



A Cascading Failure Simulator to Estimate Blackout Risk for Electric Power Infrastructure

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Project Goal

To develop a high performance cascading failure simulator which will provide grid operators with a real time estimate of blackout risk



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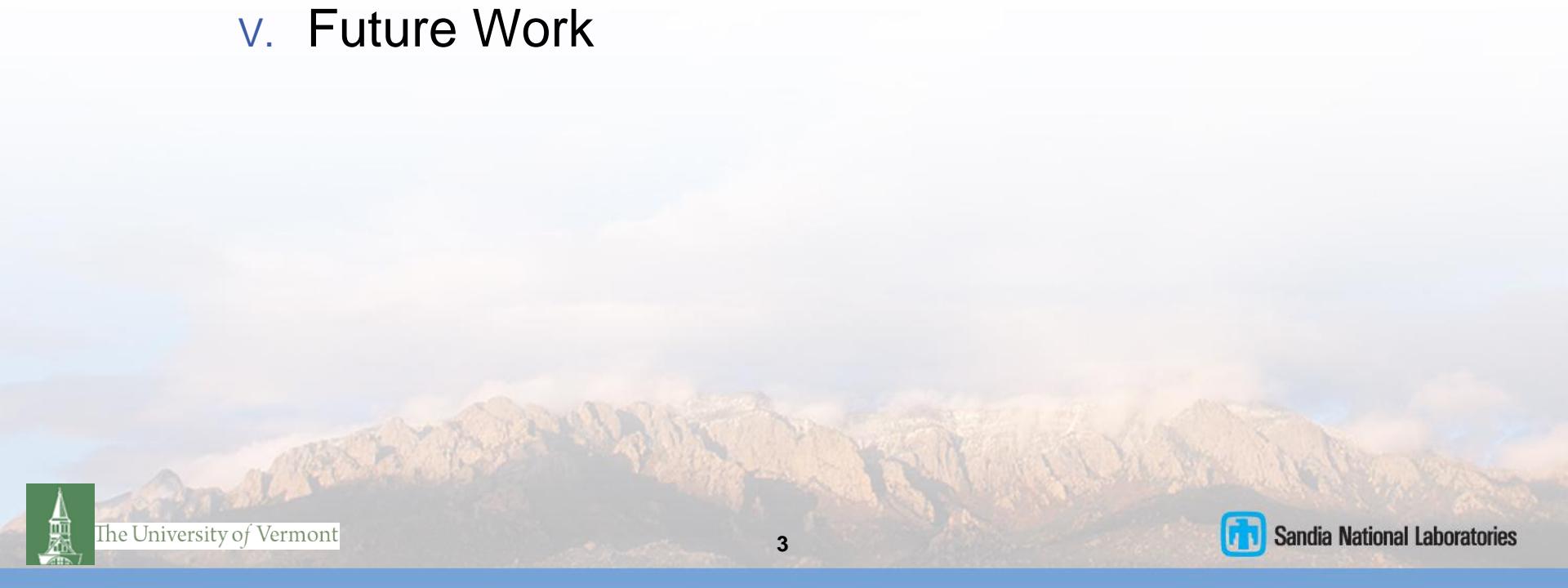


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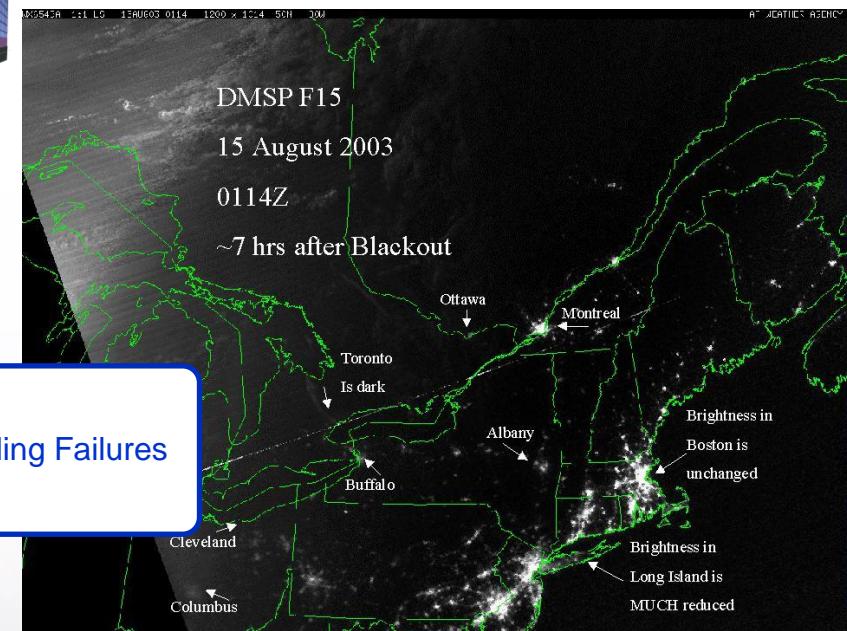
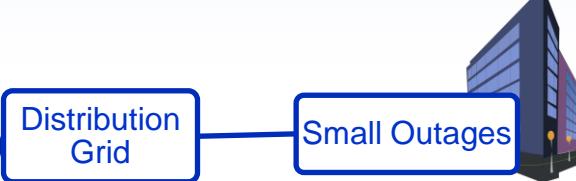


