

Quantum Mechanics of Neutrino Oscillations - Hand Waving for Pedestrians

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Abstract

Why Hand Waving? All calculations in books describe oscillations in time. But real experiments don't measure time. Hand waving is used to convert the results of a "gedanken time experiment" to the result of a real experiment measuring oscillations in space. Right hand waving gives the right answer; wrong hand waving gives the wrong answer. Many papers use wrong handwaving to get wrong answers. This talk explains how to do it right and also answers the following questions:

1. A neutrino which is a mixture of two mass eigenstates is emitted with muon in the decay of a pion at rest. This is a "missing mass experiment" where the muon energy determines the neutrino mass. Why are the two mass states coherent?
2. A neutrino which is a mixture of two mass eigenstates is emitted at time $t=0$. The two mass eigenstates move with different velocities and arrive at the detector at different times. Why are the two mass states coherent?
3. A neutrino is a mixture of two overlapping wave packets with different masses moving with different velocities. Will the wave packets eventually separate? If yes, when?

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I. INTRODUCTION

A. History and Dedication

This analysis of the basic physics of flavor oscillations began in 1981, when Israel Dostrovsky, then working on the gallium-germanium chemistry for a solar neutrino experiment, invited me to give a series of talks at Brookhaven in a language that chemists would understand. The notes of these lectures [1] were later expanded into lecture notes for a course in quantum mechanics [2] and then given further in a talk at a GALLEX collaboration meeting [3]. Meanwhile the gallium collaboration moved to Grand Sasso to become GALLEX. Dostrovsky has continued as one of the leaders in the collaboration, while his pioneering chemistry developed for the separation and detection of tiny numbers of germanium atoms produced by neutrinos in tons of gallium has been used by both GALLEX and SAGE.

It is a pleasure to dedicate this talk to my friend and colleague Israel Dostrovsky on the occasion of his 80th birthday.

B. Problems in the description and treatment of flavor oscillations

Flavor oscillations are observed when a source creates a particle which is a mixture of two or more mass eigenstates, and a different mixture is observed in a detector. Such oscillations have been observed in the neutral kaon and B-meson systems and seem now also to occur in neutrino experiments.

A flavor eigenstate with a sharp momentum is a mixture of mass eigenstates with different energies. It will oscillate in time with a well-defined oscillation period. A flavor eigenstate with a sharp energy is a mixture of mass eigenstates with different momenta. It will oscillate in space with a well-defined oscillation wave length. Many calculations describe “gedanken” experiments which begin with states having either a sharp momentum or a sharp energy.

They require some recipe for applying the results to a real experiment [3–7] which is always performed with wave packets having neither sharp momenta nor sharp energies.

Considerable confusion has arisen in the description of such experiments in quantum mechanics [3,4], with questions arising about time dependence and production reactions [5], and defining precisely what is observed in an experiment [6]. Combining features of both the space and time oscillations can lead to double counting.

This issue has been clarified [8] by showing that in all oscillation experiments the size of the neutrino source is so much smaller than the distance between source and detector that the problem reduces to the propagation of a linear combination of waves emitted from a point source with well defined relative phases at the source. This wave picture uniquely determines the relative phases at the detector, gives all the right answers, and justifies the hand-waving used in all the standard treatments. The particle picture is more complicated because all momentum conservation relations must take into account the uncertainty in the total momentum of the system resulting from the small source size, which is orders of magnitude larger than the tiny momentum differences between mass eigenstates.

C. The basic quantum mechanics of flavor oscillations

Treatments combining classical particle and classical wave descriptions are often inconsistent with quantum mechanics and violate uncertainty principles. It is inconsistent to describe a neutrino to be both a classical point-like particle following a classical path in space-time and also a classical wave with a definite frequency and wave length and a phase which is a well defined function of space-time. The neutrino emitted in a weak interaction is a wave packet described by a quantum-mechanical wave function, not a classical point-like particle which travels between source and detector in a well-defined time. The neutrino wave passes the detector during a finite time interval. Its amplitude at the position of the detector defines the probability of observing the neutrino at the detector as a function of time. The flavor structure observed at the detector depends upon the relative phases of the

mass eigenstate waves at the detector and upon the overlaps between them.

The assumption that the mass eigenstate is simultaneously a particle which arrives at the detector at a definite time and also a wave with a well defined phase violates basic principles of quantum mechanics. A pulse short enough to define a time interval exactly has no well-defined frequency and no well-defined phase. A pulse long enough to define a phase exactly must contain many wave lengths in space and many periods in time. The physical neutrino in an oscillation experiment is described by a wave with such adequate lengths in space and time. The wave defines a probability amplitude for its observation at the detector. The exact time of detection, the exact value of the time interval between emission and detection and the proper time interval are therefore not predicted precisely and are given by a probability distribution. This quantum-mechanical fluctuation in time for the detection of a neutrino with well determined energy is just the well-known “energy-time uncertainty relation” which makes it impossible to define a phase and also a time interval which introduces uncertainty in energy and frequency.

However, the flavor change at the detector; i.e. the change in the relative phase of the mass eigenstates, is negligible during the time period when the neutrino may be detected. The exact transit time of the neutrino from source to detector is subject to unpredictable quantum-mechanical fluctuations, but the flavor observed at the detector is well defined. Thus neutrino oscillations can be observed in space and not in time in practical experiments where the position of the source in space is well defined.

II. DIFFERENT TYPES OF FLAVOR OSCILLATIONS

A. $K^0 - \bar{K}^0$ Oscillations

The first examples of flavor oscillations observed were in the production of neutral kaons as flavor eigenstates K^0 and $\bar{K}^0$ propagating in space as the nearly degenerate unstable mass eigenstates K_L and K_S states which decayed with long and very unequal lifetimes. They

were detected many ways - including both decays and interactions. The mass eigenstates have very different lifetimes and are detectable by this lifetime difference; i.e. by waiting until the K_S has decayed to get a pure K_L beam. Their propagation in space as mass eigenstates K_L and K_S induces oscillations between the flavor eigenstates K^0 and $\bar{K}^0$ which are observable by measurements at different points in space.

B. $B^0 - \bar{B}^0$ Oscillations

These two nearly degenerate unstable bound states have short and very nearly equal lifetimes. They are produced as flavor eigenstates and detected in practice only by weak decays, where there are many decay modes. The short lifetimes make it impossible to detect them by their strong interactions as flavor eigenstates B^0 and $\bar{B}^0$. Their propagation in space as mass eigenstates induces flavor oscillations which are detected by observing their decays at different space points.

C. Neutrino Oscillations

Here we have two or three nearly degenerate stable elementary particles which propagate without decay. They are produced and detected as flavor eigenstates. There is no possible direct detection of the mass eigenstates. If the flavor eigenstates are not mass eigenstates, their propagation in space as linear combinations of mass eigenstates induces flavor oscillations.

III. RIGHT AND WRONG TREATMENTS OF FLAVOR OSCILLATIONS

A. Common Wisdom

WRONG!

K^0 at Rest - Propagates in Time

$$|K^o(t)\rangle = a(t)e^{-iE_L t} |K_L\rangle + b(t)e^{-iE_S t} |K_S\rangle \quad (3.1)$$

$$\langle K^o | K^o(t) \rangle = a(t)e^{-iE_L t} \langle K^o | K_L \rangle + b(t)e^{-iE_S t} \langle K^o | K_S \rangle \quad (3.2)$$

$$\langle K^o | K^o(t) \rangle = a(t)a^*(o)e^{-iE_L t} + b(t)b^*(o)e^{-iE_S t} \quad (3.3)$$

Probability of finding K^o oscillates in time.

Oscillation frequency given by interference between

States of same momentum, different energies.

But nobody ever measures TIME!

All flavor oscillation experiments measure DISTANCES.

Oscillation wave length given by interference between

States of same energy, different momenta.

B. Correct Treatment

Nobody ever measures TIME!

All flavor oscillation experiments measure DISTANCES.

K^o at Source - Propagates in Space

$$|K^o(x)\rangle = a(x)e^{-ip_L x} |K_L\rangle + b(x)e^{-ip_S x} |K_S\rangle \quad (3.4)$$

$$\langle K^o | K^o(x) \rangle = a(x)e^{-ip_L x} \langle K^o | K_L \rangle + b(x)e^{-ip_S x} \langle K^o | K_S \rangle \quad (3.5)$$

$$\langle K^o | K^o(x) \rangle = a(x)a^*(o)e^{-ip_L x} + b(x)b^*(o)e^{-ip_S x} \quad (3.6)$$

Probability of finding K^o oscillates in space.

Oscillation wave length given by interference between

States of same energy, different momenta.

WHY SAME ENERGY?

Gives Right Answer

But how do we know it's right?

IV. PARADOXES IN CLASSICAL TREATMENTS OF OSCILLATIONS

A. Problems - Why Are States with Different Masses Coherent?

1. Energy-momentum kinematics

Consider the example of a pion decay at rest into a neutrino and muon, $\pi \rightarrow \mu\nu$. The energy E_π and the momentum p_π of the pion are:

$$E_\pi = M_\pi; \quad p_\pi = 0 \quad (4.1)$$

where M_π denotes the pion mass. Conservation of energy and momentum then determine the energies and momenta E_ν , E_μ , p_ν and p_μ of the neutrino and muon,

$$E_\nu = M_\pi - E_\mu; \quad p_\nu = -p_\mu \quad (4.2)$$

The mass of the neutrino M_ν is then determined by the relation

$$M_\nu^2 = (M_\pi - E_\mu)^2 - p_\mu^2 \quad (4.3)$$

This is just a “Missing Mass” experiment. The value of M_ν is uniquely determined and there can be no interference between states of different mass.



