



# **Coupled Ionic and Electronic Heat Transport at the Nanoscale**

**N.A. Modine, R.E. Jones, D.L. Olmsted,  
J.A. Templeton, and G.J. Wagner**  
**Sandia National Laboratories**

**R.M. Hatcher**  
**Lockheed Martin Advanced Technology  
Laboratories**

**M.J. Beck**  
**University of Kentucky**

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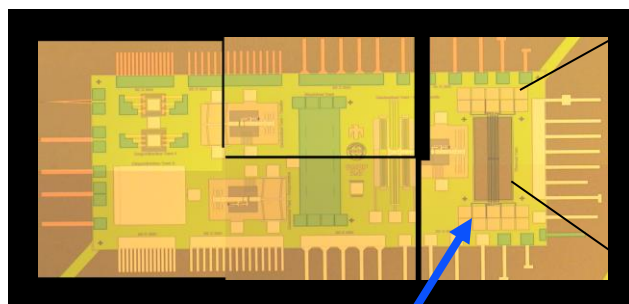
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# Novel Experimental Work at CINT – Thermal Transport at the Nanoscale

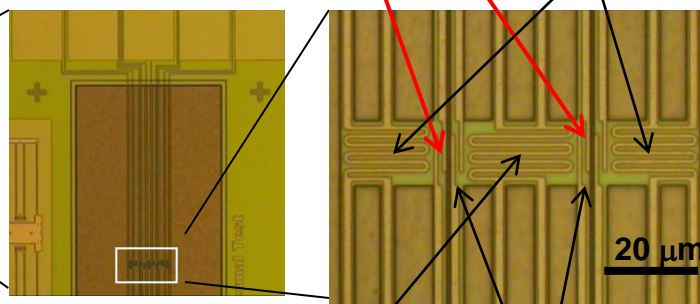
**Experimental Goal: Methods to sensitively measure thermal, electrical & thermoelectric properties simultaneously for single nanostructures**

**Cantilever Array  
Discovery Platform 2.0**



J. Sullivan

**NW transport structure**



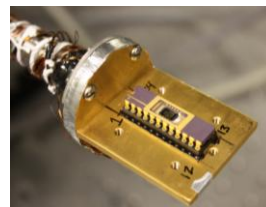
Differential thermal transport

Thermometers

Heater

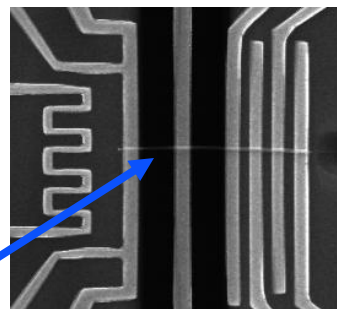
4-probe transport

**In situ TEM to probe structure**



J. Huang

**Test structure**



Si NW

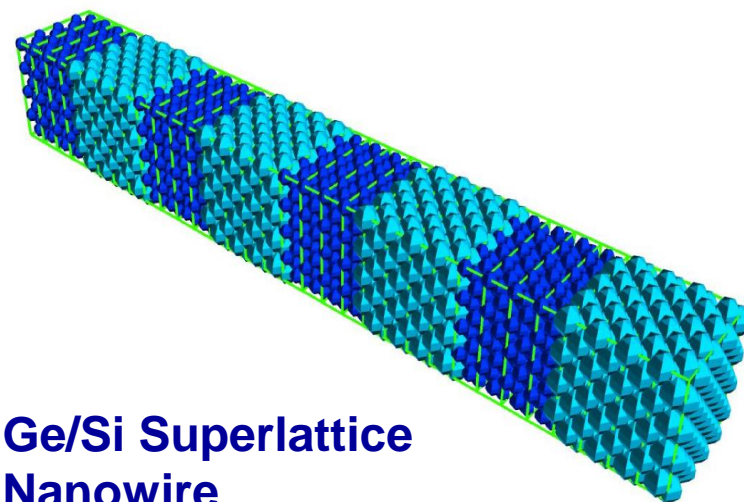
T. Harris, J. Huang



# Why Are Electrons Important to Thermal Behavior?

Material	Thermal Conductivity [w/K-m]			Heat Capacity [J/K-kg]			Debye T [K]	e-p coupling [W/K-m <sup>3</sup> ]
Cu	401	11.2	244	385	394	10.8	345	2.6 X 10 <sup>17</sup>
Si	148	120	0.02	705	888	0.0002	645	1.0 X 10 <sup>11</sup>

- Both **phonons** and **electrons** contribute to thermal properties
- At macroscale, electron-phonon equilibrium gives aggregate thermal properties
- This is not necessarily true at the nanoscale!