Outline

- I. Cascading Failures in the Electric Power Grid
- II. Modeling a Power System with DAEs
- III. Risk Estimation
- IV. Solving a System of DAEs
- V. Future Work



I. Cascading Failures in the Electric Power Grid



“Risk is the measure of the probability and severity of adverse effects” (*Lowrance, 1976*)

$$\text{Risk}_i = \text{Pr}(i) \times c(i)$$

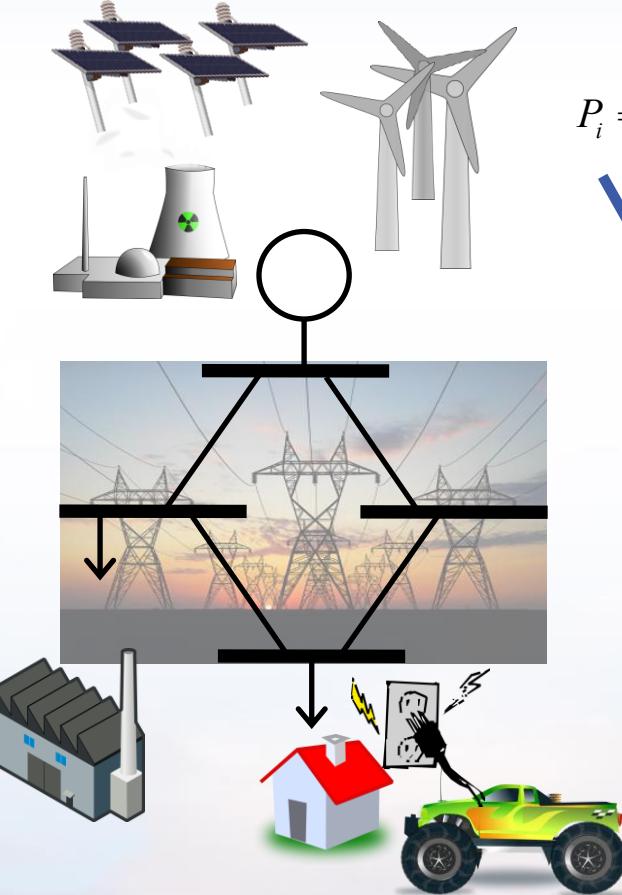


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II. Modeling a Power System with Differential-Algebraic Equations



$$P_i = \frac{|E'_a||V_a|}{X'_d} \sin d + \frac{|V_a|^2}{2} \left(\frac{1}{X_q} - \frac{1}{X'_d} \right) \sin 2d$$

$$M\ddot{d} + D\dot{d} + P_i(d) = P_m$$

$$\dot{x}(t) = f(x(t), y(t), t)$$

$$0 = g(x(t), y(t), t)$$

$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos q_{ik} + B_{ik} \sin q_{ik})$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \sin q_{ik} - B_{ik} \cos q_{ik})$$

