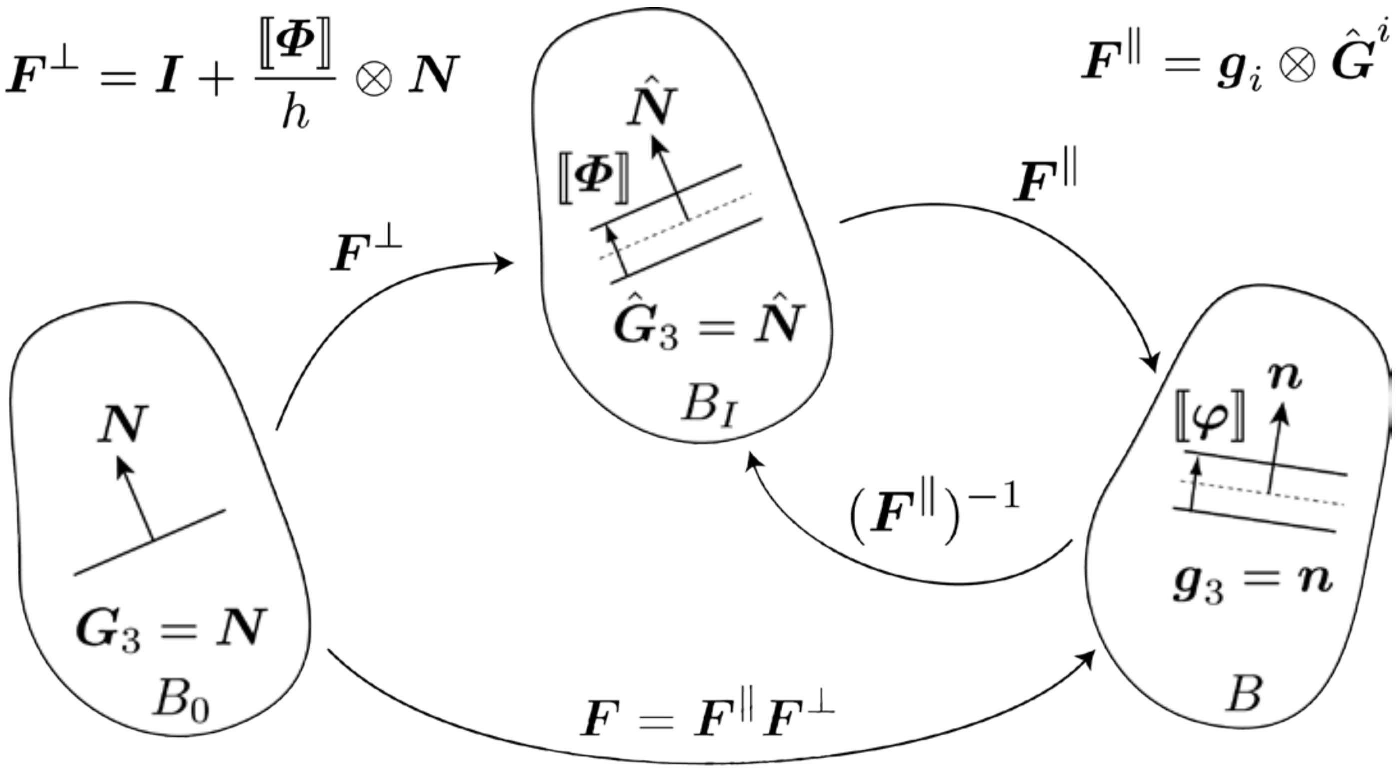
$$\boldsymbol{x}^+ = \boldsymbol{Q}\boldsymbol{x} \qquad \boldsymbol{F}^+ = \boldsymbol{Q}\boldsymbol{F} \qquad \boldsymbol{g}_i^+ = \boldsymbol{Q}\boldsymbol{g}_i \qquad [\![\boldsymbol{\varphi}]\!]^+ = \boldsymbol{Q}[\![\boldsymbol{\varphi}]\!]$$

$$(\boldsymbol{F}^{\perp})^+ = \boldsymbol{I} + \frac{\boldsymbol{Q}[\![\boldsymbol{\varphi}]\!]}{h} \otimes \boldsymbol{N} \qquad (\boldsymbol{F}^{\parallel})^+ = \boldsymbol{g}_i^+ \otimes \boldsymbol{G}^i = \boldsymbol{Q}\boldsymbol{F}^{\parallel}$$

Objectivity

$$(\boldsymbol{F})^+ = \boldsymbol{Q}\boldsymbol{F}^{\parallel} + \frac{\boldsymbol{Q}\boldsymbol{F}^{\parallel}\boldsymbol{Q}[\![\boldsymbol{\varphi}]\!]}{h} \otimes \boldsymbol{N}$$

An intermediate configuration



$$\hat{G}_A \parallel G_A \longrightarrow F = F^{\parallel} + \frac{[\![\varphi]\!]}{h} \otimes N$$

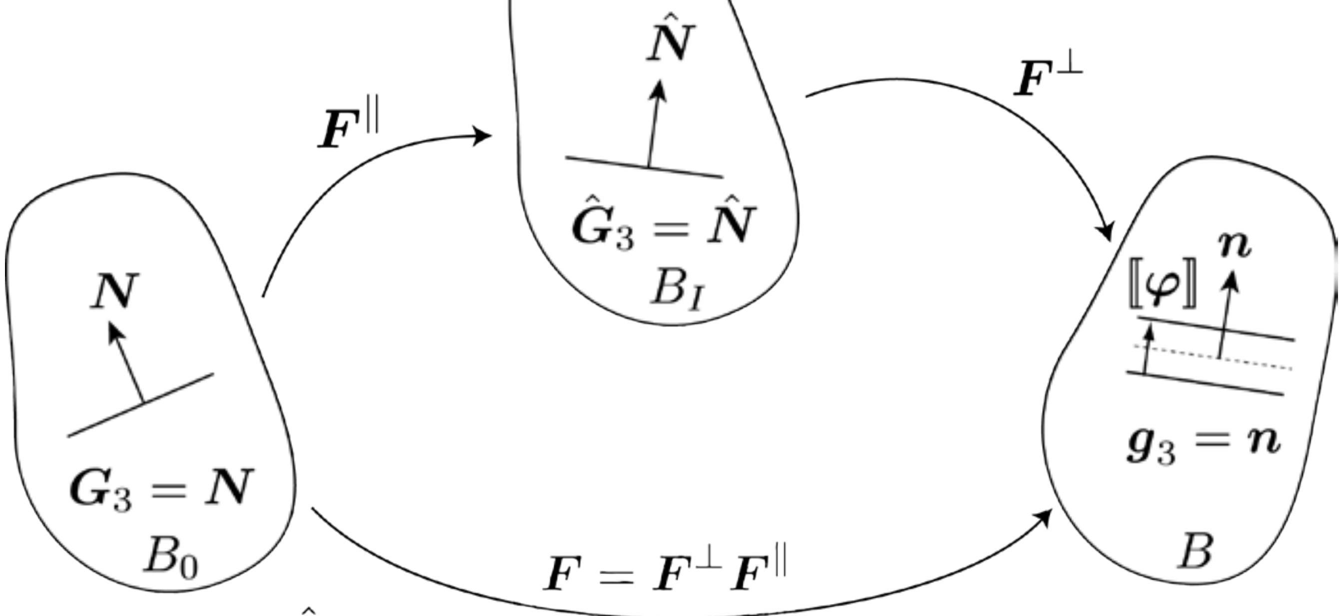
additive decomposition!

The jump is pushed backwards $[\![\Phi]\!] = (F^{\parallel})^{-1}[\![\varphi]\!]$

Retain definition of membrane def. grad. $F^{\parallel} = g_i \otimes G^i$

The order is not unique

$$\boldsymbol{F}^{\|} = \hat{\boldsymbol{G}}_A \otimes \boldsymbol{G}^A \qquad \boldsymbol{F}^{\perp} = \boldsymbol{I} + \frac{[\![\varphi]\!]}{h} \otimes \bar{\boldsymbol{N}}$$



$$\bar{\boldsymbol{N}} = \hat{\boldsymbol{N}} \frac{d\hat{A}}{\hat{J}dA} \qquad \boldsymbol{g}_i \parallel \hat{\boldsymbol{G}}_i \longrightarrow \boldsymbol{F} = \boldsymbol{F}^{\|} + \frac{[\![\varphi]\!]}{h} \otimes \boldsymbol{N}$$

additive decomposition!

The normal used for construction $\quad \bar{\boldsymbol{N}} = (\boldsymbol{F}^{\|})^{-T}\boldsymbol{N}$

Retain definition of membrane def. grad. $\quad \boldsymbol{F}^{\|} = \boldsymbol{g}_i \otimes \boldsymbol{G}^i$

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Benefits of minor change in the kinematics

$$\boldsymbol{F} = \boldsymbol{F}^{\|} + \frac{[\![\varphi]\!]}{h} \otimes \boldsymbol{N}$$

Configuration, Additive decomposition

$$\begin{aligned} P^D &= \sum_{\pm} \int_{B_0^\pm} \boldsymbol{P} \cdot \dot{\boldsymbol{F}} \, dV + \int_{S_0} \boldsymbol{P} \cdot \dot{\boldsymbol{F}} h \, dS \\ &= \sum_{\pm} \int_{B_0^\pm} \boldsymbol{P} \cdot \dot{\boldsymbol{F}} \, dV + \int_{S_0} \boldsymbol{P} \cdot \left[\dot{\boldsymbol{F}}^{\|} + \dot{\boldsymbol{F}}^{\perp} \right] h \, dS \\ &= \sum_{\pm} \int_{B_0^\pm} \boldsymbol{P} \cdot \dot{\boldsymbol{F}} \, dV + \int_{S_0} \left[h \boldsymbol{P} \cdot \dot{\boldsymbol{F}}^{\|} + \boldsymbol{P}\boldsymbol{N} \cdot [\![\dot{\varphi}]\!] \right] dS \end{aligned}$$

Conjugacy

$$\boldsymbol{x}^+ = \boldsymbol{Q}\boldsymbol{x} \qquad \boldsymbol{F}^+ = \boldsymbol{Q}\boldsymbol{F} \qquad \boldsymbol{g}_i^+ = \boldsymbol{Q}\boldsymbol{g}_i \qquad [\![\varphi]\!]^+ = \boldsymbol{Q}[\![\varphi]\!]$$