Ge/Si Superlattice  
Nanowire



# Two Temperature Model (TTM)

- Second moment of Boltzmann equation
- Two temperatures: phonon  $\theta_p$  and electron  $\theta_e$
- Two coupled, diffusive systems

$$c_p \frac{\partial \theta_p}{\partial t} = \nabla \cdot (k_p \nabla \theta_p) - g(\theta_p - \theta_e) + r_p(\vec{x}, t)$$

$$c_e \frac{\partial \theta_e}{\partial t} = \nabla \cdot (k_e \nabla \theta_e) - g(\theta_e - \theta_p) + r_e(\vec{x}, t)$$

- Where  $c$  is heat capacity,  $k$  is conductivity,  $r$  is a heat source, and  $g$  is electron-phonon exchange



# Phenomenology

- The spatially uniform solution gives a time scale

$$\tau_{ep} = \frac{c_e c_p}{g(c_e + c_p)} \qquad \theta_e - \theta_p = (\theta_e - \theta_p)_{t=0} \exp\left(-\frac{t}{\tau_{ep}}\right)$$

- The steady-state solution gives a length scale

$$\lambda_{ep} = \sqrt{\frac{k_e k_p}{g(k_e + k_p)}} \qquad \theta_e - \theta_p = (\theta_e - \theta_p)_{x=0} \exp\left(-\frac{x}{\lambda_{ep}}\right)$$

Material	$\tau_{ep}$	$\lambda_{ep}$
Cu	0.36 ps	6.41 nm
Si	5.8 ps	494 nm

**Below these time and length scales, it is not safe to assume that phonons and electrons are in equilibrium!**



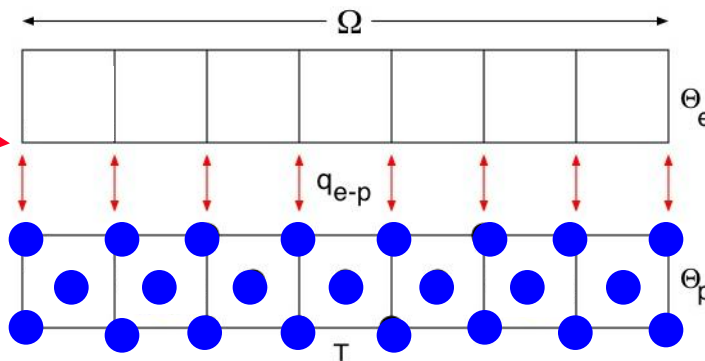
# Our Modeling Approach: MD for Phonons, PDE for Electrons

## PDE Model of Electrons

(Electrical and Thermal Conduction, Thermoelectric Coupling, Joule Heating)

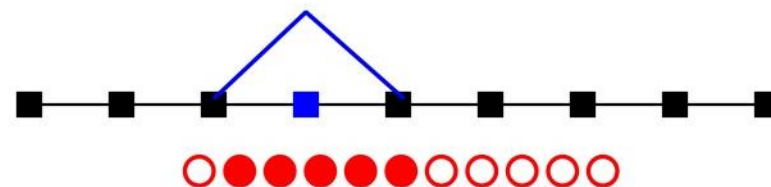
## MD Model of Phonons

(Phonon Confinement, Ballistic Transport, Scattering Mechanisms)



- Use finite elements to solve PDE for electronic temperature  $\theta_e$
- Obtain local ionic temperature  $\theta_p$  from MD velocities

$$\theta_p \equiv \sum_{I,\alpha} \frac{m_\alpha}{3k_B} \tilde{N}_{I\alpha} \langle v_\alpha \cdot v_\alpha \rangle$$

