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SEG 2011 San Antonio

ExxonMobil Upstream Research

Viscoelastic orthorhombic full wavefield inversion

Development of multiparameter inversion methods

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Society of Exploration Geophysicists Annual Meeting

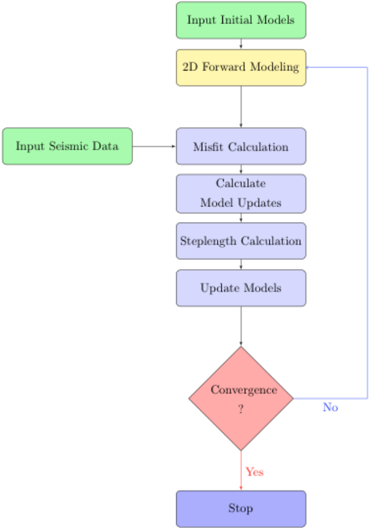
Broadband Seismic Post Convention Workshop

3:30pm Friday September 23, 2011

Introduction to the problem

- In full wavefield inversion, seismic data are used to reconstruct physical parameters of the earth
  - wave velocities
  - attenuation
  - anisotropy
  - density
  - permeability, porosity
- Adjoint-based optimization minimizes the difference between measured and synthesized seismic data
- Industry challenges desire inversion results for a larger number of earth parameters
- Environments where attenuation is important: gas clouds, hydrate-bearing sediments, gas packets, fracture zones

Full wavefield inversion



Introduction to the problem

- How attenuation relates to low frequencies
  - Q (quality factor): the number of wavelengths a wave can propagate before its amplitude decreases by $e^{-\pi}$
  - Attenuation is responsible for absorption and dispersion of the wavefield
  - Properly accounting for accurate attenuation reduces processing artifacts due to:
    - inaccurate filtering
    - erroneously attributing the change in frequency/phase/amplitude to another Earth model parameter
  - Viscoelastic inversion
    - cycle-skipping
    - low shear-wave velocities require low frequency wavefield modelling in order to satisfy stability constraints

Outline

- Viscoelastic full waveform inversion in isotropic and orthorhombic media
  - wave equation is not self-adjoint
  - inaccuracies exist in rheological models used to approximate attenuation (Q)
  - seismic data has low sensitivity to attenuation model changes
  - computationally demanding
- Formulation for full wavefield inversion
  - viscoelastic orthorhombic wave equation
  - adjoint wave equation
- Viscoelastic wavefield modelling
  - modelling a quasi frequency-independent Q
- Sensitivity analysis
- Full wavefield inversion example using a 2-D slice from the SEAM Phase I earth model without salt

Stress-strain relations

Stress Tensor (Elastic):

$$\sigma_{ij} = \mathbf{c}_{ijkl}\epsilon_{kl}$$

where $\mathbf{c}_{ijkl}$ is a fourth-order tensor containing the elastic moduli, and has the properties

$$\mathbf{c}_{ijkl}(\mathbf{x}, t)_{t<0} = 0$$

$$\mathbf{c}_{ijkl} = \mathbf{c}_{klij} = \mathbf{c}_{jikl} = \mathbf{c}_{ijlk}$$

Strain Tensor:

$$\begin{aligned}\epsilon_{kl} &= \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right) \\ &= \frac{1}{2}(u_{k,l} + u_{l,k})\end{aligned}$$

Stress Tensor (Viscoelastic):

$$\begin{aligned}\sigma_{ij}(t) &= \int_{-\infty}^{+\infty} \dot{\psi}_{ijkl}(\mathbf{x}, t-\tau)\epsilon_{kl}d\tau \\ &= \dot{\psi}_{ijkl} * \epsilon_{kl}\end{aligned}$$