2. Space-time measurements

Consider a neutrino created at the space-time point $(x = 0, t = 0)$ with momentum p . It is detected at the position of a detector, $(x = x_d)$. The time of detection, $t_d = x_d/v$ depends upon the velocity of the neutrino. If the neutrino is a linear combination of two mass eigenstates with masses m_1 and m_2 , they will have different velocities,

$$v_1 = \frac{p}{m_1}; \quad v_2 = \frac{p}{m_2} \quad (4.4)$$

They will therefore arrive at the detector with different arrival times,

$$t_1 = \frac{x_d \cdot m_1}{p} \quad t_2 = \frac{x_d \cdot m_2}{p} \quad (4.5)$$

The detector will therefore detect either one or the other. There will be no coherence between mass eigenstates, no interference and no oscillations.

B. Solutions - Wave-particle duality provides coherence

1. Common Feature of all Flavor Oscillation Experiments

The flavor-oscillating particle is produced as a flavor eigenstate by a localized source in space. It is detected at a large distance (x_d) compared to the source size (x_s). If the flavor eigenstate is produced with a sharp energy and is a linear combination of mass eigenstates with masses m_1 and m_2 , they have momenta p_1 and p_2 . Space oscillations arise from interference between p_1 and p_2 .

The uncertainty principle requires a momentum uncertainty in the particle wave-packet $\delta p_W \approx \hbar/x_s$. This will also produce an uncertainty in the energy. Coherence between mass eigenstate waves will occur if the momentum difference between the different mass eigenstates *with the same energy*, $|p_1 - p_2|_E$ is much smaller than momentum uncertainty in the wave packet $|p_1 - p_2|_E \ll \delta p_W$ and give rise to spatial oscillations.

2. Lipkin's Principle - *If you can measure it you can measure it!*

PROOF

Any sensible experiment must have an oscillation wave length λ much larger than source size.

$$\lambda \approx \frac{\hbar}{|p_1 - p_2|_E} \gg x_s \quad (4.6)$$

The momentum uncertainty must then be much larger than the momentum difference between the mass eigenstates.

$$\delta p_W \approx \frac{\hbar}{x_s} \gg \frac{\hbar}{\lambda} \approx |p_1 - p_2|_E \quad (4.7)$$

Thus any sensible experiment will have $p_1 - p_2$ coherence.

Note that this implies that the initial state of any realistic flavor oscillation experiment does not have a sharp four-momentum. The quantum-mechanical fluctuations in this four-momentum required by the uncertainty principle are always much larger than the four-momentum differences between the different mass eigenstates which produce oscillations. They are therefore also much larger than any four-momentum differences between the states of other particles recoiling against these mass eigenstates. Thus any possible effects like induced oscillations which use four-momentum conservation to obtain a precise knowledge of the recoil momentum are destroyed by these quantum-mechanical four-momentum fluctuations.

V. RIGHT AND WRONG WAYS TO TREAT FLAVOR OSCILLATIONS

A. THE RIGHT WAY

1. *The Problem*

A particle with definite flavor is created at a source. This particle is a linear combination of mass eigenstate waves ~~with~~ with amplitudes and phases determined by the mixing dynamics. The mass eigenstates propagate independently with no interactions (we exclude the MSW interactions for the present) in a manner described by the Schroedinger or Dirac equation. The relative phases of different mass eigenstate waves change during propagation in space.

The problem is to calculate the flavor of the particle measured at a remote detector which depends upon the relative phases of the mass eigenstates at that point.

2. *The Solution*

1. Solve the free Schroedinger or Dirac Equation. This solution is trivial with no need for

fancy field theory or Feynman diagrams. The presence of mixtures of noninteracting mass states provide no problem.

2. Introduce the proper initial conditions at the source. This means defining a wave packet whose behavior in space and time describe the real experiment.
3. Get the answer for what is observed at the detector by evaluating the solution of the propagation equations at the detector.

3. The Question

WHY DOESN'T ANYONE DO THIS?

B. WHAT EVERYONE DOES INSTEAD - HAND WAVING!

1. Solve the wrong problem - Flavor oscillations in time. Nobody measures oscillations in time.
2. Obtain a correct but useless irrelevant answer - the frequency of oscillations in time.
3. Handwave to convert the irrelevant answer to the wrong problem into the answer to the right problem; to convert the frequency of oscillations in time to the wave length of oscillations ~~in~~ space.
4. Right hand waving by using $x = vt$ and choosing the right value for v gives the right answer.
5. Wrong hand waving gives the wrong answer.
6. All results in textbooks and in papers used by experimenters and phenomenologists to analyze data have used the right hand waving and get the right answer
7. The literature is still flooded with papers using the wrong hand waving, publishing wrong answers, and confusing many people.

VI. REAL & GEDANKEN ν -OSCILLATION EXPERIMENTS

A mixture of two or more mass eigenstates is created by a source and a different mixture is observed in a detector. If the initial state is a flavor eigenstate with a sharp momentum the mass eigenstates have different energies and oscillations in time are observed with a well-defined oscillation period. If the initial state is a flavor eigenstate with a sharp energy, the mass eigenstates have different momenta and oscillations in space are observed with a well-defined oscillation wave length. Experiments always measure oscillations in space; whereas conventional wisdom describes oscillations in time.

We now show in a simple example how the description of a time-dependent non-experiment can lead to ambiguities and confusion. Consider neutrino oscillations in one dimension with two mass eigenstates. We assume a 45° mixing angle for convenience so that the states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ are equal mixtures with opposite relative phase of the mass eigenstates denoted by $|\nu_1\rangle$ and $|\nu_2\rangle$ with masses denoted respectively by m_1 and m_2 .

$$|\nu_e\rangle = (1/\sqrt{2})(|\nu_1\rangle + |\nu_2\rangle); \quad |\nu_\mu\rangle = (1/\sqrt{2})(|\nu_1\rangle - |\nu_2\rangle) \quad (6.1)$$

A. The Gedanken Time Experiment

Consider the “non-experiment” often described in which a ν_e is produced at time $t=0$ in a state of definite momentum p . The energies of the ν_1 and ν_2 components denoted by E_1 and E_2 will be different and given by

$$E_1^2 = p^2 + m_1^2; \quad E_2^2 = p^2 + m_2^2 \quad (6.2)$$

Let $|\nu_e(t)\rangle$ denote this linear combination of $|\nu_1\rangle$ and $|\nu_2\rangle$ with energies E_1 and E_2 which is a pure $|\nu_e\rangle$ at $t = 0$. The $|\nu_e\rangle$ and $|\nu_\mu\rangle$ components of this wave function will oscillate as a function of t in a manner described by the expression

$$\left| \frac{\langle \nu_\mu | \nu_e(t) \rangle}{\langle \nu_e | \nu_e(t) \rangle} \right|^2 = \left| \frac{e^{iE_1 t} - e^{iE_2 t}}{e^{iE_1 t} + e^{iE_2 t}} \right|^2 = \tan^2 \left(\frac{(E_1 - E_2)t}{2} \right) = \tan^2 \left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)} \right) \quad (6.3)$$

This is a “non-experiment” or “gedanken experiment”. To compare this result with a real experiment which measures space oscillations the gedanken time dependence must be converted into a real space dependence. Here troubles and ambiguities arise and the need for hand-waving.

1. Handwaving - Method A

One can simply convert time into distance by using the relation

$$x = vt = \frac{p}{E} \cdot t \quad (6.4)$$

where v denotes the velocity of the ν meson. This immediately gives

$$\left| \frac{\langle \nu_\mu | \nu_e(t) \rangle}{\langle \nu_e | \nu_e(t) \rangle} \right|^2 = \tan^2 \left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)} \right) \approx \tan^2 \left(\frac{(m_1^2 - m_2^2)x}{4p} \right) \quad (6.5)$$

where the small differences between p_1 and p_2 and between E_1 and E_2 are neglected.

2. Handwaving - Method B

However, one can also argue that the ν_1 and ν_2 states with the same momentum and different energies also have different velocities, denoted by v_1 and v_2 and that they therefore arrive at the point x at different times t_1 and t_2 ,

$$x = v_1 t_1 = \frac{p}{E_1} \cdot t_1 = v_2 t_2 = \frac{p}{E_2} \cdot t_2 \quad (6.6)$$

One can then argue that the correct interpretation of the time-dependent relation for measurements as a function of x is

$$\left| \frac{\langle \nu_\mu | \nu_e(x) \rangle}{\langle \nu_e | \nu_e(x) \rangle} \right|^2 = \left| \frac{e^{iE_1 t_1} - e^{iE_2 t_2}}{e^{iE_1 t_1} + e^{iE_2 t_1}} \right|^2 = \tan^2 \left(\frac{(E_1 t_1 - E_2 t_2)}{2} \right) = \tan^2 \left(\frac{(m_1^2 - m_2^2)x}{2p} \right) \quad (6.7)$$

This differs from the relation (6.5) by a factor of 2 in the oscillation wave length. If one does not consider directly the result of a real experiment but only the two different interpretations of the gedanken experiment, it is not obvious which is correct. Questions also arise regarding the use of phase velocity or group velocity in eqs.(6.5) and (6.7)

B. The real experiment - measurement directly in space

All this confusion is avoided by the direct analysis of use of the result of the real experiment. In an experiment where a ν_e is produced at $x=0$ in a state of definite energy E , the momenta of the ν_1 and ν_2 components denoted by p_1 and p_2 will be different and given by

$$p_1^2 = E^2 - m_1^2; \quad p_2^2 = E^2 - m_2^2 \quad (6.8)$$

Let $|\nu_e(x)\rangle$ denote this linear combination of $|\nu_1\rangle$ and $|\nu_2\rangle$ with momenta p_1 and p_2 which is a pure $|\nu_e\rangle$ at $x = 0$. The $|\nu_e\rangle$ and $|\nu_\mu\rangle$ components of this wave function will oscillate as a function of x in a manner described by the expression

$$\left| \frac{\langle \nu_\mu | \nu_e(x) \rangle}{\langle \nu_e | \nu_e(x) \rangle} \right|^2 = \left| \frac{e^{ip_1 x} - e^{ip_2 x}}{e^{ip_1 x} + e^{ip_2 x}} \right|^2 = \tan^2 \left(\frac{(p_1 - p_2)x}{2} \right) \approx \tan^2 \left(\frac{(m_1^2 - m_2^2)x}{4p} \right) \quad (6.9)$$

These are just the normal neutrino oscillations, and the results agree with those (6.5) obtained by handwaving A.