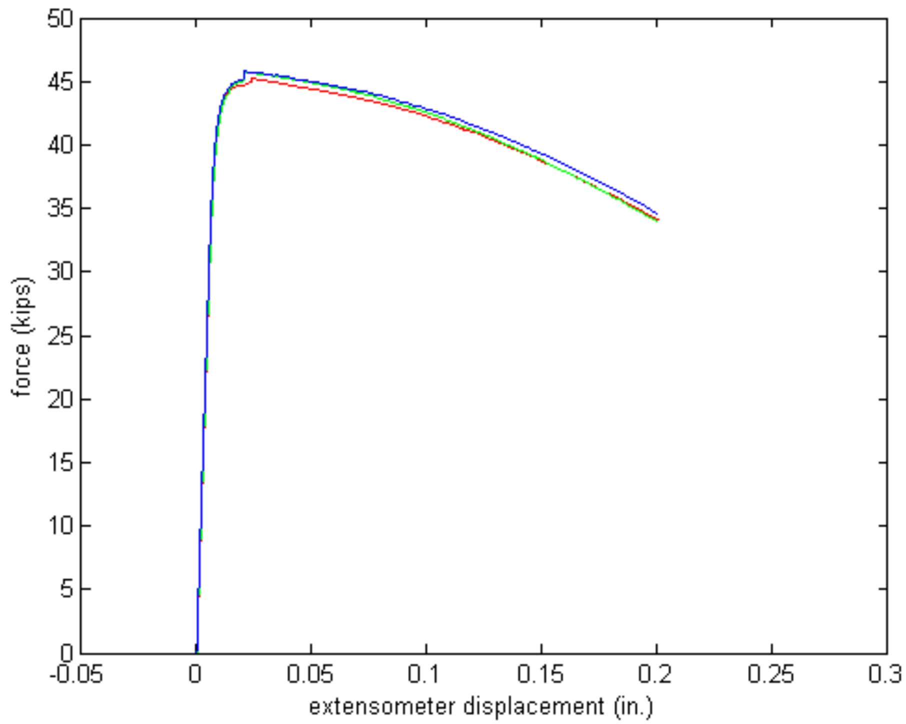
$$(\boldsymbol{F}^{\|})^+ = \boldsymbol{g}_i^+ \otimes \boldsymbol{G}^i = \boldsymbol{Q}\boldsymbol{F}^{\|}$$

Objectivity

$$(\boldsymbol{F})^+ = \boldsymbol{Q}\boldsymbol{F}^{\|} + \frac{\boldsymbol{Q}[\![\varphi]\!]}{h} \otimes \boldsymbol{N} = \boldsymbol{Q}\boldsymbol{F}$$

Case study for PH 13-8 H950

We obtained a data point from a PH 13-8 H977 test in which they used Bridgman correction factors (knowing the notch dimensions) to infer that the stress at a true strain of 0.316 was 1750 MPa.



$$\dot{\epsilon_p} = f\{\sinh[\frac{\bar{\sigma}}{(1-\phi)(\kappa+Y)} - 1]\}^n$$

$$\dot{\kappa} = [H - R_d\kappa]\dot{\epsilon_p}$$

f	*4.52 x 10*^{4}
Y	*1600 MPa*
n	*0.386*
H	*492 MPa*
R_d	*1.0 x 10*$^{-4}$

In this model, there is no recovery. Because f is big and n small, they do not affect the response. Hence, we really only fit two parameters, the yield stress, Y, and the hardening, H. Can two parameters get us there?

NOTE: For ease, the constitutive behavior is assumed to be isotropic, linear hardening.

Damage model and length scale

The only parameters that remain are the damage exponent, *m*, in the evolution of damage ϕ and the characteristic length scale *h* used to normalize the gap vector δ to yield a deformation gradient for the localization element. We lock down *h* at 30 μm and fit *m* with the plane-strain fracture toughness. For damage to evolve, we much select an initial porosity, ϕ_0. We choose to initialize the porosity to 1 x 10^{-4}.

$$\dot{\phi} = \{\frac{1}{(1-\phi)^m} - (1-\phi)\} \sinh[\frac{2(2m-1)}{2m+1} \frac{<p>}{\bar{\sigma}}]\dot{\epsilon}_p$$

Cocks and Ashby, 1980

$\frac{<p>}{\bar{\sigma}}$ triaxiality governing void growth

$$\boldsymbol{F} = \boldsymbol{F}^{\|}\boldsymbol{F}^{\perp} \qquad \boldsymbol{F}^{\perp} = \boldsymbol{I} + \frac{\boldsymbol{\phi}}{h} \otimes \boldsymbol{N}$$

Yang, Mota, Ortiz, 2005

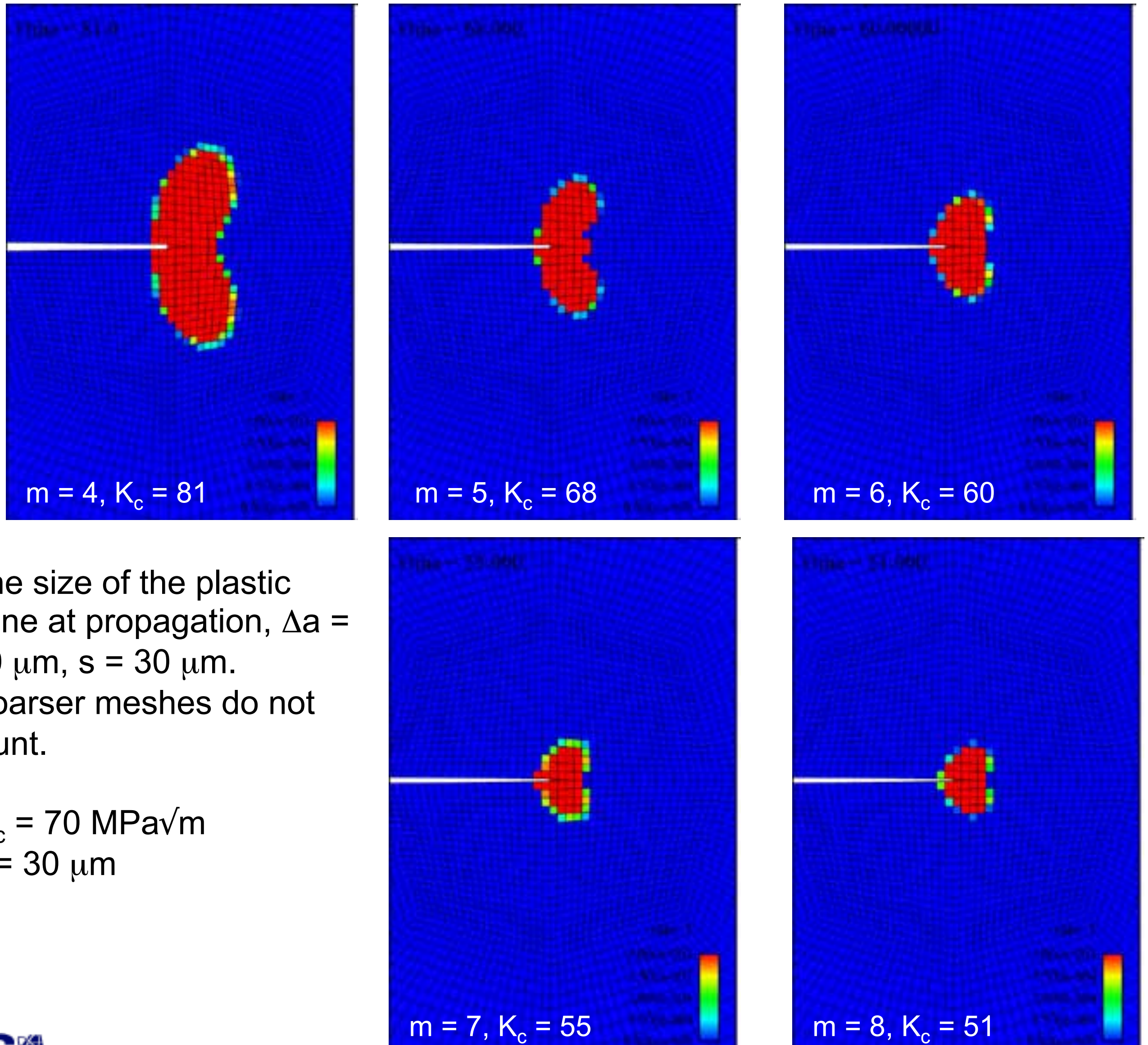
$\boldsymbol{\phi}$ gap vector for surface separation

$\boldsymbol{N}$ normal to mid-plane in the reference configuration

h characteristic length scale - 30 μm

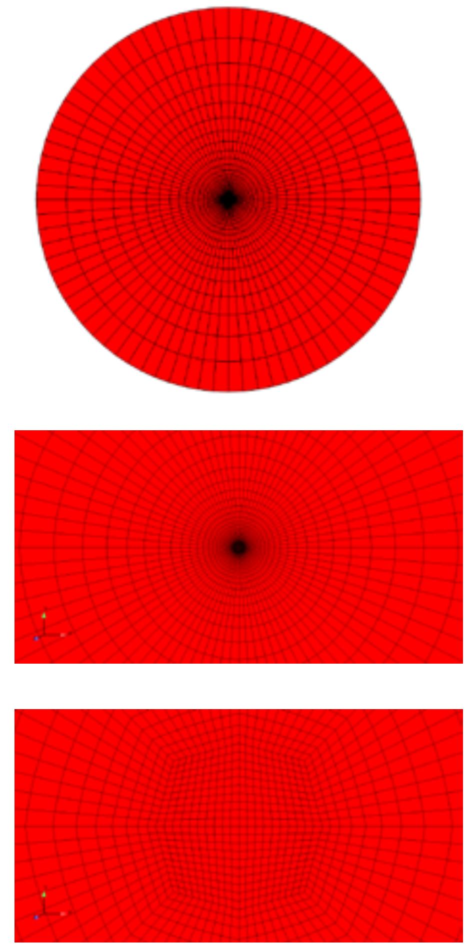
NOTE: In the original work of Cocks and Ashby, m is the power-law creep exponent. Just as in plasticity, we use the functional form of Cocks and Ashby to “fit” m.

Resolution and lumping dissipation



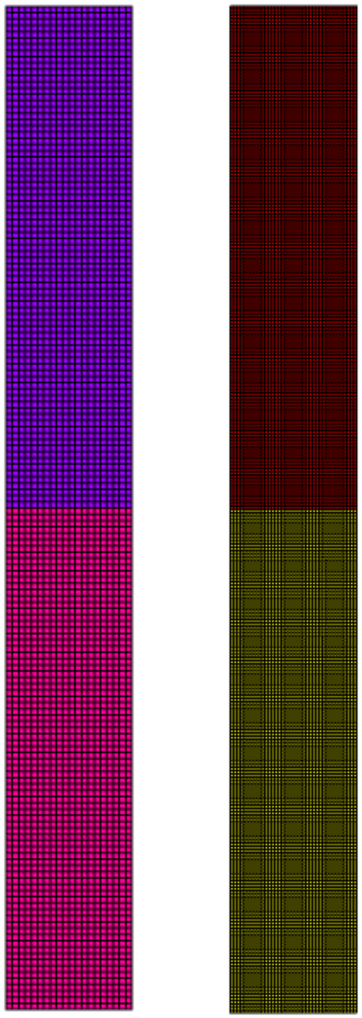
The size of the plastic zone at propagation, Δa = 60 μm, s = 30 μm. Coarser meshes do not blunt.