Viscoelastic Modelling

The GMB-EK rheological model is used to model attenuation, which defines the viscoelastic modulus

$$M(\omega) = \mathbf{c}_{ijkl} \left[1 - \sum_{\ell=1}^{n} \alpha_\ell \frac{\omega_\ell}{\omega_\ell + i\omega} \right]$$

and the relaxation function

$$\begin{aligned} \psi(t) &= \mathcal{F}^{-1} \left\{ \frac{M(\omega)}{i\omega} \right\} \\ &= \mathbf{c}_{ijkl} \left[1 - \sum_{\ell=1}^{n} \alpha_\ell (1 - e^{-\omega_\ell t}) \right] H(t) \end{aligned}$$

where $H(t)$ is the Heaviside unit step-function. The viscoelastic stress-strain relationship simplifies to

$$\sigma_{ij}(t) = \mathbf{c}_{ijkl} \epsilon_{kl}(t) - \mathbf{c}_{ijkl} \sum_{\ell=1}^{n} \alpha_\ell \omega_\ell \left[e^{-\omega_\ell t} * \epsilon_{kl}(t) \right]$$

where n is the number of memory variables.

Viscoelastic Modelling

Denoting material-independent functions

$$\phi_\ell(t) = \omega_\ell e^{-\omega_\ell t} * \epsilon_{kl}(t),$$

the viscoelastic stress-strain relationship simplifies to

$$\sigma_{ij}(t) = \mathbf{c}_{ijkl}\epsilon_{kl}(t) - \mathbf{c}_{ijkl}\sum_{\ell}^{n}\alpha_\ell\phi_\ell(t)$$

2D Elastic stress-strain relation:

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{zz}\\ \sigma_{xz}\end{pmatrix} = \begin{pmatrix}c_{11} & c_{13} & c_{15}\\ c_{13} & c_{33} & c_{35}\\ c_{15} & c_{35} & c_{55}\end{pmatrix}\begin{pmatrix}\epsilon_{x,x}\\ \epsilon_{z,z}\\ \epsilon_{x,z}\end{pmatrix}$$

2D Viscoelastic stress-strain relation:

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{zz}\\ \sigma_{xz}\end{pmatrix} = \begin{pmatrix}\dot{\psi}_{11} & \dot{\psi}_{13} & \dot{\psi}_{15}\\ \dot{\psi}_{13} & \dot{\psi}_{33} & \dot{\psi}_{35}\\ \dot{\psi}_{15} & \dot{\psi}_{35} & \dot{\psi}_{55}\end{pmatrix} * \begin{pmatrix}\epsilon_{x,x} - \sum_i \alpha_i\phi_i\\ \epsilon_{z,z} - \sum_i \alpha_i\phi_i\\ \epsilon_{x,z} - \sum_i \alpha_i\phi_i\end{pmatrix}$$

(Visco)elastic Modulus

Isotropic (2D): Requires 2 unique c_{ij}'s

$$
\begin{aligned}
c_{11} &= c_{33} \\
c_{33} &= (\lambda + 2\mu) = \pi \\
c_{55} &= \mu \\
c_{13} &= \lambda = (c_{33} - 2c_{55}) \\
c_{15} &= c_{35} = 0
\end{aligned}
$$

VTI (2D): Requires 4 unique c_{ij}'s

$$
\begin{aligned}
c_{11} &= c_{33} * (1 + 2\varepsilon) \\
c_{33} &= (\lambda + 2\mu) = \pi \\
c_{55} &= \mu \\
c_{13} &= \sqrt{(c_{33} - c_{55})^2 + [2\delta c_{33}(c_{33} - c_{55})]} \\
c_{15} &= c_{35} = 0
\end{aligned}
$$

2D Isotropic Viscoelastic Equations

Stresses:

$$t_{xx} = c_{33}(u_{x,x} + u_{z,z}) - 2c_{55}u_{z,z} + \sum_{i=1}^{n} \phi_{xx_i}$$

$$t_{zz} = c_{33}(u_{x,x} + u_{z,z}) - 2c_{55}u_{x,x} + \sum_{i=1}^{n} \phi_{zz_i}$$

$$t_{xz} = c_{55}(u_{z,x} + u_{x,z}) + \sum_{i=1}^{n} \phi_{xz_i}$$

$$\eta_i = \omega_i e^{-\omega_i t}$$

Memory variables:

$$\phi_{xx_i} = c_{33}\alpha_i^{33}\left[\eta_i(t) * (u_{x,x} + u_{z,z})\right] - 2c_{55}\alpha_i^{55}\left[\eta_i(t) * u_{z,z}\right]$$

$$\phi_{zz_i} = c_{33}\alpha_i^{33}\left[\eta_i(t) * (u_{x,x} + u_{z,z})\right] - 2c_{55}\alpha_i^{55}\left[\eta_i(t) * u_{x,x}\right]$$

$$\phi_{xz_i} = c_{55}\alpha_i^{55}\left[\eta_i(t) * (u_{z,x} + u_{x,z})\right]$$

