



# **A Cascading Failure Simulator to Estimate Blackout Risk for Electric Power Infrastructure**

**July 21, 2011**

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# Project Goal

To develop a high performance cascading failure simulator which will provide grid operators with a real time estimate of blackout risk





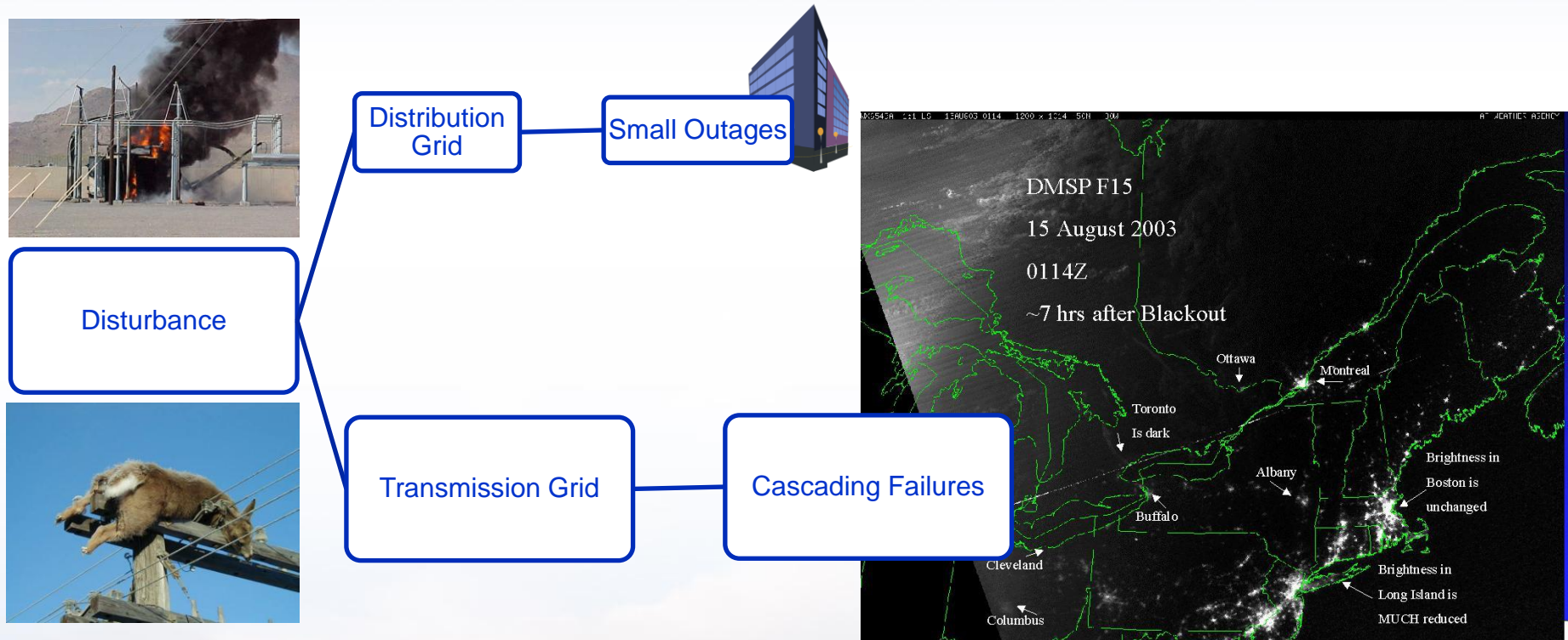
# Outline

- I. Cascading Failures in the Electric Power Grid
- II. Modeling a Power System with DAEs
- III. Risk Estimation
- IV. Solving a System of DAEs
- V. Future Work