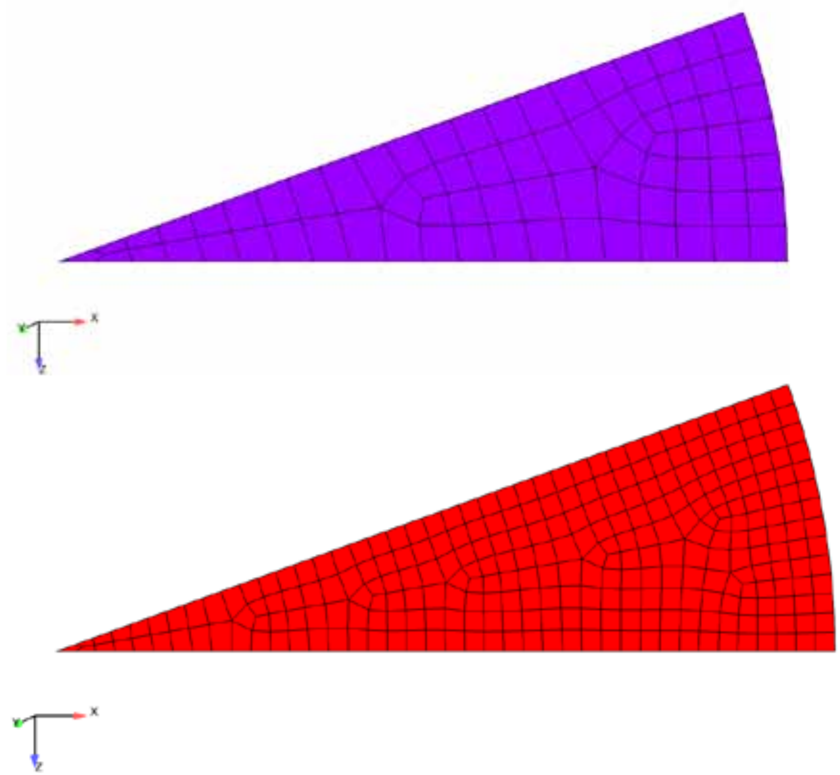
K_{Ic} = 70 MPa$\sqrt{m}$
h = 30 μm



K-field boundary condition

Probing properties through tensile necking



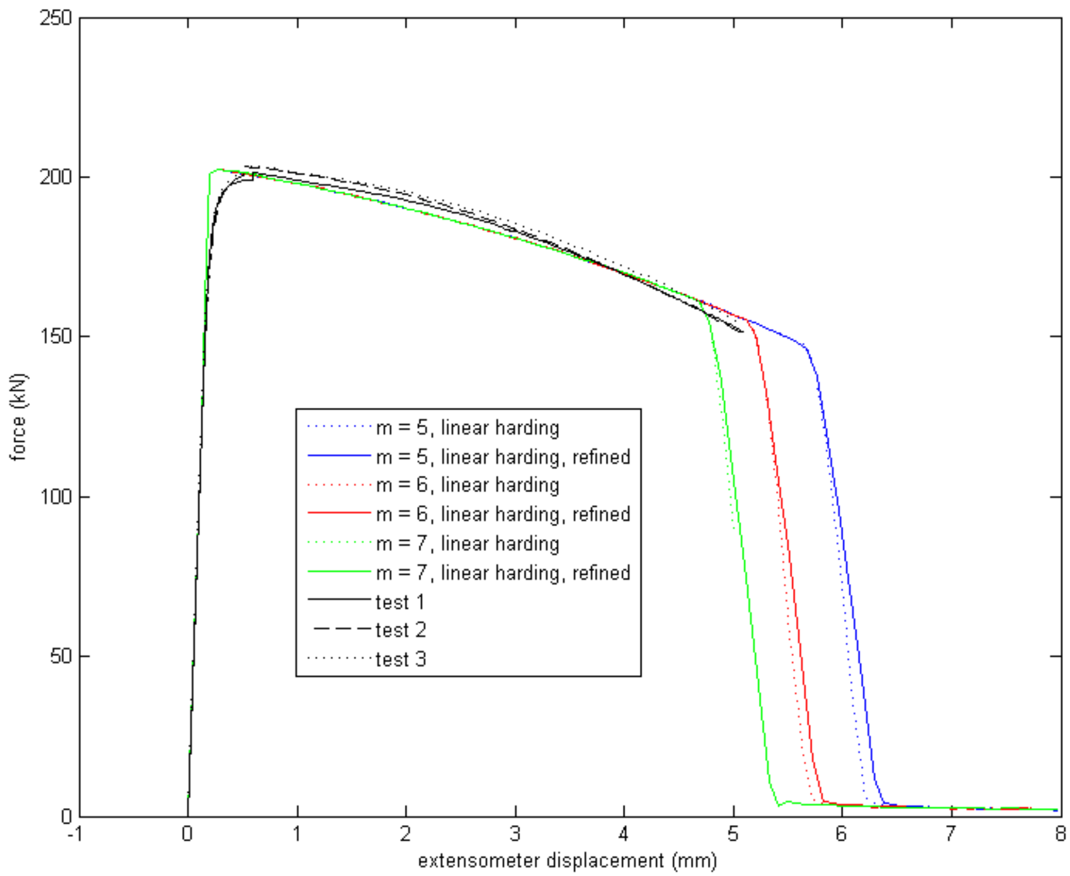


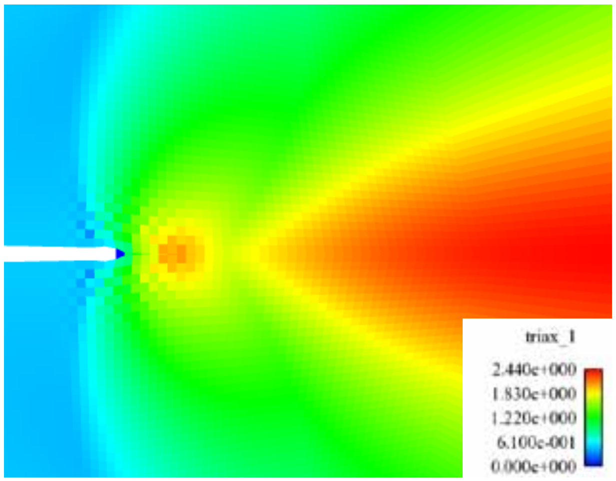
Bar radius: 12.7 mm
Bar length: 50.8 mm
Gauge section: 25.4 mm
Coarse element size ~ 635 μm
Finer element size ~ 318 μm

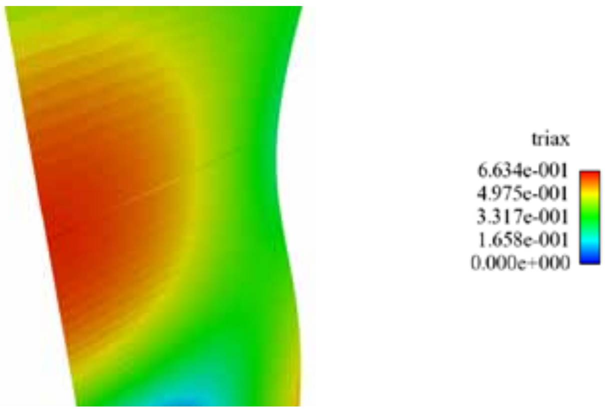
NOTE: A 2.54 μm taper is enforced over ½ the gauge section to control the necking process.

Global and local information

The simple linear hardening model does quite well to match the slope during the necking process. However, given that we have one curve with no measurements of the neck, this is not too bad. We assert that we did not tune the failure model to the smooth necking simulation.







Load-displacement is convergent

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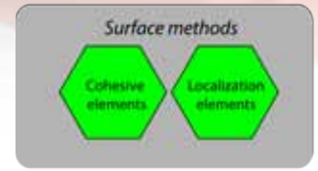
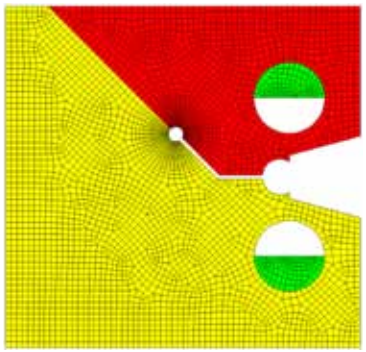
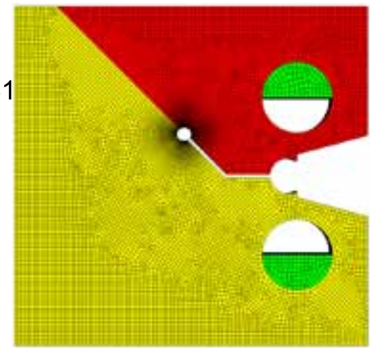


Damage is convergent

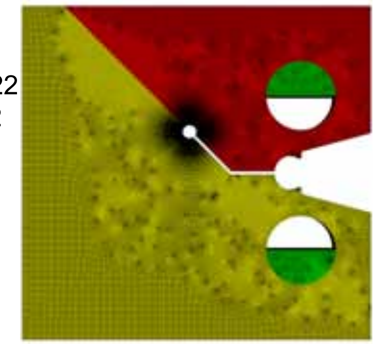
Mesh: 02
Label: Medium
Nodes: 29,535
Elem: 23,673
s ~ 120 mm

