SAND2012-0424C

Module 1: Nuclear Energy Fundamentals (Week 2/Day 1)

The Need for Electricity, How Electricity is Produced, and Nuclear Power Introduction

Gulf Nuclear Energy Infrastructure Institute – 2012 Fundamentals Course

Dr. David Boyle
Texas A&M University

Module 1/Week 2:

- **Nuclear and Reactor Physics**

Week 2 Learning Objectives:

- Be able to explain how nuclear power works and why it is needed
- Be able to explain why nuclear power works
- Be able to explain why fission products and radiation are major issues for nuclear power
- Be able to explain the role of neutron interactions in nuclear power

Today's Primary Learning Objective

Primary Day 1 Learning Objective:

- Be able to explain how nuclear power works and why it is needed

Take away from these lectures:

- Nuclear reactions generate thermal energy which can be used to create steam; this steam can be used to drive a turbine-generator to produce electricity. Electricity production benefits the standard of living of the country.

Module 1: Nuclear Energy Fundamentals (Week 2/Day 1)

Lecture #1: Why We Need Electricity

Dr. David Boyle

Lecture Outline

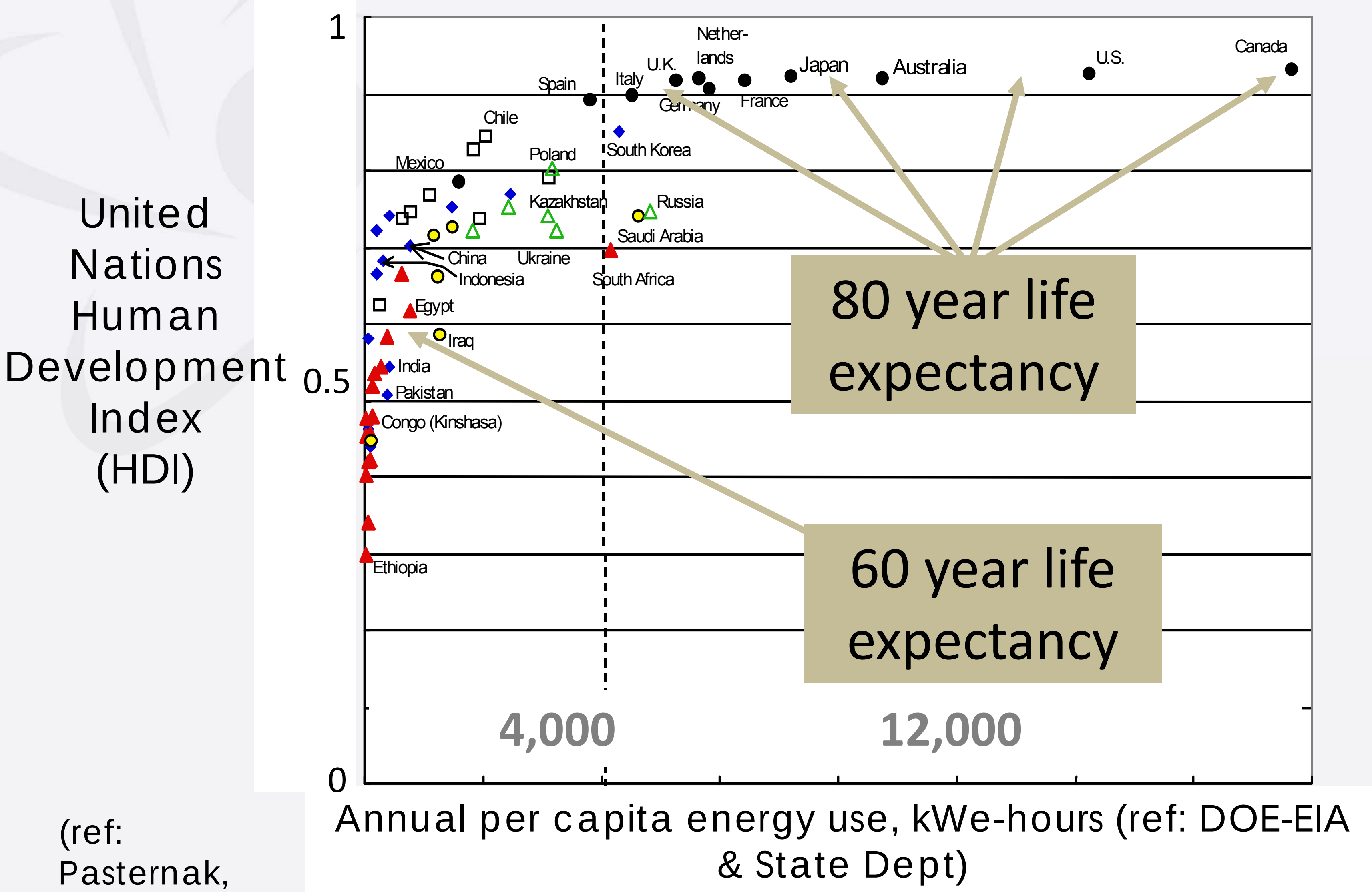
1. Electricity Basics
2. Rankine Cycle
3. Electricity Production Alternatives
4. Nuclear Steam Supply System

Why do we need electricity?

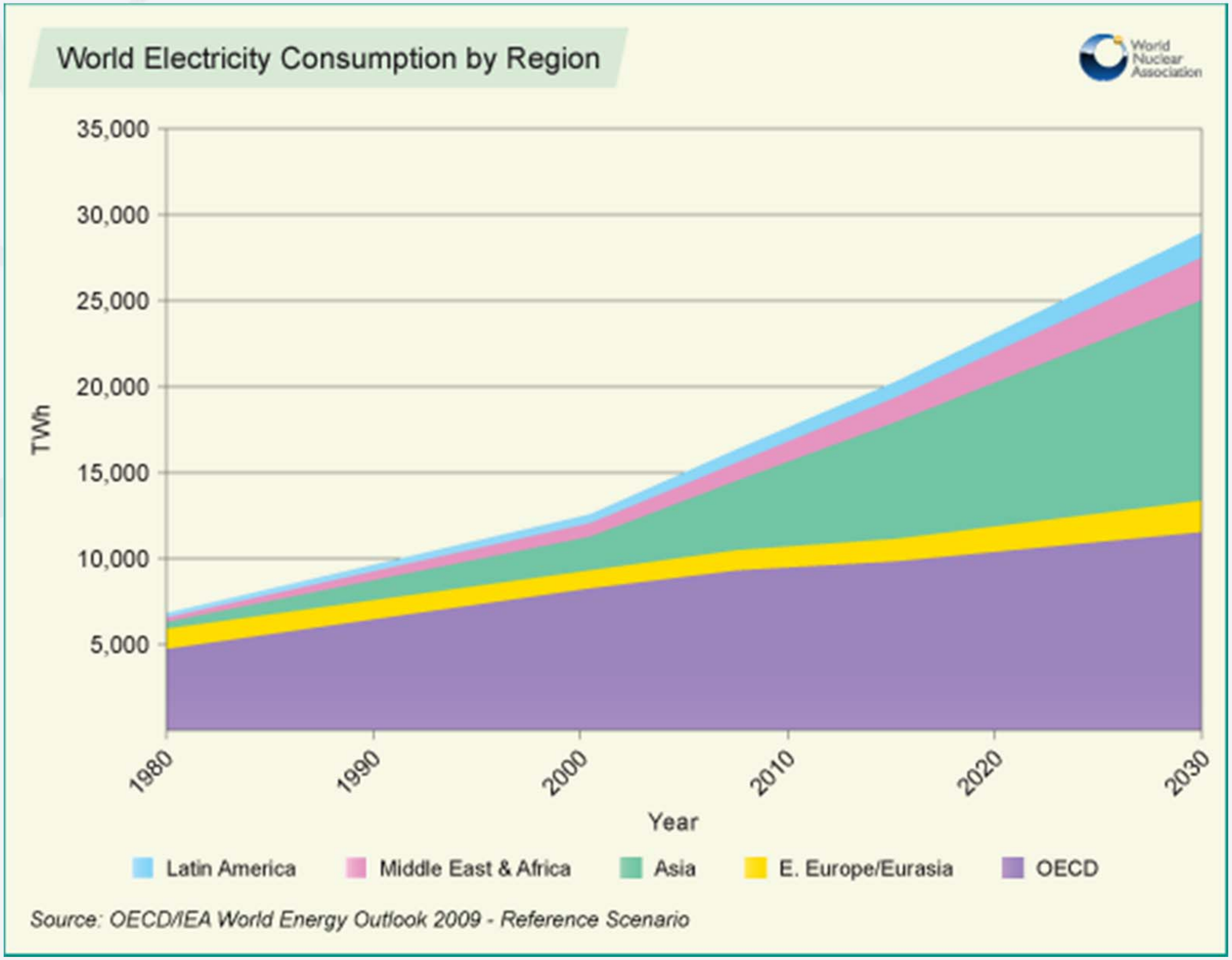
Source: knol.google.com

Electricity Consumption

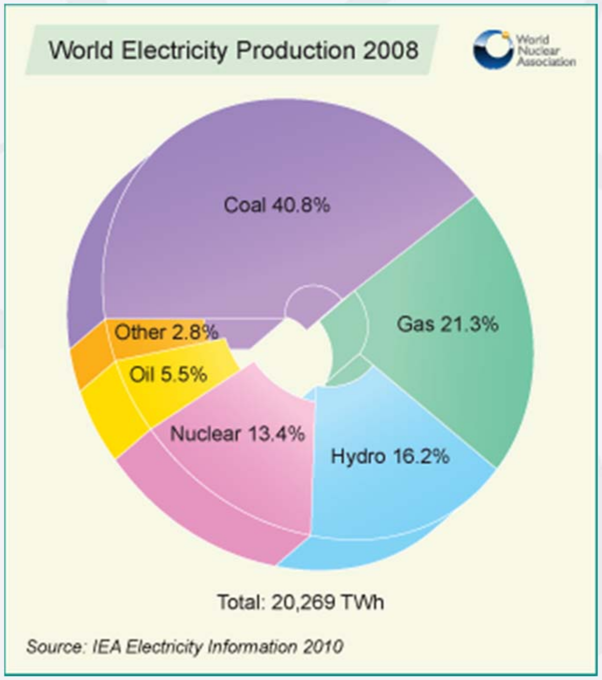
Increases with human development
Then increases more.

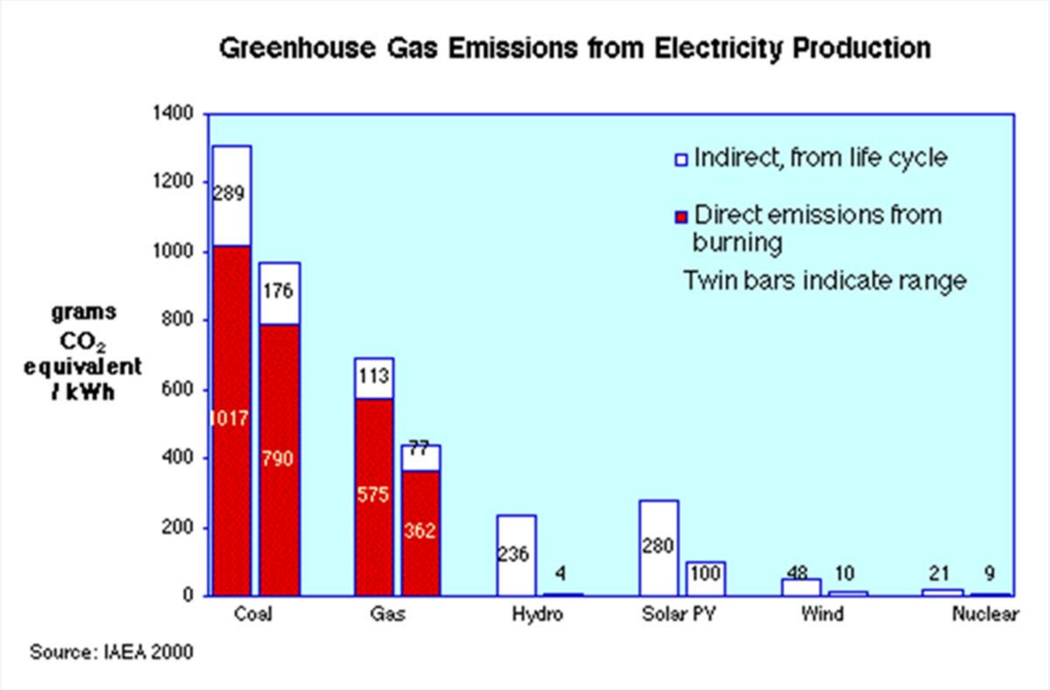


(ref: Pasternak, Global 2001)

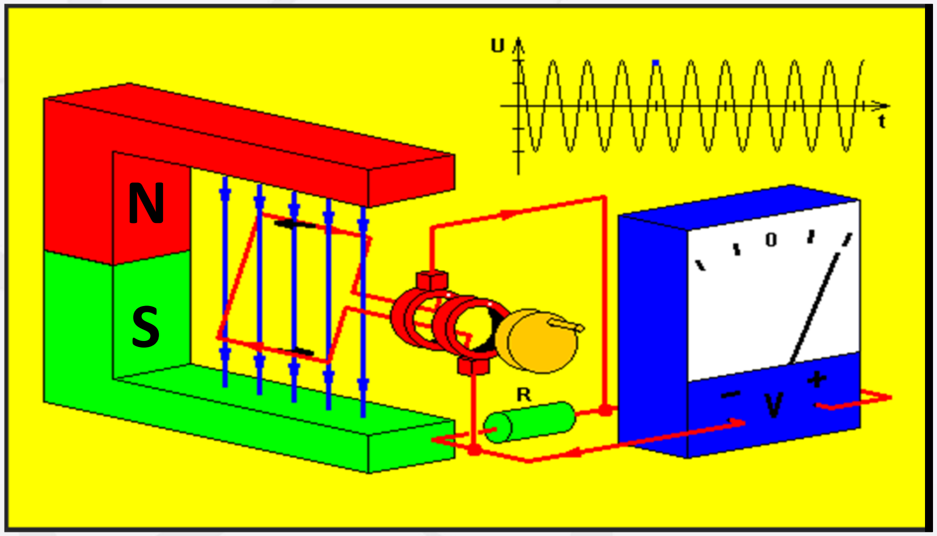

World Electricity Consumption by Region
World Nuclear Association
35,000
30,000
25,000
20,000
15,000
10,000
5,000
TWh
1980
1990
2000
2010
2020
2030
Year
Latin America
Middle East & Africa
Asia
E. Europe/Eurasia
OECD
Source: OECD/IEA World Energy Outlook 2009 - Reference Scenario

Sources of Electricity and Their Associated Greenhouse Gas Burdens

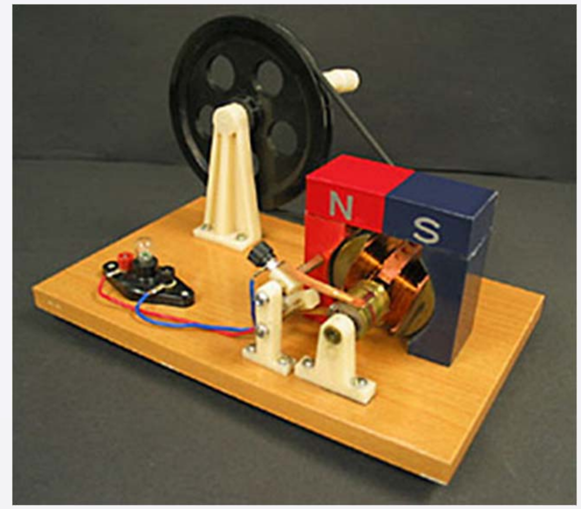




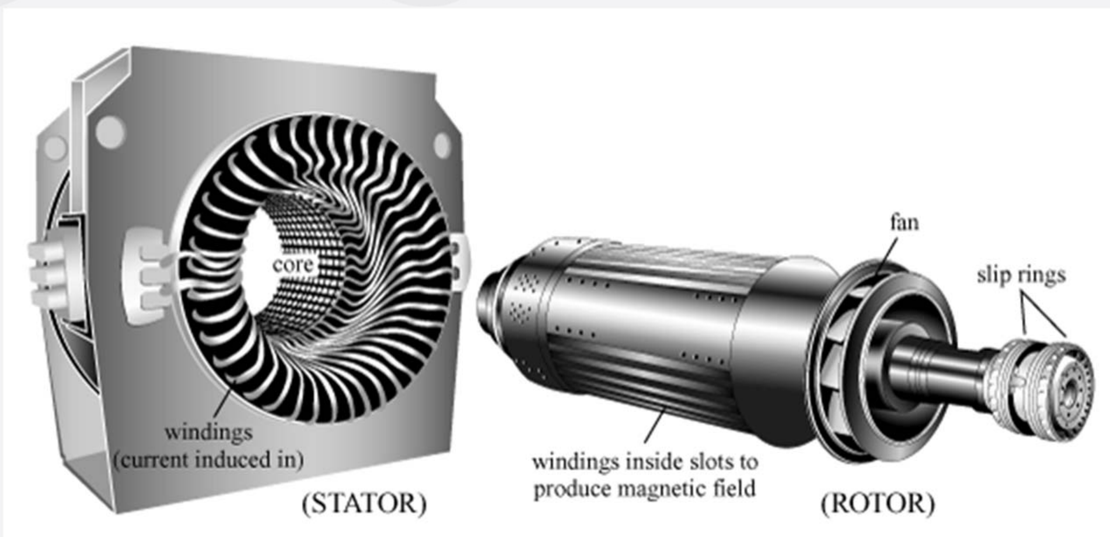
How do We Generate Electric Power?