# I. Cascading Failures in the Electric Power Grid

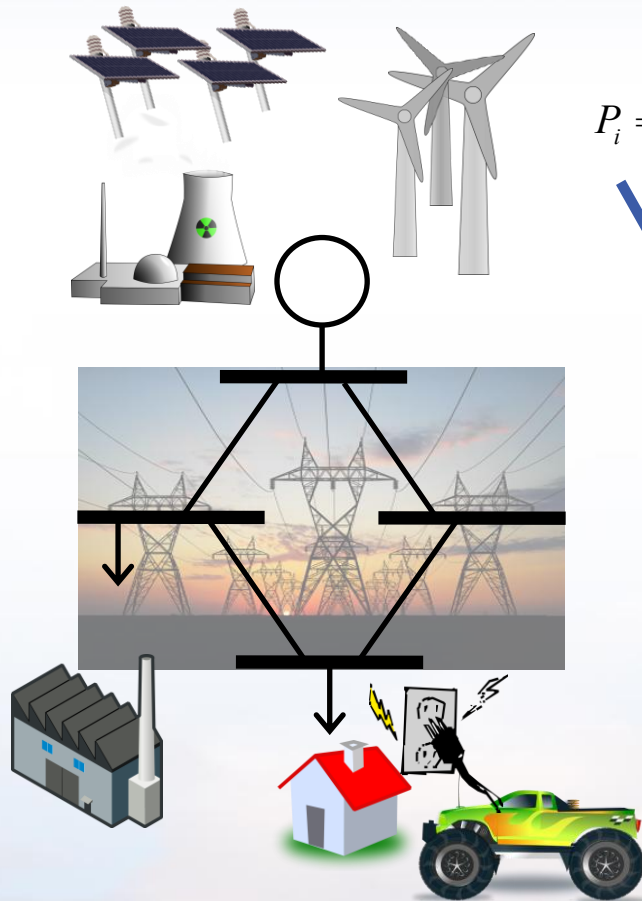


“Risk is the measure of the probability and severity of adverse effects” (*Lowrance, 1976*)

$$\text{Risk}_i = \text{Pr}(i) \times c(i)$$



## II. Modeling a Power System with Differential-Algebraic Equations



$$P_i = \frac{|E'_a||V_a|}{X'_d} \sin d + \frac{|V_a|^2}{2} \left( \frac{1}{X_q} - \frac{1}{X'_d} \right) \sin 2d$$