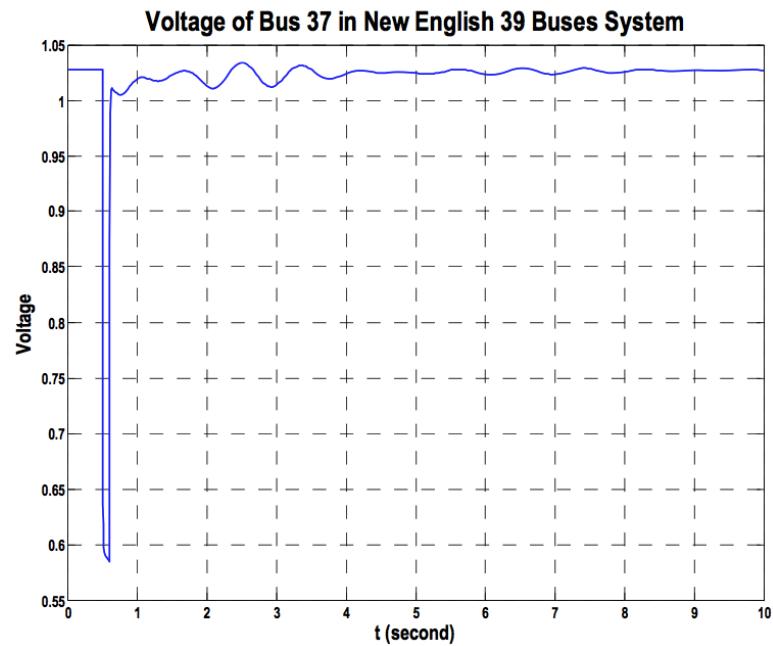
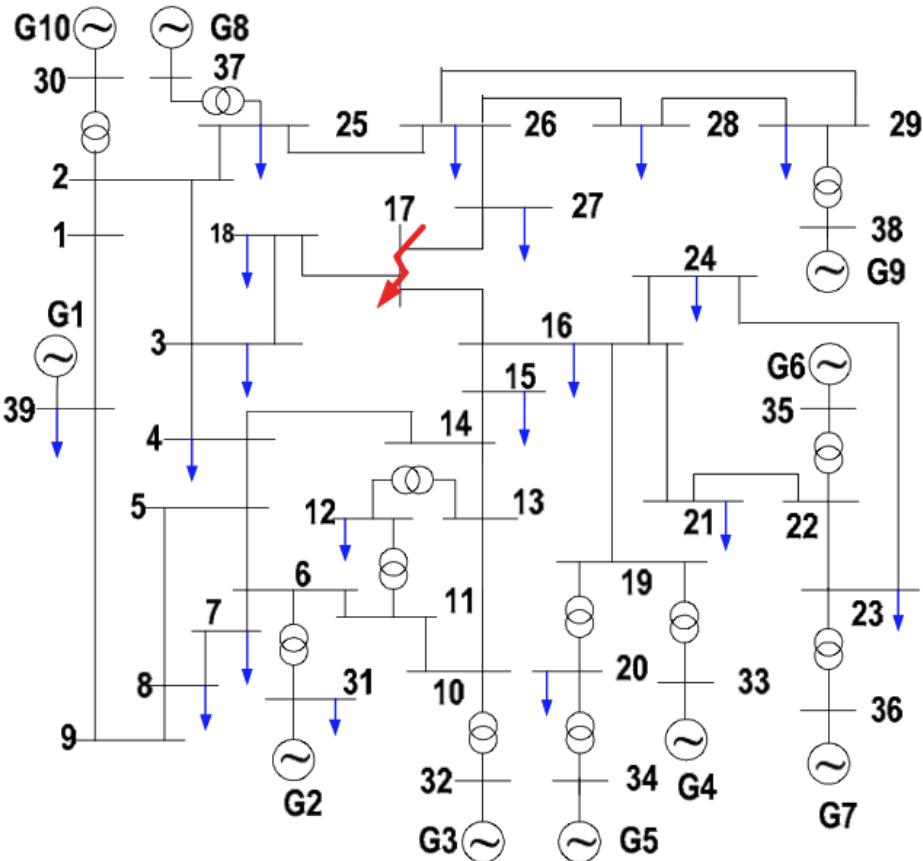
$$P_i = \sum_{k=1}^n P_{base} |V_i|^g$$