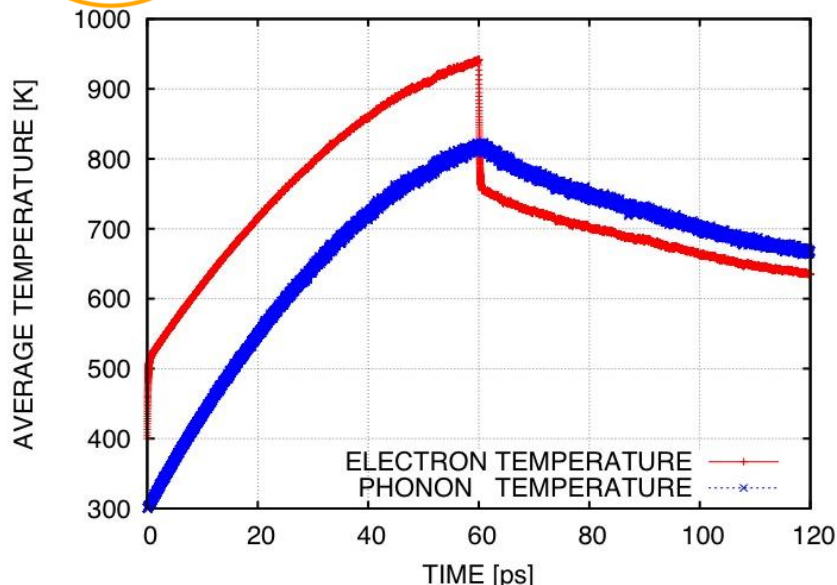


- Ionic thermostats enforce Two Temperature Model coupling

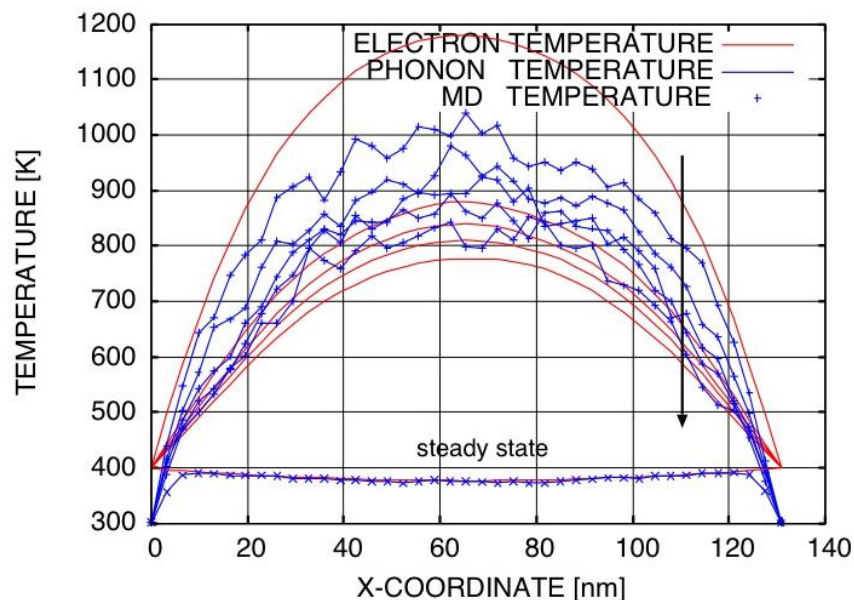
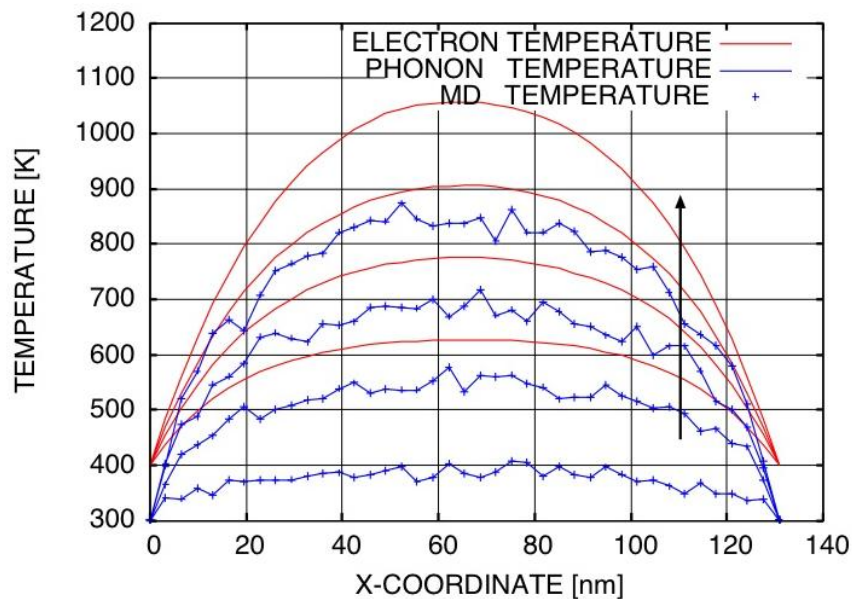