$$M\ddot{d} + D\dot{d} + P_i(d) = P_m$$

$$\dot{x}(t) = f(x(t), y(t), t)$$

$$0 = g(x(t), y(t), t)$$

$$P_i = \sum_{k=1}^n \dot{a} |V_i| |V_k| (G_{ik} \cos q_{ik} + B_{ik} \sin q_{ik})$$

$$Q_i = \sum_{k=1}^n \dot{a} |V_i| |V_k| (G_{ik} \sin q_{ik} - B_{ik} \cos q_{ik})$$

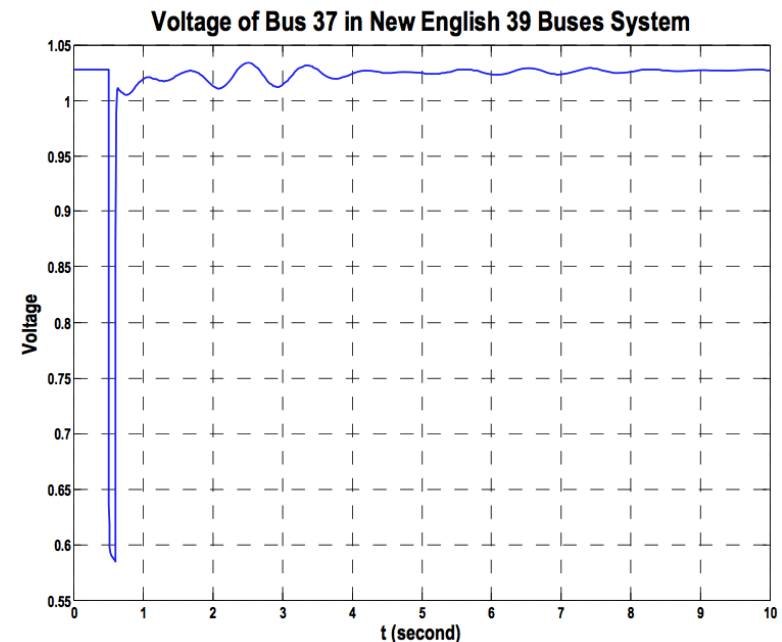
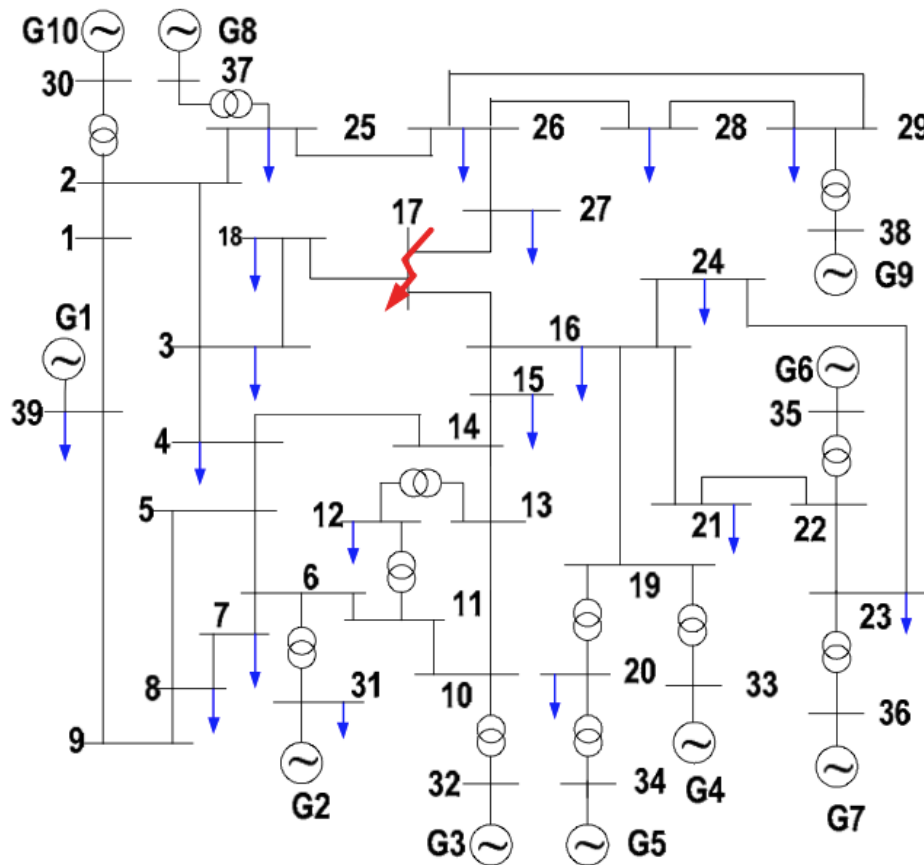
$$P_i = \sum_{k=1}^n \dot{a} P_{base} |V_i|^g$$