2D VTI Viscoelastic Equations

Stresses:

$$t_{xx} = c_{11} u_{x,x} + c_{13} u_{z,z} + \sum_{i=1}^{n} \phi_{xx_i}$$

$$t_{zz} = c_{13} u_{x,x} + c_{33} u_{z,z} + \sum_{i=1}^{n} \phi_{zz_i}$$

$$t_{xz} = c_{55} (u_{z,x} + u_{x,z}) + \sum_{i=1}^{n} \phi_{xz_i}$$

Memory variables:

$$\phi_{xx_i} = c_{11} \alpha_i^{11} \left[\eta_i(t) * u_{x,x} \right] + c_{13} \alpha_i^{13} \left[\eta_i(t) * u_{z,z} \right]$$

$$\phi_{zz_i} = c_{13} \alpha_i^{13} \left[\eta_i(t) * u_{x,x} \right] + c_{33} \alpha_i^{33} \left[\eta_i(t) * u_{z,z} \right]$$

$$\phi_{xz_i} = c_{55} \alpha_i^{55} \left[\eta_i(t) * (u_{z,x} + u_{x,z}) \right]$$

Modelling attenuation

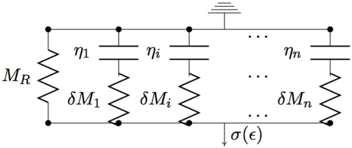
Q^{-1} is defined

$$Q^{-1}(\omega) = \sum_{i=1}^{n} \alpha_i \left[\frac{\omega_i \omega + \omega_i^2 Q^{-1}(\omega)}{\omega_i^2 + \omega^2} \right]$$

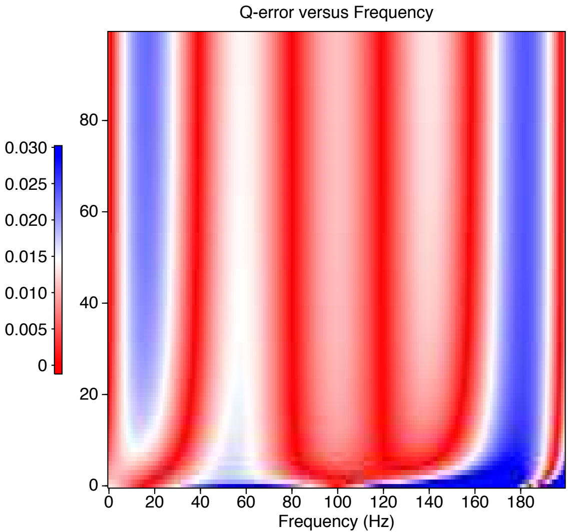
$$\mathbf{A} = \frac{\omega_i \omega + \omega_i^2 Q^{-1}(\omega)}{\omega_i^2 + \omega^2}$$