We immediately note the analogous implications for all experiments measuring flavor oscillations. Calculations for neutrino oscillations in time describe non-experiments. Times are never measured in the laboratory; distances are measured. When correlated decays of two mesons will be measured in an asymmetric B factory, the points in space where the two decays will be measured in the laboratory, not the time difference which appears in many calculations.

When a ν_e is produced at $x=0$ with energy E , its mass eigenstates propagate in space and their relative phase changes produce $|\nu_e\rangle$ and $|\nu_\mu\rangle$ oscillations in space. The simple argument using handwaving A is right. The treatment is completely relativistic and needs no discussion of time dependence or "proper times".

But is the use of a sharp energy really correct?

C. Another Approach with Different E and Different p

The interference has also been considered [5] between two states having both different E and different p produced at the point $x = 0, t = 0$.

$$\left| \frac{\langle \nu_\mu | \nu_e(x, t) \rangle}{\langle \nu_e | \nu_e(x, t) \rangle} \right| = \left| \frac{e^{i(E_1 t - p_1 x)} - e^{i(E_2 t - p_2 x)}}{e^{i(E_1 t - p_1 x)} + e^{i(E_2 t - p_2 x)}} \right| = \tan \left(\frac{(E_1 - E_2)t - (p_1 - p_2)x}{2} \right) \quad (6.10)$$

We now find that we can get the same result as the above treatment with a sharp energy (6.9) if we choose the time that the wave appears at the detector as the time after traveling with the mean group velocity $\langle v_{gr} \rangle$,

$$t = \frac{x}{\langle v_{gr} \rangle} = x \cdot \frac{E_1 + E_2}{p_1 + p_2} \quad (6.11)$$

$$\left| \frac{\langle \nu_\mu | \nu_e(x) \rangle}{\langle \nu_e | \nu_e(x) \rangle} \right| = \tan \left(\frac{[(E_1^2 - E_2^2) - (p_1^2 - p_2^2)]x}{2(p_1 + p_2)} \right) = \tan \left(\frac{(m_1^2 - m_2^2)}{2(p_1 + p_2)} \right) \cdot x \quad (6.12)$$

This result is simply interpreted in the wave picture. Eq. (6.10) holds at all points in space and time, and is due to the difference in the *phase* velocities of the two mass eigenstate waves. To apply this to the detector, we substitute the position of the detector and the time at which the neutrino is detected. There is only a single time, not two times as in eq.(6.7) obtained by Handwaving B. Although the centers of the wave packets move apart, the neutrino is detected for both wave packets at the same single time.

However, one can question the use of the expression value (6.11) determined by the mean group velocity. Since the wave packets pass the detector during a finite time interval, the detection time t to be substituted into eq. (6.10) can be *any time* during which the wave amplitude is finite at the detector. There is therefore a spread δt in the detection time which will give rise to a spread in the relative phase $\delta\phi$ between the two mass eigenstates.

$$\delta\phi = \frac{(E_1 - E_2)}{2} \cdot \delta t \approx \frac{(E_1 - E_2)}{2\delta E} \quad (6.13)$$

where $\delta E = 1/\delta t$ is the spread in energy required by the uncertainty principle for a wave packet restricted in time to an interval δt . We thus see that the uncertainty $\delta\phi$ will be of order unity and wash out all oscillations unless the energy difference $E_1 - E_2$ between the two interfering mass eigenstates is much smaller than the energy spread in the wave packet. We are therefore reduced to case described by eq. (6.9) and the necessity for use of a sharp energy to render oscillations observable.

The use of sharp energies has been justified [8,9] and is discussed in detail below. First we review carefully what is known in a realistic neutrino oscillation experiment and what cannot be known because of quantum mechanics and the uncertainty principle.

VII. WHAT DO WE KNOW ABOUT FLAVOR OSCILLATIONS

A. A General Guide to knowledge

My Father Used to Tell Me

“If you would know what you don’t know,
You would know more than you know”

Quantum Mechanics Tells Us

You can’t know everything

If you know the position of a neutrino source, you don’t know its momentum

Guide to Flavor Oscillations

Use what you can know

Don’t cheat by pretending you know what you can’t know

Examples of What We Can’t Know

The total momentum of a neutrino source in any experiment

The momentum of muon, Λ or other particle recoiling against a mass eigenstate

Exact center-of-mass system for fixed target experiment

Neutrino transit time from source to detector

All these are smeared by the uncertainty principle

B. What do we really know and really not know?

We know there is a neutrino source

We know the position of the source
 We know the flavor of the neutrino emitted by the source
 We do not know the time of emission!
 We do not know the momentum of the source

We know there is a neutrino detector
 We know the position of the detector
 We know the sensitivity of the detector to neutrino flavor
 We do not know the time of detection!

All books cheat by pretending we know what you can't know

C. RECOIL is a RED HERRING! RECOILS are unobservable

Recoil momenta of muons, Λ 's etc. given only by probability distributions
 Oscillations of recoil particles completely washed out by quantum-mechanical fluctuations

D. TIME is a RED HERRING! Nobody measures TIME!

Solar Neutrino Experiments
~~✗~~ Atmospheric Neutrino Experiments
 Reactor Neutrino Experiments
 Accelerator Neutrino Experiments
 None of them measure TIME!
 Nobody wants to measure TIME!

Nobody would know what to do with a TIME measurement!

VIII. THE KINEMATICS OF FIXED TARGET EXPERIMENTS

The complete description of a flavor oscillation experiment requires knowledge of the density matrix for the flavor-mixed state. This depends upon the production mechanism

and possible entanglements with other degrees of freedom as well as on other dynamical factors which are often ignored.

One example of such a generally ignored dynamical factor is the force on a proton in a fixed-target experiment. This proton is not free. To keep it in a solid target it must be constrained by some kind of effective potential with characteristic lattice energies like Debye temperatures. This energy scale is of the order of tens of millivolts and not at all negligible in comparison with mass differences between flavor eigenstates. In a simple potential model the proton is initially in some energy level with a well defined total energy. But there are large variations in its potential and kinetic energies. Thus the kinetic energy and momentum of the proton are not sharply defined. The bound proton is not strictly on shell and arguments of Galilean and Lorentz invariance and separation of center-of-mass motion may not hold for the kinematics of the production process if the degrees of freedom producing the binding are neglected.

Consider for example the reaction

$$\pi^- + p \rightarrow K^0 + \Lambda \quad (8.1)$$

If the energies and momenta of the pion beam, the target proton, and the outgoing Λ are known, the energy, momentum and mass of the outgoing kaon are determined by energy and momentum conservation. If, however, the energy and momentum of the target proton differ by small amounts δE and $\delta \vec{p}$ from the values for a free proton at rest, the squared mass of the kaon determined from conservation laws is given to first order in the small quantity $\delta \vec{p}$ by

$$M_K^2 = M_K(o)^2 + \delta M_K^2 \quad ; \quad \delta M_K^2 \approx -2\delta \vec{p} \cdot (\vec{p}_\pi - \vec{p}_\Lambda) \quad (8.2)$$

where $M_K(o)$ denotes the value of the kaon mass that is obtained from the conservation laws when δE and $\delta \vec{p}$ are neglected and we note that δE is of second order in $\delta \vec{p}$ and can be neglected to this approximation. Let us assume that the target proton is bound in a solid with a characteristic frequency ω ; e.g. the Debye or Einstein temperature of a crystal. This then sets the scale of the kinetic energy of the bound proton. Thus

$$|\delta\vec{p}| = O(\sqrt{M_p \cdot \omega}) \quad ; \quad \delta M_K^2 = O(\sqrt{M_p \cdot \omega}) \cdot |\vec{p}_\pi - \vec{p}_\Lambda| \quad (8.3)$$

Since ω is of order 10^{-2} ev., while M_K , M_p , $\vec{p}_\pi$ and $\vec{p}_\Lambda$ are all of order 1 GeV, we see that $|\delta\vec{p}|$ and δM_K are of order 3 KeV. This is so much larger than the mass difference 3×10^{-6} ev. that any discussion of detecting recoil effects due the kaon mass difference is simply ridiculous. Since the momentum of the center of mass in this experiment has an uncertainty of 3 KeV due to the continuous exchange of momentum between the target proton and the forces binding it to the target, one cannot define a center-of-mass system for the beam and proton and ignore the rest of the target. Galilean and Lorentz transformations are clearly not valid at the scale of the kaon mass difference, without also transforming the macroscopic target to the moving frame.