$$Q_i = \sum_{k=1}^n \dot{a} Q_{base} |V_i|^g$$

## II. Modeling a Power System with Differential-Algebraic Equations

Test case: New England 39 buses, 10 generators

Event: Fault on bus 17 lasting for 0.1 seconds



McCalley et al. 2010, Siemens PSS/E

## II. Modeling a Power System with Differential-Algebraic Equations

Test case: New England 39 buses, 10 generators

Event: Fault on bus 17 lasting for 0.1 seconds

T1: 0 – 0.5 sec

2 LU decompositions

12 solutions of linear systems

T2: 0.5 – 0.6 sec

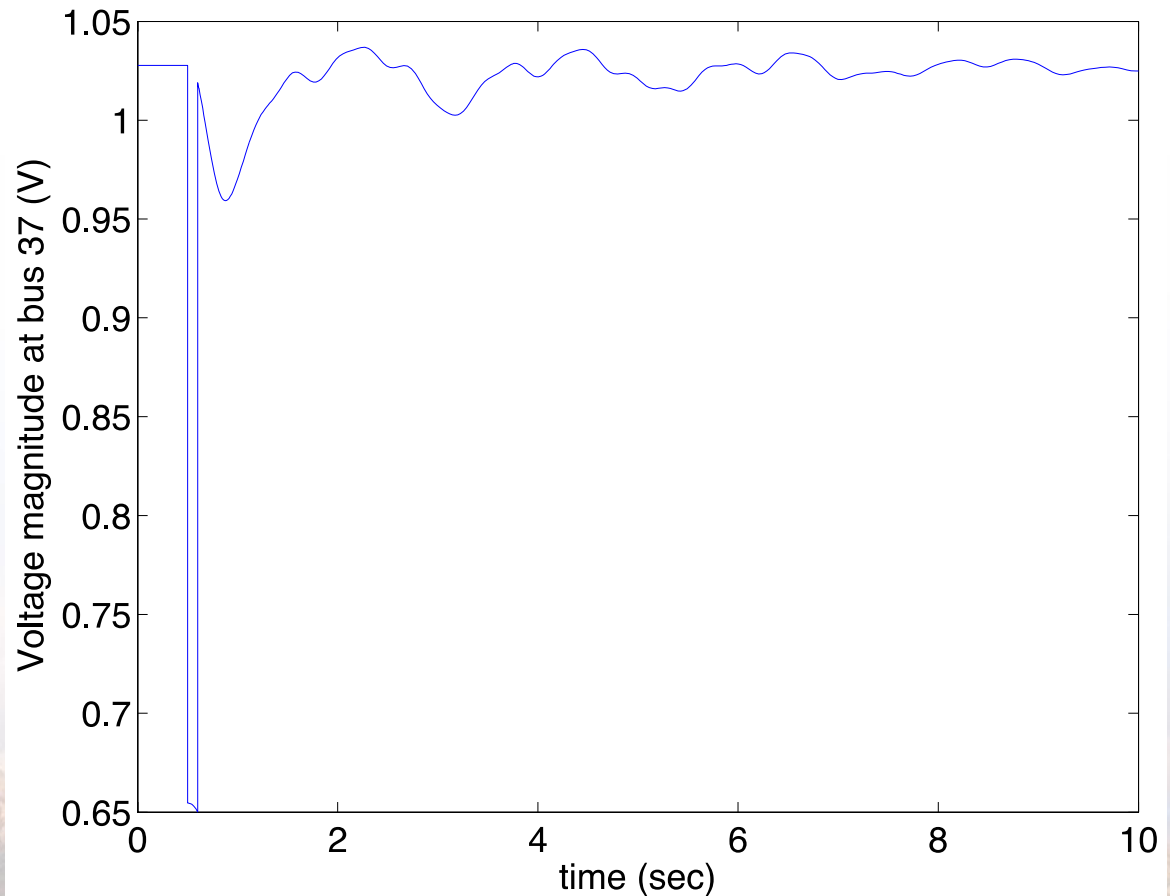
6 LU decompositions

20 solutions of linear systems

T3: 0.6 – 10 sec

91 LU decompositions

737 solutions of linear systems





### III. Risk Estimation

- Importance Sampling (IS) is a method that improves the estimation of rare events by using modified sampling densities, which are constructed to improve the estimation of a statistical response of interest.

$$E(r(X)) = \int r(x) f_X(x) dx \qquad \hat{E}_n(r(X)) = \frac{1}{n} \sum_{i=1}^n r(x_i)$$

$$E(r(X)) = \int r(x) \frac{f(x)}{h(x)} h(x) dx = \hat{E}_h \left[ r(X) \frac{f(X)}{h(X)} \right] = \frac{1}{n} \sum_{i=1}^n \frac{r(x_i) f(x_i)}{h(x_i)}, x_i \sim h(X)$$





# IV. Solving a System of DAEs

## DAE

$$f(t, x, \dot{x}) = 0$$

$$\dot{x} = \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}$$

## Nonlinear Algebraic Equation

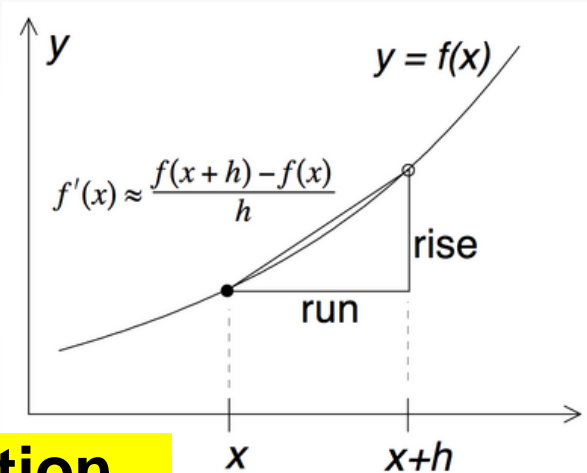
$$f\left(t, x, \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}\right) = 0$$