http://www.generatorguide.net/howgeneratorworks.html

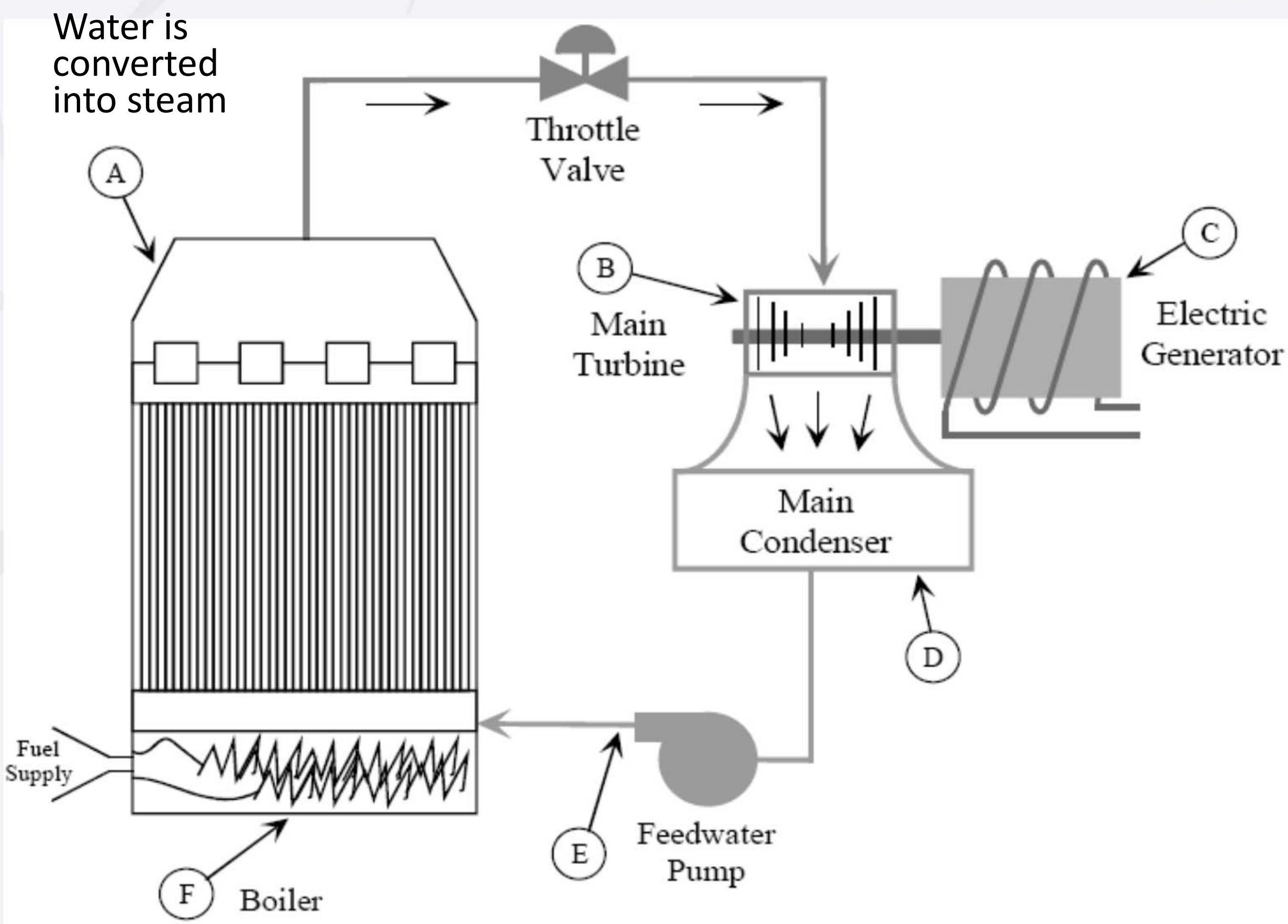


Source: www. hk-phy.org



Source: www. hk-phy.org

Fossil Fuel Steam Plant



Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC
http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

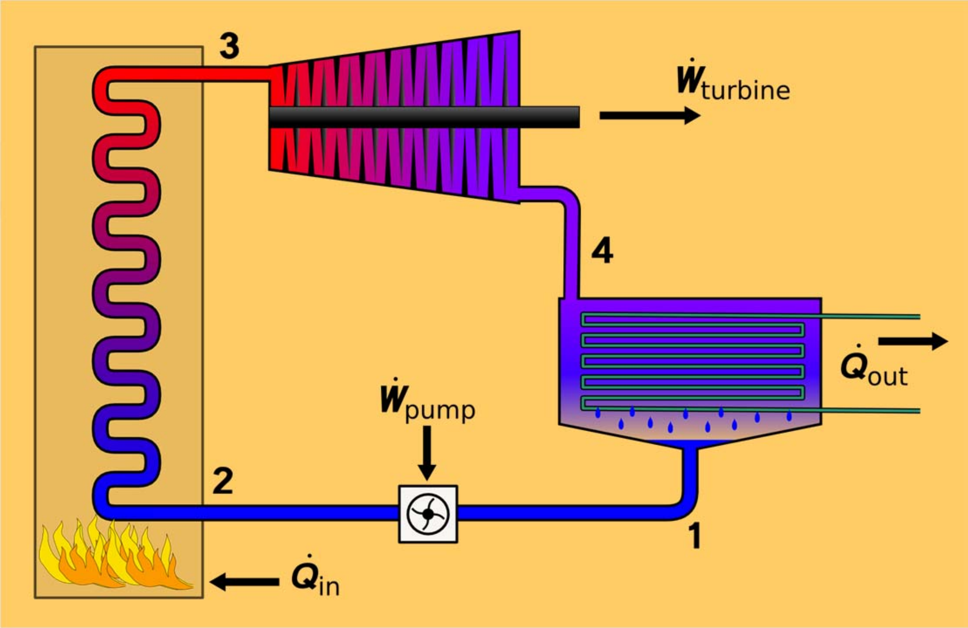
Rankine Cycle

1. The Rankine cycle converts heat (thermal energy) into mechanical energy, which we then convert to electrical energy by using the turbine shaft to drive a generator.
2. The heat is supplied externally to a closed loop containing the "working fluid" (usually water).
3. The Rankine cycle generates about 80% of all electric power used throughout the world, including virtually all solar thermal, biomass, coal, and nuclear power plants.
4. The Rankine cycle is the fundamental thermodynamic underpinning of the steam engine.
5. It is named after William John Macquorn Rankine, a Scottish polymath.

Source: Wikipedia

Processes in the Rankine Cycle

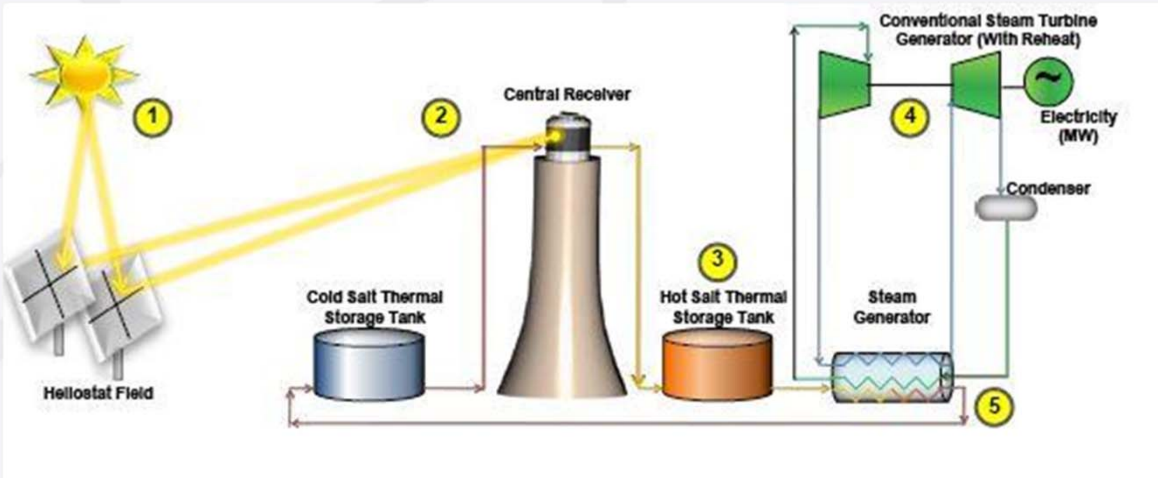
- **Process 1-2**: Working fluid pumped from low to high pressure. As the fluid is a liquid at this stage, the pump requires little input energy.
- **Process 2-3**: High pressure liquid enters a boiler where it is heated at constant pressure by an external heat source to become a dry saturated vapor.



Source: Andrew.Ainsworth at the English Wikipedia project

- **Process 3-4**: Dry saturated vapor expands through a turbine, generating power. This expansion process decreases the temperature and pressure of the vapor, and some condensation may occur.
- **Process 4-1**: Wet vapor then enters a condenser where it is condensed at a constant pressure to become a saturated liquid.

Concentrating Solar Power Plant



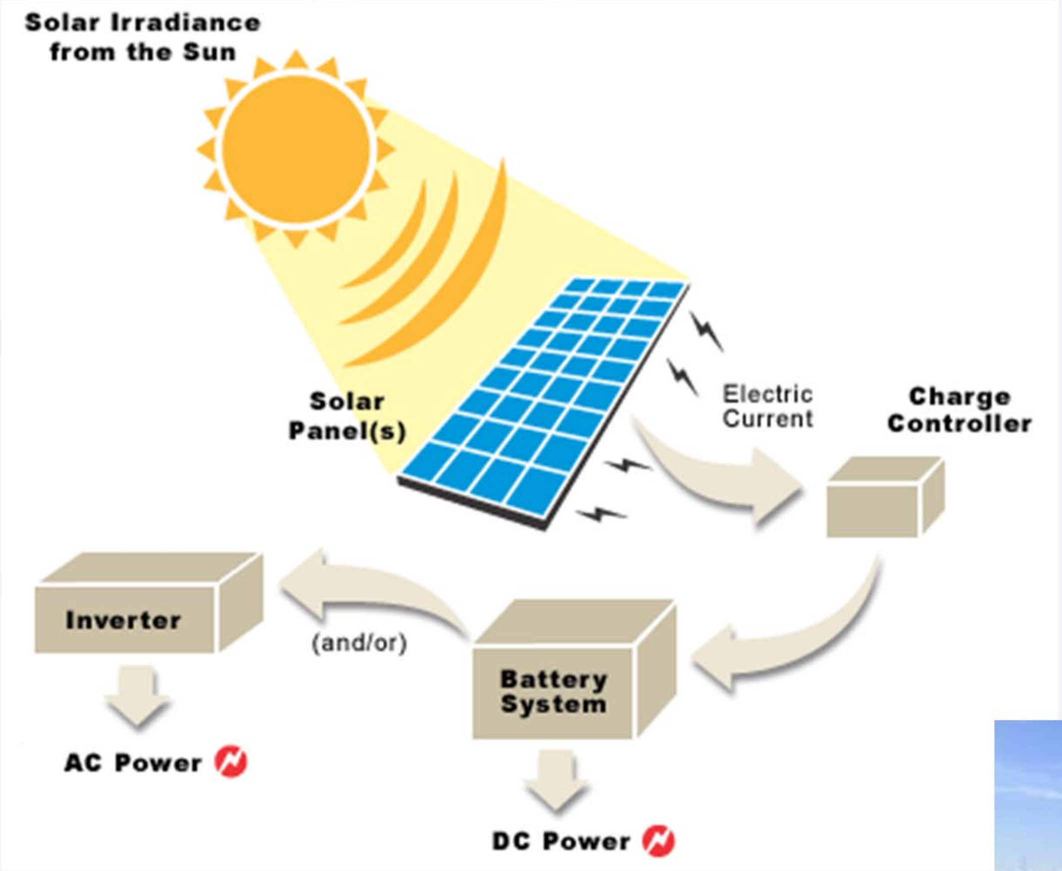
Process Flow Description

1. Sunlight is concentrated and directed from a large field of heliostats (mirrors) to a receiver on a tall tower.
2. Molten salt from the cold salt tank is pumped through the receiver where it is heated to 1050 °F (566 °C).
3. The heated salt from the receiver is stored in the hot salt tank.
4. Molten salt is pumped from the hot salt tank through a steam generator that creates steam, which drives a steam turbine, generating electricity.
5. Cold salt at 525°F (288 °C) flows back to the cold salt tank and is re-used.