$$Q^{-1}(\omega) = \mathbf{A}\alpha$$

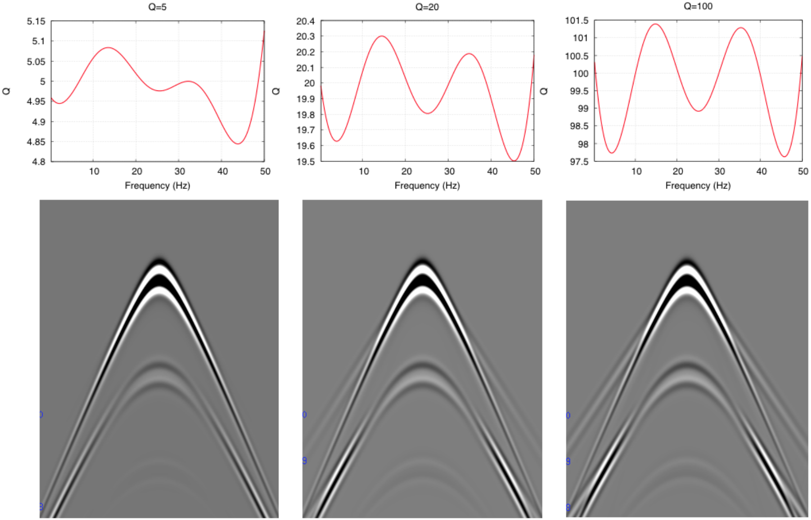
$$\alpha = \left[\mathbf{A}^T \mathbf{A}\right]^{-1} \mathbf{A}^T Q^{-1}$$



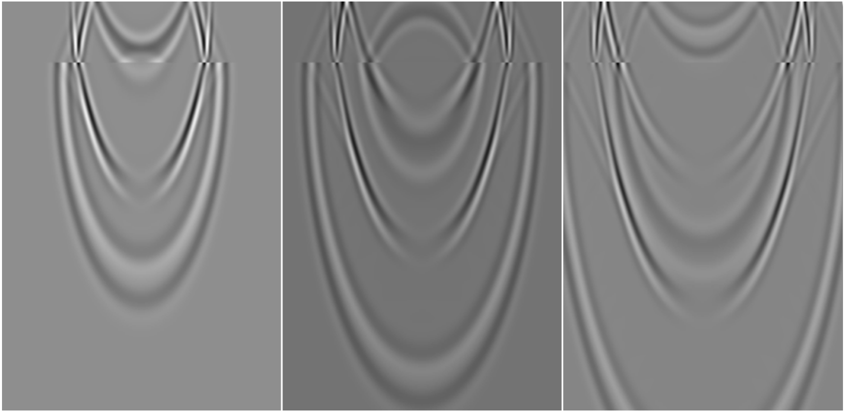
Modelling attenuation



Modelling attenuation



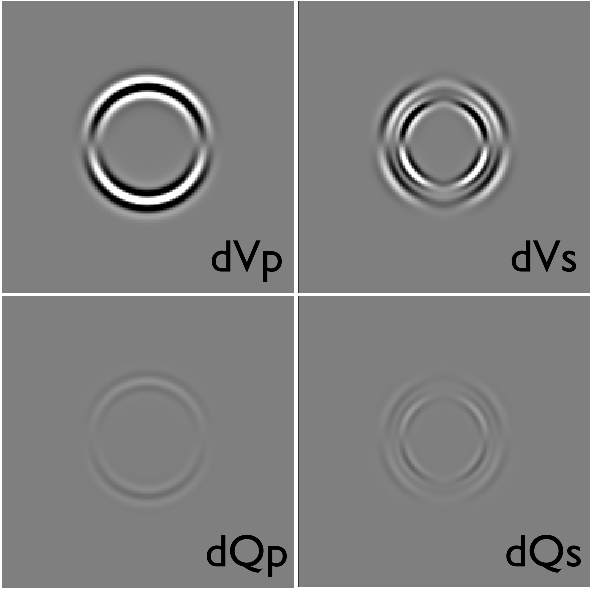
2D Viscoelastic VTI Wavefield Propagation