$$Q_i = \sum_{k=1}^n Q_{base} |V_i|^g$$

II. Modeling a Power System with Differential-Algebraic Equations

Test case: New England 39 buses, 10 generators

Event: Fault on bus 17 lasting for 0.1 seconds



McCalley et al. 2010, Siemens PSS/E



II. Modeling a Power System with Differential-Algebraic Equations

Test case: New England 39 buses, 10 generators

Event: Fault on bus 17 lasting for 0.1 seconds

T1: 0 – 0.5 sec

2 LU decompositions

12 solutions of linear systems

T2: 0.5 – 0.6 sec

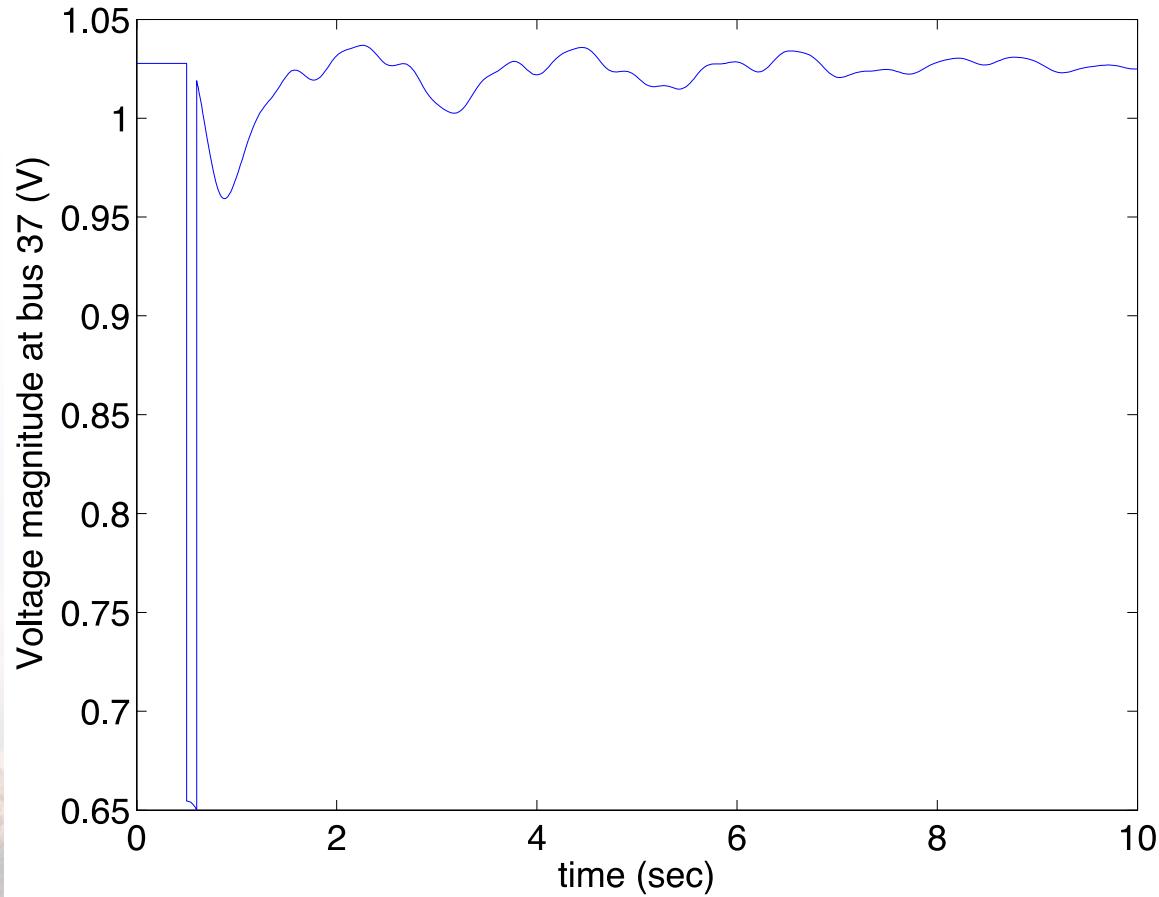
6 LU decompositions

20 solutions of linear systems

T3: 0.6 – 10 sec

91 LU decompositions

737 solutions of linear systems





III. Risk Estimation

- Importance Sampling (IS) is a method that improves the estimation of rare events by using modified sampling densities, which are constructed to improve the estimation of a statistical response of interest.

$$E(r(X)) = \int r(x) f_X(x) dx \quad \hat{E}_n(r(X)) = \frac{1}{n} \sum_{i=1}^n r(x_i)$$

$$E(r(X)) = \int r(x) \frac{f(x)}{h(x)} h(x) dx = \hat{E}_h \left[r(X) \frac{f(X)}{h(X)} \right] = \frac{1}{n} \sum_{i=1}^n \frac{r(x_i) f(x_i)}{h(x_i)}, x_i \sim h(X)$$



IV. Solving a System of DAEs

DAE

$$f(t, x, \dot{x}) = 0$$

$$\dot{x} = \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}$$

Nonlinear Algebraic Equation

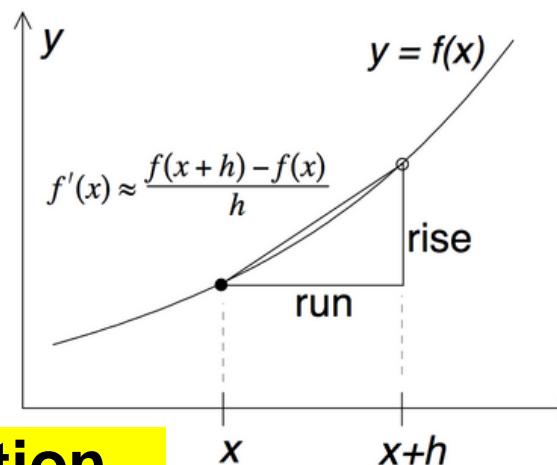
$$f\left(t, x, \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}\right) = 0$$