Source: Politics in the Zeros

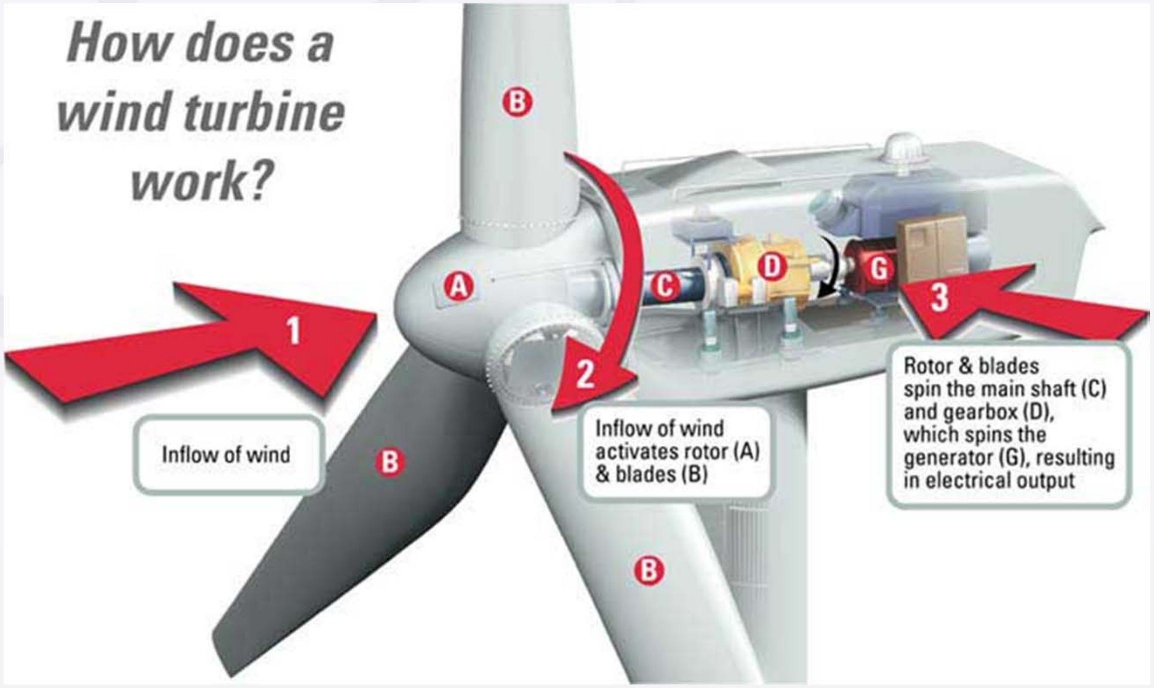
Source: Sandia National Labs

Generating Electricity Using Photovoltaics



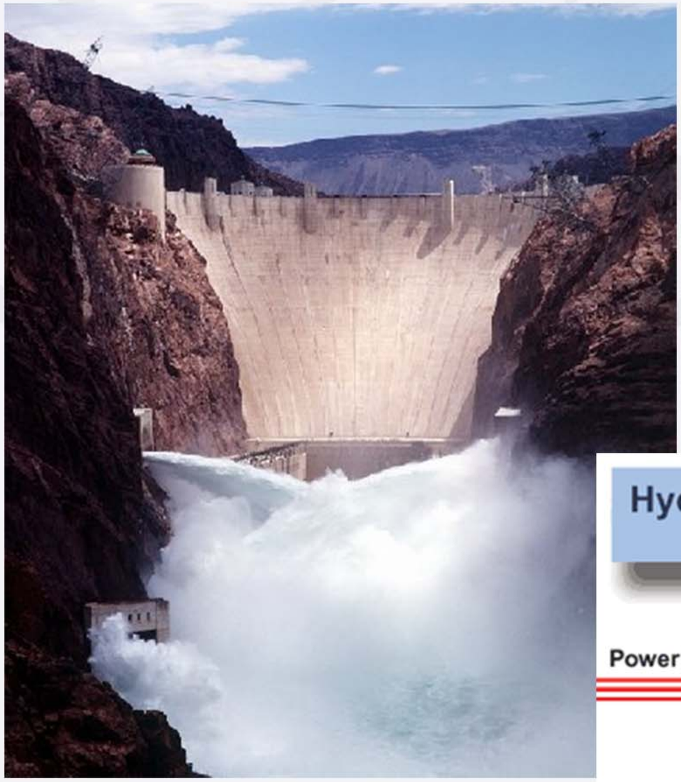
Source: Solar System Facts

Generating Electricity From Wind Power

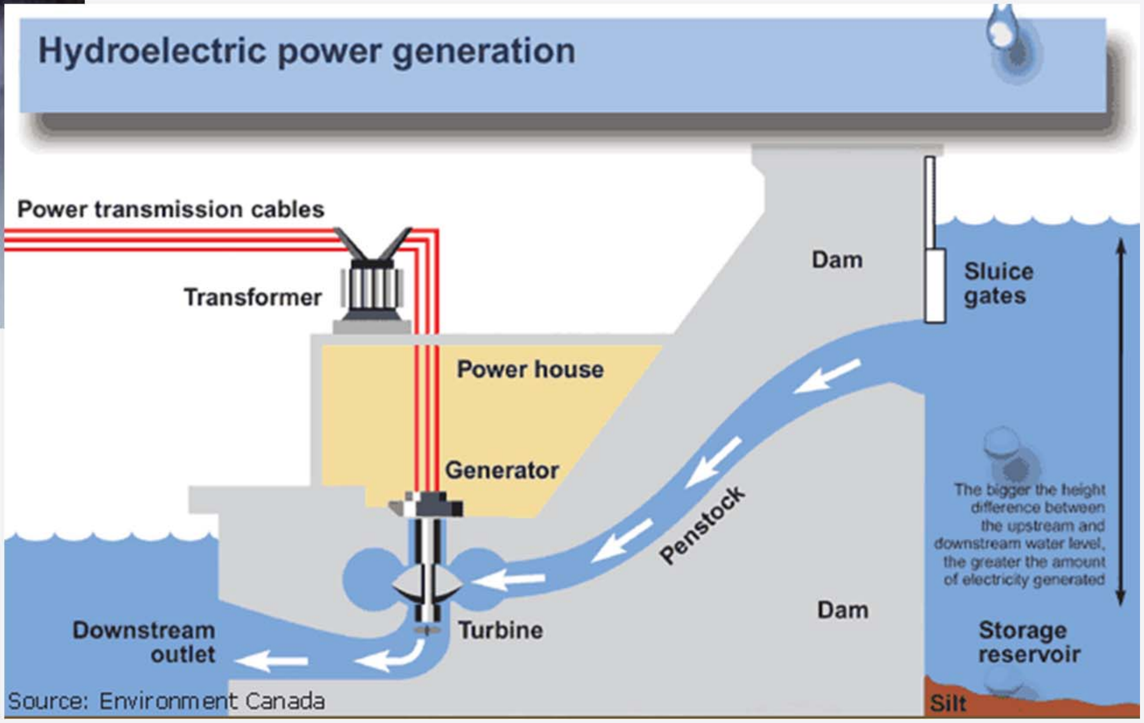


Source: Chromebook, Interesting Engineering Facts

Generating Electricity From Water Power

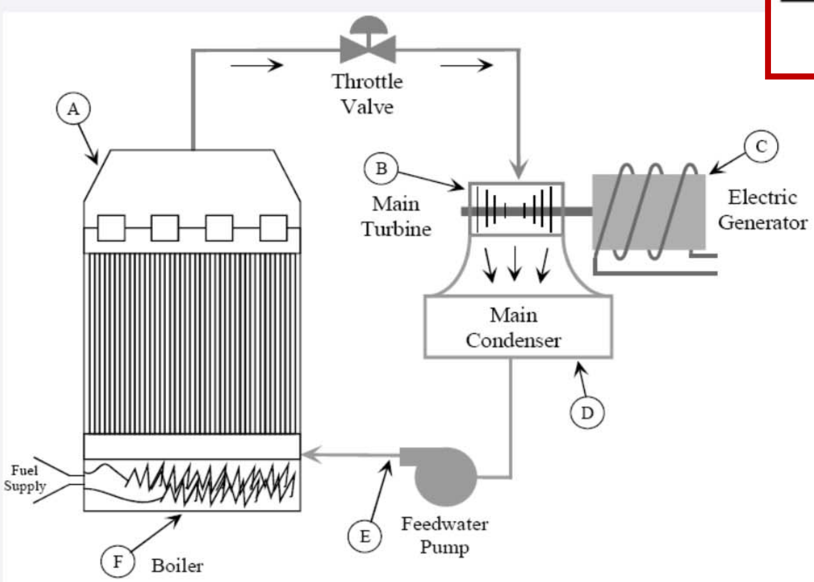


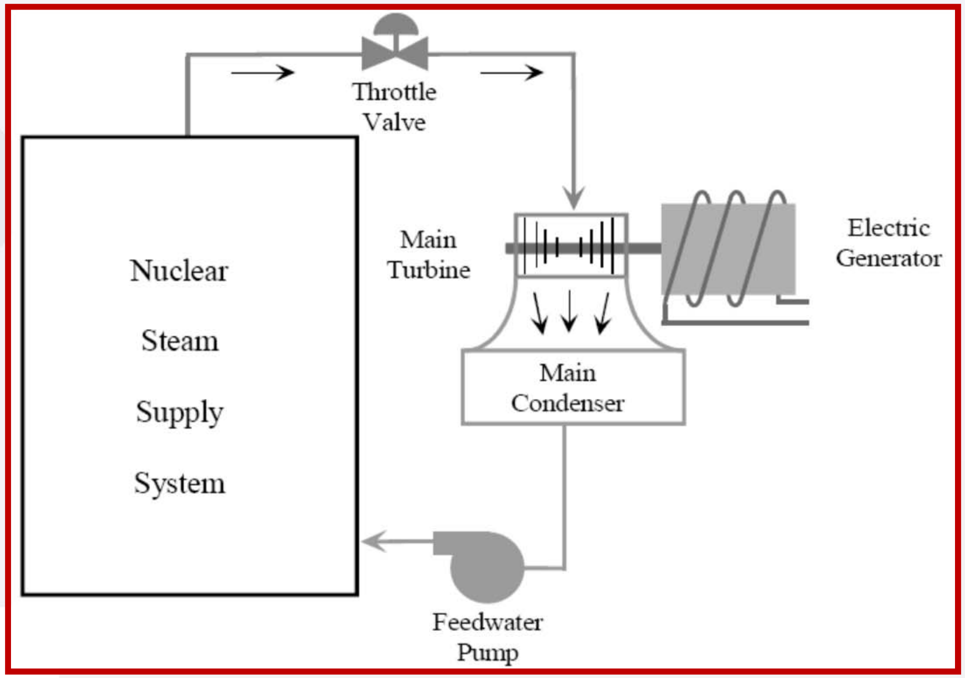
Source: Famous Landmarks Image Gallery



Nuclear Fueled Steam Plant

Fossil Fueled Steam Plant





Nuclear Fueled Steam Plant

Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC
http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

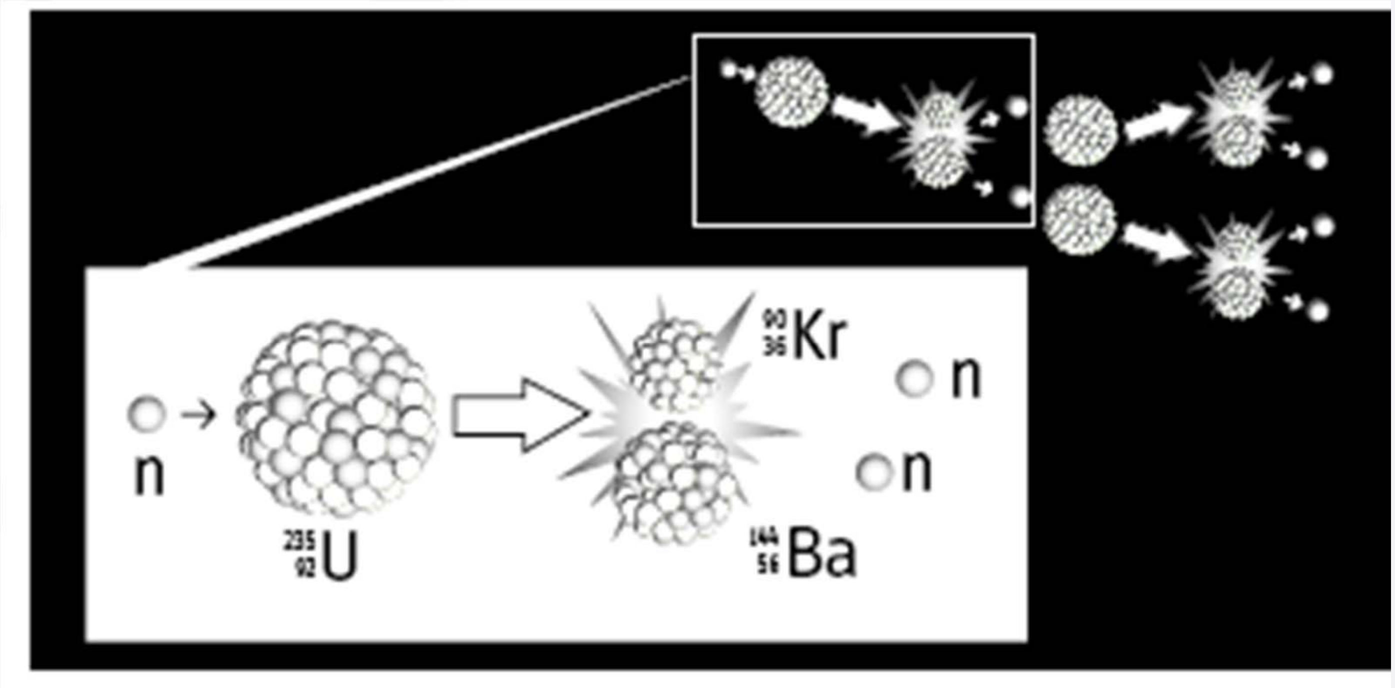
Nuclear Steam Supply System

- In nuclear power plants, the Nuclear Steam Supply System serves the same function as the steam boiler in a fossil-fuel plant.

- Nuclear Steam Supply System (NSSS) includes
  - nuclear reactor
  - steam generator
  - and other components necessary to produce high pressure steam

- Steam is used to drive steam turbines coupled to an electric generator
  - steam is always delivered to the turbines as dry as possible
  - Pumps and condenser return feed water to steam generator

Nuclear Fission Energy ($E = mc^2$)