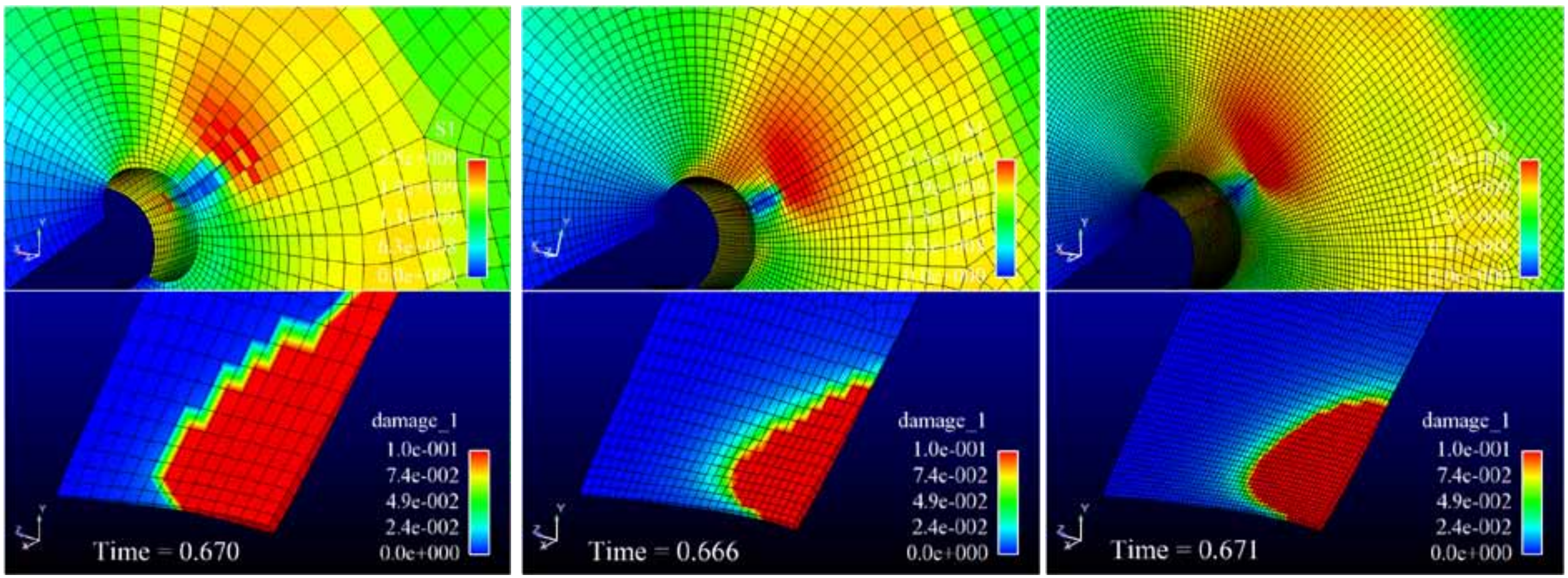


Mesh: 03
Label: Fine
Nodes: 141,831
Elem: 125,865
s ~ 60 μm



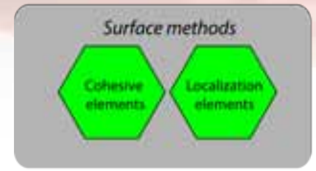
Mesh: 04
Label: Finest
Nodes: 1,079,622
Elem: 1,015,812
s ~ 30 μm

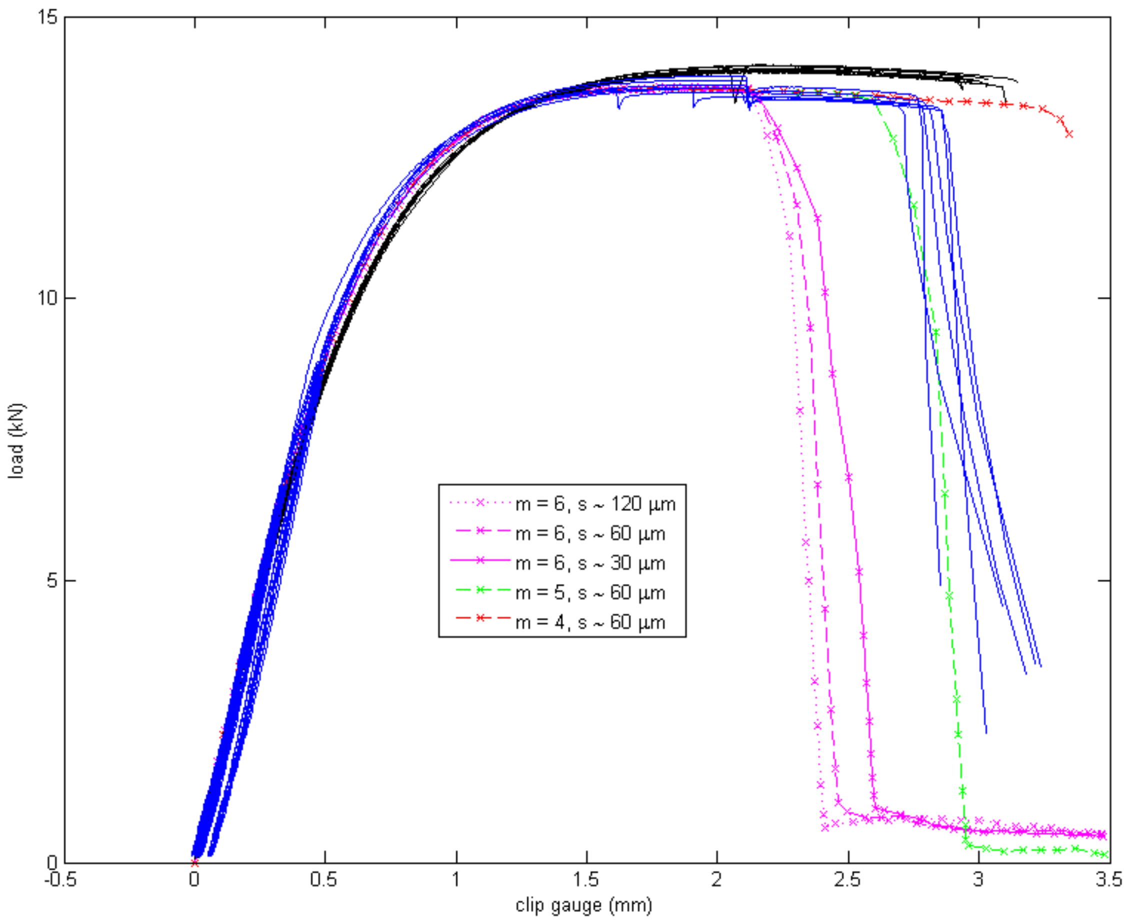




Although at slightly different times, the evolution of damage is comparable for 03 & 04.

Load-displacement is convergent





Boyce's lab:

- *Load line rate is 0.0127 mm/s*

Cordova's lab

- *Load line rate before 2.03 mm is 0.0027 mm/s*
- *Load line rate after 2.03 mm is 0.00025 mm/s*

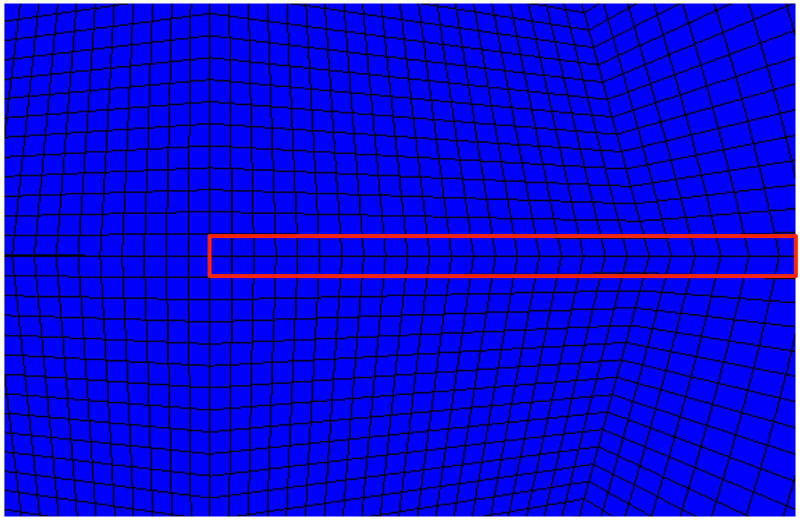
Element size on the order of h

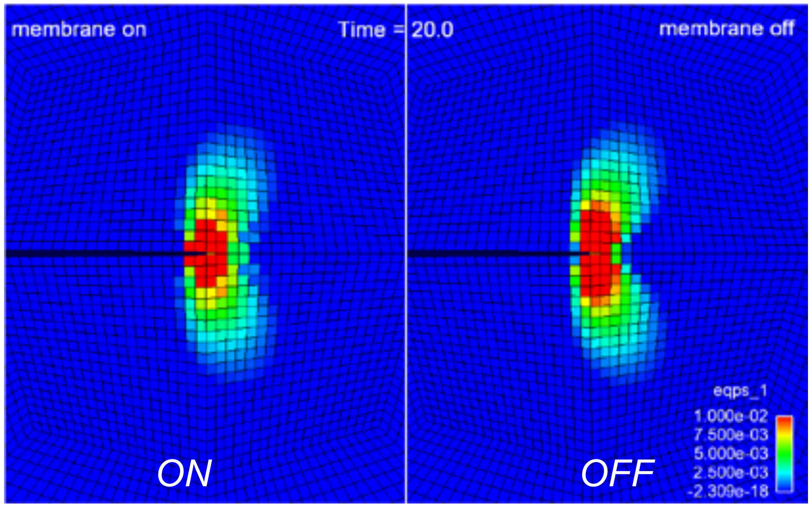
Localization elements with no volume have membrane forces that scale with h.

What if the mesh size s is order h?

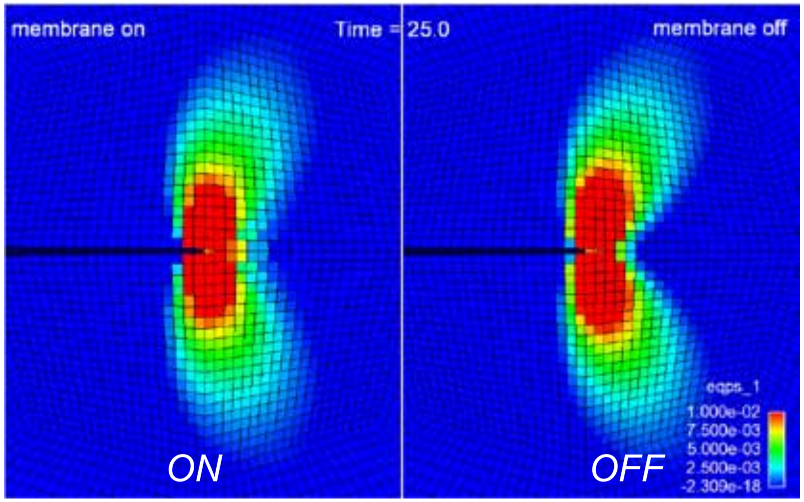
For ductile metals, this is the norm, the *plastic zone size ~ process zone size*

material: 2024-T3
yield stress: 375 MPa, K_{Ic} ~ 30 MPa√m
mesh size s: 30 μm, h = 60 μm





K = 20 MPa√m



K = 25 MPa√m

NOTE: The force calculation for membrane forces = off is “cohesive”.