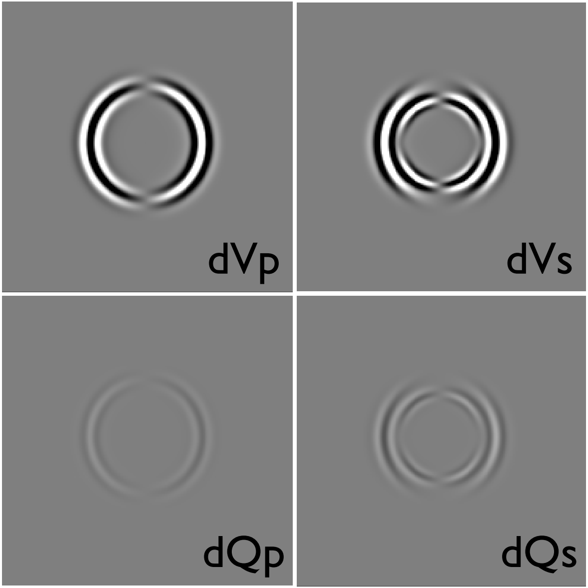
Data Sensitivity to Model Perturbations

- The inversion is driven by differences between the measured and synthesized seismic data
- What happens when the seismic data does not "feel" a change in the subsurface
- How does this impact:
  - inversion performance/convergence
  - compute time
  - accuracy
  - resolution
  - what is the utility of multi-parameter inversion

Data Sensitivity to Model Perturbations



Sensitivity of seismic data to Earth model perturbations



Viscoelastic (isotropic) inversion

- Model updates are derived from the adjoint wave equation
- Apply model gradient terms for C33, C55, Q33 (Qp), and Q55 (Qs) models

- Experience shows that attenuation is not easily inverted for at depth
- Select a shallow model in order to have results to interpret
- Invert for Earth models that the seismic data has higher sensitivity towards (velocities)
- Invert for attenuation models following an initial inversion of velocities, maintaining the Q-models fixed
- Use and L2 norm for the velocity inversion
- Use a L1 norm for the attenuation inversion

Continuous gradient formulae

For viscoelastic inversion, invert for 4 parameters:

$$\mathbf{g}_{c_{33}} = -\sum_{shots}\int_0^t dt\left[\overleftarrow{\epsilon_{ij}}(\delta_{ij}\delta_{kl})\overrightarrow{\epsilon_{kl}}\right] = -\sum_{shots}\int_0^t dt\left[\overleftarrow{\epsilon_{ii}}\overrightarrow{\epsilon_{kk}}\right]$$

$$\mathbf{g}_{c_{55}} = -\sum_{shots}\int_0^t dt\left[\overleftarrow{\epsilon_{ij}}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-2\delta_{ij}\delta_{kl})\overrightarrow{\epsilon_{kl}}\right]$$

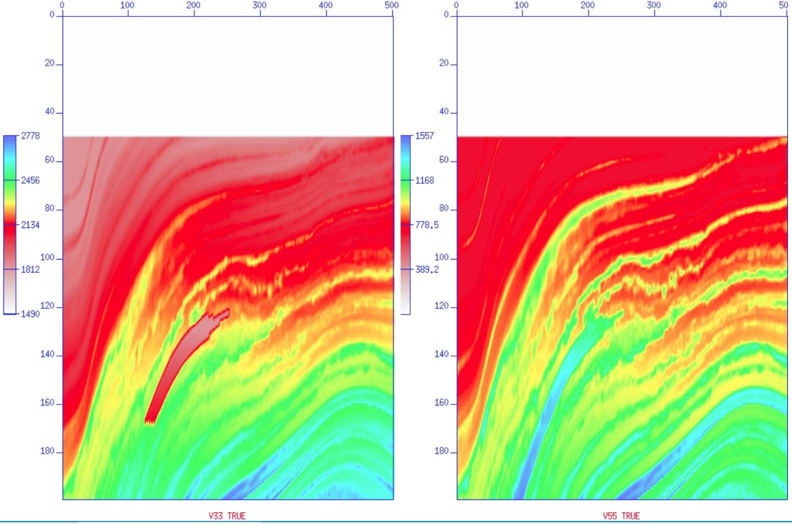
$$\mathbf{g}_{c^C_{33i}} = -\sum_{shots}\int_0^t dt\left[\overleftarrow{\epsilon_{ij}}(\delta_{ij}\delta_{kl})[\eta_i * \overrightarrow{\epsilon_{kl}}]\right] = -\sum_{shots}\int_0^t dt\left[\overleftarrow{\epsilon_{ii}}[\eta_i * \overrightarrow{\epsilon_{kk}}]\right]$$