# Example 1: Cu Nanowire with Electrons Uniformly Heated by 60 ps Pulse



- Small electron heat capacity
  - Rapid electron response
- Large phonon heat capacity - Slower phonon response
- Different profiles for electron and phonon temperatures

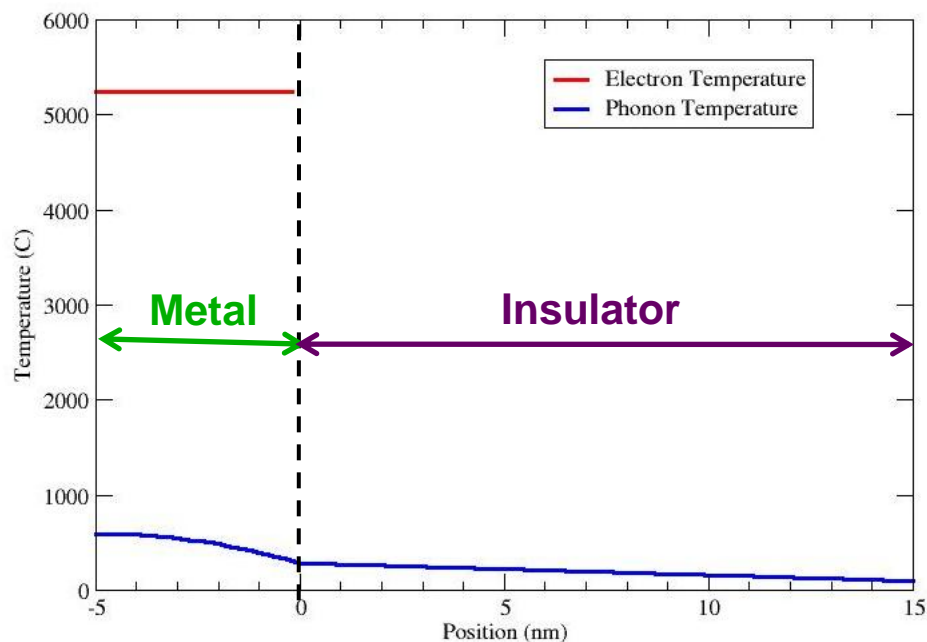




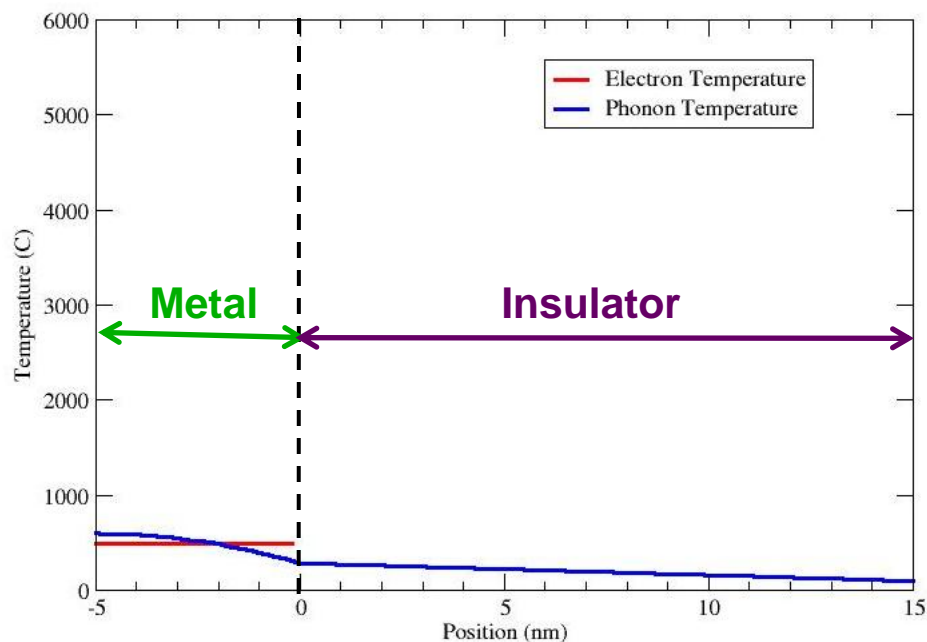
# Large Electron-Phonon Temperature Differences in Steady-State Systems