Newton-Iteration

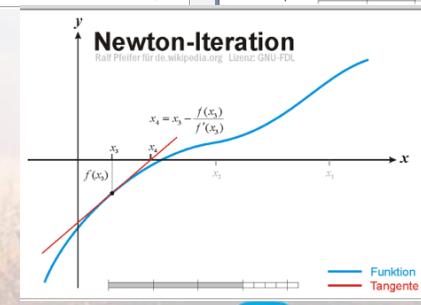
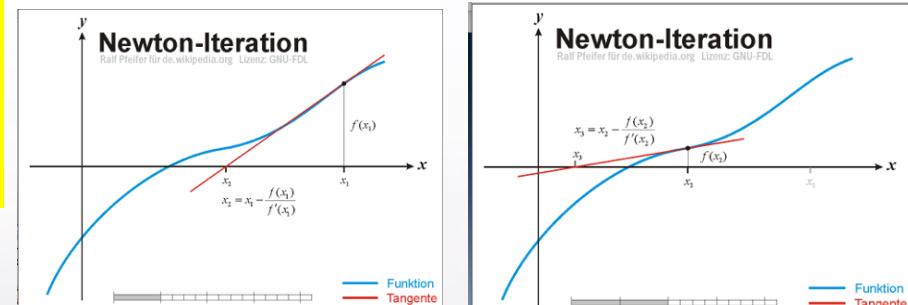
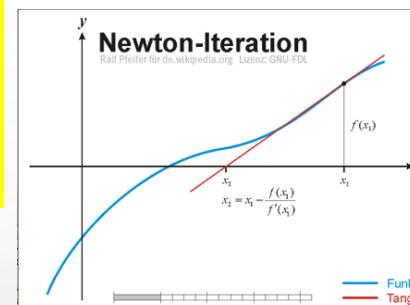
System of Linear Equations

$$(x_{n+1} - x_n) = -J(x_n)^{-1} F(x_n)$$

$$x = A^{-1}b$$



step advance could
aving hundreds
tions!



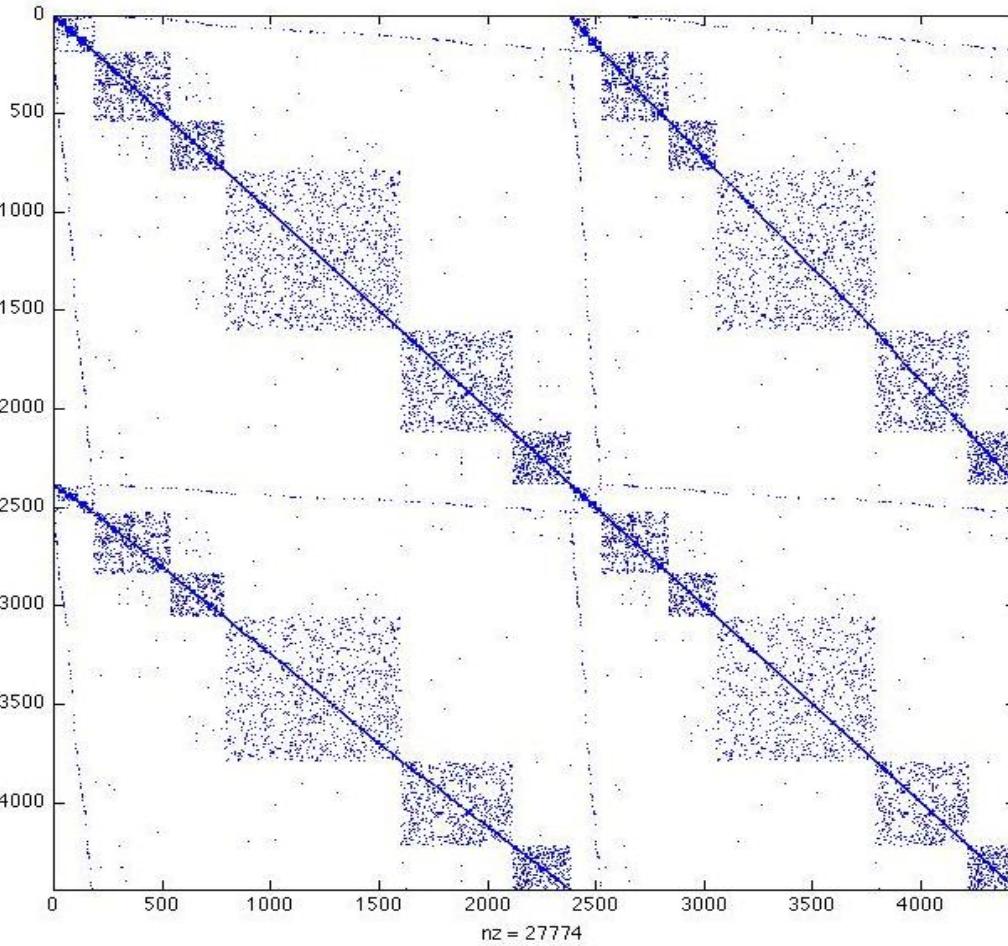
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IV. Solving a System of DAEs

Power Flow Jacobian, Polish Power Grid



- **Sparse, only 0.001% of entries are non-zero**
- **4439×4439 (relatively small)**