Fissioning 1 kg. U-235
Equivalent to Burning
3,000 tons Coal

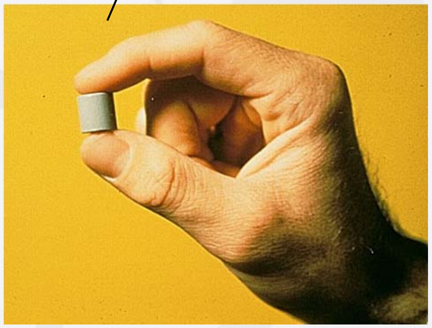
<u>Uranium in nature</u>

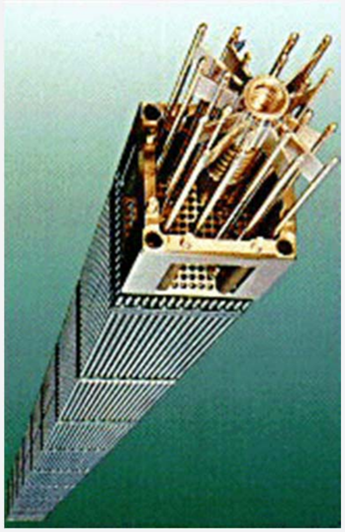
99.3 percent U-238

0.72 percent U-235

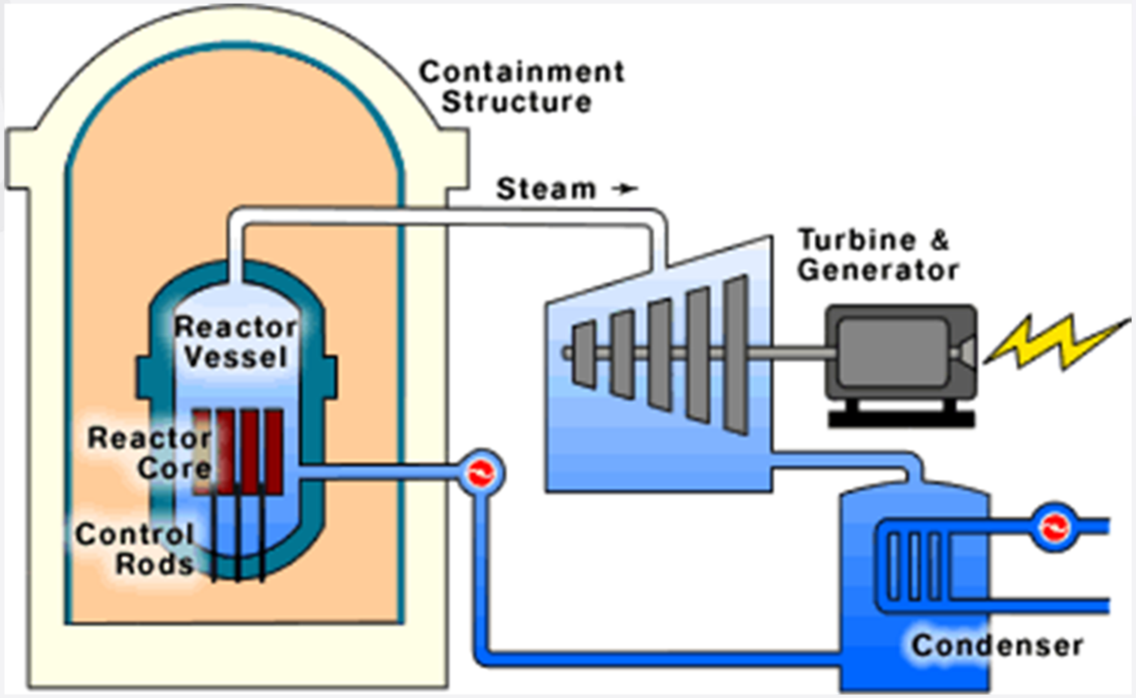
Where the Heat Comes From

UO_2 fuel pellet. Enriched to 3-5 percent U-235

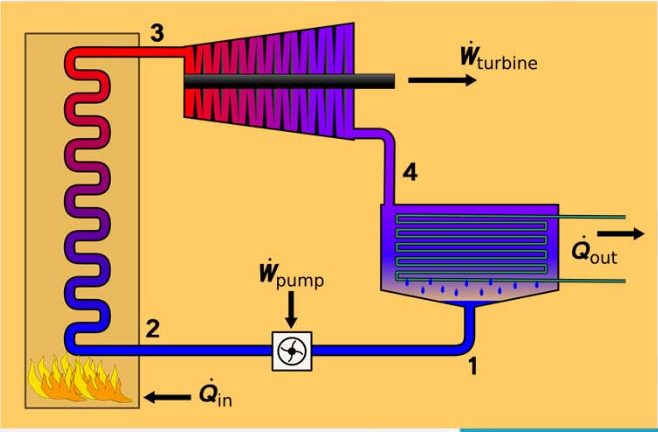


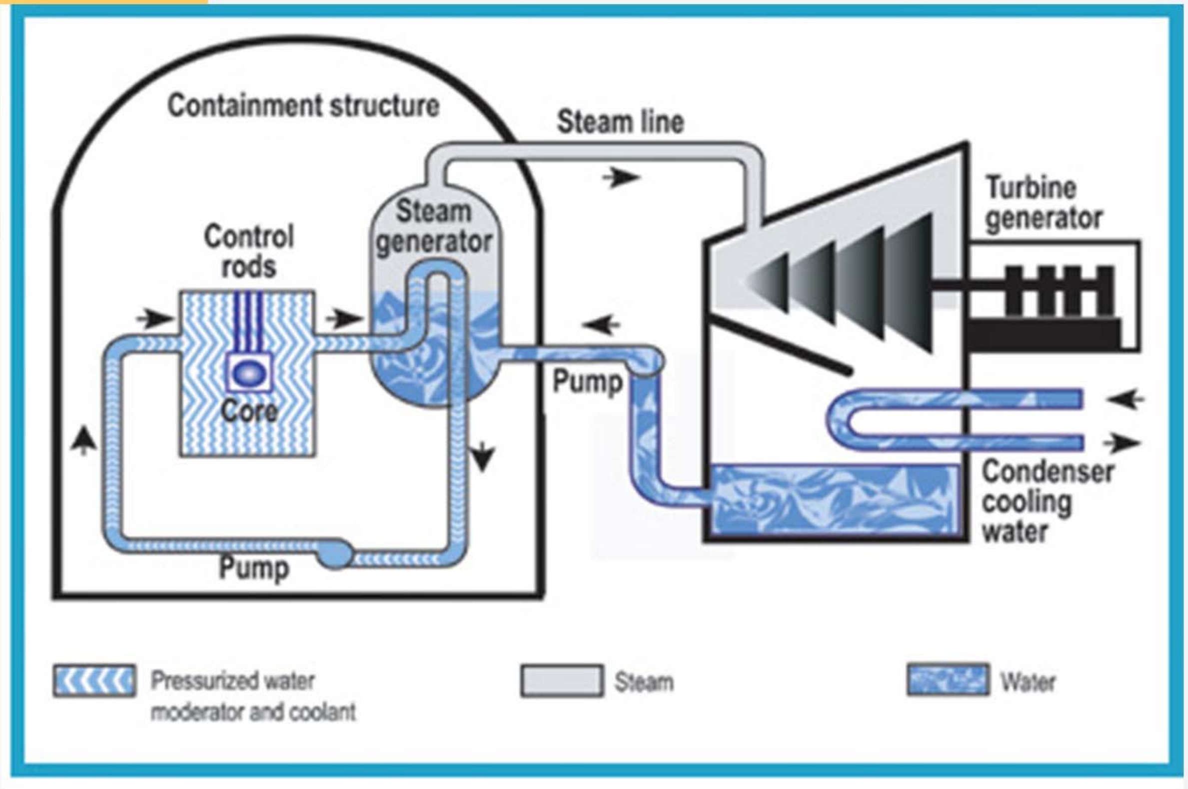


Typical Nuclear Power System



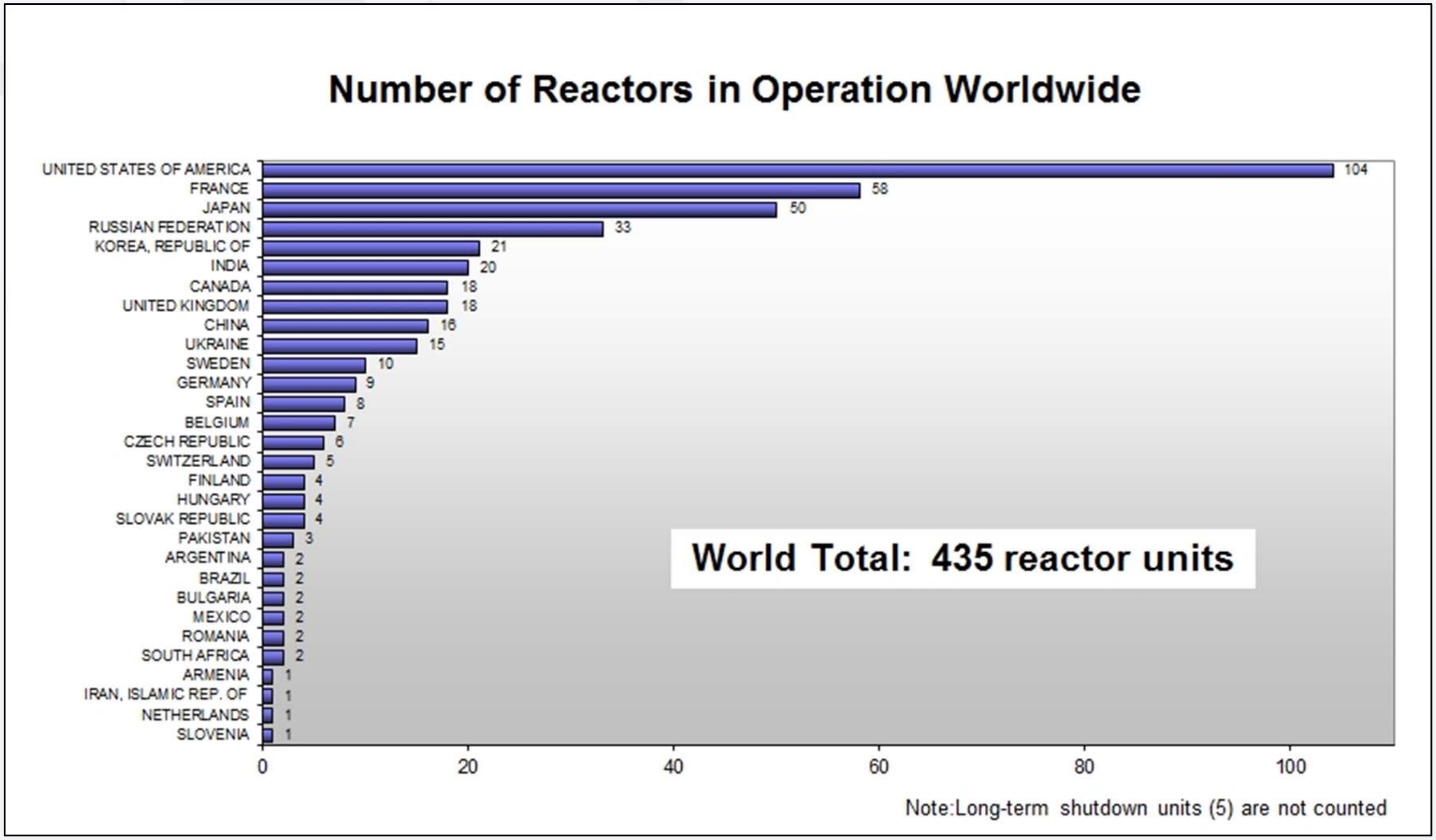
Rankine Cycle with Nuclear Steam Supply System (NSSS)



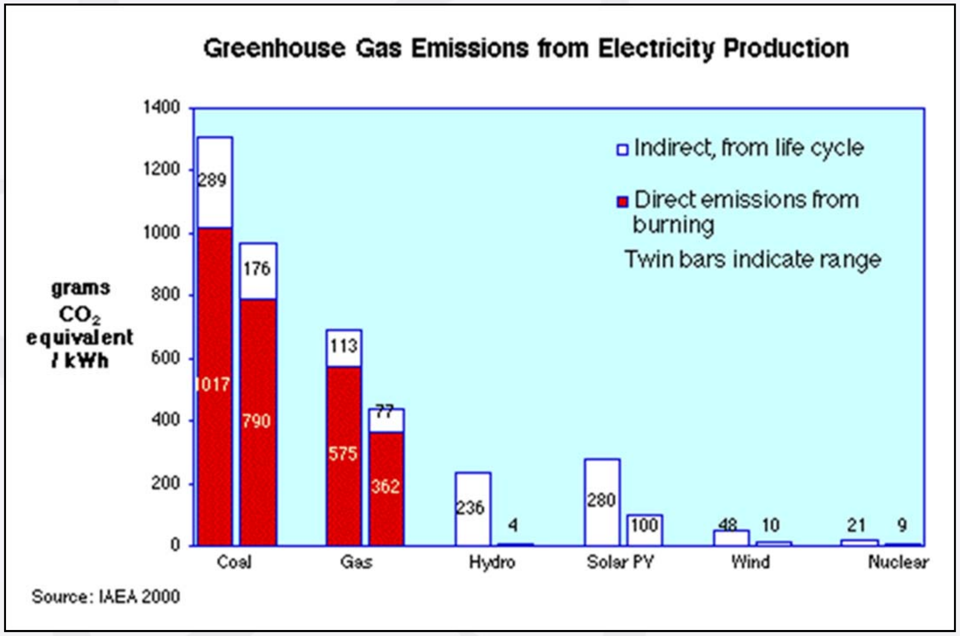


Current Status of Nuclear Power

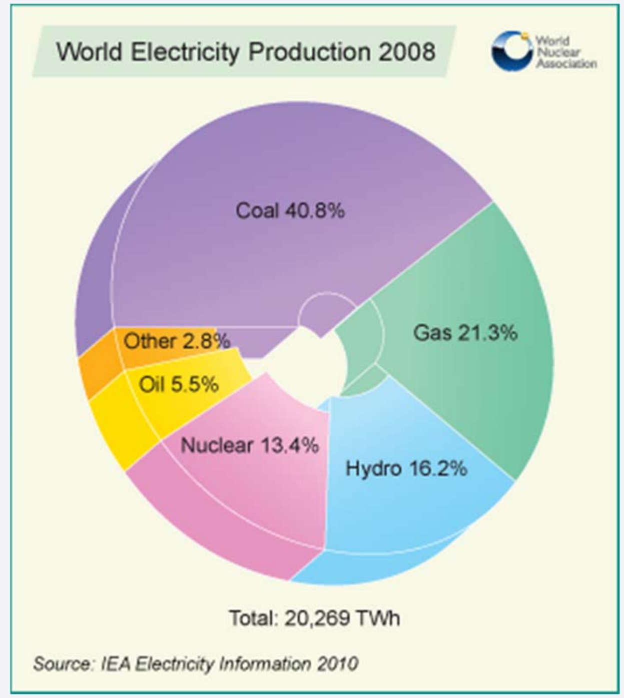
435 nuclear power plants in operation, providing 17% of total electricity production



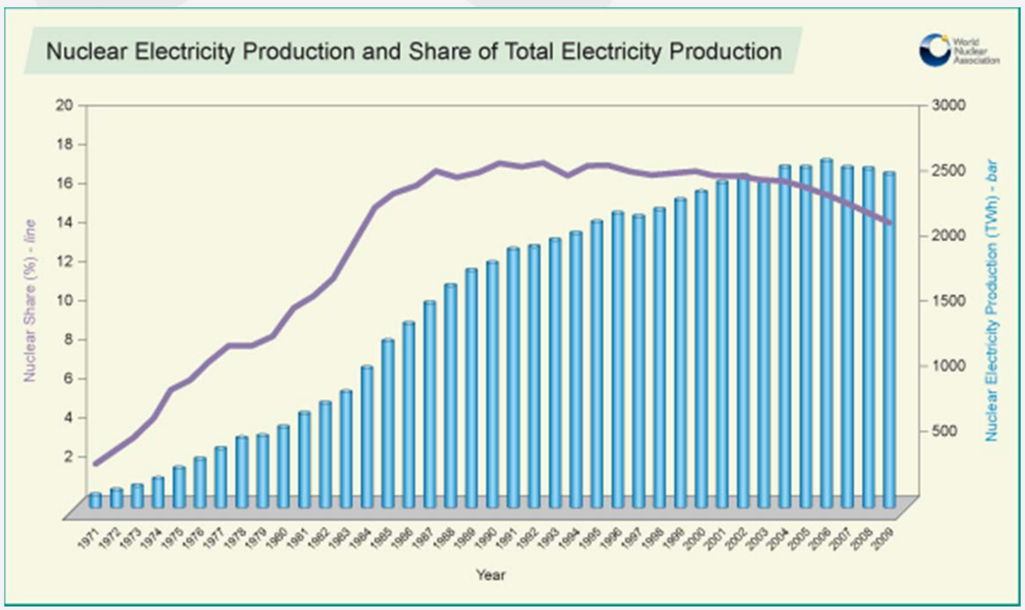
Source: IAEA – Nuclear Power Information



Greenhouse Gas Emissions from Electricity Production
grams CO_2 equivalent / kWh
1400
1200
1000
800
600
400
200
0
Indirect, from life cycle
Direct emissions from burning
Twin bars indicate range
289
1017
176
790
113
575
77
362
236
4
280
100
48
10
21
9
Coal
Gas
Hydro
Solar PV
Wind
Nuclear
Source: IAEA 2000



World Electricity Production 2008
World Nuclear Association
Coal 40.8%
Gas 21.3%
Other 2.8%
Oil 5.5%
Nuclear 13.4%
Hydro 16.2%
Total: 20,269 TWh
Source: IEA Electricity Information 2010



Nuclear Electricity Production and Share of Total Electricity Production
World Nuclear Association
Nuclear Share (%) - line
Nuclear Electricity Production (TWh) - bar
20
18
16
14
12
10
8
6
4
2
3000
2500
2000
1500
1000
500
Year

Nuclear Reactors

- **Nuclear reactors rely on a controlled nuclear chain reaction to supply energy**
- **This energy can provide:**
  - Heat
  - Electricity
  - Radiation
- **Three main reactor categories:**
  - Power
  - Research (e.g., medical isotope production, basic physics research, nuclear fuels testing)
  - Propulsion

Module 1: Nuclear Energy Fundamentals (Week 2/Day 1)

Lecture #2: Types and Components of Nuclear Power Plants

Dr. David Boyle

Lecture Outline

- Types of Nuclear Reactors
- Pressurized vs. Boiling Water Reactors
- Nuclear Power Plant Components

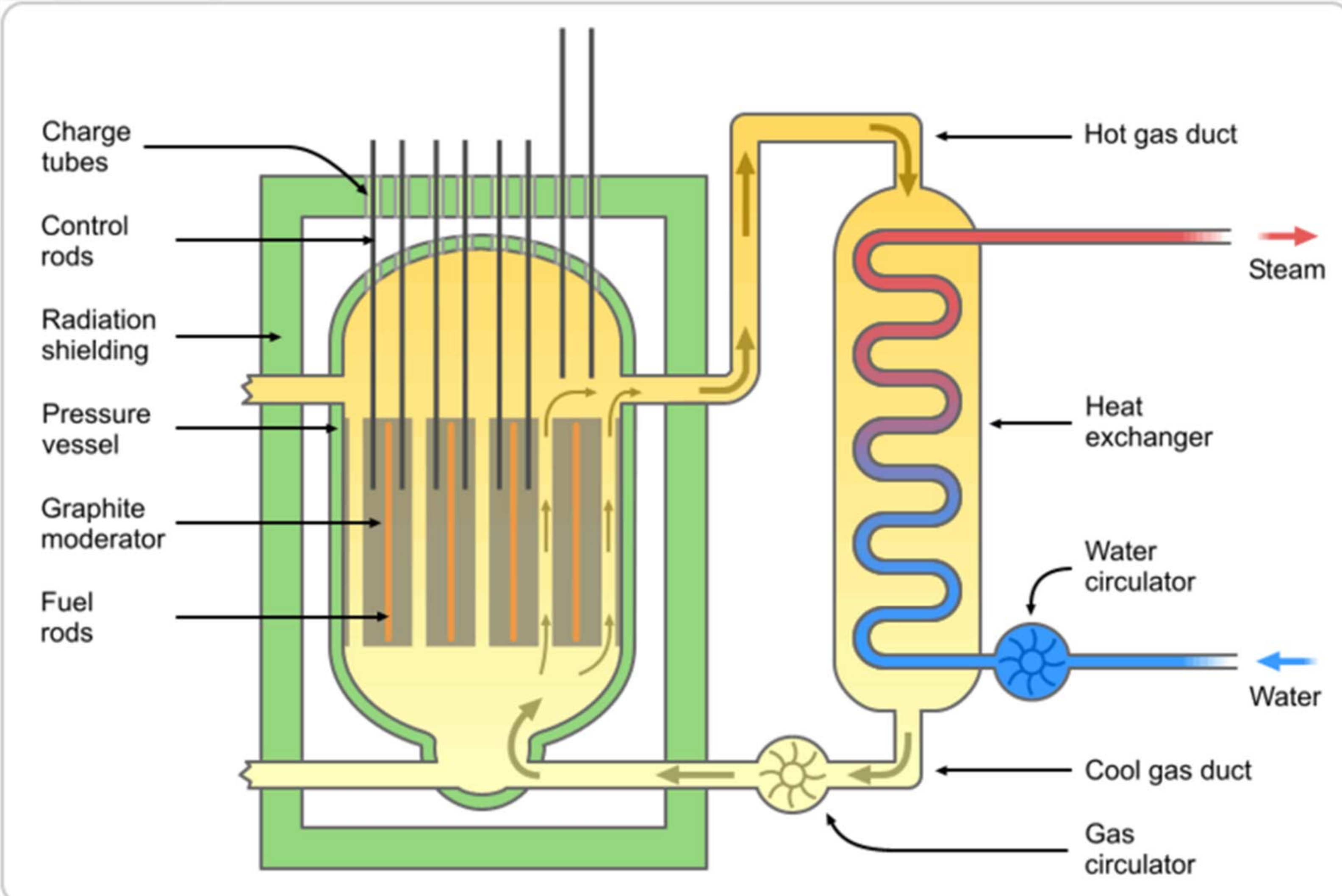
Types of Nuclear Power Reactors

- **Power Reactors are classified by**
  - neutron energy
    - Thermal reactors: majority of neutron interactions involve thermal neutrons (energy < 1 eV)
    - Fast reactors: majority of neutron interactions involve fast neutrons (energy > 1 MeV)
  - Type of moderator
  - Type of coolant
  - Type of fuel