In the language of the parton model the target proton might be considered as a parton moving in a sea of "brown muck". Measurements of energy and momentum of incoming and outgoing particles then determine the energy and momentum distribution of the "parton" proton in the initial state. However, this does not work for the same reason that the parton model cannot describe the photoelectric effect in which an electron is ejected from an inner shell by the absorption of a photon. One must understand the dynamics of the binding and know the bound state wave function and the ionization energy to predict the results of a photoelectric experiment. Knowing the momentum distribution of the electron "parton" is not enough. Similarly describing the finite momentum spread of a target proton by a momentum distribution is not enough to enable prediction of the results of an experiment using the reaction (8.1) to the accuracy required for the determination of the kaon mass difference. One must know a wave function or density matrix as well as an ionization or dissociation energy in order to take subtle coherence effects and energy conservation into account.

If however, one is only interested in determining the kaon mass difference and not in the precise measurements of recoil momenta on that scale, a detailed knowledge of the bound state wave function is not necessary. One only needs to know that the bound state

wave function in momentum space is sufficiently wide to produce full coherence between components of the same energy with different mass and different momenta. The measured oscillation wave length then determines the mass difference to the same precision with which the wave length is determined. There is no need to measure momenta at the kilovolt level. This is shown in detail below.

The required coherence is between states of the same energy and different momenta, rather than vice versa. That energy and momentum conservation are not on the same footing is seen here as the same physics that describes the photoelectric effect and describes bouncing a ball elastically against the earth with energy conservation and no momentum conservation. In each case the relevant degrees of freedom are in interaction with a very large system which can recoil with arbitrary momentum and negligible kinetic energy.

IX. WHAT IS MEASURED IN REAL NEUTRINO OSCILLATION EXPERIMENTS

A. A single mass state passes a detector

NEUTRINO INCIDENT ON DETECTOR IS A WAVE!

Has finite length - passes detector in finite time interval

Square of amplitude at time t gives probability of detection

DETECTION TIME WITHIN WAVE PACKET UNPREDICTABLE!

Time of detection generally not measured

Precise time measurement gives no useful information!

B. Two overlapping mass states pass detector

NEW INGREDIENT: Neutrino flavor depends on relative phase

Still finite length - finite time interval

Square of amplitude at time t gives probability of detection

DETECTION TIME WITHIN WAVE PACKET UNPREDICTABLE!

Relative phase changes with space and time in packet

Negligible phase change with time at fixed detector!

DETECTION TIME WITHIN WAVE PACKET STILL USELESS!

X. AN OPTICAL GUIDE TO NEUTRINO OSCILLATIONS

A. A Faraday-rotated optical beam

As an instructive electromagnetic analog to quantum mechanical particle flavor oscillations consider the propagation of a Faraday-rotated polarized optical beam. We examine the case where a source emits vertically polarized light through a medium in which a magnetic field produces Faraday rotations. The parameters are chosen so that the plane of polarization is rotated by 90° between the source and detector. The light then reaches the detector horizontally polarized. Because of the presence of the medium, the light travels with phase and group velocities which are different from c . The states of right and left handed circular polarization are analogous to the neutrino mass eigenstates, which propagate unchanged through space. The states of plane polarization are analogous to neutrino flavor eigenstates which undergo oscillations while propagating in space. In this picture one can consider neutrino flavor as an intrinsic degree of freedom described by $SU(n)$ rotations in an abstract space where n is the number of flavors.



1. A classical wave picture

In a classical wave picture the light is a coherent linear combination of left-handed and right handed circularly polarized light beams which travel with slightly different velocities. The tiny velocity difference produces a change in the relative phase of left-handed and right handed components and rotates the plane of polarization.

2. Quantum photon picture

But light is quantized and consists of photons. What happens to a single vertically-polarized photon? Will it arrive horizontally polarized at the detector? The left-handed and right-handed components have different velocities and will arrive at the detector at different times.

This is a standard quantum-mechanical problem occurring whenever a beam of polarized particles passes through a field which would classically rotate the direction of polarization. Sometimes the components remain coherent and rotate the polarization. Sometimes they split to produce a Stern-Gerlach experiment.

3. Back to classical wave picture

For more intuition upon when there is coherence and when there is Stern-Gerlach we consider a classical source emitting classical pulses of finite length. They are therefore not monochromatic; there is a chromatic aberration that fuzzes the polarization. There is a classical uncertainty principle known to every electronic engineer. To define the time of a short pulse to a precision δt one needs a finite band width $\delta \nu$ which satisfies the classical uncertainty principle $\delta \nu \cdot \delta t \approx O(1)$.

The two pulses with left and right circular polarization have different velocities and gradually move apart. During the separation period there is a coherent overlap region with plane polarization and incoherent forward and backward zones with opposite circular polarizations.

4. Back to quantum photon picture

We now can quantize this picture and see that a photon can be detected either in the overlap region or in the forward or backward zones. A photon produced in the overlap region is horizontally polarized; a photon produced in the forward or backward zones is

circularly polarized. The amplitude at the detector at time t gives probability of detecting a photon at time t . For quantized waves, Planck introduces $E = h\nu$ to get the quantized uncertainty relation $\delta E \cdot \delta t \approx O(h)$. But the uncertainty between frequency and time and between position and wave-length are already there in the wave picture. It is the quantum-mechanical wave-particle duality that makes these into uncertainties between energy and time and between position and momentum.

B. A Faraday-Rotated Polarized Radar Pulse

To get a quantitative picture let us consider the propagation of a plane polarized microwave radar pulse through a medium containing a magnetic field in which Faraday rotations occur. Let the difference in velocities between the left-handed and right-handed polarization states be tiny, of order one part per million,

$$\frac{\delta v}{v} = 10^{-6} \quad (10.1)$$

This velocity difference introduces a relative phase shift between the two circularly polarized waves observed as a rotation of the plane of polarization between the transmitter and receiver. We first consider the classical wave picture and then introduce the quantum particle picture by considering individual photons.

Consider a pulse of one microsecond duration and a wave length of one centimeter traveling at very near the velocity of light. We assume that the deviations in velocity produced by the medium and magnetic field are smaller than one part per million and negligible for rough estimates. The length of the wave train or wave packet in space L_w is

$$L_w = 3 \times 10^{10} \cdot 10^{-6} = 10^4 \text{ cm.} \quad (10.2)$$

Both the size of the transmitter and the size of the receiver are small relative to the length of the wave train, which contains 10^4 wave lengths. The frequency of the microwave radiation is seen to be

$$\nu = 3 \times 10^{10} \text{ cycles} \quad (10.3)$$

or 30,000 megacycles. However, the radiation is not monochromatic. The frequency spectrum of a one microsecond pulse must have a finite band width of the order of one megacycle.

$$\delta\nu \approx 10^6 \text{ cycles} = \nu/3,000 \quad (10.4)$$

Since the velocities of the right-handed and left-handed pulses are different, the two wave packets eventually separate. If the receiver is sufficiently distant, it receives two one-microsecond pulses circularly polarized in opposite directions. We examine the interesting domain when the distance between transmitter and receiver is sufficiently small so that the overlap between the two circularly polarized wave packets is essentially 100%; e.g. if the centers of the wave packets have separated by 10 cm. which is negligible compared to the 100 meter lengths of the packets but sufficiently large so that the plane of polarization has undergone 10 complete Faraday rotations between the transmitter and receiver. If polarization measurements are made between the transmitter and receiver, 10 oscillations will be observed over this distance.

Since $\delta\nu/\nu = 10^{-6}$, ten oscillations will be observed after the waves have traversed a distance of ten million wave lengths; i.e. 100 kilometers. The oscillation wave length will be ten kilometers. The transit time of the wave will be

$$\delta t = \frac{10^7}{3 \cdot 10^{10}} = (1/3) \cdot 10^{-3} \text{ sec.} \quad (10.5)$$

or (1/3) millisecond.

The description in quantum mechanics is seen by examining the case where the transmitter is sufficiently weak and the receiver sufficiently sensitive so that individual photons can be counted in the receiver. The one microsecond pulse observed at the detector is seen as individual photons whose time of arrival at the detector are equally distributed over the one microsecond interval. There is thus a fluctuation of one microsecond in the times of arrival of an individual photon. This gives an uncertainty in the transit time of 3 parts per thousand. In any calculation of the velocity of the photon from the measured time of arrival

after it traversing a distance of 100 kilometers, the uncertainty of the arrival time produces an uncertainty in the velocity of 3 parts per thousand. This is enormous compared to the resolution of one part per million required to distinguish between the velocities of the two circularly polarized components. In principle one could measure the velocity difference by measuring the centroid of the arrival time distribution with sufficient precision. In practice this is out of the question.

The photons arriving at the receiver remain coherent mixtures of the two circularly polarized states. The polarization observed at the detector is just the polarization defined by the classical Faraday rotation; i.e. the relative phase of the two circularly polarized components arising from their traveling at different velocities. The exact time of arrival of an individual photon plays no role here. The quantum-mechanical uncertainty in the time arrival arising from the finite time duration of the pulse makes it impossible to determine the velocity of the photon to the precision needed to distinguish between the velocities of the two circularly polarized components.

If the detector is 10,000 kilometers or 10^9 cm. from the source, the centers of the two waves will have separated by 10^3 cm or $(1/10)L_w$. The probability for observing a photon will now have spread to an interval of 1.1 microsecond, The photons detected in the central 0.9 microseconds of this interval will still have the polarization defined by the classical Faraday rotation. The first and last intervals of 0.1 microseconds will now be left-handed and right-handed circularly polarized. As the distance is increased, the circularly polarized leading and trailing edges of the wave becomes greater until the wave separates into two one-microsecond pulses circularly polarized in opposite directions. .