$$\mathbf{g}_{c^C_{55i}} = -\sum_{shots}\int_0^t dt\left[\overleftarrow{\epsilon_{ij}}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-2\delta_{ij}\delta_{kl})[\eta_i * \overrightarrow{\epsilon_{kl}}]\right]$$

where the superscript C denotes the modulus coupled with the modulus ratio α_i, such that

$$c^C_{33i} = c_{33}\alpha_i^{33}$$
$$c^C_{55i} = c_{55}\alpha_i^{55}$$

SEAM I FWI



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V33 TRUE
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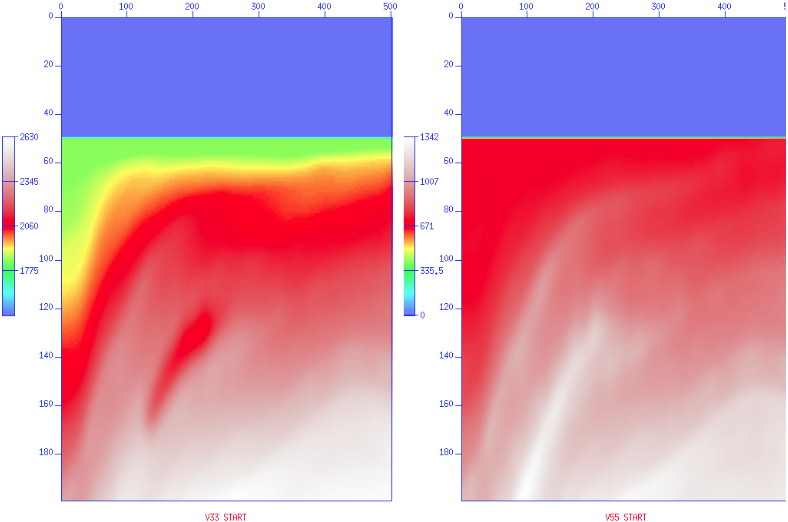
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671
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V55 START

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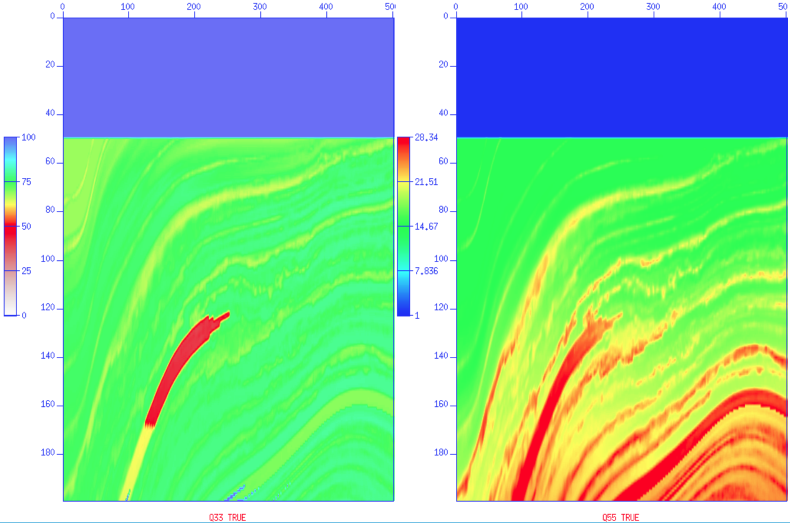

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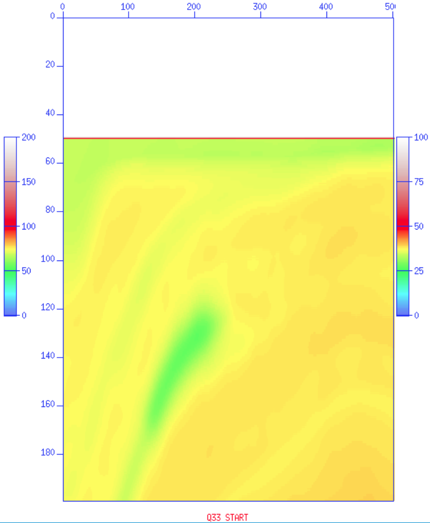