## Example 2: A Thin Metal Film on an Insulator

Uniform Heating in  
Electrons of Metal



Uniform Heating in  
Phonons of Metal







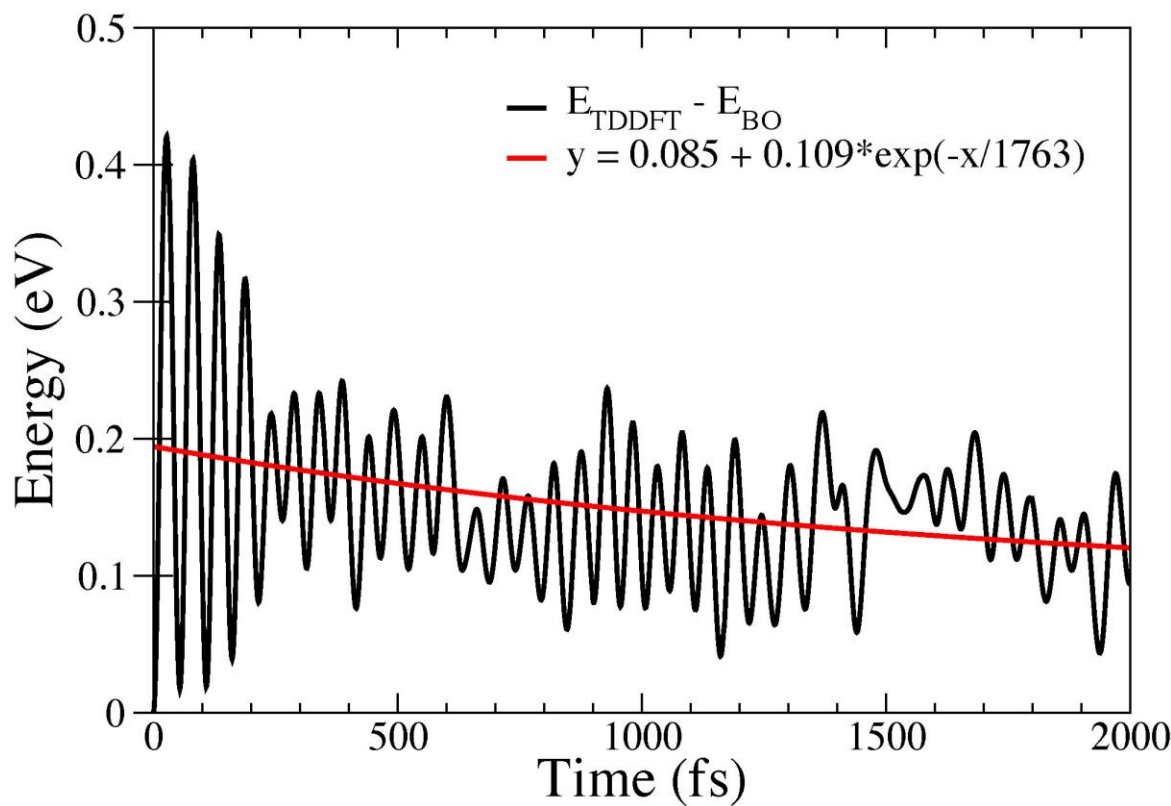
# Finding Parameters from First Principles - Time Dependent Density Functional Theory

Unlike DFT, TDDFT allows **electrons** and **ions** to exchange energy!

- Direct simulation of energy transfer between electrons and ions in Al gives time constant  $\tau_{ep} = 1.8$  ps