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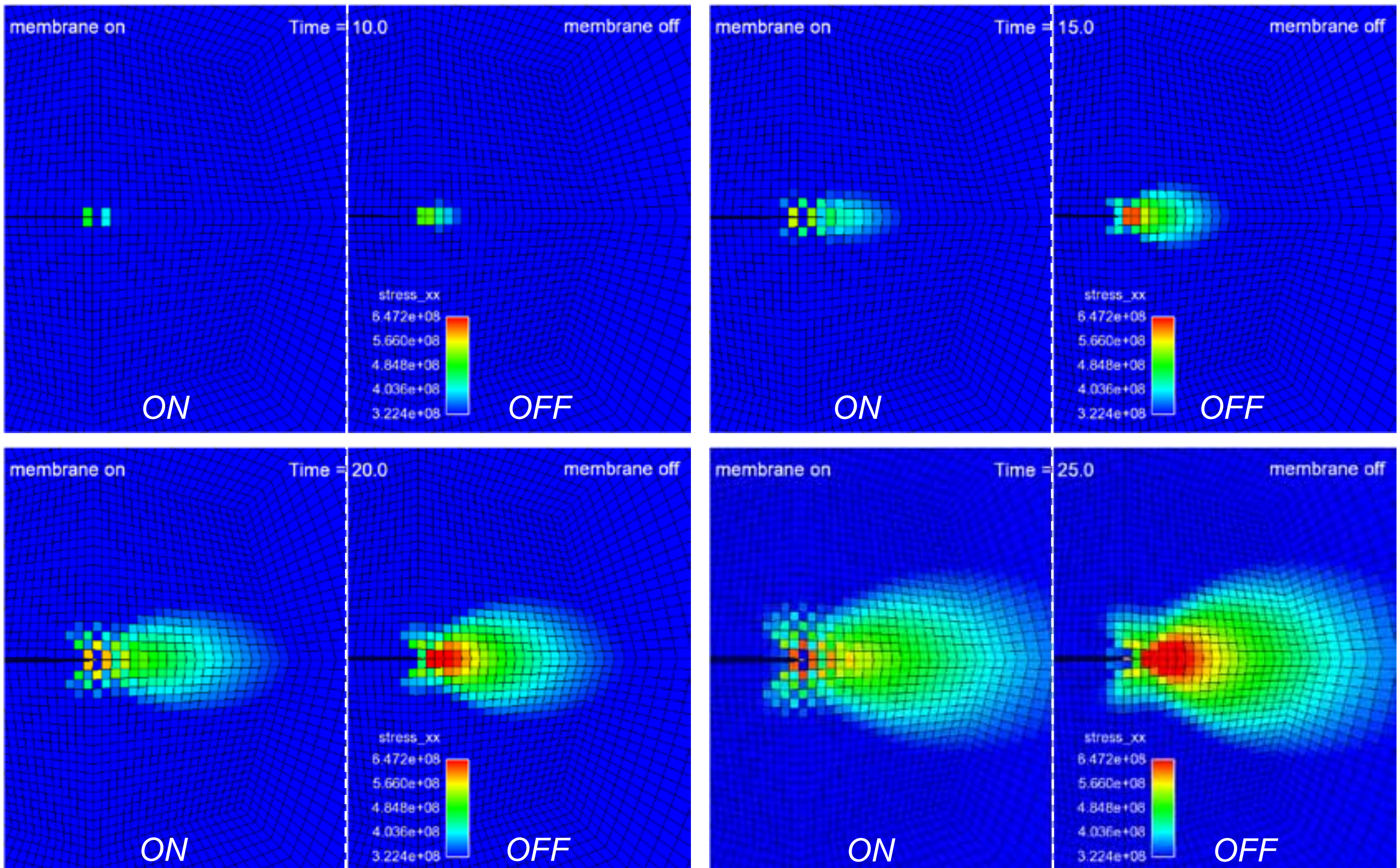
A case for no membrane forces for s~h

A case for eliminating membrane forces when the mesh size is order h

K = 10 MPa√m

K = 15 MPa√m



K = 20 MPa√m

K = 25 MPa√m

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Conclusions

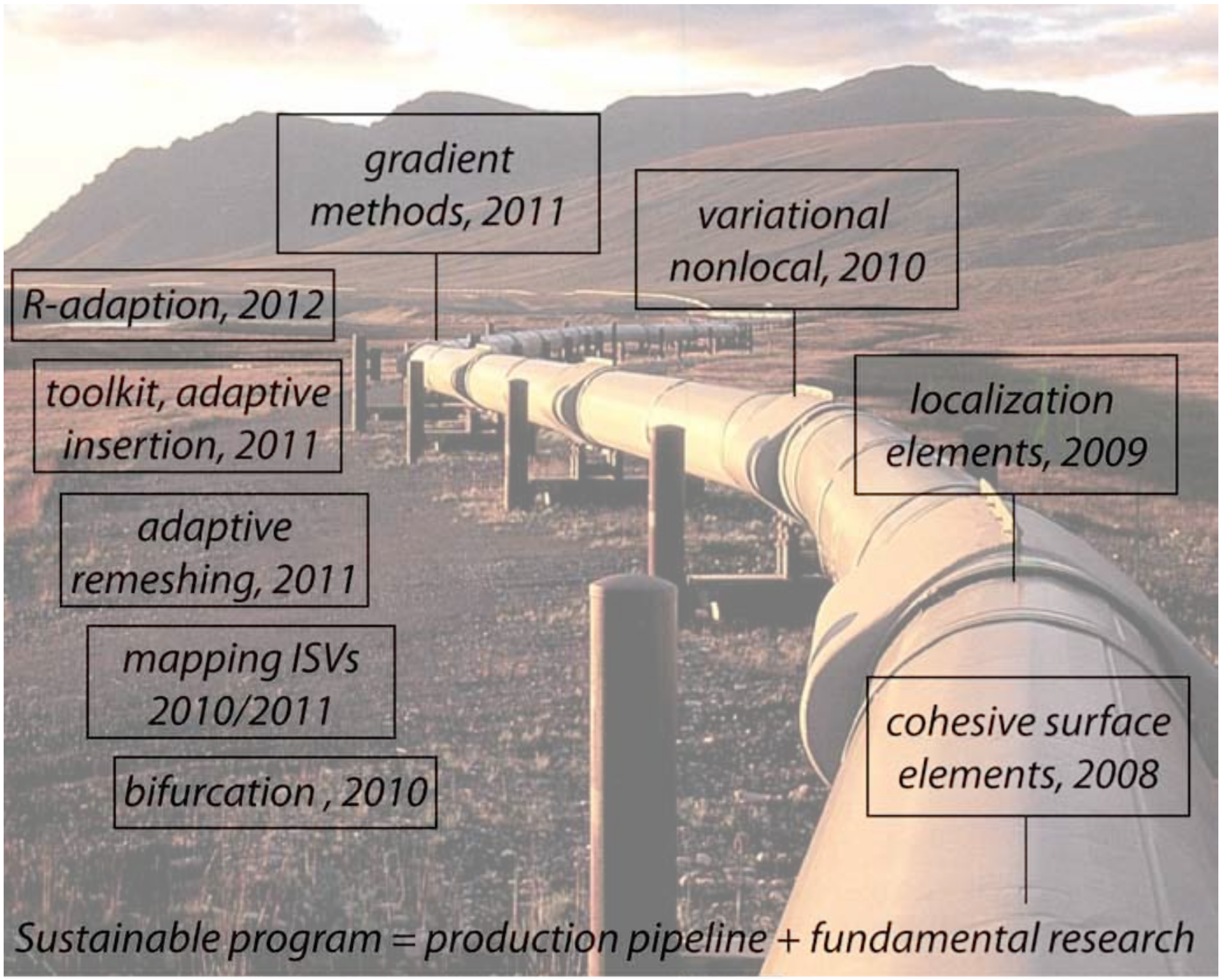
- *Developed methods have broad applicability*
- *Multiple paths towards regularization needed*
- *Localization elements provide regularization*
- *Changes to kinematics remedy issues of configuration, conjugacy, and objectivity*
- *Fitting process extrapolates plane-strain fracture toughness*
- *Case for ignoring membrane forces when element size is on order h*
- *Implemented into Sandia production code, SierraSM*

Acknowledgement: This work was supported in part through the Joint DoD/ DOE Munitions Technology Development Program.

Extra Slides

Methods pipeline for SierraSM

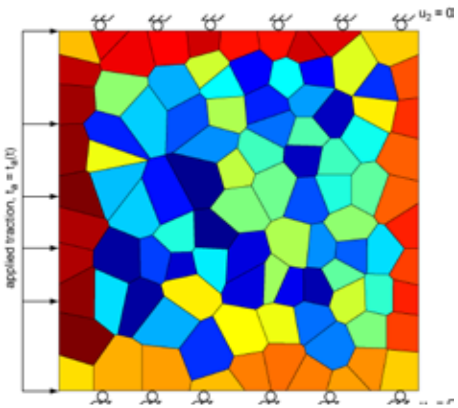
Basic research in engineering science is earned, not given. Production tools enable research.

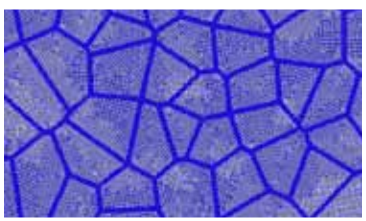


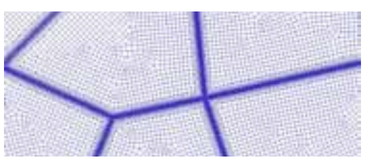
http://www.knowledgerush.com/kr/encyclopedia/Alaska_Pipeline/

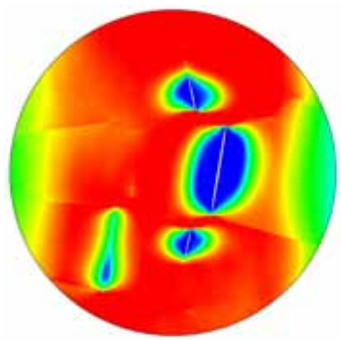
Cohesive approaches

Cohesive methods work well when we can lump all dissipation into surface separation. Ductile fracture challenges this approach (2 model problem, easily incorporating triaxiality)