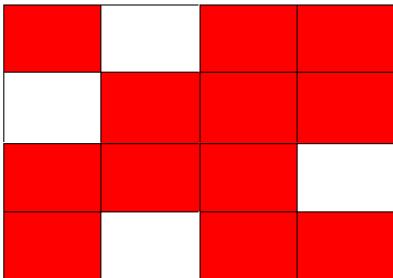
IV. Solving a System of DAEs

Solving $Ax = B$

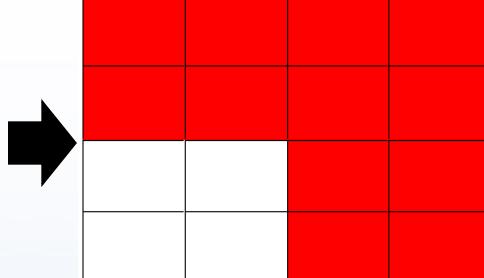
Symbolic Factorization

Rearrange A to make LU Decomposition easier

Original A



Ordered A



Numeric Factorization

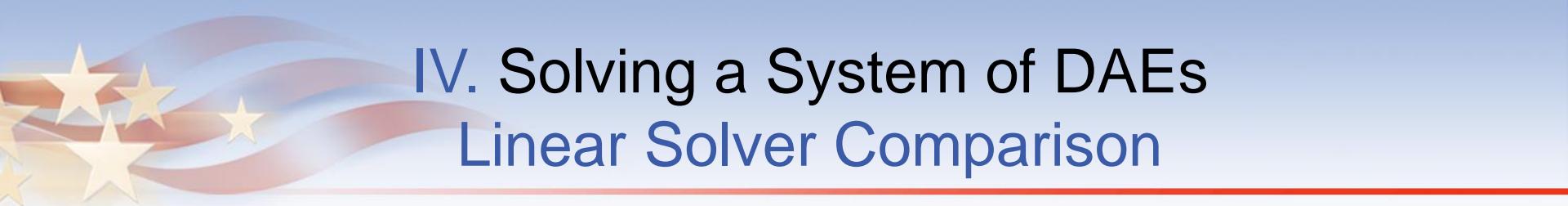
Decompose A into LU Matrices and solve

A diagram illustrating the LU decomposition of matrix A. On the left is a 4x4 matrix A with a red and white checkerboard pattern. To its right is a column vector x in yellow. An equals sign follows, and to the right of that is a column vector b in blue. This represents the equation $Ax = b$.

A diagram illustrating the LU decomposition of matrix A. On the left is a 4x4 matrix A with a red and white checkerboard pattern. To its left is a 4x4 matrix L with a red and white checkerboard pattern. To the right of A is a 4x4 matrix U with a red and white checkerboard pattern. An equals sign follows, and to the right of that is a 4x4 matrix A. This represents the decomposition $LU = A$.

A diagram illustrating the solution process using LU decomposition. On the left is a 4x4 matrix L with a red and white checkerboard pattern. To its right is a column vector y in orange. An equals sign follows, and to the right of that is a column vector b in blue. This represents the equation $Ly = b$. To the right of this is a 4x4 matrix U with a red and white checkerboard pattern. To its right is a column vector x in yellow. An equals sign follows, and to the right of that is a column vector y in orange. This represents the equation $Ux = y$.





IV. Solving a System of DAEs

Linear Solver Comparison

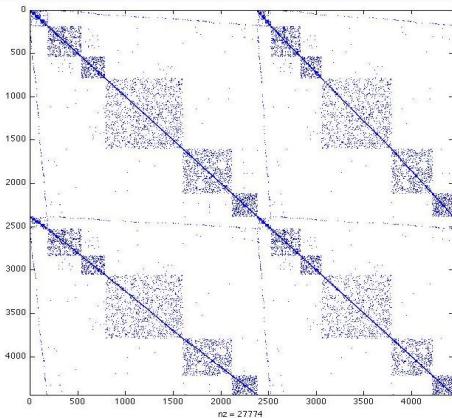
	UMFPACK	LAPACK	Default KLU	SUPERLU
<i>Symbolic Factorial Time</i>	0.0049	0	0.0029	0
<i>Numeric Factorial Time</i>	0.0197	6.40	0.0036	0.0155
<i>Total Time</i>	0.033	6.44	0.010	0.016
<i>Speed up</i>	1	0.003	5.47	1.27