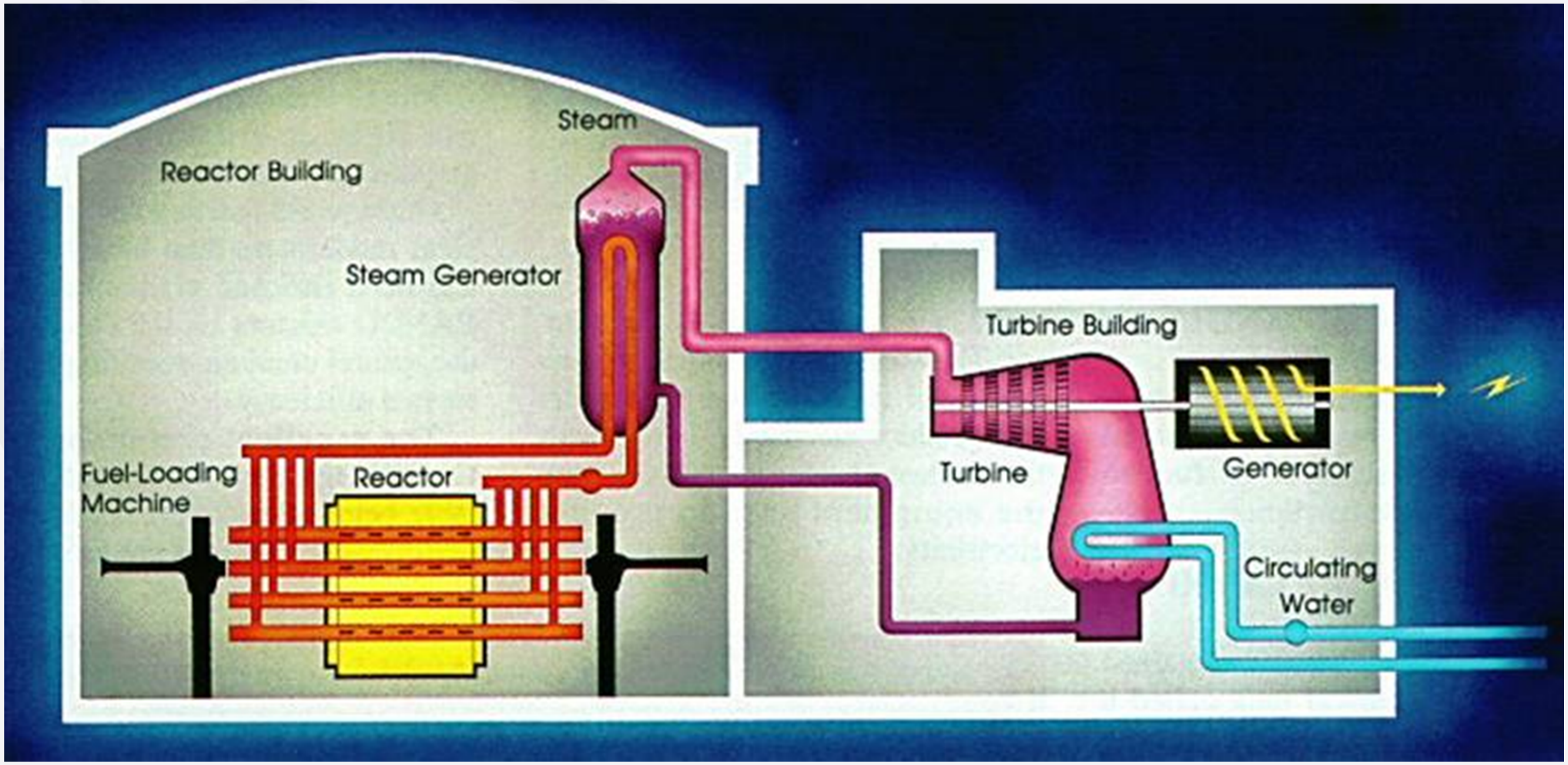
Main Types of Nuclear Power Reactors

- Gas-cooled Reactors (GCRs); may be "Thermal" or "Fast"
- Heavy Water Reactors (e.g., Canadian Deuterium-Uranium Reactor (CANDU))
- Light Water Reactors (LWRs)
  - Pressurize Water Reactor (PWR)
  - Boiling Water Reactor (BWR)

Gas-Cooled Reactor -- Magnox CO_2



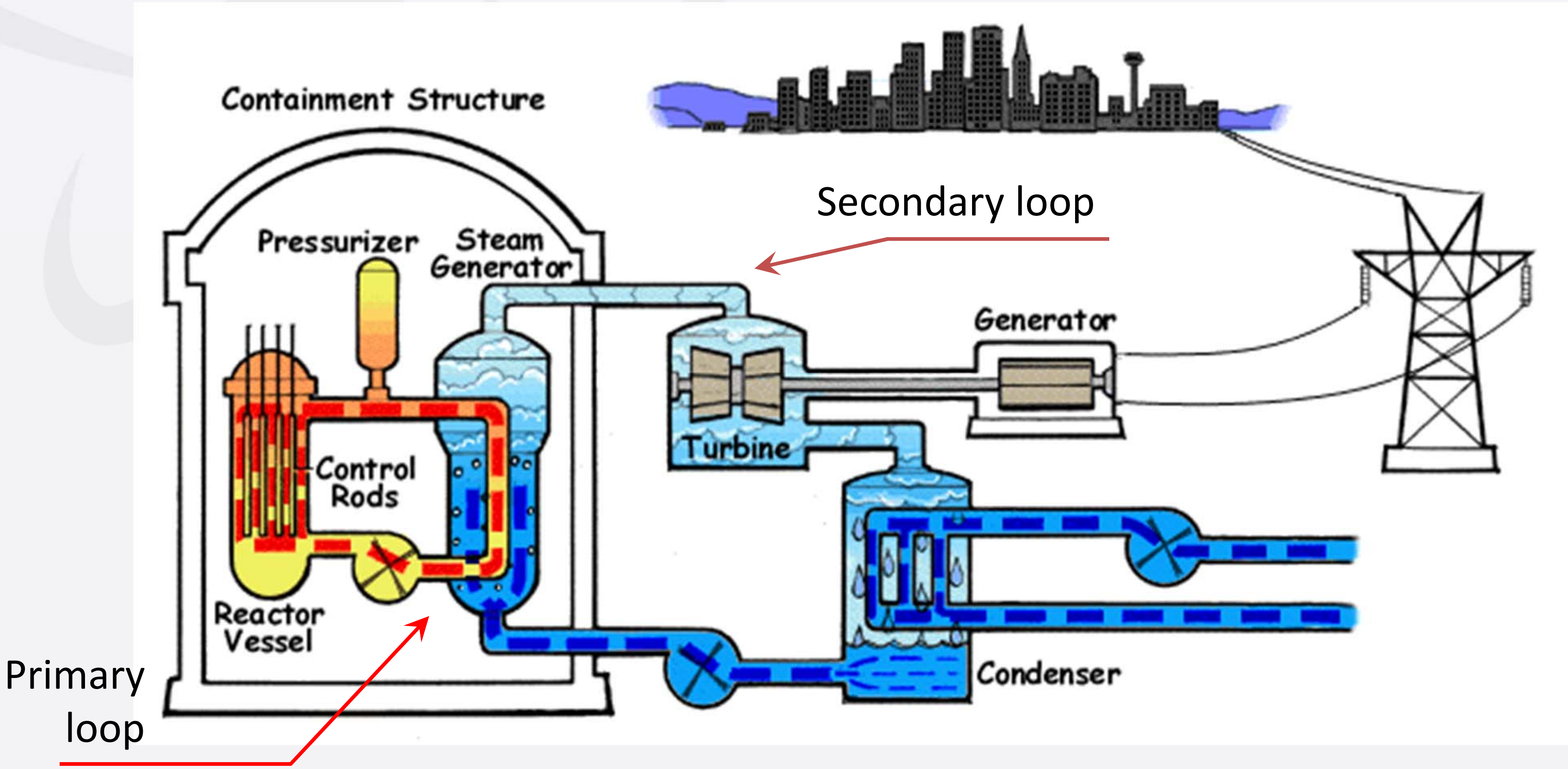
Canadian Deuterium-Uranium Reactors



Source: Unknown

Pressurized Water Reactors

- Pressurized water heats, but does not boil
- Two major systems used to convert the heat into electrical power: primary system and secondary system



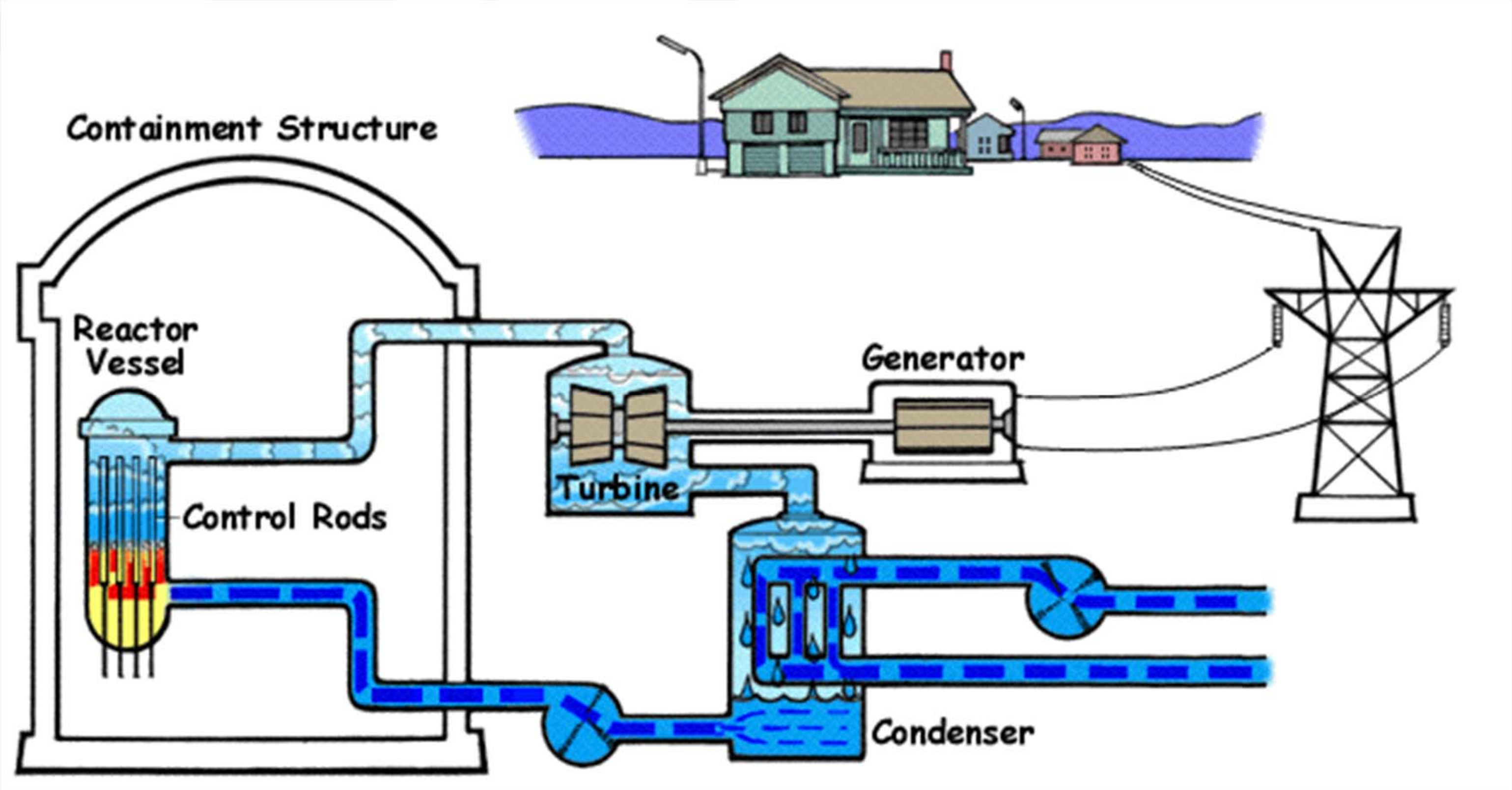
Source: NRC http://www.nrc.gov/reading-rm/basic-ref/students/animated-pwr.html

Pressurized Water Reactors (continued)

- Primary system (primary loop) transfers the heat from the fuel to the steam generator
- Secondary system (secondary loop) transfers the steam formed in the steam generator to the main turbine
  - the turbine is attached to the electrical generator
- Then the steam is routed to the main condenser
  - cool water circulates through the tubes in the condenser
  - the excess heat is removed and the steam condenses to water which is pumped back to the steam generator
- Water from the reactor and the water in the steam generator never mix
  - most of the radioactivity stays in the reactor area

Boiling Water Reactors (BWR)

- BWRs boil the water directly in the core: water is converted to steam, and then recycled back into water by the condenser, to be used again to remove heat from the core.



Boiling Water Reactor Plant (continued)

- Inside the BWR vessel, a steam water mixture is produced when very pure water (reactor coolant) moves upward through the core absorbing heat.
  - The major difference in the operation of a BWR from other nuclear systems is the steam void formation in the core.
- The steam-water mixture leaves the top of the core and enters the two stages of moisture separation, where water droplets are removed before the steam is allowed to enter the steam line.

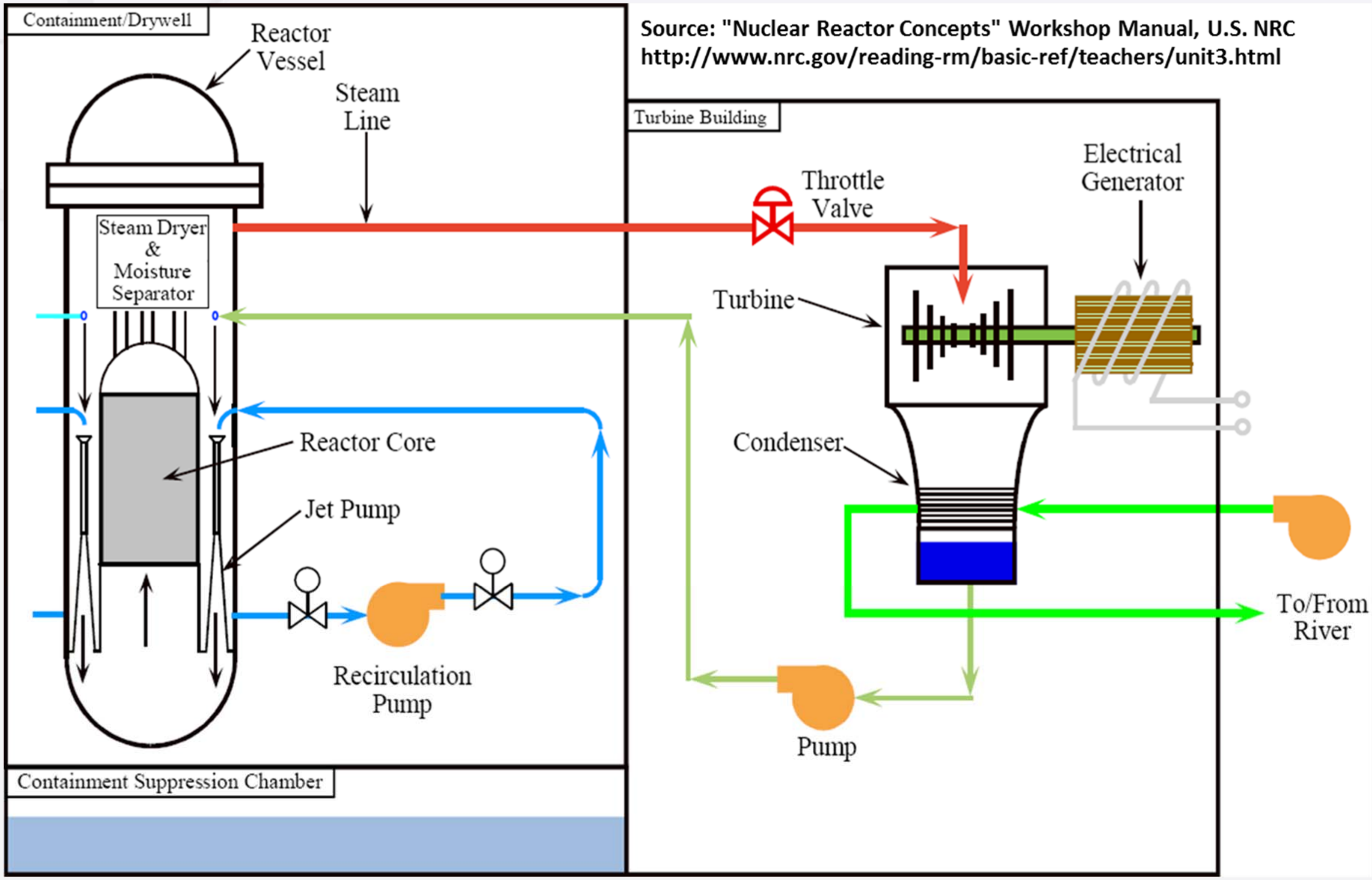
Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC
http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

Boiling Water Reactor Plant (continued)

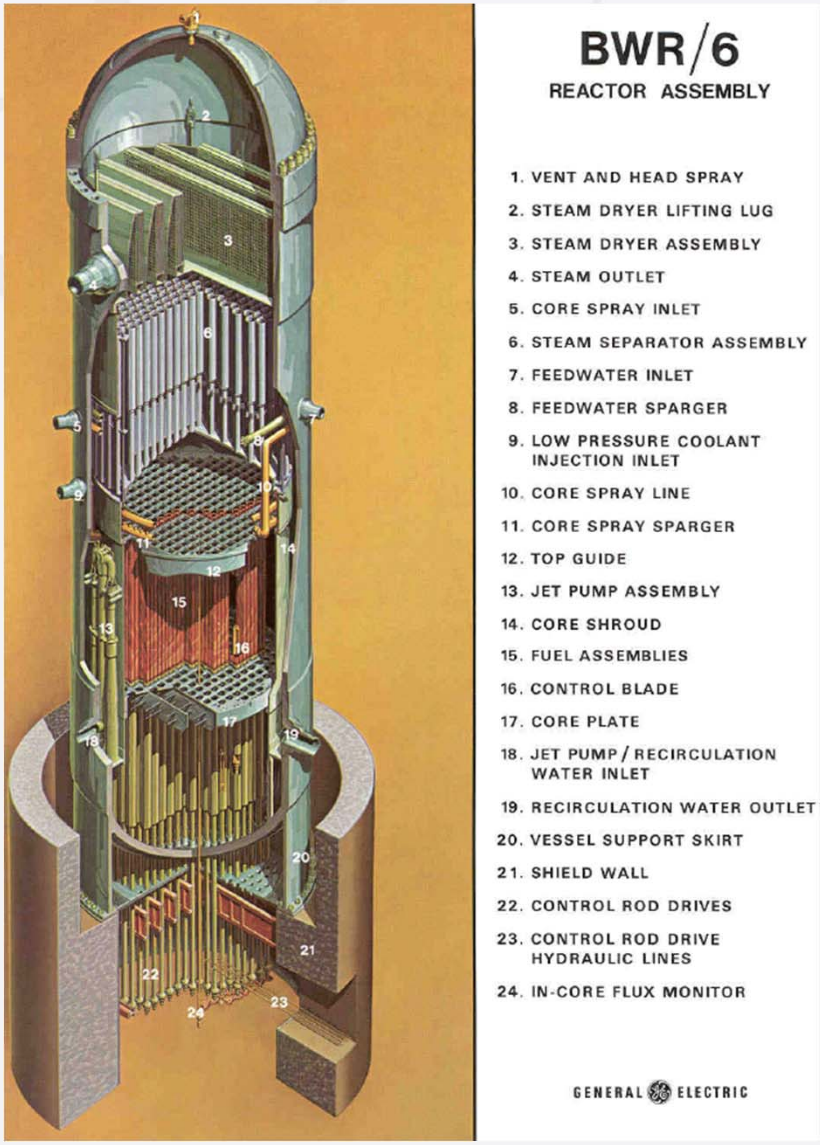
- The steam line directs the steam to the main turbine causing it to turn the turbine and the attached electrical generator.
- The low-pressure steam exiting the turbine enters the condenser where it is condensed into water. The resulting water is pumped out of the condenser with a series of pumps and back to the reactor vessel.
- The recirculation pumps and jet pumps allow the operator to vary coolant flow through the core and change reactor power.

Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC
http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

Boiling Water Reactor Plant



BWR: Reactor Vessel Assembly



- The reactor vessel assembly consists of the reactor vessel and its internal components, including
  - the core support structures,
  - core shroud,
  - moisture removal equipment, and
  - jet pump assemblies.

Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

Nuclear power plants in commercial operation (Source: http://www.world-nuclear.org/info/inf32.html)

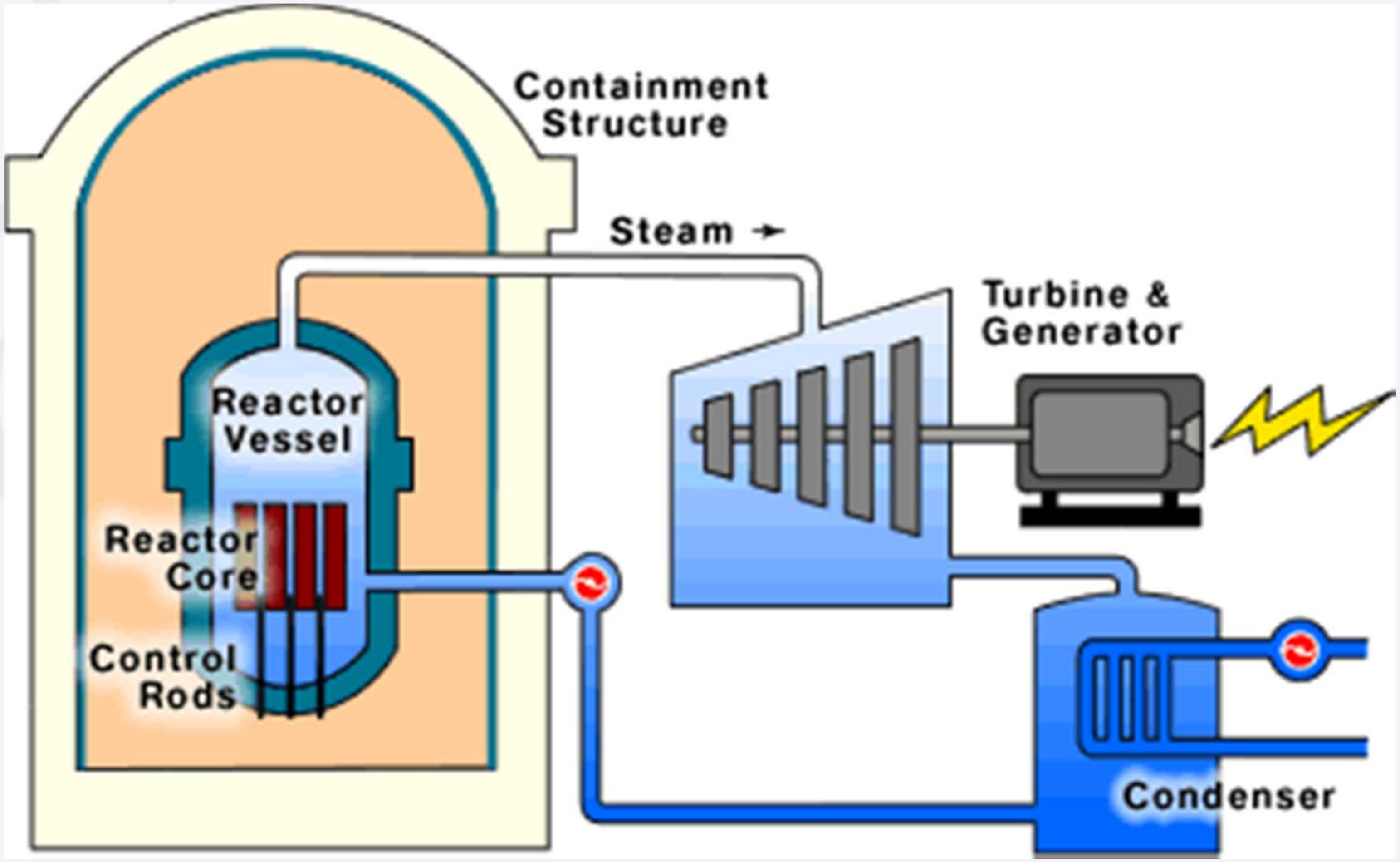
Reactor type	Main Countries	Number	GWe	Fuel	Coolant	Moderator
Pressurised Water Reactor (PWR)	US, France, Japan, Russia, China	265	251.6	enriched UO_2	water	water
Boiling Water Reactor (BWR)	US, Japan, Sweden	94	86.4	enriched UO_2	water	water
Pressurised Heavy Water Reactor 'CANDU' (PHWR)	Canada	44	24.3	natural UO_2	heavy water	heavy water
Gas-cooled Reactor (AGR & Magnox)	UK	18	10.8	natural U (metal), enriched UO_2	CO_2	graphite
Light Water Graphite Reactor (RBMK)	Russia	12	12.3	enriched UO_2	water	graphite
Fast Neutron Reactor (FBR)	Japan, France, Russia	4	1.0	PuO_2 and UO_2	liquid sodium	none
Other	Russia	4	0.05	enriched UO_2	water	graphite
	TOTAL	441	386.5			

GWe = capacity in thousands of megawatts (gross)

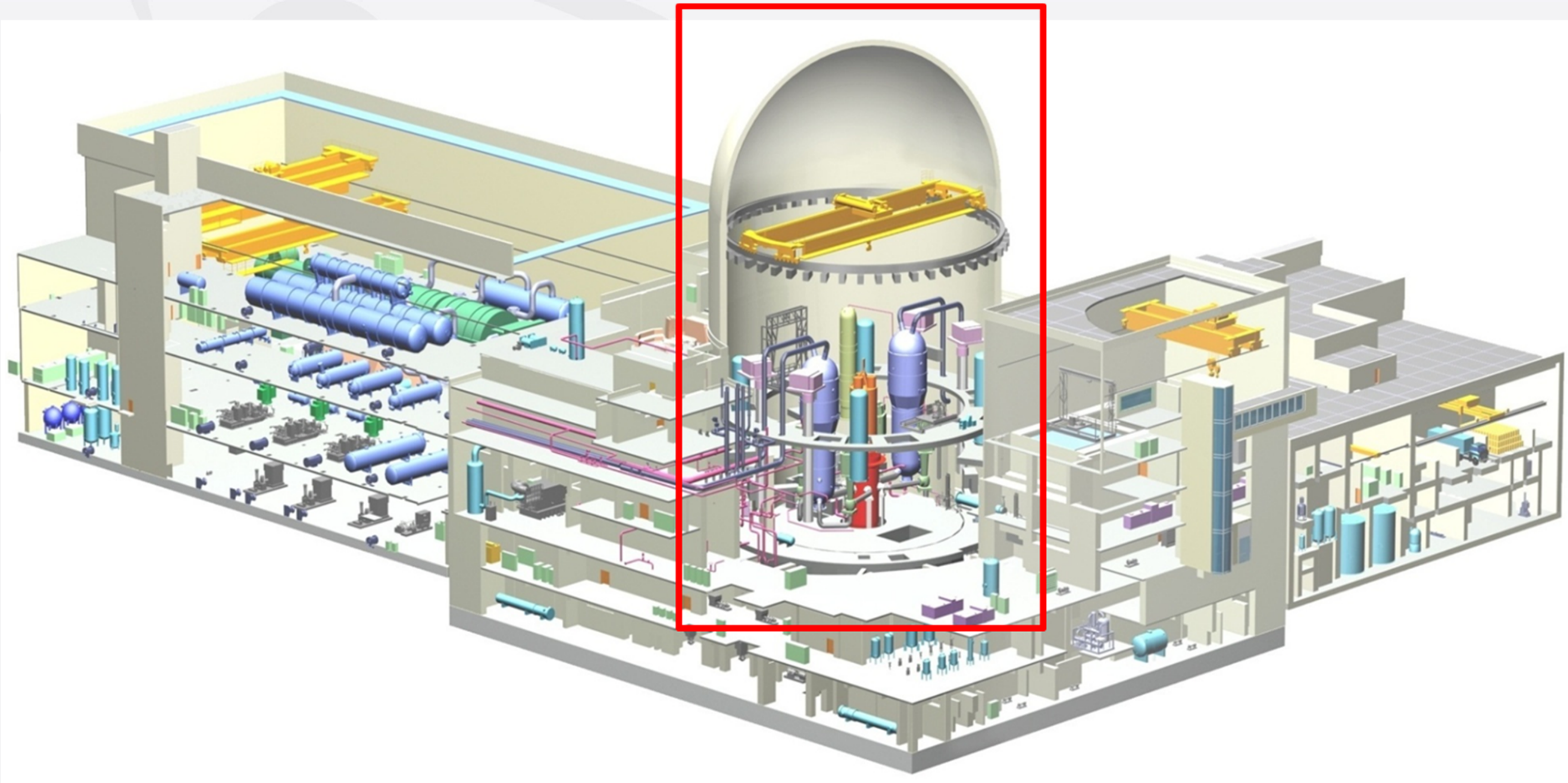
Source: Nuclear Engineering International Handbook 2008

For reactors under construction: see paper "Plans for New Reactors Worldwide" at http://www.world-nuclear.org/info/inf17.html

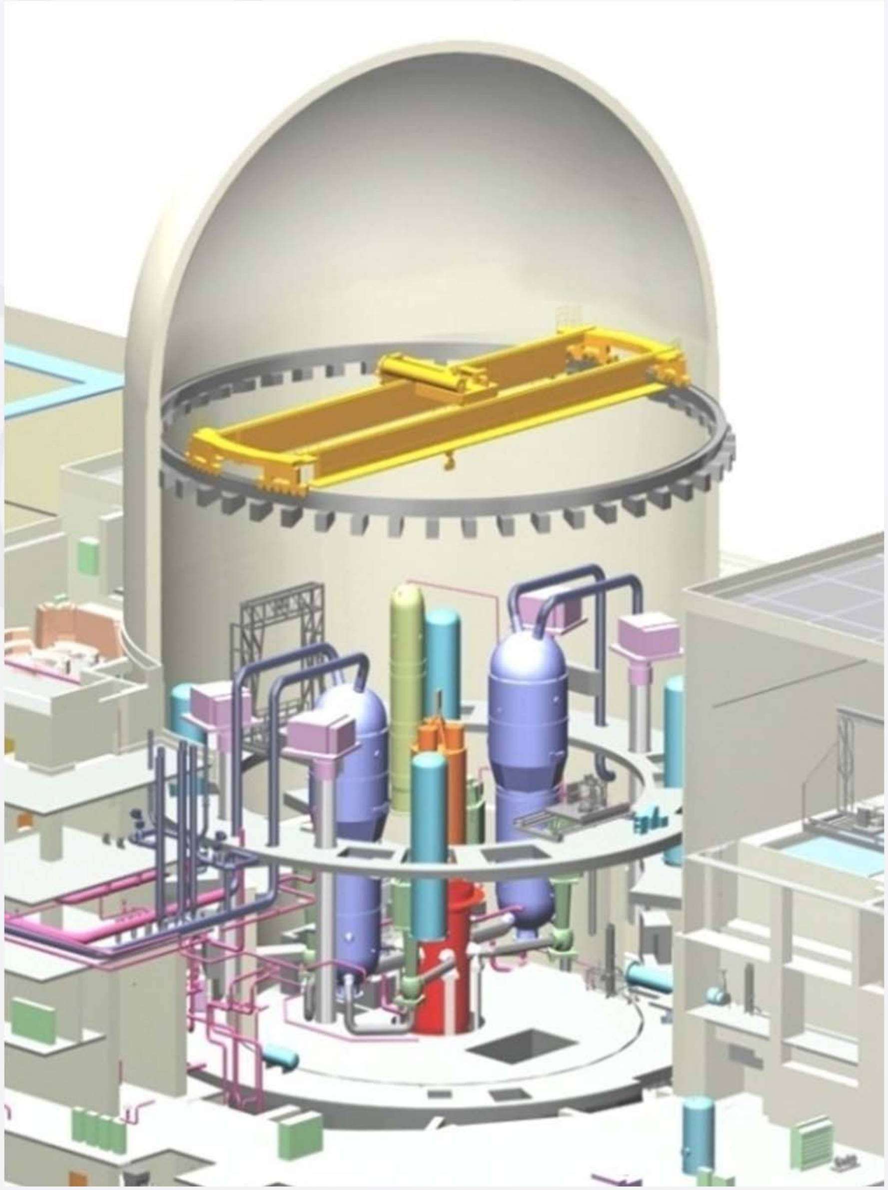
How does this typical nuclear power system make electricity?



APR1400 General Arrangement

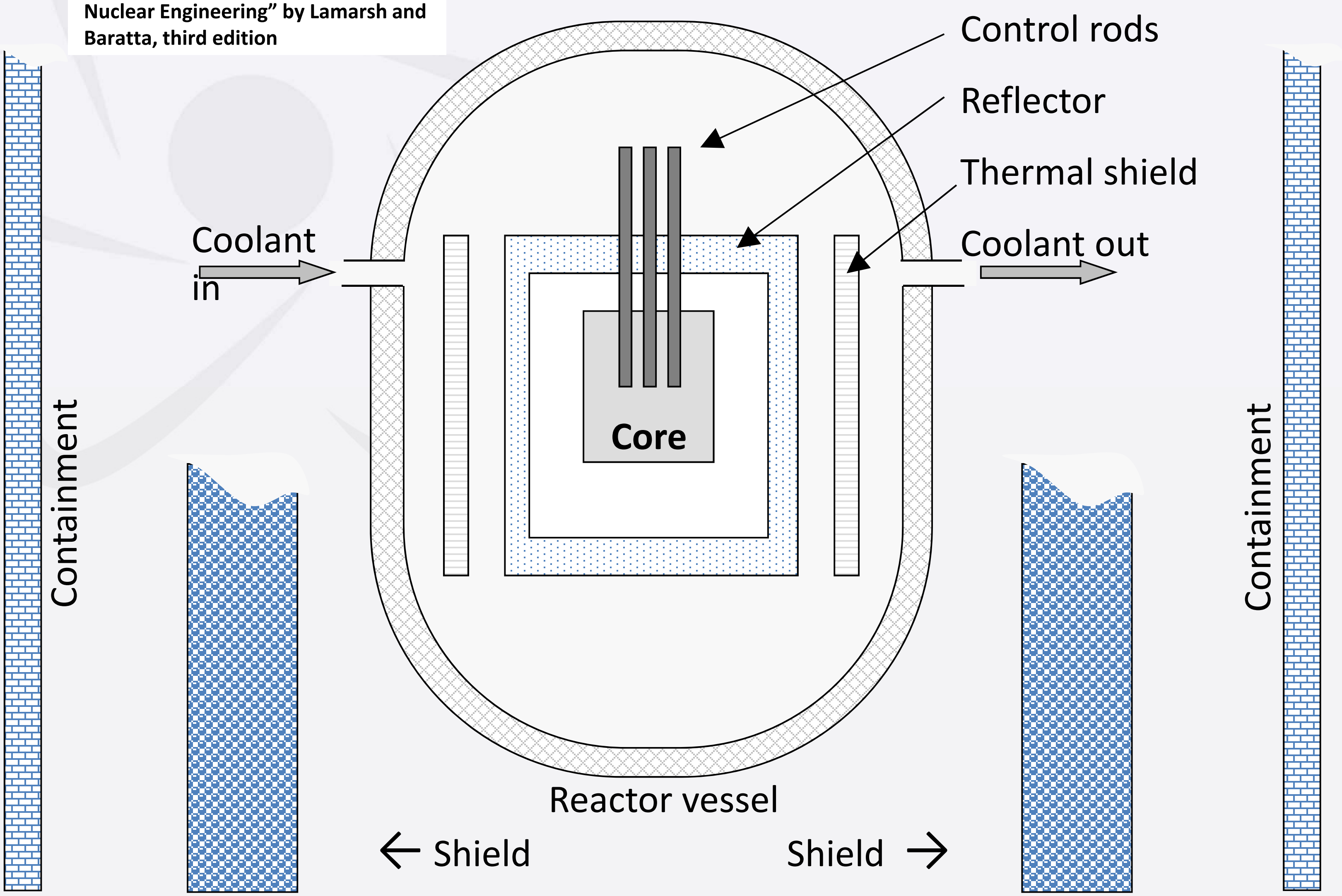


APR1400 Nuclear Steam Supply System in Containment



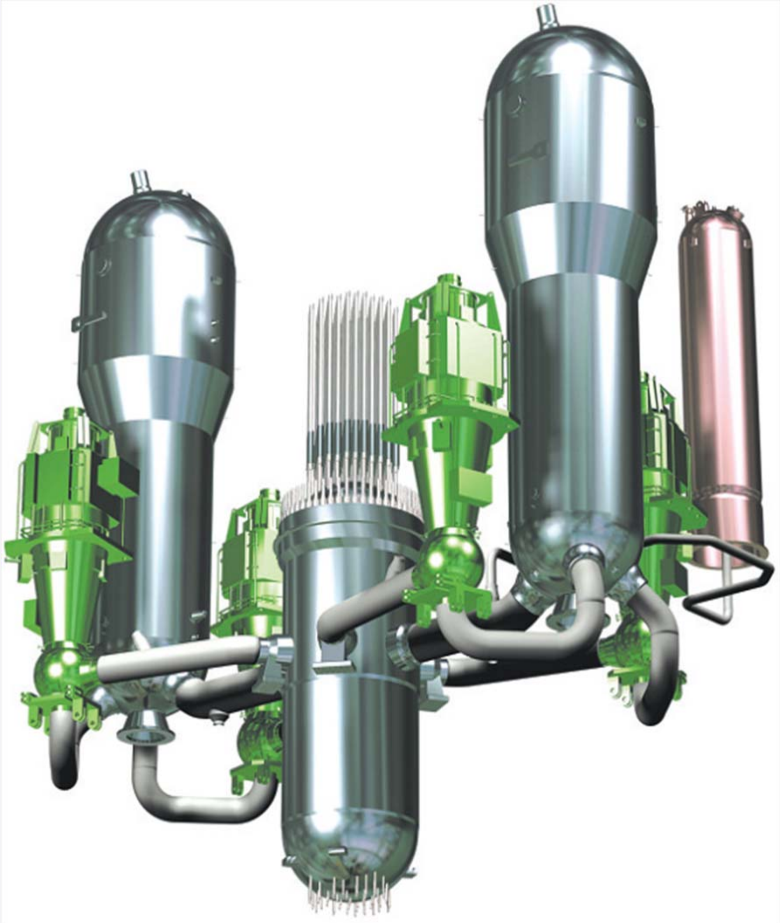
Elements of Nuclear Reactor Heat Source

Adapted from "Introduction to Nuclear Engineering" by Lamarsh and Baratta, third edition