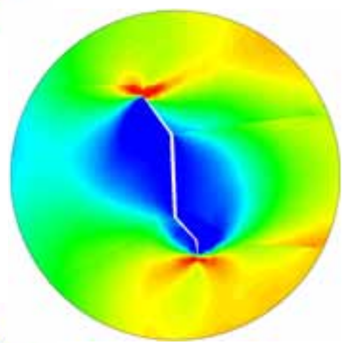




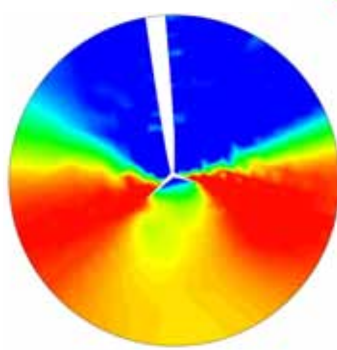




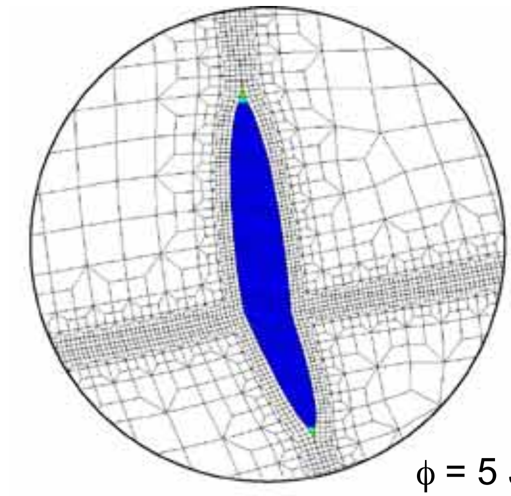
crack initiation



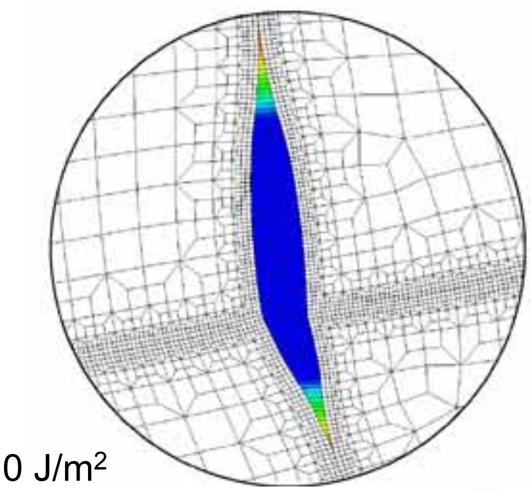
crack propagation



crack interaction



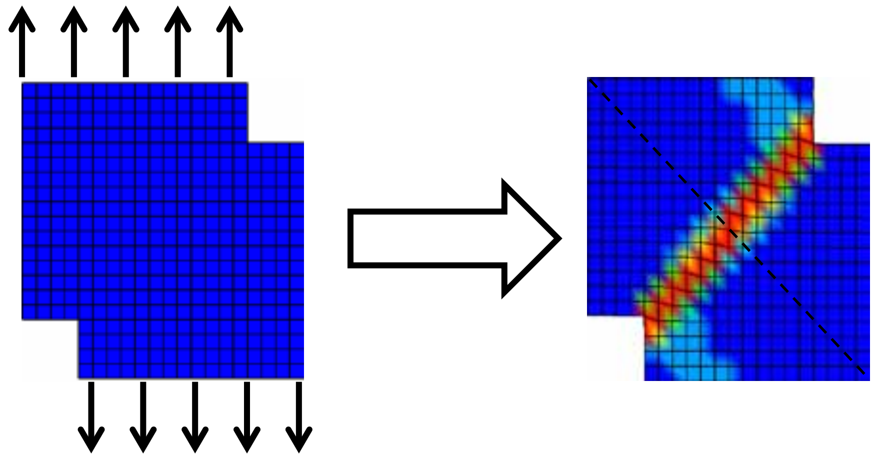
ϕ = 5 J/m^2



ϕ = 10 J/m^2

Nonlocal approaches

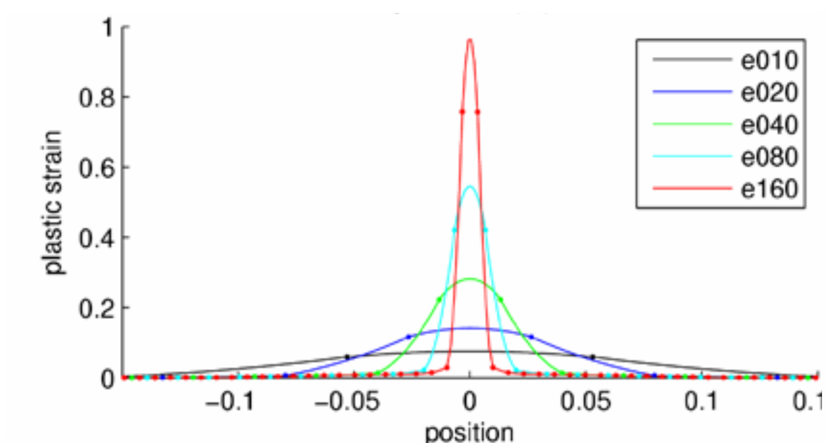
Nonlocality promising but we would like to simplify the numerics, simplify behavior at boundaries, and ensure scalability for massively parallel simulations.

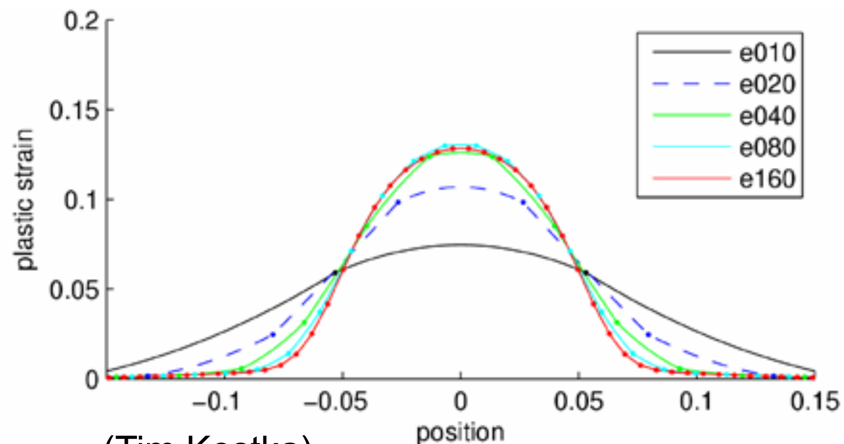


$$\dot{\varepsilon}(\underline{x}) = \frac{\sum_{i=1}^{N} w(|\underline{x}_i - \underline{x}|)\,\dot{\varepsilon}(\underline{x}_i)\,V_i}{\sum_{i=1}^{N} w(|\underline{x}_i - \underline{x}|)\,V_i}$$

$$w(\rho) = \left[1 + \left(\frac{\rho}{L}\right)^{p}\right]^{-q}$$

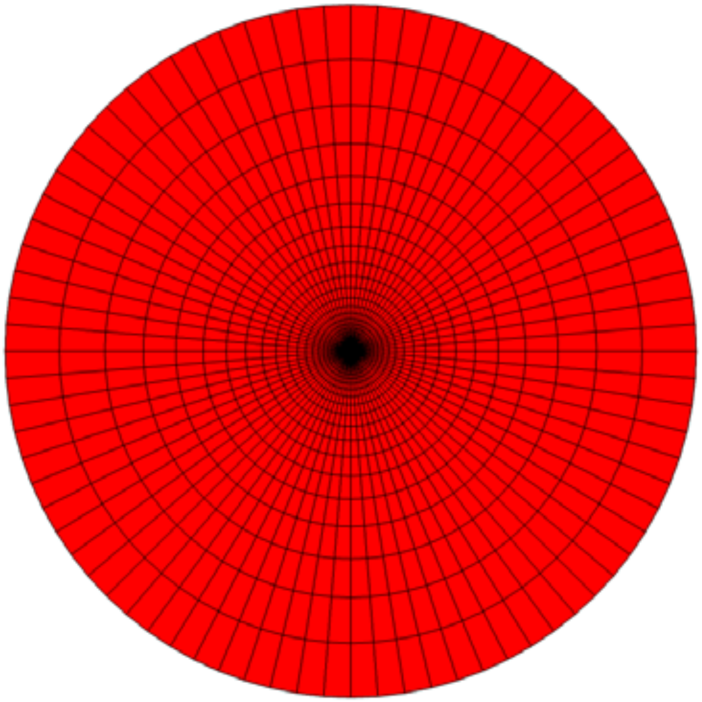
(LS-Dyna, single point integration)





(Tim Kostka)

K-field simulations

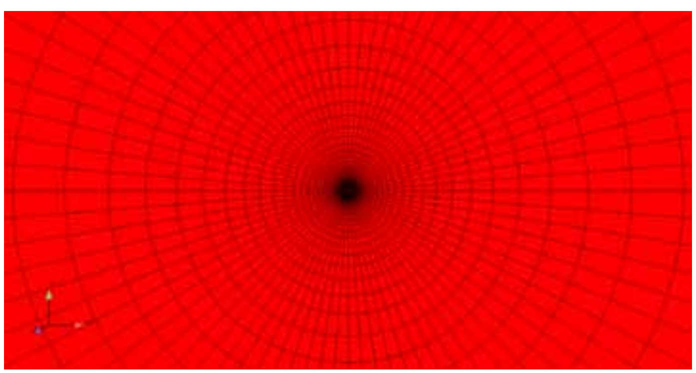


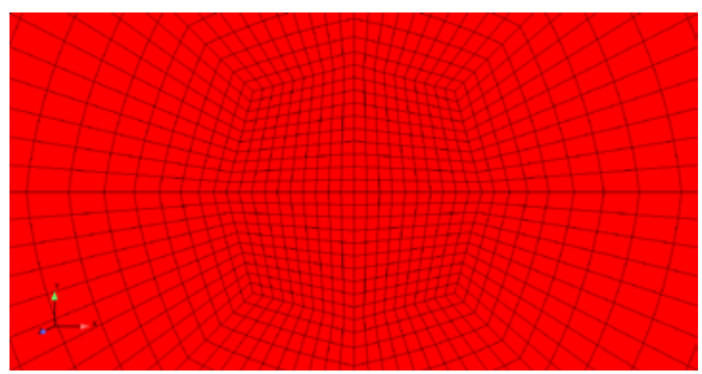
Given a node set on the boundary, we can apply a displacement field that enforces mode I, K_I, and mode II, K_{II}, stress intensity factors in plane-strain.

$$u_1 = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}}\left[1-2\nu+\sin^2\frac{\theta}{2}\right]\cos\frac{\theta}{2}+\frac{K_{II}}{\mu}\sqrt{\frac{r}{2\pi}}\left[2-2\nu+\cos^2\frac{\theta}{2}\right]\sin\frac{\theta}{2}$$

$$u_2 = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}}\left[2-2\nu-\cos^2\frac{\theta}{2}\right]\sin\frac{\theta}{2}+\frac{K_{II}}{\mu}\sqrt{\frac{r}{2\pi}}\left[-1+2\nu+\sin^2\frac{\theta}{2}\right]\cos\frac{\theta}{2}$$

Given K_{Ic} = 70 MPa√m and σ_y = 1.25 GPa, the plastic zone size r_p is roughly 0.5 mm (500 μm). We set the radius of the disk r_d to be 150 mm. r_d/r_p ~ 300, small-scale yielding. For small-scale yielding, we can use the K-field boundary condition to imply parameters governing the evolution of damage (m).