KLU - Developed by Tim Davis, Sandia Contractor for Xyce Circuit Simulator

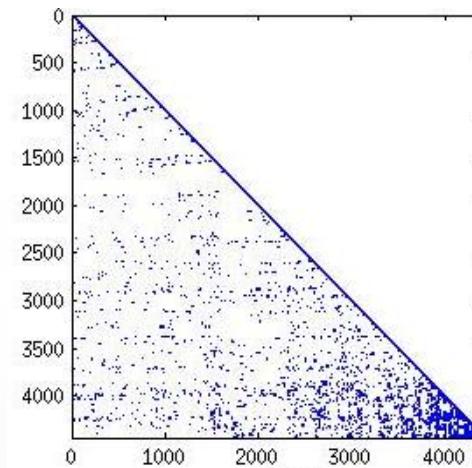
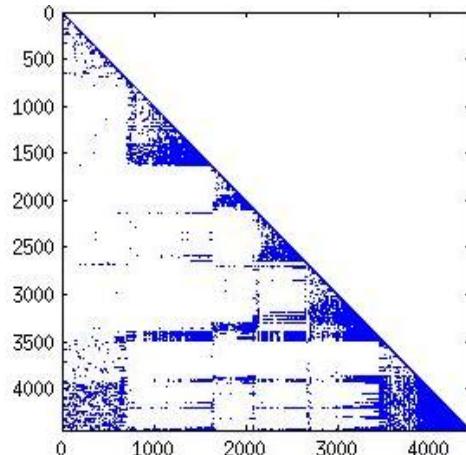


IV. Solving a System of DAEs

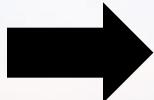
Evaluating KLU Algorithms



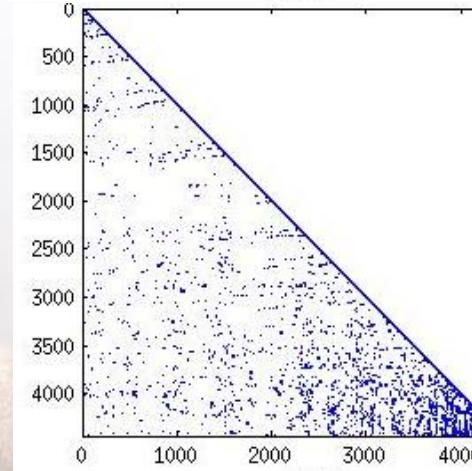
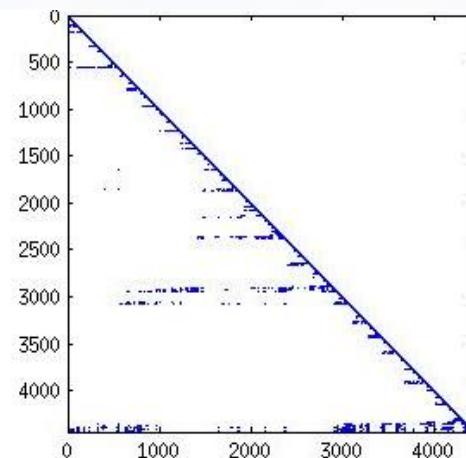
Non-zero structure of 4 Jacobian Decompositions



**6x less memory
allocation (more
zeros)**



**64x less floating
point operations**





IV. Solving a System of DAEs

DAE

$$f(t, x, \dot{x}) = 0$$

Nonlinear Algebraic Equation

$$f\left(t, x, \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}\right) = 0$$

System of Linear Equations

$$(x_{n+1} - x_n) = -J(x_n)^{-1}F(x_n)$$

$$x = A^{-1}b$$

System of Linear Equations

Completed – Using KLU solver with Block Triangular Form and Approximate Minimum Degree Ordering

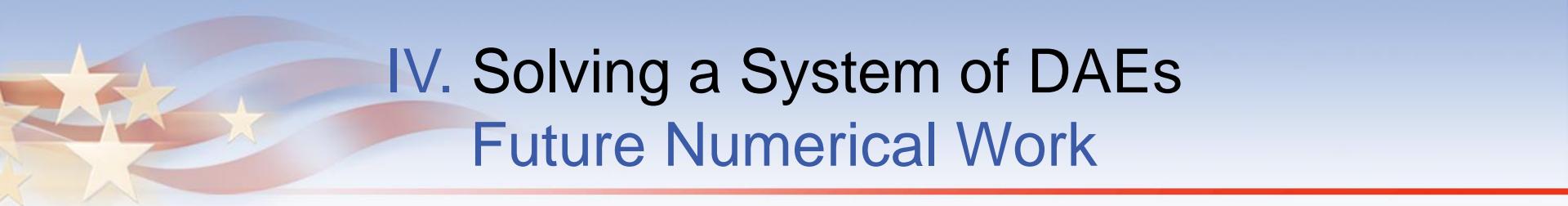


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Trilinos



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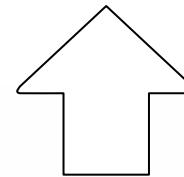
IV. Solving a System of DAEs

Future Numerical Work

DAE

$$f(t, x, \dot{x}) = 0$$

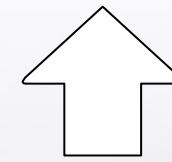
Next: Optimize DAE solver from *Sandia's Trilinos Packages*