## Newton-Iteration

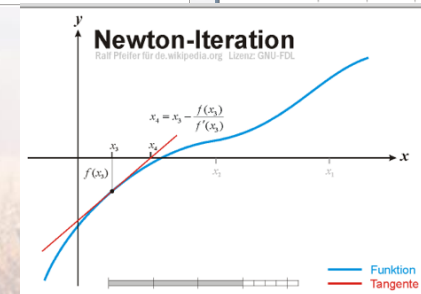
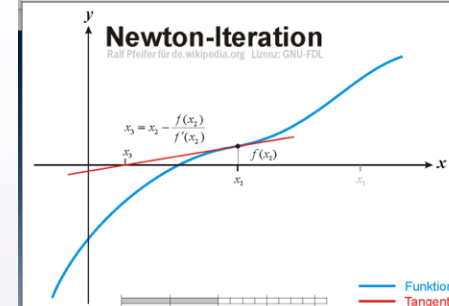
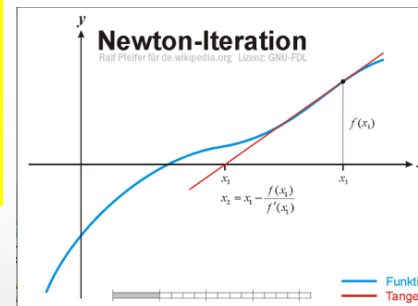
## System of Linear Equations

$$(x_{n+1} - x_n) = -J(x_n)^{-1} F(x_n)$$

$$x = A^{-1}b$$

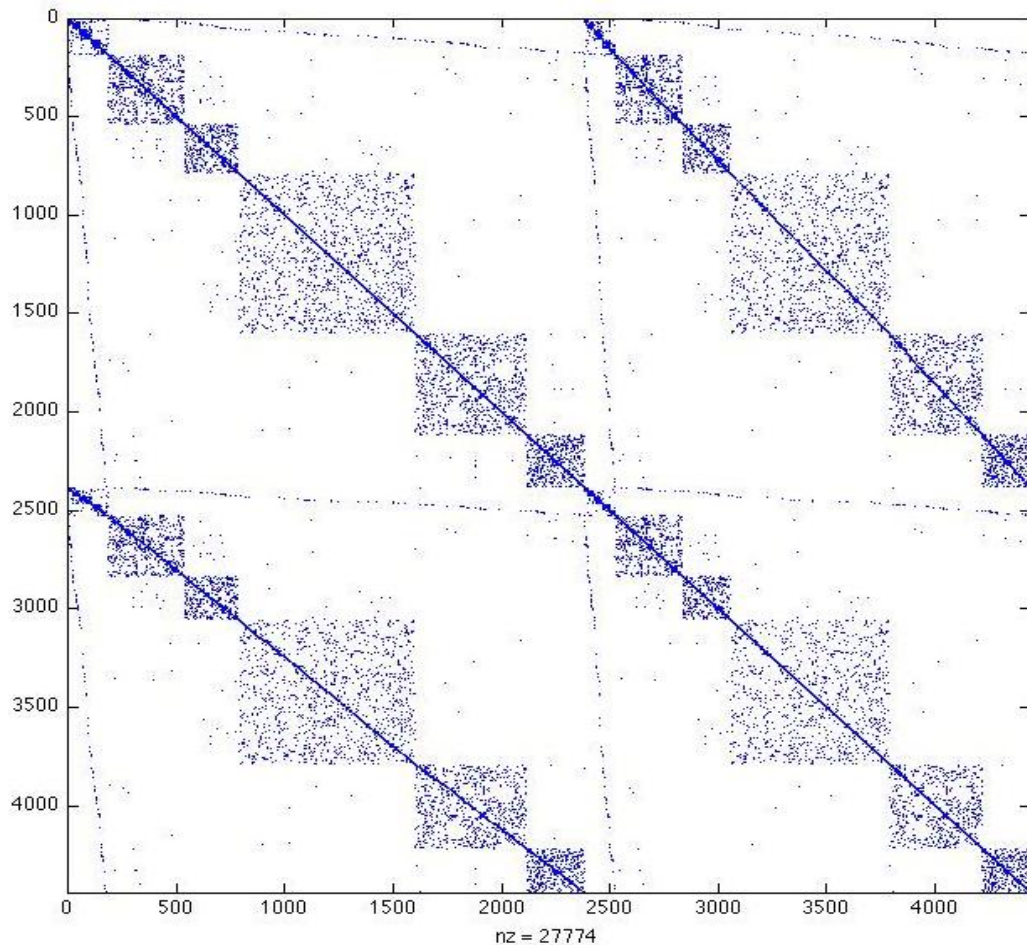


step advance could  
be taking  
hundreds of  
iterations!