$$\tau_{ep} = \frac{c_e c_p}{g(c_e + c_p)}$$

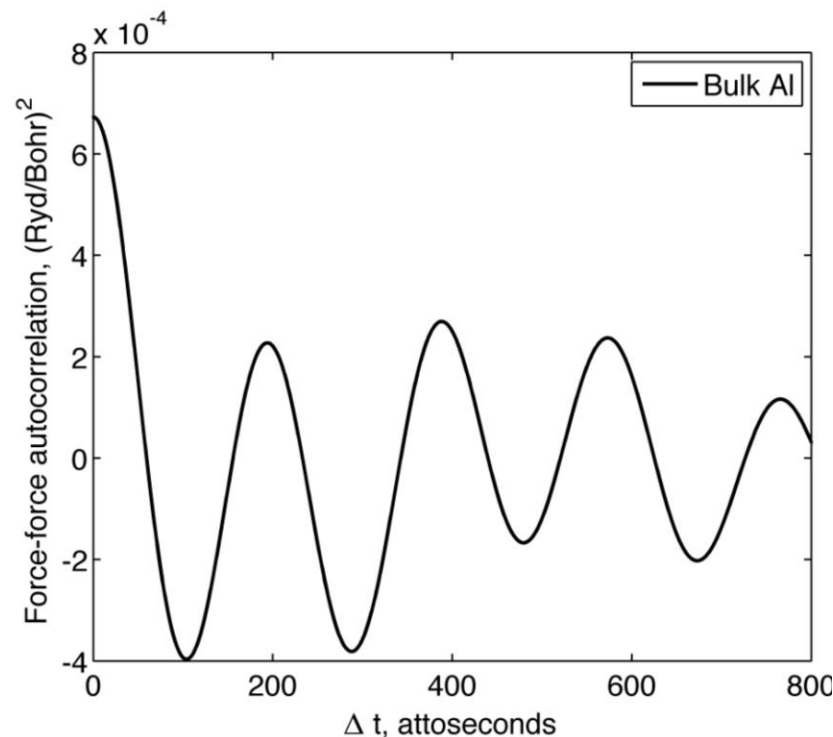
- Good agreement with 1.5-2.0 ps equilibration time from experiment (Kandyla, Shih, and Mazur, 2007)





# Working on TDDFT-Based Green-Kubo Approach to Improve Efficiency

- **Explicit energy flow is computationally expensive**
- **Green-Kubo approach gives same quantities from steady state fluctuations**
- **Electron-Phonon coupling obtained from fluctuations in the ionic force**
- **Can calculate a wide variety of quantities with same approach**



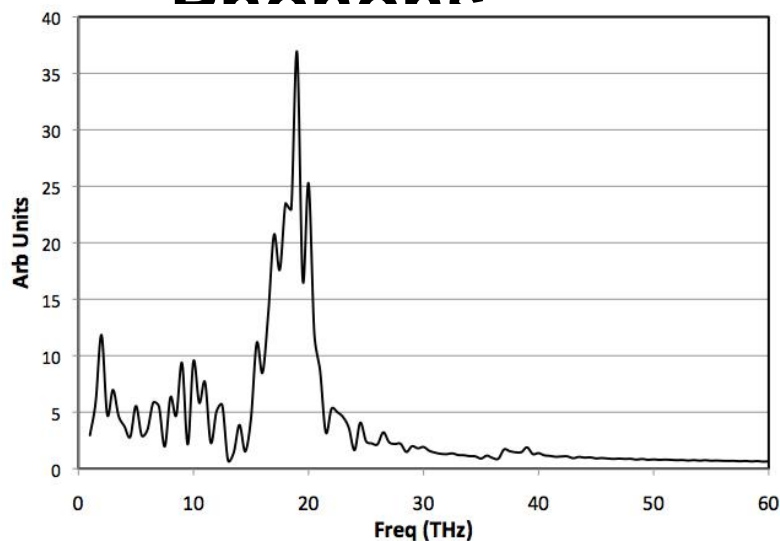
**Problem: Autocorrelation functions oscillate strongly!**



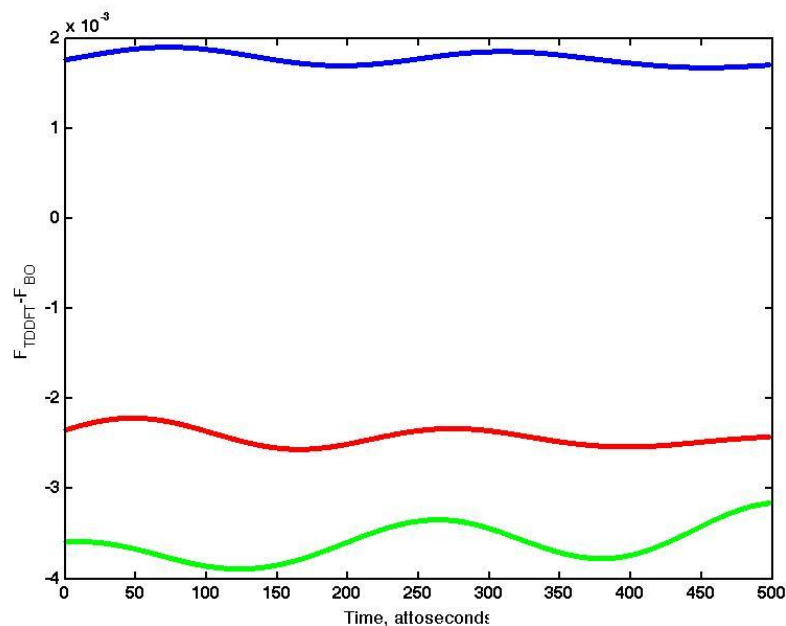
# What are the fluctuations in TDDFT?