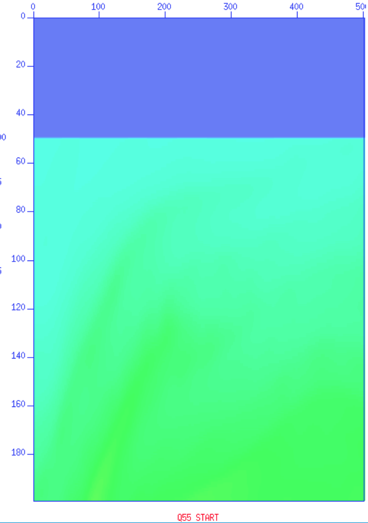

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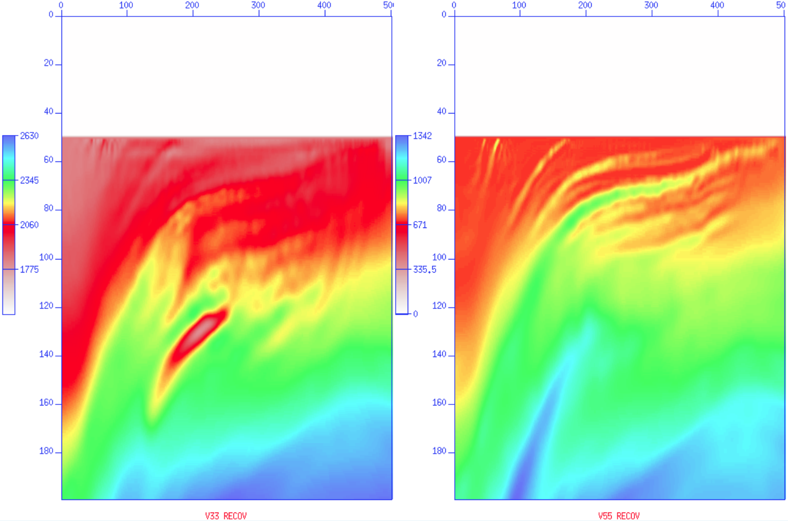

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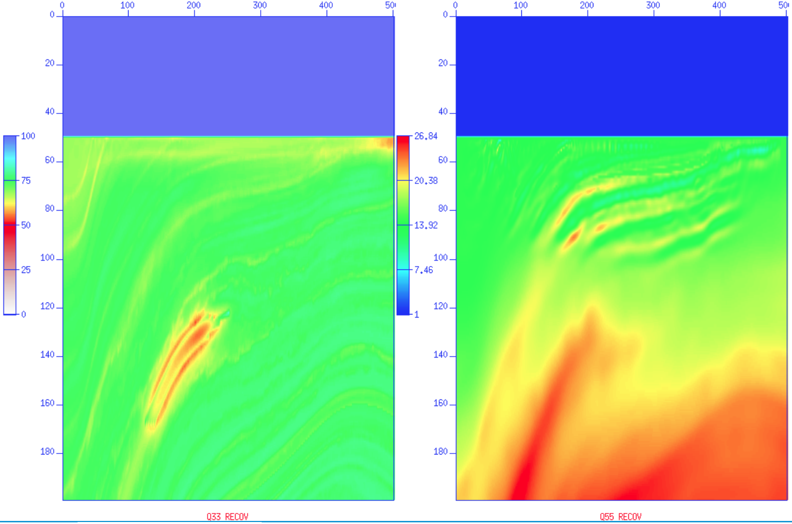
V33 RECOV
V55 RECOV

SEAM I FWI



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7.46
1
Q33 RECOV
Q55 RECOV

Discussion

- Evaluate the range of applicability of this approach
- Evaluate if viscoelastic (isotropic or orthorhombic) inversion is realistic/feasible at this time
- Development of efficient code: understanding what short-cuts may be made, what constraints can be implemented:
    - Understanding the mechanism
    - Development of preconditioners and regularizations for multiparameter (multi-scale) inversion
    - Resolution analysis