Loop configuration

- 1 Reactor vessel
- 1 Pressurizer
- 2 Steam generators
- 4 Recirculating Coolant Pumps
- 2 Hot legs, 4 Cold legs

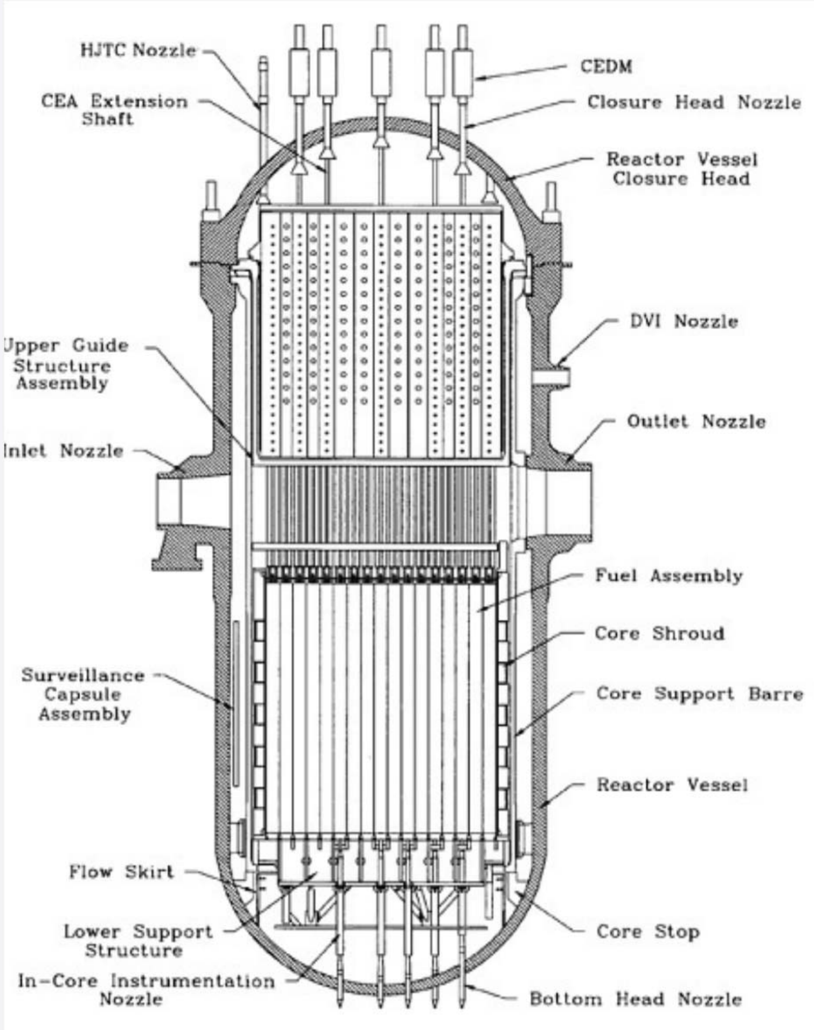


NSSS – Reactor Vessel

4 Inlet nozzles, 2 Outlet nozzles and 4 DVI nozzles

Provides barrier to fission product release

Part of coolant system pressure boundary



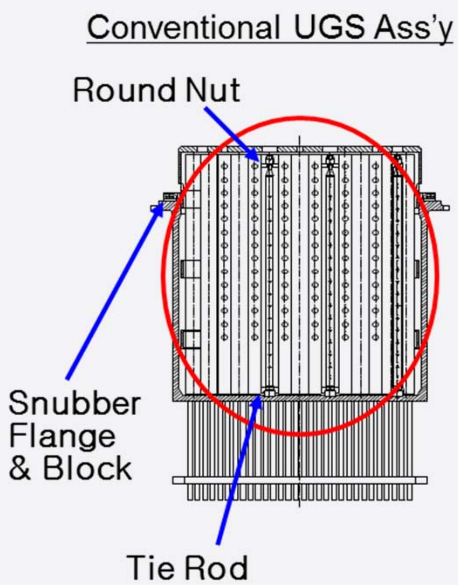
NSSS – Reactor Vessel Internals

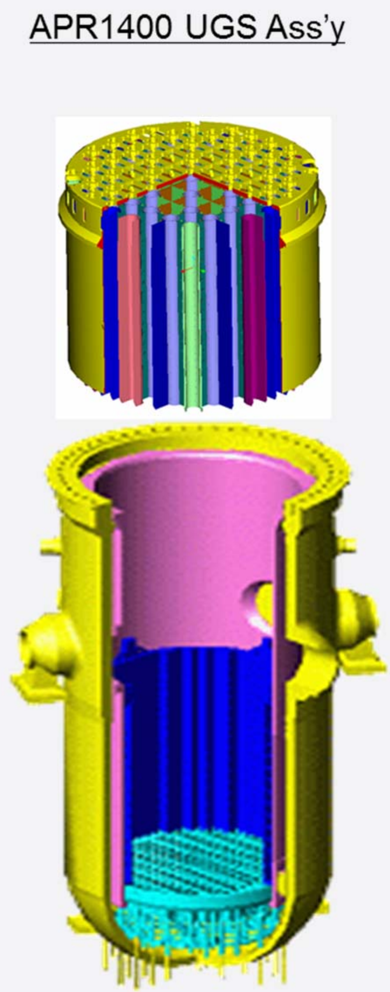
Integrated Inner Barrel Assembly (IBA)

- IBA is welded to UGS Upper Flange
- Elimination of Tie Rod, Round Nut, Snubber Flange & Block

Integrated Lower Internal Assembly

- Core support barrel + Core shroud + Lower support structure

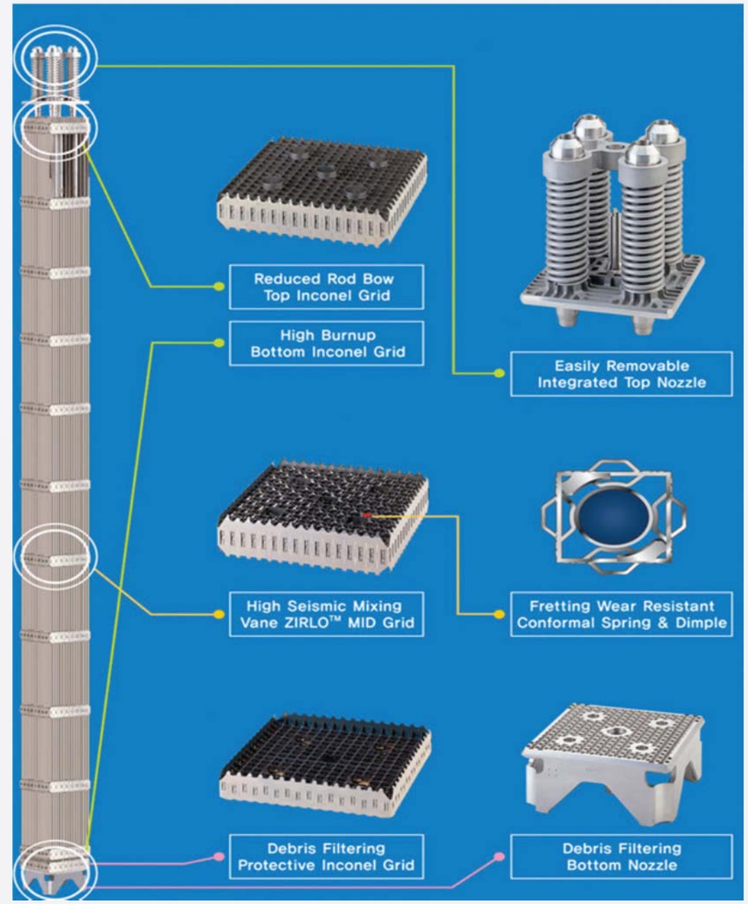




* UGS : Upper Guide Structure

NSSS – Fuel Assembly-Plus7

- Increased thermal margin of greater than 10 %
  - High thermal performance mixing vanes in all mid-grids
- High burnup of 55,000 Megawatt-Days per Metric Ton of Uranium (MWD/MTU)
  - Optimized fuel rod dimensions
  - Zirlo alloy as fuel clad
- Improved neutron economy
  - Axial blankets at ends of pellet region
  - Optimized fuel rod diameter
- Increased seismic resistance
  - Increased mid-grid buckling strength
  - (Straight grid straps and optimized grid height)
- Improved the resistance capacity for fretting wear
  - Conformal spring and dimple
- Debris-Filter Bottom Nozzle



NSSS – Steam Generator (SG)

SG tube design

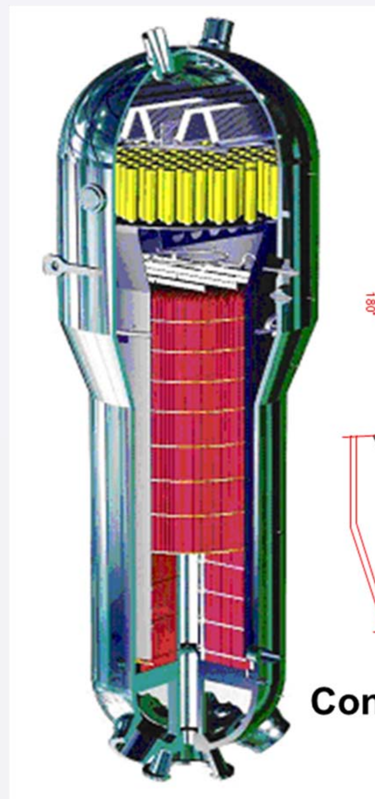
- No. of tubes per SG : 13,102
- Plugging margin : 10 %
- Material : Inconel 690

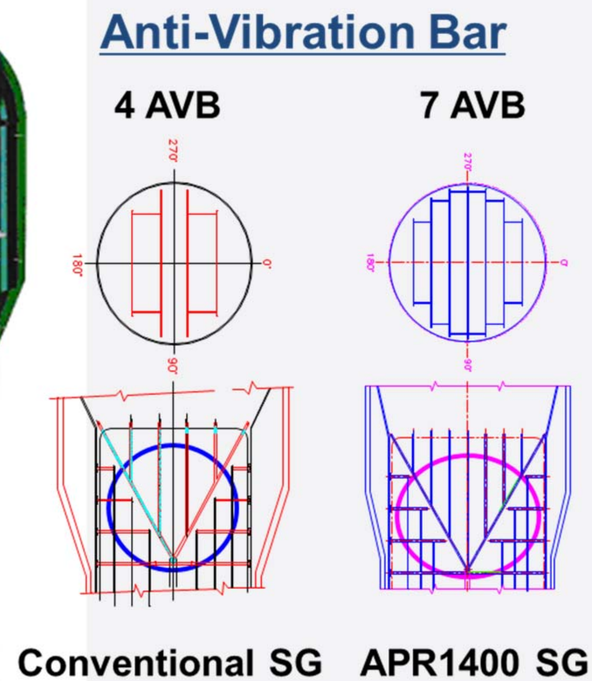
Improved upper tube support bar and plate

- To reduce flow-induced vibration

Modified primary outlet nozzle angle

- To improve stability of mid-loop operation





NSSS – Pressurizer

Design values

- Total Free volume : 2,400 ft^3 (67.9 m^3)
- Coolant volume at full power : 1,100 ft^3 (31.1 m^3)
- Heater capacity : 2,400 kW

Increased pressurizer volume

- Enhance capability against RCS transients

Pilot Operated Safety Relief Valve (POSRV)

- 4 POSRVs
- Function both over-pressure protection and safety depressurization
- Reliable valve operation without chattering and leakage
- Low valve stuck-open susceptibility

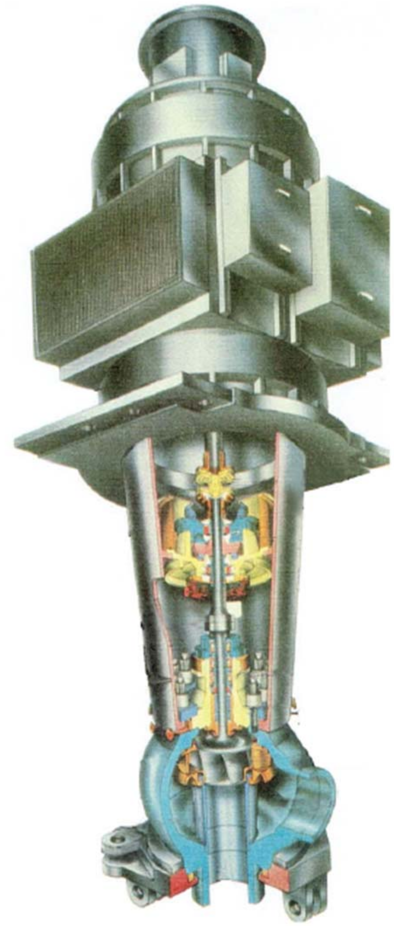
NSSS – Reactor Coolant Pump (RCP)

Type

- Vertical bottom suction
- Horizontal discharge
- Single stage impeller
- Motor-driven centrifugal pump
- Speed : 1,190 rpm

Shaft seal assembly

- Two face-type mechanical seals
  - Reduce the RCS pressure to volume control tank pressure
- Third face-type low-pressure vapor seal
  - Withstand RCS pressure when RCP is stopped



Reactor Coolant Pump

Turbine & Generator

Turbine

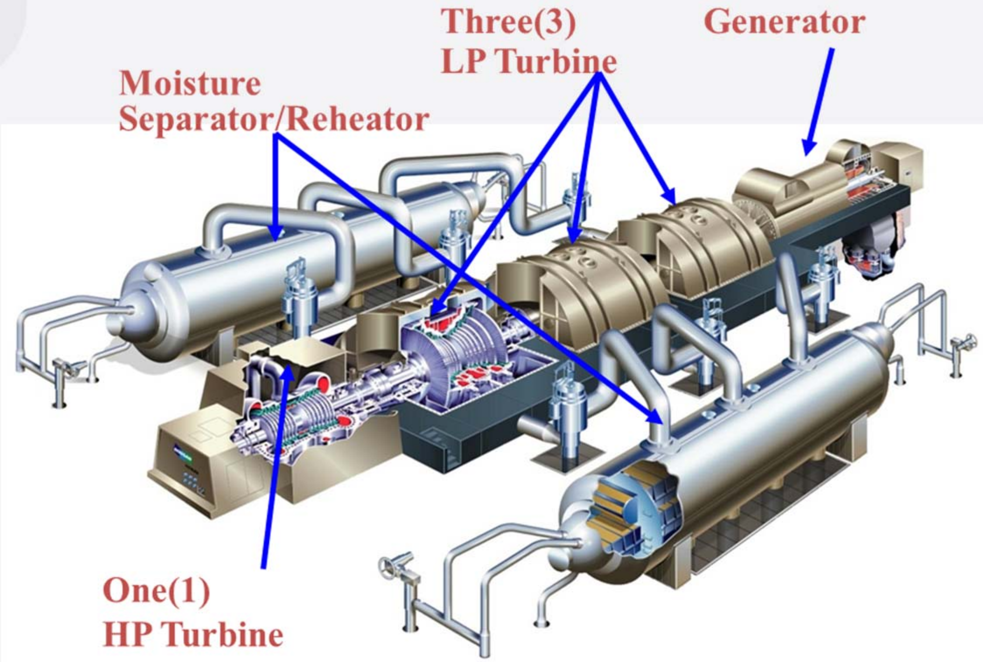
Number : 1 double flow HP TBN, 3 double flow LP TBN