# IV. Solving a System of DAEs

## Power Flow Jacobian, Polish Power Grid



- **Sparse, only 0.001% of entries are non-zero**
- **4439 x 4439 (relatively small)**



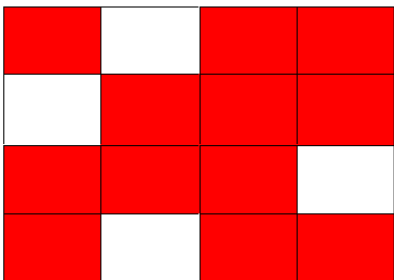
# IV. Solving a System of DAEs

## Solving $Ax = B$

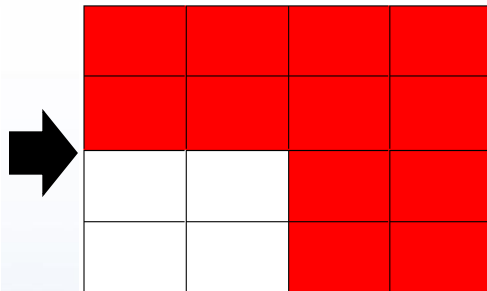
### Symbolic Factorization

*Rearrange A to make LU Decomposition easier*

Original A

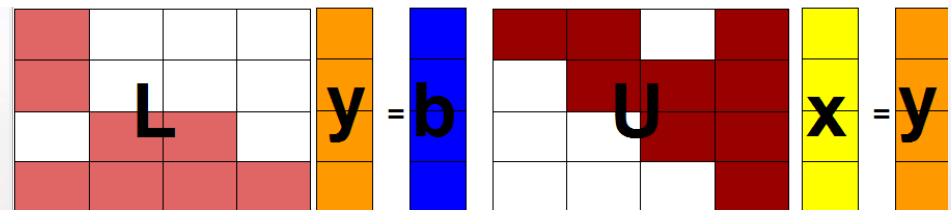
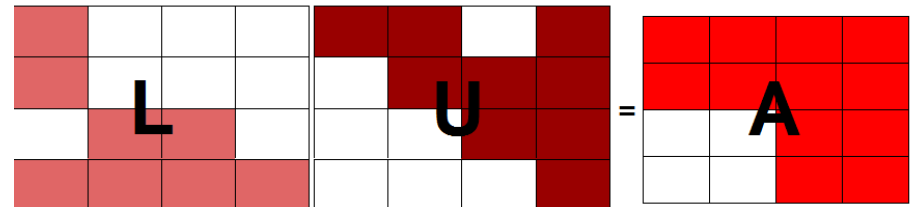
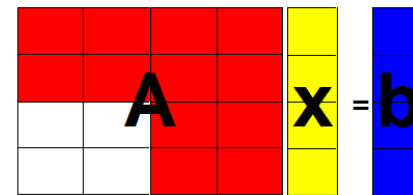


Ordered A



### Numeric Factorization

*Decompose A into LU Matrices and solve*



## IV. Solving a System of DAEs

### Linear Solver Comparison

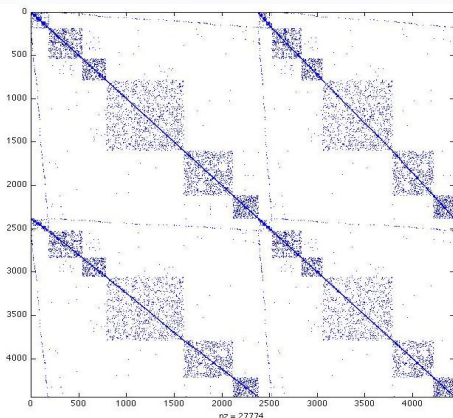
	UMFPACK	LAPACK	Default KLU	SUPERLU
<i>Symbolic Factorial Time</i>	0.0049	0	0.0029	0
<b><i>Numeric Factorial Time</i></b>	0.0197	6.40	<b>0.0036</b>	0.0155
<i>Total Time</i>	0.033	6.44	0.010	0.016
<b><i>Speed up</i></b>	1	0.003	<b>5.47</b>	1.27