Nonlinear Algebraic Equation

$$f\left(t, x, \frac{\sum_{i=0}^q \alpha_{n,i} y_{n-i}}{\Delta t}\right) = 0$$

Newton-Raphson non-linear solver
Completed – Using Trilinos NOx



System of Linear Equations

$$(x_{n+1} - x_n) = -J(x_n)^{-1}F(x_n)$$

$$x = A^{-1}b$$

System of Linear Equations

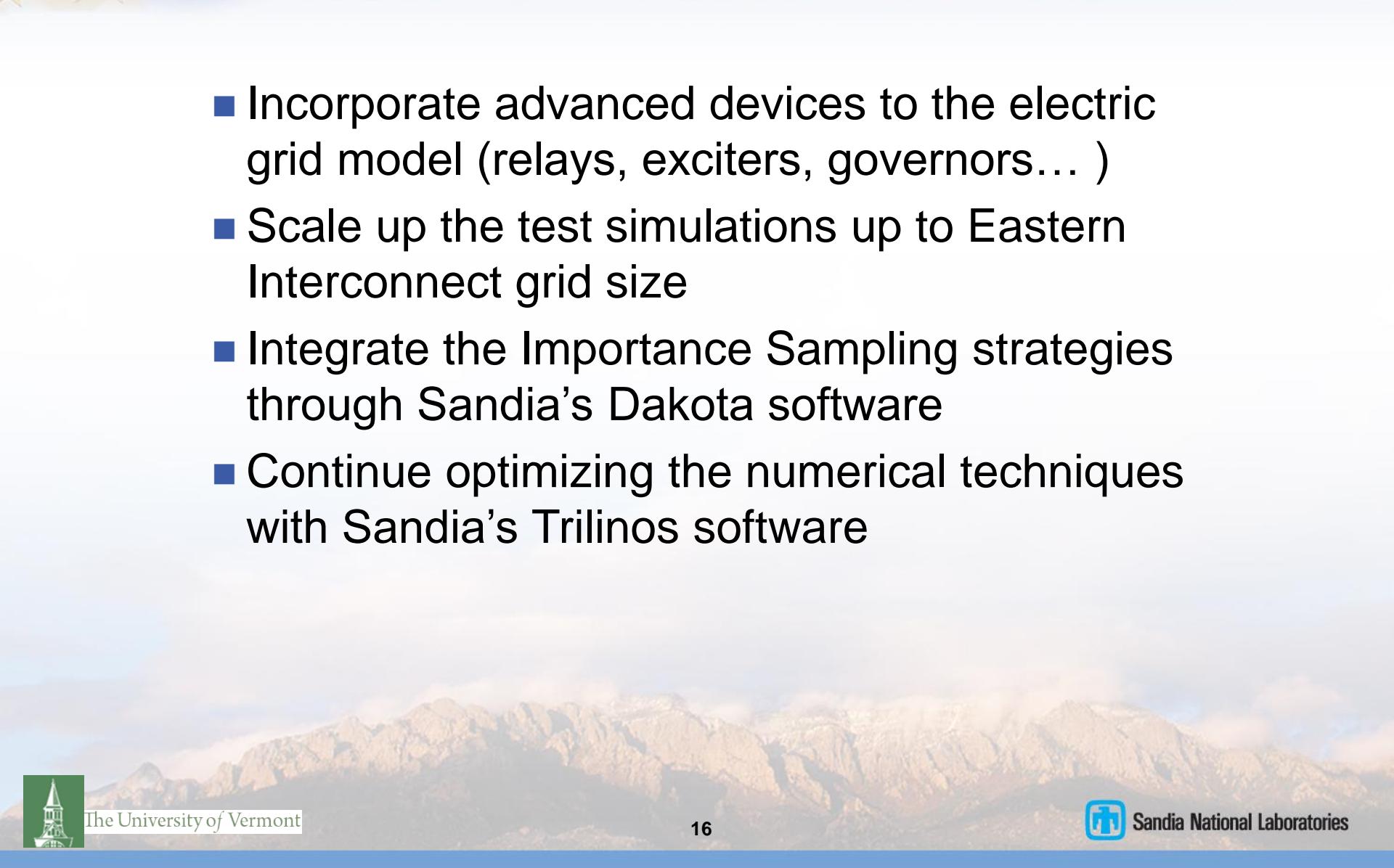
Completed – Using KLU solver with Block Triangular Form and Approximate Minimum Degree Ordering





V. Future Work

- Incorporate advanced devices to the electric grid model (relays, excitors, governors...)
- Scale up the test simulations up to Eastern Interconnect grid size
- Integrate the Importance Sampling strategies through Sandia's Dakota software
- Continue optimizing the numerical techniques with Sandia's Trilinos software





Thanks for your attention Questions? Comments?