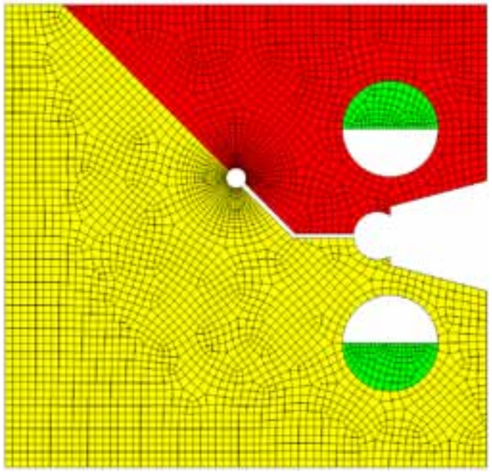


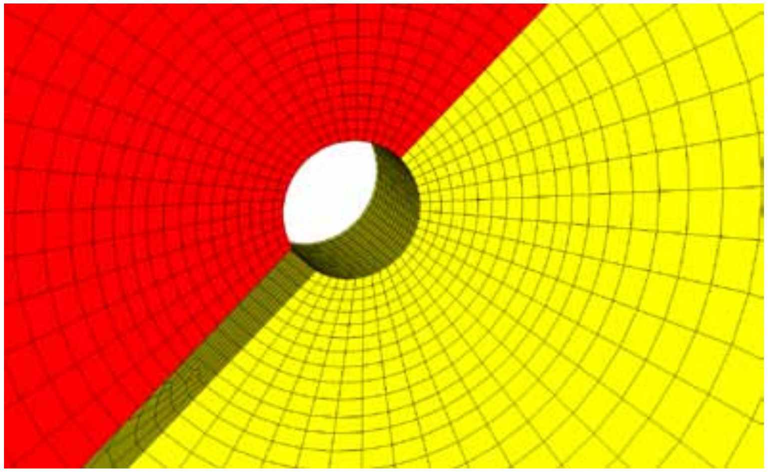


s = 60 μm

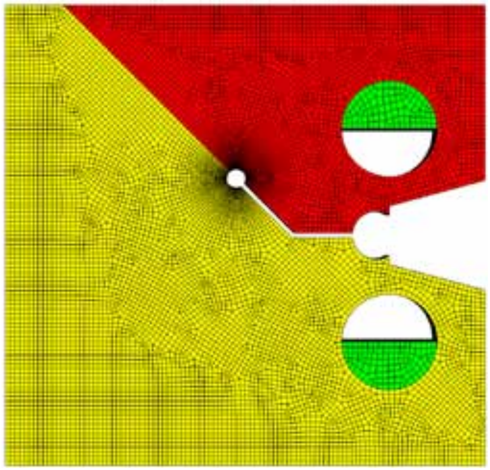
Discretization in length and time

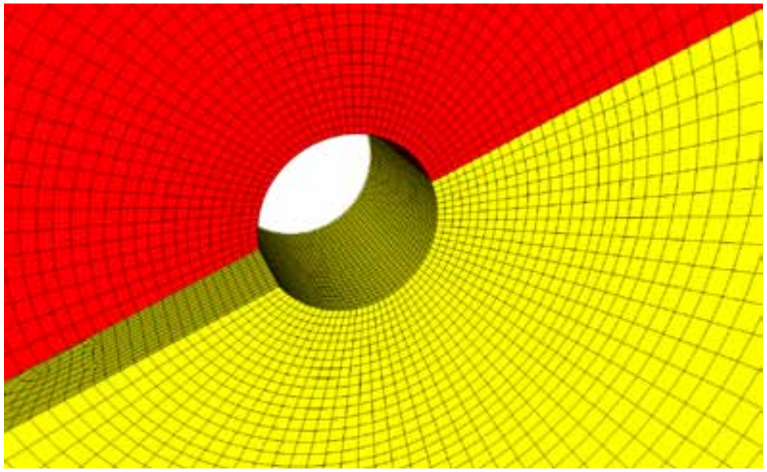
Mesh: 02
Label: Medium
Nodes: 29,535
Elements: 23,673
s ~ 120 μm





Mesh: 03
Label: Fine
Nodes: 141,831
Elements: 125,865
s ~ 60 μm



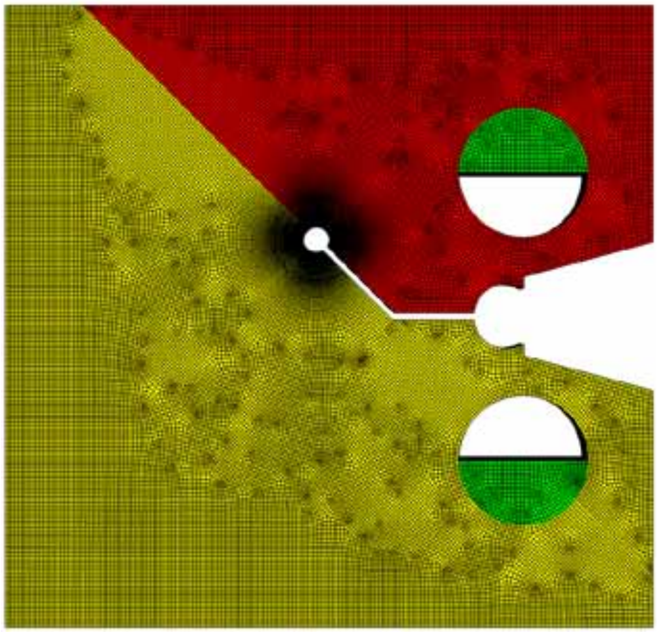


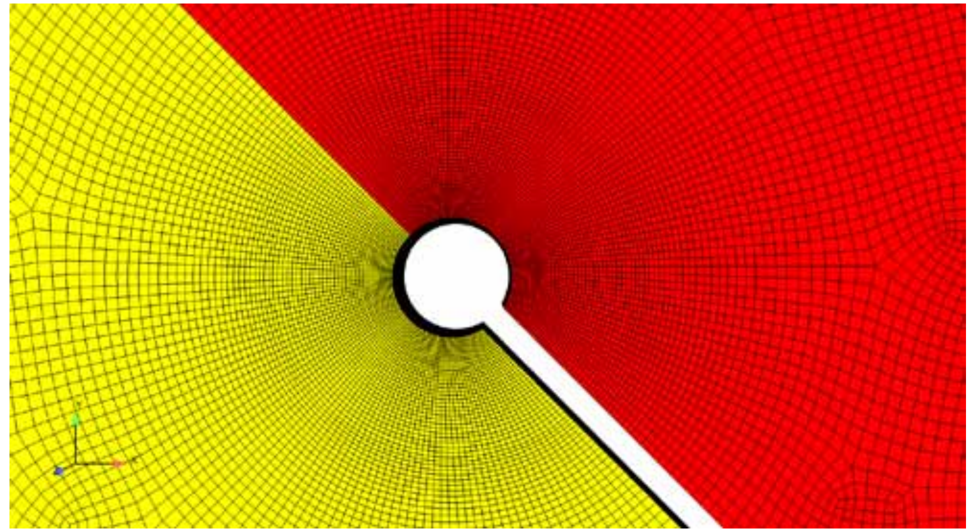
Time discretization: A minimum of 500 time steps were employed with adaptive time stepping (1000 steps specified).

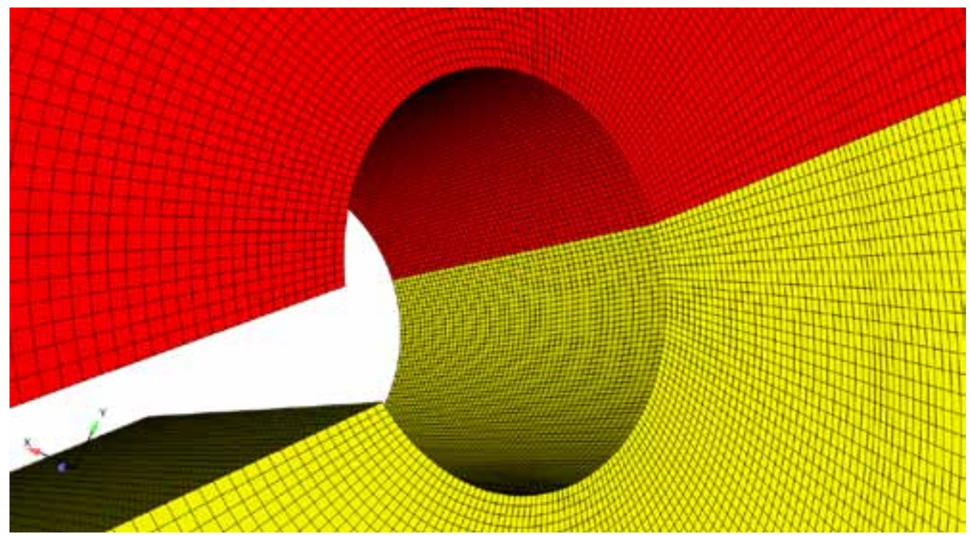
NOTE: Stiff, elastic plugs used to apply the pinned, line load. Contact could be used but typically aids instability during unloading (propagation).