The essential feature of this description is the necessity to create a wave train which contains a large number of cycles. This allows the different components of the wave packet traveling with different velocities to separate by a small number of cycles without appreciably affecting the overlap between these components. This is also the essential feature of any flavor oscillation experiment where a source creates a wave packet containing a sufficiently large number of cycles so that displacements of a few cycles between the packets of different

mass eigenstates traveling with different velocities produce a relative phase shift at the detector of the order of one cycle without appreciably affecting the overlap between the wave packets. Exact measurements of transit times between source and detector play no role, as they are subject to quantum-mechanical fluctuations arising from the condition that the length of the wave packet must contain a sufficient number of cycles to enable the definition of a phase and a frequency.

The above optical analog is easily taken over into the description of particle flavor oscillations. The flavor eigenstates are analogous to spin polarization eigenstates, and the neutrino oscillations are describable as rotations in some abstract flavor-spin space. The fact that all experiments in which oscillations can be measured involve sources which are very small in comparison with the oscillation wave length enable a description in which waves are emitted from a point source with a definite polarization state in this flavor-spin state.

XI. A UNIVERSAL BOUNDARY CONDITION APPROACH

A. Resolution of Confusion

We have noted that the proper solution for the flavor oscillation problem is simply to solve the free Schroedinger or Dirac equation and introduce the proper initial conditions at the source. The reason why nobody ever does this is because the initial conditions at the source are generally very complicated and not known. This is the reason for the general procedure of solving gedanken experiments and hand waving.

However, it has now been shown [8,9] that it is not necessary to know all details of the initial conditions in order to obtain the desired results. Much confusion has been resolved [8] by noting and applying one simple general feature of all practical experiments. The size of the source is small in comparison with the oscillation wave length to be measured, and a unique well-defined flavor mixture is emitted by the source; e.g. a ν_e in a ν oscillation experiment. The particles emitted from the source must leave the source before their flavor

begins to oscillate. They are therefore described by a wave packet which satisfies a simple general boundary condition: the probability amplitude for finding a particle having the wrong flavor; e.g. a ν_μ at the source must vanish for all times. There should be no flavor oscillations at the source.

This boundary condition requires factorization of the flavor and time dependence at the position of the source. Since the energy dependence is the Fourier transform of the time dependence, this factorization also implies that the flavor dependence of the wave packet is independent of energy at the position of the source. In a realistic oscillation experiment the relative phase is important when the oscillation length is of the same order as the distance between the source and the detector. In that case this flavor-energy factorization holds over the entire distance between the source and detector. The boundary condition then determines the relative phase of components in the wave function with different mass having the same energy and different momenta. Thus any flavor oscillations observed as a function of the distance between the source and the detector are described by considering only the interference between a given set of states having the same energy. All questions of coherence, relative phases of components in the wave function with different energies and possible entanglements with other degrees of freedom are thus avoided.

Many formulations describe flavor oscillations in time produced by interference between states with equal momenta and different energies. These “gedanken” experiments have flavor oscillations in time over all space including the source. The ratio of the wave length of the real spatial oscillation to the period of the gedanken time oscillation has been shown [8] to be given by the group velocity of the wave packet.

B. Explicit Solution of Oscillation Problem

We now present a rigorous quantitative treatment of the above argument and show how the results of a flavor oscillation experiment can be predicted without solving all the problems of production, time behavior and coherence. If oscillations are observable, the dimensions

of the source must be sufficiently small in comparison with the distance to the detector and the oscillation wave length to be measured so that the particle leaves the source with its original flavor. The distance traversed by the particle in leaving the source is too small in comparison with the oscillation wave length for any significant flavor change to occur. It is therefore a good approximation to consider the outgoing wave to be produced by a point source at the origin. The wave length in space of the oscillation can then be shown to be completely determined by the propagation dynamics of the outgoing wave in space and the boundary condition that the probability of observing a particle of the wrong flavor at the position of the source at any time must vanish for all times. Note that the exact time in which the particle is produced is not necessarily determined. The wave packet describing the particle must generally have a finite spread in time at the source position. But whenever it is produced in time, it leaves the source in space still with its original flavor.

We choose for example a neutrino oscillation experiment with a source of electron neutrinos. The neutrino wave function for this experiment may be a very complicated wave packet, but a sufficient condition for our analysis is to require it to describe a pure ν_e source at $x = 0$; i.e. the probability of finding a ν_μ or ν_τ at $x = 0$ is zero.

This boundary condition requires factorization of the flavor and time dependence at the position of the source. Since the energy dependence is the Fourier transform of the time dependence, this factorization also implies that the flavor dependence of the wave packet is independent of energy at the position of the source.

We write the neutrino wave function as an expansion in energy eigenstates satisfying the condition that it must avoid spurious flavor oscillations at the source and therefore be a pure ν_e state at $\vec{x} = 0$ for a finite length of time.

$$\psi = \int g(E) dE e^{-iEt} \cdot \sum_{i=1}^3 c_i e^{i\vec{p}_i \cdot \vec{x}} |\nu_i\rangle \quad ; \quad \sum_{i=1}^3 c_i \langle \nu_i | \nu_\mu \rangle = \sum_{i=1}^3 c_i \langle \nu_i | \nu_\tau \rangle = 0 \quad (11.1)$$

where $|\nu_i\rangle$ denote the three neutrino mass eigenstates and the coefficients c_i are energy-independent. The momentum of each of the three components is determined by the energy and the neutrino masses. The propagation of this energy eigenstate, the relative phases of

its three mass components and its flavor mixture at the detector are completely determined by the energy-momentum kinematics for the three mass eigenstates.

The flavor mixture at the detector given by substituting the detector coordinate into Eq. (11.1) can be shown to be the same for all the energy eigenstates except for completely negligible small differences. For example, for the case of two neutrinos with energy E and mass eigenstates m_1 and m_2 the relative phase of the two neutrino waves at a distance x is:

$$\phi_m(x) = (p_1 - p_2) \cdot x = \frac{(p_1^2 - p_2^2)}{(p_1 + p_2)} \cdot x = \frac{\Delta m^2}{2p} \cdot x \approx - \left(\frac{\partial p}{\partial(m^2)} \right)_E \Delta m^2 \cdot x \quad (11.2)$$

where $\Delta m^2 \equiv m_2^2 - m_1^2$, we have assumed the free space relation between the masses, m_i energy E and momenta: $p_i^2 = E^2 - m_i^2$, noted that $|m_2 - m_1| \ll p \equiv (1/2)(p_1 + p_2)$ and kept terms only of first order in $m_2 - m_1$. This result is seen to agree with eq. (6.5) obtained by the use of handwaving A.

Thus we have a complete solution to the oscillation problem and can give the neutrino flavor as a function of the distance to the detector by examining the behavior of a single energy eigenstate. Flavor-energy factorization enables the result to be obtained without considering interference effects between different energy eigenstates. All such interference is time dependent and required to vanish at the source, where the flavor is time independent. This time independence also holds at the detector as long as there is significant overlap between the wave packets for different mass states. The only information needed to predict the neutrino oscillation wave length is the behavior of a linear combination of the three mass eigenstates having the same energy and different momenta. Same energy and different momenta are relevant rather than vice versa because the measurement is in space, not time, and flavor-time factorization holds in a definite region in space.

We now note that this solution (11.2) enables a simple rigorous justification of handwaving A to first order in the mass difference $m_2 - m_1$. The standard relativistic energy-momentum relation gives the following relation between the change in energy or momentum with mass when the other is fixed,

$$\left(\frac{2E\partial E}{\partial(m^2)}\right)_p = -\left(\frac{2p\partial p}{\partial(m^2)}\right)_E = 1. \quad (11.3)$$

Thus if

$$x = \frac{p}{E} \cdot t \quad (11.4)$$

$$\left(\frac{\partial p}{\partial(m^2)}\right)_E \cdot x = \left(\frac{E}{p} \cdot \frac{\partial E}{\partial(m^2)}\right)_p \cdot x = \left(\frac{\partial E}{\partial(m^2)}\right)_p \cdot t \quad (11.5)$$

C. Generalization to cases with external fields

The above treatment is now easily generalized to include cases where flavor-independent external fields can modify the relation (11.1), but where the mass eigenstates are not mixed by these fields, e.g. a gravitational field. The relation between energy, momentum and mass is described by an arbitrary dispersion relation

$$f(E, p, m^2) = 0 \quad (11.6)$$

where the function f can also be a slowly varying function of the distance x . In that case, the momentum p for fixed E is also a slowly varying function of x and the x -dependence of the phase shift $\phi(x)$ is now expressed by generalizing Eq. (11.2) to a differential equation

$$\frac{\partial^2 \phi(x)}{\partial x \partial(m^2)} = -\left(\frac{\tilde{\partial}^2}{\partial(m^2)}\right)_E = \frac{1}{v} \cdot \left(\frac{\partial E}{\partial(m^2)}\right)_p = \frac{1}{v} \cdot \frac{\partial^2 \phi(t)}{\partial t \partial(m^2)}, \quad v \equiv \left(\frac{\partial E}{\partial p}\right)_{(m^2)} \quad (11.7)$$

where we note that the result can also be expressed in terms of the change in energy with m^2 for constant momentum, $\left(\frac{\partial E}{\partial(m^2)}\right)_p$, instead of vice versa and the group velocity v , and can also be expressed in terms of the time-dependence of the phase shift measured at constant position. We thus have generalized the justification (11.5-11.4) of handwaving A to the case of a nontrivial dispersion relation by using the group velocity of the wave.

Considerable confusion has arisen in the description of flavor-oscillation experiments in quantum mechanics [4,3], with questions arising about time dependence and production reactions [5], defining precisely what exactly is observed in an experiment [6], and relations

between gedanken and real experiments [7]. Despite all these difficulties the expression (11.7) is seen to provide an unambiguous value for the oscillation wave length in space and also a rigorous recipe justifying Handwaving A for obtaining this oscillation wave length from the period of oscillation calculated for a “gedanken” experiment which measures a gedanken oscillation in time. Note that the group velocity and not the phase velocity enters into this relation.