KLU - Developed by Tim Davis, Sandia Contractor for Xyce Circuit Simulator

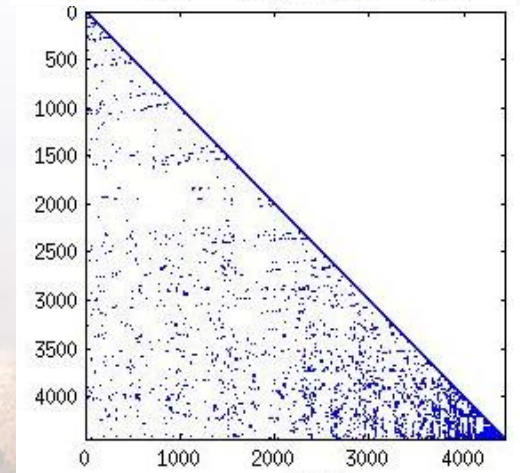
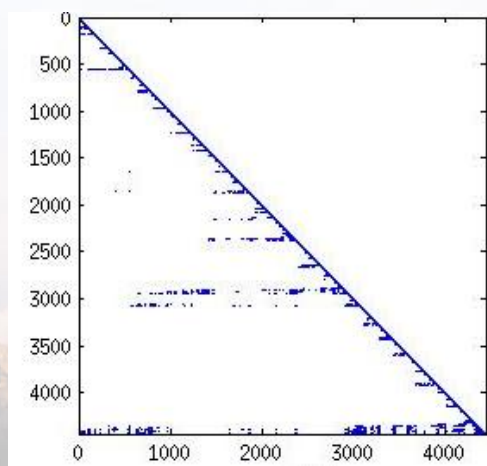
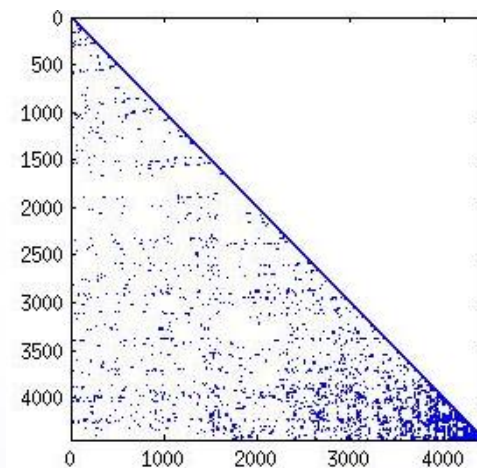
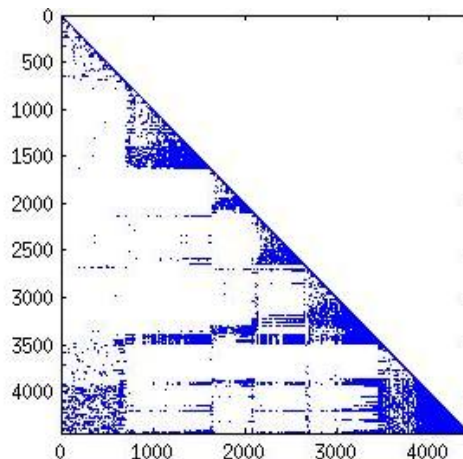




# IV. Solving a System of DAEs Evaluating KLU Algorithms



*Non-zero structure of 4 Jacobian Decompositions*



**6x less memory  
allocation (more  
zeros)**



**64x less floating  
point operations**



## IV. Solving a System of DAEs

### DAE

$$f(t, x, \dot{x}) = 0$$

### Nonlinear Algebraic Equation

$$f\left(t, x, \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}\right) = 0$$

### System of Linear Equations

$$(x_{n+1} - x_n) = -J(x_n)^{-1} F(x_n)$$
$$x = A^{-1}b$$

System of Linear Equations

*Completed – Using KLU solver with Block  
Triangular Form and Approximate  
Minimum Degree Ordering*



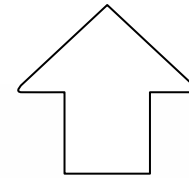
# IV. Solving a System of DAEs

## Future Numerical Work

### DAE

$$f(t, x, \dot{x}) = 0$$

Next: Optimize DAE solver from *Sandia's Trilinos Packages*



### Nonlinear Algebraic Equation

$$f\left(t, x, \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}\right) = 0$$

Newton-Raphson non-linear solver  
*Completed – Using Trilinos NOx*



### System of Linear Equations

$$(x_{n+1} - x_n) = -J(x_n)^{-1} F(x_n)$$
$$x = A^{-1}b$$

System of Linear Equations  
*Completed – Using KLU solver with Block Triangular Form and Approximate Minimum Degree Ordering*





## V. Future Work

- Incorporate advanced devices to the electric grid model (relays, exciters, governors... )
- Scale up the test simulations up to Eastern Interconnect grid size
- Integrate the Importance Sampling strategies through Sandia's Dakota software
- Continue optimizing the numerical techniques with Sandia's Trilinos software





# Thanks for your attention

## Questions? Comments?