More refinement

Mesh: 04
Label: Finest
Nodes: 1,079,622
Elements: 1,015,812
s ~ 30 μm

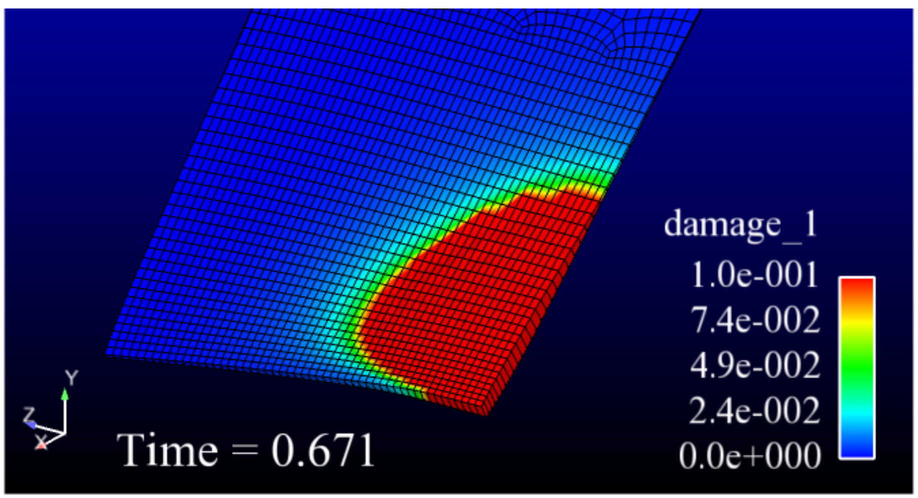




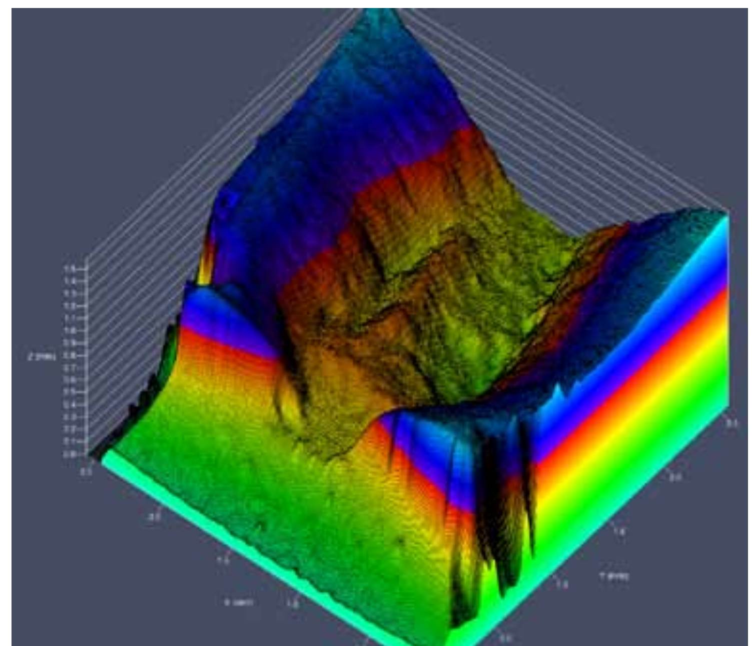


Life is complicated

(Cordova)



Shear lips, anyone? anyone? Not good.



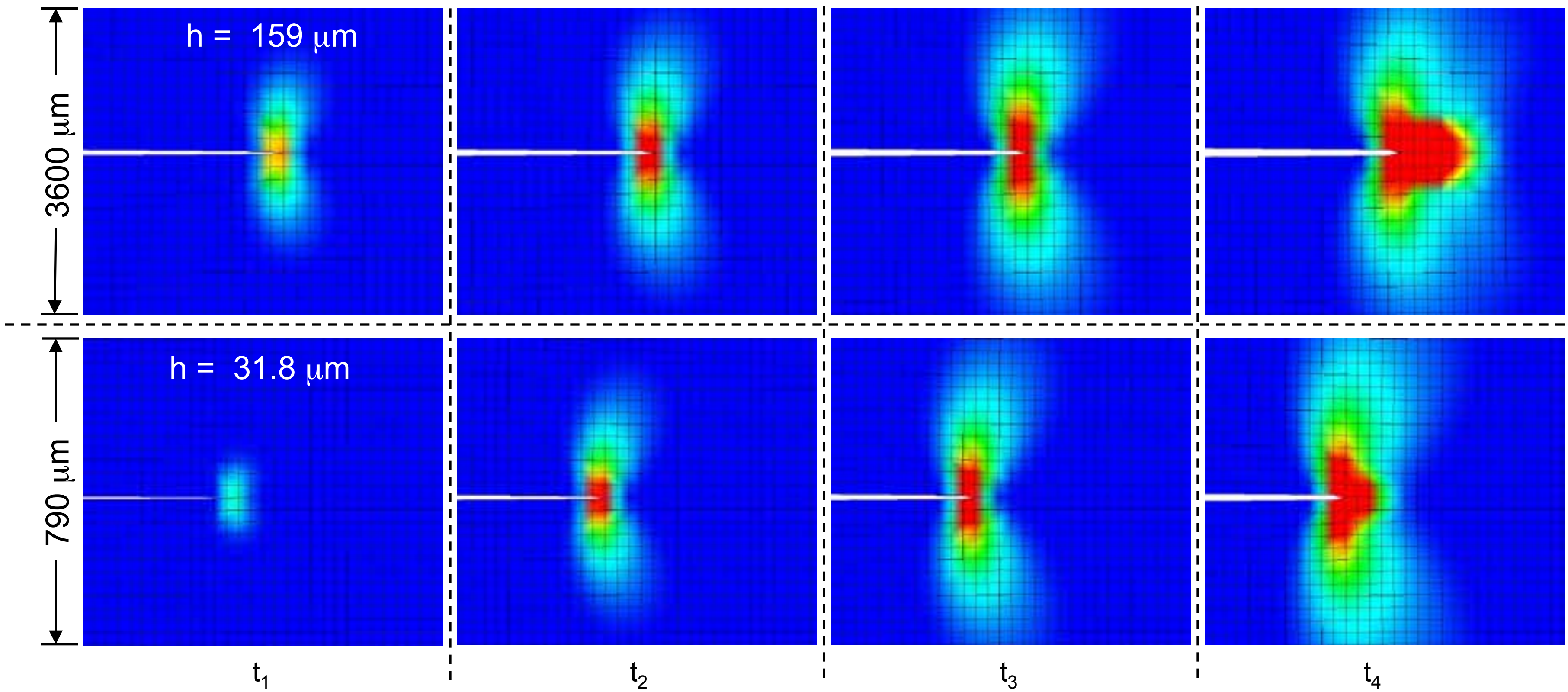
(Boyce)

Planned studies & improvements

- Improvements
  - Revised multiplicative decomposition (implemented)
  - Membrane forces on/off (implemented)
  - Objective velocity gradient update (implemented)
  - Stable time step estimate (to do)
- Planned studies
  - Weld Failure on the W-87 (Kostka, Templeton)
  - Membrane forces on X-Prize (Foulk, Emery, Boyce)
  - Implicit/explicit dynamics V&V study (Dike)
  - Adaptive remeshing (Emery, Veilleux)
  - Applicability to crack-band methods (MLEP-fail) (Veilleux)

Seeking a length scale for plasticity

Because the plastic zone size is tied to damage, plasticity is also mesh dependent



$$r_p = \frac{K^2}{4\pi\sigma_{ys}^2}\left[\frac{3}{2} + (1-2\nu)^2\right]$$

K_{1c} = 28.8 MPa√m (D. Dawson) r_p = 1390 μm (major axis)

Note: Contours of equivalent plastic strain, 0.0 to 2.0%. Time t_4 taken at propagation.

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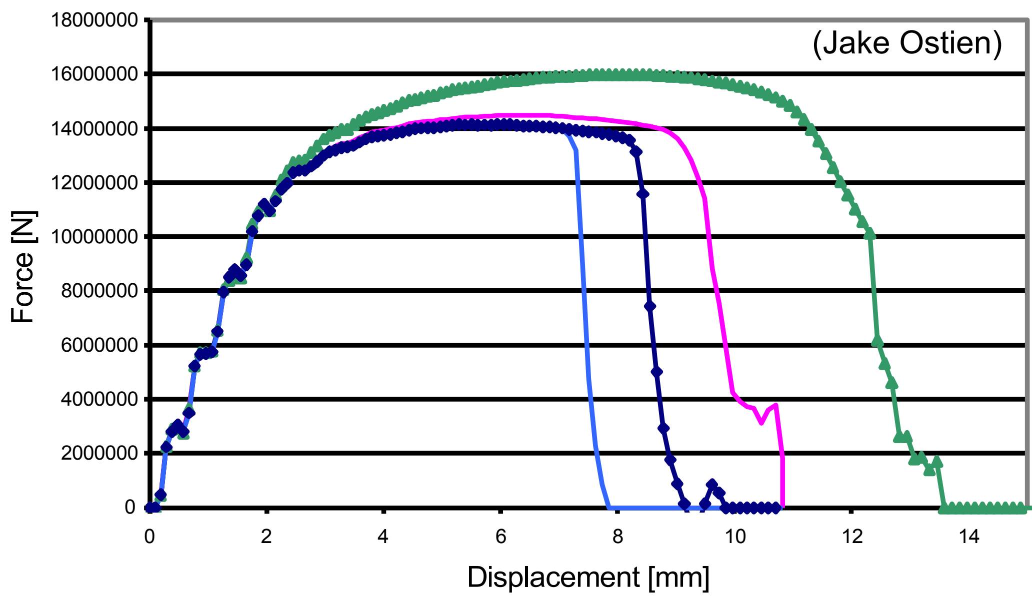
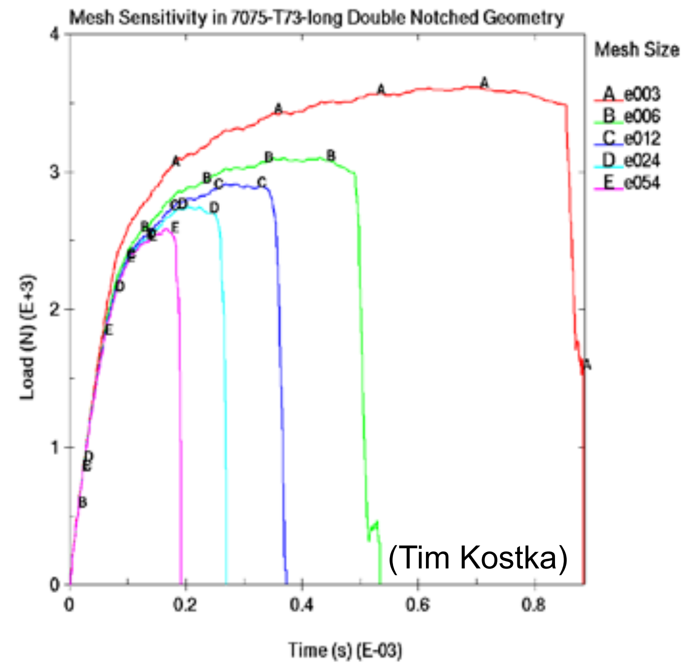
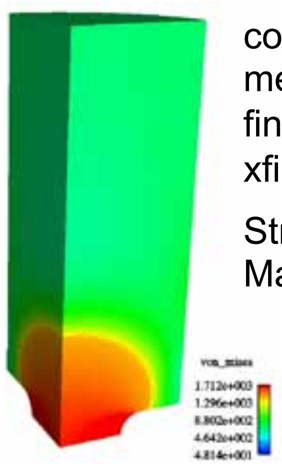
Mesh dependence under notched tension







coarse – 752 elements
med – 6016 elements
fine – 48128 elements
xfine – 385024 elements

Strain rate = 50/s
Material = A286

- *Specimens of various notched radii for "fitting"*
- *The results depend on the mesh size*
- *The fitted damage parameters are convoluted*
- *Goodness of the model is not known*
- *The issue stems from the governing PDEs*

Finite elements is just a method for solving a partial differential equation. The real issue is that we are using a local model without a length scale to solve a localization problem. It corrupts what we hold sacred...

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