The extension to propagation in a medium which mixes mass eigenstates e.g. by the MSW effect is straightforward in principle, but more complicated in practice and not considered here. The dispersion relation (11.6) must be generalized to be a nontrivial flavor-dependent 3×3 matrix whose matrix elements depend upon x .

The exact form of the energy wave packet described by the function $g(E)$ is irrelevant here. The components with different energies may be coherent or incoherent, and they may be “entangled” with other degrees of freedom of the system. For example, for the case where a neutrino is produced together with an electron in a weak decay the function $g(E)$ can also be a function $g(\vec{p}_e, E)$ of the electron momentum as well as the neutrino energy. The neutrino degrees of freedom observed at the detector will then be described by a density matrix after the electron degrees of freedom have been properly integrated out, taking into account any measurements on the electron. However, none of these considerations can introduce a neutrino of the wrong flavor at the position of the source.

Since the momenta p_i are energy-dependent the factorization does not hold at finite distance. At very large values of x the wave packet must separate into individual wave packets with different masses traveling with different velocities [4,10]. However, for the conditions of a realistic oscillation experiment this separation has barely begun and the overlap of the wave packets with different masses is essentially 100%. Under these conditions the flavor-energy factorization introduced at the source is still an excellent approximation at the detector. A detailed analysis of the separation process is given below.

The $\nu_e - \nu_\mu$ states with the same energy and different momenta are relevant rather than vice versa because the measurement is in space, not time, and flavor-time factorization holds

in a definite region in space.

In a realistic oscillation experiment the phase is important when the oscillation length is of the same order as the distance between the source and the detector. In that case this flavor-energy factorization holds over the entire distance between the source and detector. The boundary condition then determines the relative phase of components in the wave function with different mass having the same energy and different momenta. Thus any flavor oscillations observed as a function of the distance between the source and the detector are described by considering only the interference between a given set of states having the same energy. All questions of coherence, relative phases of components in the wave function with different energies and possible entanglements with other degrees of freedom are thus avoided.

XII. DETAILED ANALYSIS OF A PION DECAY EXPERIMENT $\pi \rightarrow \mu\nu$

We now consider an example of neutrino oscillations where the neutrinos are produced by a $\pi \rightarrow \mu\nu$ decay from a pion brought to rest in a beam dump and we consider the pion and muon wave functions in detail.

We first note that the pion is not free and is not at rest. It is still interacting with the charged particles in the beam dump which have brought it almost to rest. In the approximation where it is moving in the mean field of the other charges, its wave function can be the ground state of motion in this effective potential. In this case its energy E_π is discrete and uniquely defined, while its momentum will be just the zero-point or fermi momentum described by a wave packet in momentum space,

$$|\pi\rangle = \int g(\vec{p}_\pi) d\vec{p}_\pi |\pi(\vec{p}_\pi)\rangle \quad (12.1)$$

The decay is described by a weak interaction which commutes with the total momentum of the system. Thus we can consider the decay of each individual momentum component of eq.(12.1) separately. We assume that the width of the wave packet in momentum space is

sufficiently small so that we can neglect the relativistic variation of the pion lifetime over the wave packet.

The energy, momentum and mass of the muon, denoted by (E_μ, p_μ, m_μ) and of the three mass eigenstates of the neutrino, denoted by (E_i, p_i, m_i) where $i = 1, 2, 3$ are related by energy and momentum conservation:

$$E_i = E_\pi - E_\mu; \quad \vec{p}_i = \vec{p}_\pi - \vec{p}_\mu \quad (12.2)$$

$$E_i^2 = p_i^2 + m_i^2; \quad E_\mu^2 = p_\mu^2 + m_\mu^2 \quad (12.3)$$

These relations differ from the corresponding relations for the decay of a free pion because E_π is a constant, independent of p_π . It is determined by the binding potential and the energy change in the beam dump resulting from the removal of the pion. Since the final state of the beam dump is not measured, the results of the incoherent averaging over all final states is included by using the average energy change in the beam dump in E_π in eq. (12.2)

The final neutrino-muon wave function thus has the form:

$$|(\mu, \nu)_f\rangle = e^{-iE_\pi t} \cdot \int g(\vec{p}_\pi) d\vec{p}_\pi \int d\vec{p}_\mu \sum_{i=1}^3 \int d\vec{p}_i c_i e^{i\vec{p}_i \cdot \vec{x}_\nu} \cdot \delta(E_\pi - E_\mu - E_i) \cdot \delta(\vec{p}_\pi - \vec{p}_\mu - \vec{p}_i) \cdot |\mu(\vec{p}_\mu), \nu_i(\vec{p}_i)\rangle \quad (12.4)$$

where we have expressed the spatial dependence of the neutrino wave function explicitly but left the spatial dependence of the muon wave function in the wave function $|\mu(\vec{p}_\mu), \nu_i(\vec{p}_i)\rangle$, $|\nu_i\rangle$ denote the three neutrino mass eigenstates and the coefficients c_i are left free and determined by the condition that the neutrino must be a pure ν_μ at the point $x_\nu = 0$ where the pion decays.

The result of any experiment is obtained by taking the expectation value of an operator O_{exp} describing the measurement with the above wave function. Since the muon and neutrino have separated by the time a measurement is made, we assume that the operator factorizes into a product of two operators O_μ and O_ν acting on the muon and neutrino respectively,

$$O_{exp} = O_\mu \cdot O_\nu \quad (12.5)$$

We now assume that the muon operator O_μ commutes with the muon momentum.

$$[O_\mu, \vec{p}_\mu] = 0. \quad (12.6)$$

This expression thus holds for any measurement in which the muon is not detected as well as those where it is detected by an operator which commutes with its momentum. The experimental result is therefore given by the expression

$$\begin{aligned} R_{exp} = \langle \psi(\mu, \nu) | O_{exp} | \psi(\mu, \nu) \rangle &= \sum_{i=1}^3 \sum_{j=1}^3 \int \int \int \int d\vec{p}_\mu d\vec{p}_\pi d\vec{p}'_\pi d\vec{p}'_j g^*(\vec{p}_\pi) g(\vec{p}'_j) \cdot \\ &c_i^* \cdot c_j e^{i(\vec{p}_j - \vec{p}_i) \cdot \vec{x}_\nu} \cdot \delta(E_\pi - E_\mu - E_i) \delta(E_i - E'_j) \delta(\vec{p}_\pi - \vec{p}_\mu - \vec{p}_i) \delta(\vec{p}'_j - \vec{p}_\mu - \vec{p}'_i) \\ &\langle \mu(\vec{p}_\mu), \nu_i(\vec{p}_\pi - \vec{p}_\mu) | O_\mu \cdot O_\nu | \mu(\vec{p}_\mu), \nu_j(\vec{p}'_\pi - \vec{p}'_\mu) \rangle \end{aligned} \quad (12.7)$$

We thus again obtain the result that the only interference terms that need be considered are those between neutrino states having the same energy. The crucial ingredient here is the unexpected relation between energy and momentum of the stopped pion, which is not free. This is closely analogous to the physics of the Mössbauer effect, where the relation between energy and momentum for a nucleus bound in a lattice is crucially different from that for a free nucleus. This resemblance between the treatment of recoil momentum transfer in flavor oscillation phenomena and in the Mössbauer effect has been pointed out [11] in the example of experiments measuring the $K_L - K_S$ mass difference by observing the regeneration of a K_L beam as a function of the distance between two regenerators. The coherence required depends upon the impossibility of detecting the individual recoils of the two regenerators resulting from the momentum transfer due to the mass difference.

XIII. A SIMPLE PEDAGOGICAL NEUTRINO OSCILLATION PUZZLE

A. Statement of the Puzzle

A pion at rest decays into a muon and neutrino. The neutrino oscillates between electron neutrino and muon neutrino. We know everything and can calculate the result of any

neutrino oscillation experiment when the source is a pion at rest. All factors of two are understood and the results agree with experiment.

How do we apply these results to a pion moving with relativistic velocity? A naive picture of the conventional time dilatations and Lorentz contractions occurring in moving systems suggests that the oscillation period goes up, because of time dilatation, but the oscillation wave length goes down because of the Lorentz contraction. Which wins? Is the oscillation in time slowed down by the time dilatation? Is the oscillation in space speeded up by the Lorentz contraction? What happens in a real experiment with Fermilab neutrinos? In a long baseline experiment?

Of course the real result is given above in eq. (11.2) and there is no ambiguity. But what is wrong with the naive picture of time dilatations and Lorentz contractions? Note that this statement of the problem separates relativity from quantum mechanics by assuming that the quantum mechanics is already solved in the pion rest frame, and that only a Lorentz transformation to a moving frame is needed.

B. Pedestrian Solution to Puzzle

Consider a 45° mixing angle with a pion at rest and a detector at just the right distance so that it detects only electron neutrinos and no muon neutrinos. For a qualitative picture of the physics, consider the Lorentz transformation to a frame moving with velocity v , and assume that the pion decay and the neutrino detection occur at the points $(x, t) = (0, 0)$ and (X, T) , where we can as a first approximation let $X = T$, with $c = 1$ and assume that the velocity v of the frame is not too large. For a one-dimensional case we immediately obtain

$$(X, T) \rightarrow (X', T') = \frac{X - vT, T - vX}{\sqrt{1 - v^2}} = (X, T) \cdot \sqrt{\frac{1 - v}{1 + v}} \quad (13.1)$$

We now note that the neutrino momentum and energy (p, E) undergo the transformation in the same approximation

$$(p, E) \rightarrow (p', E') = \frac{p - vE, E - vp}{\sqrt{1 - v^2}} = (p, E) \cdot \sqrt{\frac{1 - v}{1 + v}} \quad (13.2)$$

Thus

$$\frac{X'}{p'} = \frac{X}{p} \quad (13.3)$$

So the observed oscillation wave length and period both decrease if the neutrino is emitted backward and increase if the neutrino is emitted forward. The backward emission is not relevant to realistic experiments. The naive pictures are not relevant because the Lorentz contraction always refers to two events occurring AT THE SAME TIME in each frame, and not to the distance between THE SAME TWO EVENTS observed in different frames.