**Low  
energies:**

**Phonons**



**High energies:  
Plasmons**



**Working to separate electron-hole fluctuations**



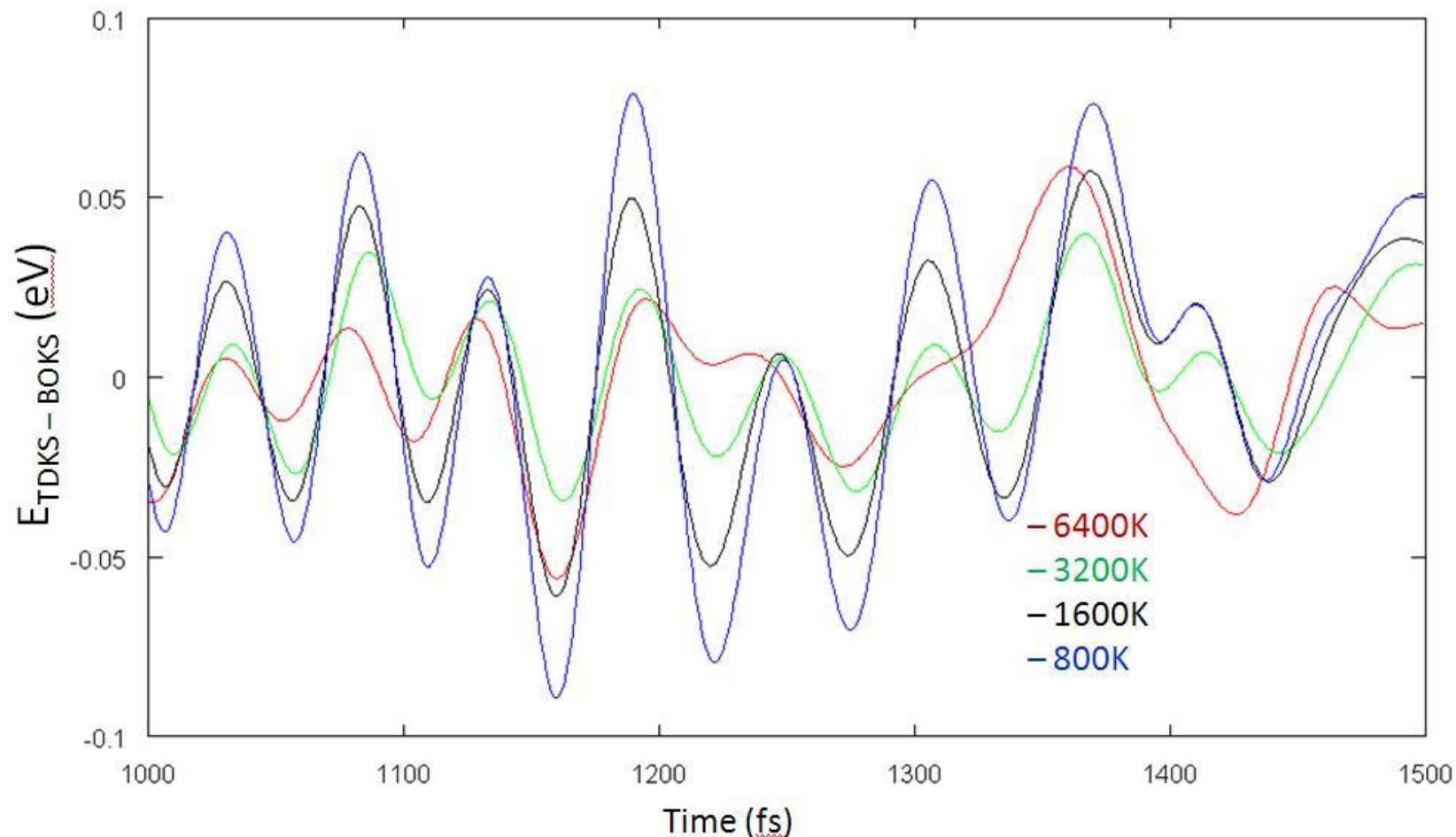
# Conclusions

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- **It is not safe to assume that the electronic and ionic temperatures are the same at the nanoscale**
- **Enhanced a MD code with a PDE-based representation of electronic heat conduction**
- **Developing a promising approach to calculating electronic properties using TDDFT**
- **Our approach is intrinsically multiphysics and multiscale**



# Why does $E_{\text{TDDFT}} - E_{\text{BO}}$ fluctuate with the phonons?



**Electronic excitations change the potential surface for ionic motion**



# Finding Parameters from First Principles - Time Dependent Density Functional Theory

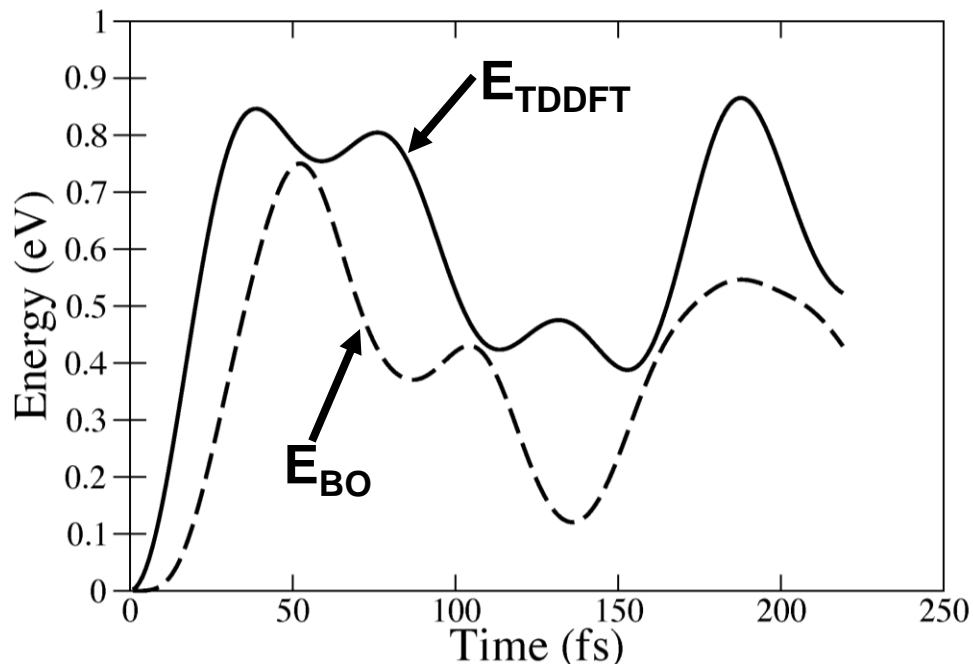
TDDFT allows **electrons** and **ions** to exchange energy!

- Initially, **ions in thermal motion** and **electrons in ground state**

- TDDFT energy  $E_{\text{TDDFT}}$  rises above Born-Oppenheimer (ground state) energy  $E_{\text{BO}}$

- $E_{\text{TDDFT}} - E_{\text{BO}}$  is instantaneous thermal energy of electrons

TDDFT Run for 32 Atoms of Al







# Current TDDFT Work

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- Understand, model, and remove phonon effects
- OR work with non-moving ions (but what about equilibration?)
- Develop better initialization to start closer to equilibrium
  - Eliminate plasmons and accentuate electron-hole pairs
- Can we get hot ions and cold electrons?



# Issues in Modeling Heat Transport at the Nanoscale

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- **Time-dependent electronic structure (e.g., TDDFT)**
  - Captures full electron and ion dynamics, BUT
  - Not feasible for most nanoscale systems
- **Partial Differential Equation (PDE) based methods**
  - Works well at macroscale, BUT
  - Misses effects of nanoscale structure on phonons (e.g., phonon confinement, ballistic transport, etc.)
- **Molecular Dynamics (MD)**
  - Explicitly represents phonons and their effects, BUT
  - Physics of electronic transport is absent