That both the wave length and period must vary in the same fashion is very clear in this approximation where the motion is on the light cone which gives $X=T$ in all frames.

Thus the ratio X/p is invariant and the expression (11.2) for the relative phase of the two neutrino waves holds also in a moving frame. Thus the result of the standard treatment is seen to hold also for neutrinos emitted in the decay of a moving pion.

We now correct for the deviation of the velocity of the neutrinos from c by writing

$$X = (p/E)T \quad (13.4)$$

Thus

$$X \rightarrow X' = \frac{X - vT}{\sqrt{1 - v^2}} = \frac{X[1 - v(E/p)]}{\sqrt{1 - v^2}} \quad (13.5)$$

and

$$p \rightarrow p' = \frac{p - vE}{\sqrt{1 - v^2}} = \frac{p[1 - v(E/p)]}{\sqrt{1 - v^2}} \quad (13.6)$$

Thus the expression (13.3) holds for the general case and the result of the standard treatment remains also when corrections for the deviations of the neutrino velocity from c are taken into account.

XIV. SPACE AND TIME IN FLAVOR OSCILLATIONS

A. Description in terms of time behavior

1. Fuzziness in Time

In a neutrino oscillation experiment there must be uncertainties in order to have coherence and oscillations. If we know that a neutrino has left a source at time $t(s)$ and has arrived at the detector at a time $t(d)$, then we know that its velocity is

$$v = \frac{x}{t(d) - t(s)} \quad (14.1)$$

where x is the distance between source and detector. We therefore know its mass and there are no oscillations.

In order to observe oscillations we cannot know exactly all the variables appearing in eq. (14.1). If oscillations are observed, there must be uncertainty somewhere. It is easy to show that the major uncertainty must be in the time $t(s)$ in which the neutrino is emitted from the source.

A detailed description of the time behavior and the need for fuzziness in time is given in ref. [8]. We summarize here the result showing quantitatively the analog with the optical case.

If the mass eigenstate wave packets leave the source with their centers together at $x = 0$ the displacement between their centers at the point x_d of the detector is

$$\delta x_c = \frac{\delta v}{v} \cdot x_d \approx \frac{\delta p}{p} \cdot x_d = \frac{\Delta m^2}{2p^2} \cdot x_d, \quad \delta v \equiv v_1 - v_2, \quad \delta p \equiv p_1 - p_2, \quad (14.2)$$

where δv and δp denote the velocity and momentum differences between the two mass eigenstates. The neutrino masses are much smaller than their energies,

$$m_i^2 = E_i^2 - p_i^2 \ll p_i^2 \quad (14.3)$$

The neutrino can be detected at the detector when any point in the wave packet passes x_d .

2. Detailed description of time behavior and time overlaps

Let $|m_1\rangle$ and $|m_2\rangle$ denote the two mass eigenstates and θ denote the mixing angle defining the flavor eigenstates denoted by $|f_1\rangle$ and $|f_2\rangle$ in terms of the mass eigenstates,

$$|f_1\rangle = \cos \theta |m_1\rangle + \sin \theta |m_2\rangle , \quad |f_2\rangle = \sin \theta |m_1\rangle - \cos \theta |m_2\rangle , \quad (14.4)$$

The wave function at the position of the detector at a time t can be written as a linear combination of the two mass eigenstates. We assume that the amplitudes denoted by $A(t)$ of the two wave packets are the same, but that they are separated in time at the detector by the time interval

$$\tau_d = \frac{x_d}{v_2} - \frac{x_d}{v_1} \approx \frac{\delta v}{v^2} \cdot x_d \approx \frac{\Delta m^2}{2p^2 v} \cdot x_d , \quad (14.5)$$

The wave function at the detector can therefore be written

$$|\Psi_d(t)\rangle = e^{i\phi_o(t)} \left[\cos \theta A(t) |m_1\rangle + \sin \theta A(t + \tau_d) e^{i\phi(\tau)} |m_2\rangle \right] , \quad (14.6)$$

where $\phi_o(t)$ is an overall phase factor and

$$\phi(\tau) = p \delta x_c = p v \tau_d \approx \frac{\Delta m^2}{2p} x_d \quad (14.7)$$

is the relative phase between the two mass eigenstates at the detector. The probability amplitudes and the ~~relative~~ probabilities that flavors f_1 and f_2 are observed at the detector are then

$$\langle f_1 | \Psi(t) \rangle = e^{i\phi_o(t)} \left[\cos^2 \theta A(t) e^{i\phi(\tau)} + \sin^2 \theta A(t + \tau_d) \right] , \quad (14.8)$$

$$\langle f_2 | \Psi(t) \rangle = e^{i\phi_o(t)} \sin \theta \cos \theta \left[A(t) e^{i\phi(\tau)} - A(t + \tau_d) \right] . \quad (14.9)$$

$$P(f_1, \tau_d) = \int dt |\langle f_1 | \Psi(t) \rangle|^2 = 1 - \frac{\sin^2(2\theta)}{2} \left[1 - O(\tau_d) \cos \phi(\tau) \right] , \quad (14.10)$$

$$P(f_2, \tau_d) = \int dt |\langle f_2 | \Psi(t) \rangle|^2 = \frac{\sin^2(2\theta)}{2} \left[1 - O(\tau_d) \cos \phi(\tau) \right] , \quad (14.11)$$

where we have normalized the amplitudes and $O(\tau_d)$ is the time overlap between the mass eigenstates,

$$\int dt |A(t)|^2 = 1, \quad O(\tau_d) \equiv \int dt A(t + \tau_d) A(t). \quad (14.12)$$

We thus see how the standard result for neutrino oscillations arises for the case where the overlap integral $O(\tau_d) \approx 1$ and how the incoherent mixture of the two mass eigenstates is approached as $O(\tau_d) \Rightarrow 0$.

B. When do Mass Eigenstate Wave Packets Separate?

Suppose a wave packet is created which is a coherent linear combination of two mass eigenstates, and the overlap of the two mass components is nearly 100%. In time both wave packets will spread, and the centers will separate. Will the separation between the centers of the packets be greater than the spreading? Will there be an eventual spatial separation between the two mass eigenstates? It is easy to see that in the extreme relativistic limit the wave packets will separate; in the nonrelativistic limit they will not. We simply need to calculate the velocities of the different components of the packet.

Let $(\delta p)_W$ denote the momentum spread within each wave packet and $(\delta p)_m$ denote the momentum difference between the components of the two mass-eigenstate wave packets with the same energy. ~~~~~

The spread in velocity within a wave packet $(\delta v)_W$ is just the difference in velocities $v = p/E$ for states with different momenta and the same mass,

$$(\delta v)_W = \frac{\partial}{\partial p} \cdot \left(\frac{p}{E} \right)_m \cdot (\delta p)_W = \frac{(\delta p)_W}{E} \cdot \frac{m^2}{E^2} \quad (14.13)$$

The difference in velocity between components in two wave packets $(\delta v)_m$ with the same energy and different mass is just the difference in velocities $v = p/E$ for states with different momenta and the same energy,

$$(\delta v)_m = \frac{\partial}{\partial p} \cdot \left(\frac{p}{E} \right)_E \cdot (\delta p)_m = \frac{(\delta p)_m}{E} \quad (14.14)$$

The ratio of the spreading velocity to the separation velocity is then given by

$$\frac{(\delta v)_W}{(\delta v)_m} = \frac{(\delta p)_W}{(\delta p)_m} \cdot \frac{m^2}{E^2} \quad (14.15)$$

In the nonrelativistic limit where $E \approx m$ the ratio of the spreading velocity to the separation velocity is just equal to the ratio of the momentum spread in the wave function $(\delta p)_W$ to the momentum difference between the two mass eigenstate wave packets. This will be much greater than unity if there is to be appreciable overlap between the two wave packets in momentum space.

$$\frac{(\delta p)_W}{(\delta p)_m} \gg 1 \quad (14.16)$$

Otherwise there will be no coherence and no spatial oscillations observed. Thus in the nonrelativistic limit two wave packets which have an appreciable overlap in momentum space will never separate.

In the relativistic case, the ratio of the spreading velocity to the separation velocity is reduced by the factor $\frac{m^2}{E^2}$. This is effectively zero in the extreme relativistic limit $E \gg m$ relevant for neutrino oscillations. Here the spreading velocity of the wave packet is negligible and the wave packets will eventually separate.

$$\frac{m^2}{E^2} \approx 0; \quad \frac{(\delta v)_W}{(\delta v)_m} \ll 1 \quad (14.17)$$

C. At what distance is coherence lost?

1. The condition on the momentum spread in the wave packet

Neutrino oscillations are always described in the relativistic limit and the wave packets corresponding to different mass eigenstates will eventually separate. Once they have separated they will arrive at a detector at different separated time intervals. The detector will see two separated probability amplitudes, each giving the probability that the detector will observe a given mass eigenstate and all coherence between the different mass eigenstates will

be lost. The question then arises when and where this occurs; i.e. at what distance from the source the coherence begin to be lost. We now examine two different approaches to this problem and find that they give the same answer.

1. The centers of the wave packets move apart with the relative velocity $(\delta v)_m$ given by eq. (14.14). Thus the separation $(\delta x)_m$ between the wave packet centers after a time t when the centers are at a mean distance x from the source is

$$(\delta x)_m = (\delta v)_m \cdot t = (\delta v)_m \cdot \frac{x}{v} = -\frac{\Delta m^2}{2pE} \cdot \frac{x}{p} = -\frac{\Delta m^2}{2p^2} \cdot x \quad (14.18)$$

The wave packets will separate when this separation distance is comparable to the length in space of the wave packet. The uncertainty principle suggests that the length of the wave packet $(\delta x)_W$ satisfies the relation

$$(\delta x)_W \cdot (\delta p)_W \approx 1/2 \quad (14.19)$$

The ratio of the separation over the length is of order unity when

$$\left| \frac{(\delta x)_m}{(\delta x)_W} \right| \approx \left| \frac{\Delta m^2}{p^2} \right| \cdot (\delta p)_W \cdot x \approx 1 \quad (14.20)$$

2. Stodolsky [9] has suggested that one need not refer to the time development of the wave packet, but only to the neutrino energy spectrum. With this approach we note that the relative phase $\phi_{\nu_2}(x)$ between the two mass eigenstate waves at a distance x from the source depends upon the neutrino momentum p_ν as defined by the relation (11.2).

Coherence will be lost in the neighborhood of the distance x where the variation of the phase over the momentum range $(\delta p)_W$ within the wave packet is of order unity. For the case of two neutrinos with energy E and mass eigenstates m_1 and m_2 the condition that the relative phase variation $|\delta \phi_m(x)|$ between the two neutrino waves is of order unity

$$|\delta \phi_m(x)| = \left| \frac{\partial \phi_m(x)}{\partial p_\nu} \right| \delta p_\nu \cdot x = \left| \frac{\Delta m^2}{2p_\nu^2} \right| (\delta p)_W \cdot x \approx 1 \quad (14.21)$$

We find that the two approaches give the same condition for loss of coherence.

2. Evaluation of the momentum spread in the wave packet

The value of the momentum spread $(\delta p)_W$ in the wave packet depends upon the production mechanism. However, we can immediately see that this can be simply estimated for all experiments in which the initial state is either a beam impinging on a solid target or a radioactive decay of a source in a solid. The momentum of the initial target or radioactive nucleus has momentum fluctuations resulting from its confinement in a lattice with a spacing of the order of angstroms. These momentum fluctuations then appear in the neutrino momentum spectrum as a result of conservation of four-momentum in the neutrino production process. One immediately sees that the momentum fluctuations are much larger than the momentum difference between the different mass eigenstates having the same energy, and that therefore the neutrino state produced at the source has full coherence between the different mass eigenstates.

The momentum spread $(\delta p)_W$ is easily calculated in any experiment where the spread is a result of the momentum spread δp_{nuc} of a nucleus in the initial state. This is just the neutrino energy change produced by the Lorentz transformation which changes the momentum of the active nucleus from zero to the finite value δp_{nuc} . The four-momentum (p, E) of the nucleus is changed by this transformation from $(0, M_{nuc})$ to $(\delta p_{nuc}, E_{nuc})$, where M_{nuc} and E_{nuc} denote the mass of the nucleus and the energy of the nucleus with momentum δp_{nuc} . The small velocity v of this Lorentz transformation is given to first order in v by

$$v \approx \frac{\delta p_{nuc}}{M_{nuc}} \quad (14.22)$$

The neutrino four-momentum is changed from (p_ν, p_ν) to $[p_\nu + (\delta p)_W, p_\nu + (\delta p)_W]$. Thus

$$(\delta p)_W = \frac{(1+v)}{\sqrt{1-v^2}} \cdot p_\nu - p_\nu \approx \frac{\delta p_{nuc}}{M_{nuc}} \cdot p_\nu \quad (14.23)$$

to first order in v . Substituting this result into the coherence condition (14.21) gives

$$|\delta \phi_m(x)| = \left| \frac{\Delta m^2}{2p_\nu} \right| \cdot \frac{\delta p_{nuc}}{M_{nuc}} \cdot x \approx 1 \quad (14.24)$$

This can be rewritten

$$x \approx \left| \frac{4p_\nu \cdot M_{nuc}}{\Delta m^2} \right| \cdot \delta x_{nuc} \quad (14.25)$$

where $\delta x_{nuc} \approx 1/(2\delta p_{nuc})$ denotes the quantum fluctuations of the position of the nucleus. This uncertainty principle relation is an exact equality for the harmonic potential generally used to describe binding in crystal lattices. Because of the very different scales of the variables appearing in eq. (14.25) we rewrite this relation expressing x in kilometers, δx_{nuc} in Angstroms, M_{nuc} in GeV, p_ν in MeV and m in electron volts. In these units eq. (14.25) becomes

$$x(km) \approx 400 \cdot \left| \frac{p_\nu(MeV) \cdot M_{nuc}(GeV)}{\Delta m(ev)^2} \right| \cdot \delta x_{nuc}(Angstroms) \quad (14.26)$$

This is seen to be a very large distance even for the case where the neutrino originates from a solid where nuclei are confined to distances of the order of Angstroms. For atmospheric and solar neutrinos, where the source is free to move in distances many orders of magnitudes larger, the decoherence distance will be even larger. This calculation confirms the result quoted in Kim and Pevner's book, chapter 9, that the coherence is lost only at astronomical distances much larger than the size of the solar system and that this coherence loss is relevant only for supernova neutrinos. Note that the present derivation avoids making assumptions like those used by Kim and Pevsner in which the neutrino is produced at time $t=0$, and which can be questioned as shown below because of the uncertainty necessary for coherence.



XV. SPACE, TIME, RELATIVITY AND QUANTUM MECHANICS

We now present a simple picture to guide intuition through all the arguments about relativity, proper time, and the equivalence of space and time. In all experiments the neutrino leaves the source as a wave packet which has a finite length in space and time. If a detector is set up to detect the neutrino at a given point in space, the wave packet passes the detector during a finite time interval. The probability of observing the neutrino at this point in space will therefore have a statistical distribution in time given by the square of the amplitude of the wave packet.

In principle, it is possible to measure time, rather than distance. This can give a photographic record of the square of the wave packet in space at a given instant of time. In principle it is possible to measure both the position in space and the exact time for each detected neutrino event. The results can be presented as a scatter plot with space position and time plotted for each event. The events for a given space position will show a time distribution over a finite interval. The events for a given time will show a space distribution over a finite interval. There is complete symmetry between space and time, and there is a statistical distribution also of proper times.

How does one get physics out of these distributions? In practice it is only the space position of the detected event that is measured, and it is known that the probability of finding a neutrino with the wrong flavor at the source must vanish. This determines the relative phase of the neutrino eigenstates as they propagate through space. This is all the information needed to give a unique interpretation for the results of any experiment.

There have been some suggestions that radioactive sources with long lifetimes can introduce additional effects due to the long lifetime. Such effects have been known and observed experimentally in electromagnetic transitions. However the neutrino is a fermion, not a boson, and its emission must be accompanied by the emission or absorption of another fermion. This change in the environment is observable and “collapses the wave function”. If we are considering a long-lived beta decay of a nucleus bound in an atom, the nuclear lifetime is irrelevant for neutrino coherence because the nucleus is interacting with the atom, and the atom knows when the charge of the nucleus has changed and an electron or positron has been emitted together with the neutrino.

The point has been repeatedly made by Stodolsky [9] that the proper formalism to treat neutrino oscillations is the density matrix, because only in this way the unavoidable interactions with the environment can be taken into account. This paper also points out that the length in time of the wave packet is irrelevant.

XVI. CONCLUSIONS

Flavor oscillations have been shown to be simply described in a wave picture, very analogous to optical polarization rotations. The flavor eigenstates are analogous to spin polarization eigenstates, and the neutrino oscillations are describable as rotations in some abstract flavor-spin space.

The simplest description begins with the detector, which is located at a definite position in space and which responds in a well-defined manner to the arrival of some mixture of neutrino mass eigenstate waves. These individual waves have traveled with different velocities from the source to the detector, but have been shown to separate very slowly under practical conditions. Thus there is almost a complete overlap at the detector except for neutrinos arriving from distances much larger than the distance between the earth and the sun; e.g. for neutrinos arriving from supernova.

The crucial parameters which determine the response of the detector are the relative phases of the mass eigenstate waves at the detector. These are determined by the initial conditions at the source and by the propagation between source and detector. The propagation is straightforward for free space and is well-defined for passage through known external fields or media with well-defined properties; e.g. MSW effects. The initial conditions at the source may be more complicated, depending upon the particular reactions in which neutrinos are produced.

The fact that all experiments in which oscillations can be measured involve sources which are very small in comparison with the oscillation wave length enables results to be easily obtained by using a universal boundary condition: the probability of finding a particle with the wrong flavor at the source must vanish. These results confirm the standard procedure of calculating oscillations in time and converting a frequency in time to a wave length in space by using the mean group velocity of the wave. That it must be the group velocity has been shown rigorously for cases where the neutrino is not free but may be subject to external fields like a gravitational field.

The role of the quantum-mechanical uncertainty principle has been shown to be crucial. Considerable care must be taken in using a particle picture with well-defined times and momenta, rather than a wave picture with times and momenta described by probability amplitudes. Most published conclusions regarding oscillations of recoil particles have been shown [12–14] to be incorrect; No such muon or Λ oscillations should be observed.

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