Type : Tandem-Compound

Turbine Speed : 1,800 rpm

Output : 1,455 MW_e

Last Stage Blade : 52 inch LSB



Generator

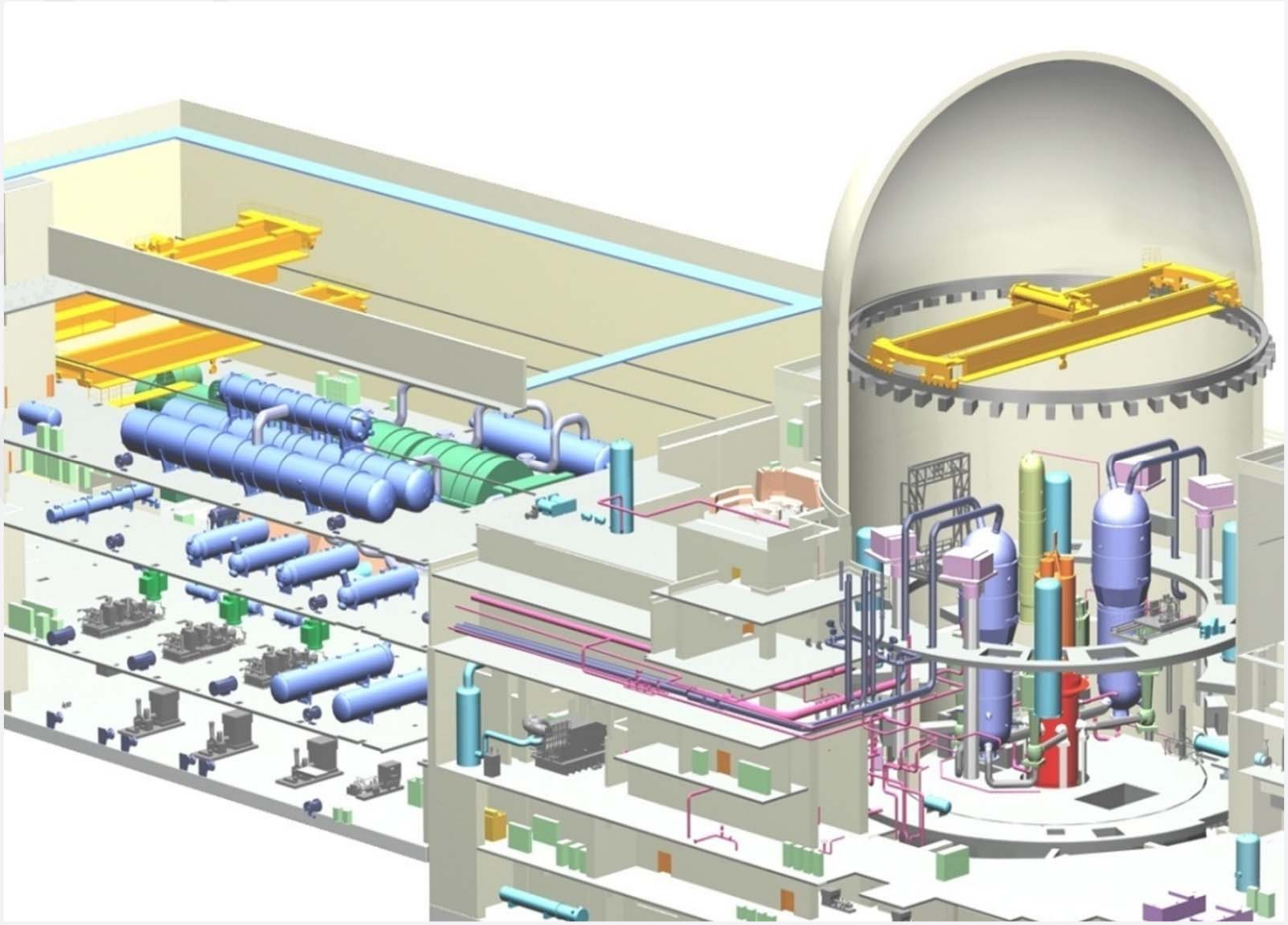
Number : 1

Type : Direct Driven
(conductor cooled)

Voltage : 24 kV, 3Phase

Frequency : 60 Hz

Reactor Power System Layout



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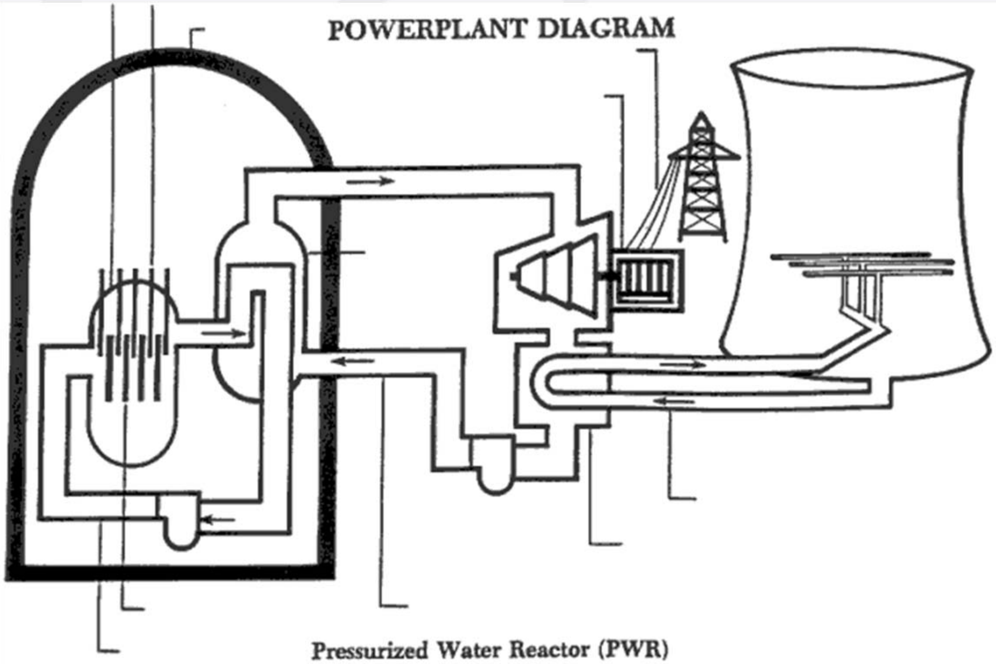
References

1. U.S. NRC "Nuclear Reactor Concepts" Workshop Manual, http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html
   - Nuclear Power for Energy Generation http://www.nrc.gov/reading-rm/basic-ref/teachers/01.pdf
   - The Fission Process and Heat Production http://www.nrc.gov/reading-rm/basic-ref/teachers/02.pdf
   - Pressurized Water Reactor Systems http://www.nrc.gov/reading-rm/basic-ref/teachers/04.pdf
2. DOE Fundamentals Handbook “Nuclear Physics and Reactor Theory”, http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v2.pdf
3. Lamarsh, J. R., and Baratta, A. J., “Introduction to Nuclear Engineering”, third edition, Prentice Hall, 2001
4. Stacey, W. M., “Nuclear Reactor Physics”, second edition, Wiley-VCH, 2007

List of Required Texts

1. U.S. NRC "Nuclear Reactor Concepts" Workshop Manual, http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html
   - Nuclear Power for Energy Generation http://www.nrc.gov/reading-rm/basic-ref/teachers/01.pdf
   - The Fission Process and Heat Production http://www.nrc.gov/reading-rm/basic-ref/teachers/02.pdf
   - Pressurized Water Reactor Systems http://www.nrc.gov/reading-rm/basic-ref/teachers/04.pdf
2. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v2.pdf

Exercise



Pressurized Water Reactor (PWR)

A. Identify the powerplant parts by writing the number of the correct powerplant part on the blank.

1. reactor
2. control rods
3. cooling water loop
4. containment building
5. steam-generator
6. turbine-generator
7. transmission lines
8. condenser
9. first water loop
10. second water loop
11. nuclear fuel

B. Color the separate loops using a different color for each loop. Use the following symbols to show what is in the loop or part of the loop.

= steam

= steam converted back to water

= cooling water

= water in first loop

Source: http://www.nrc.gov/reading-rm/basic-ref/teachers/pwr-diagram.html

Module 1: Nuclear Energy Fundamentals (Week 2/Day 2)

Major Components of a Power Reactor Core, Elementary Particles, Chart of the Nuclides

Gulf Nuclear Energy Infrastructure Institute – 2012 Fundamentals Course

Dr. David Boyle
Texas A&M University

Module 1/Week 2:

- **Nuclear and Reactor Physics**

Week 2 Learning Objectives:

- Be able to explain how nuclear power works and why it is needed
- Be able to explain why nuclear power works
- Be able to explain why fission products and radiation are major issues for nuclear power
- Be able to explain the role of neutron interactions in nuclear power

Today's Primary Learning Objective

Primary Day 2 Learning Objective:

- Why nuclear power works

Take away from this lecture:

- Nuclear physics underpins all aspects of nuclear power

Module 1: Nuclear Energy Fundamentals (Week 2/Day 2)

Lecture #1: Nuclear Physics

Dr. David Boyle

Lecture-1 Outline

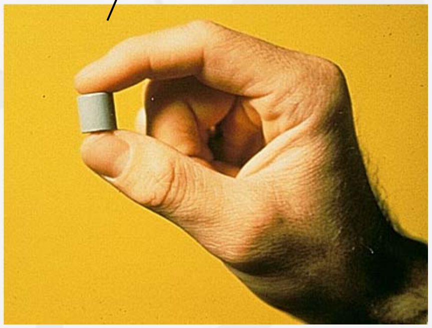
1. From nuclear power system to fundamental particles

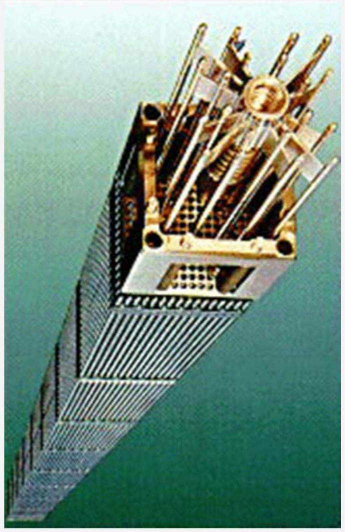
2. Intro to Nuclear physics

3. Nuclear radiation

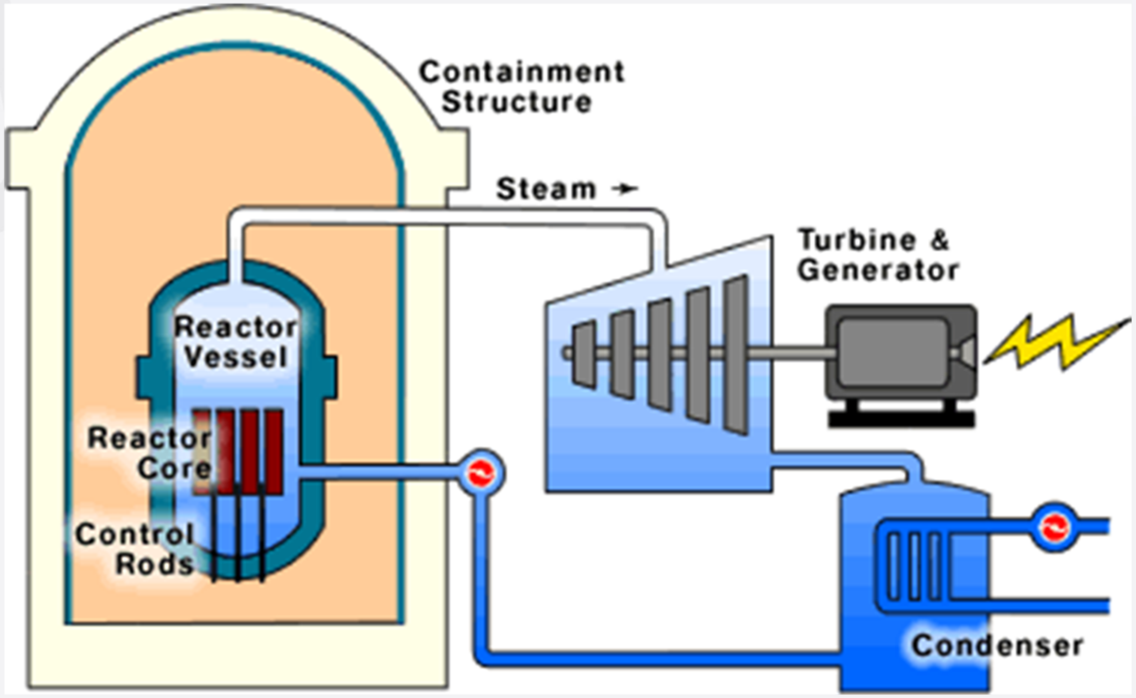
Where the Heat Comes From

UO_2 fuel pellet. Enriched to 3-5 percent U-235

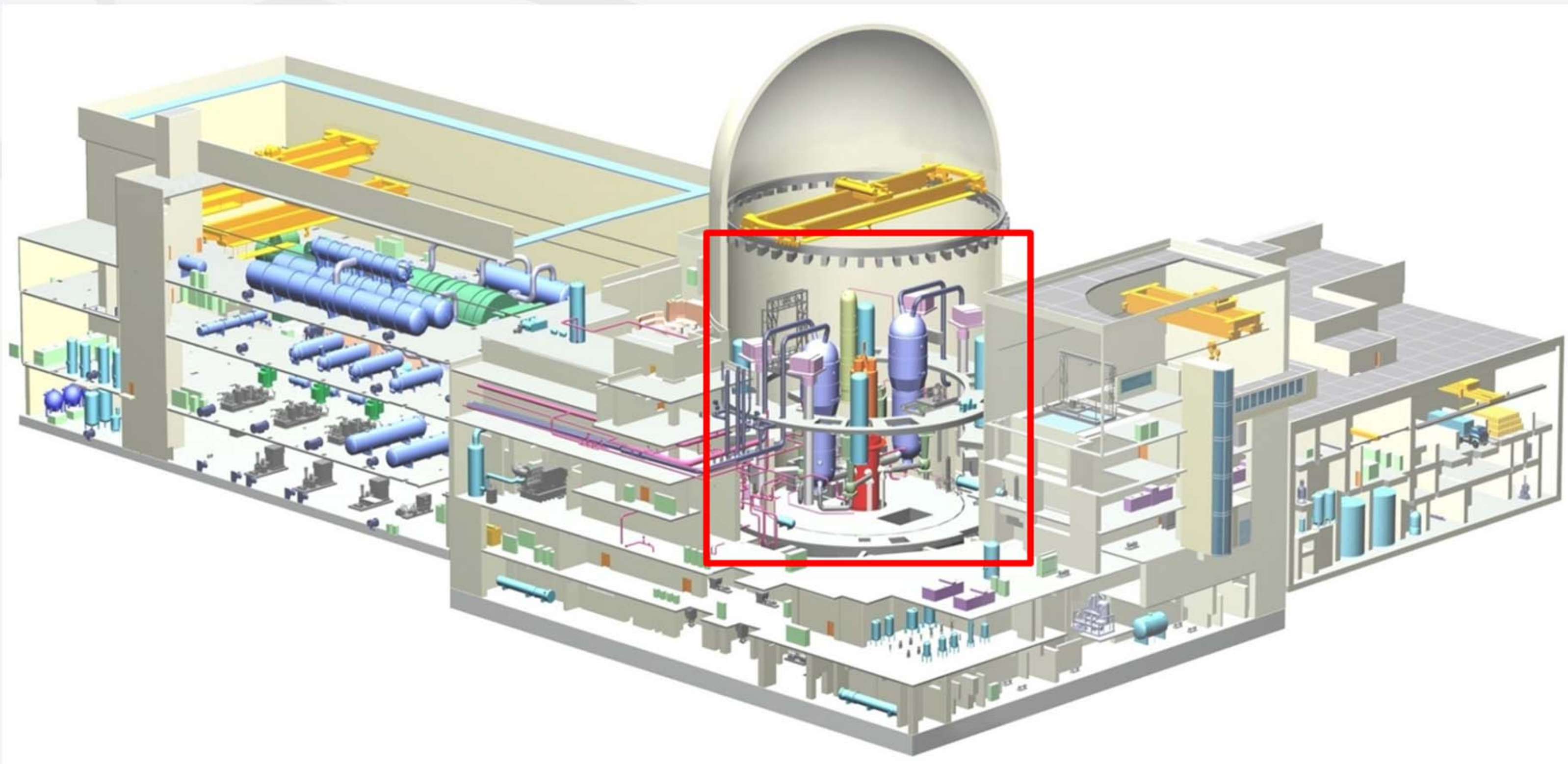




Typical Nuclear Power System



APR1400 General Arrangement



KHNP KOREA HYDRO & NUCLEAR POWER CO., LTD

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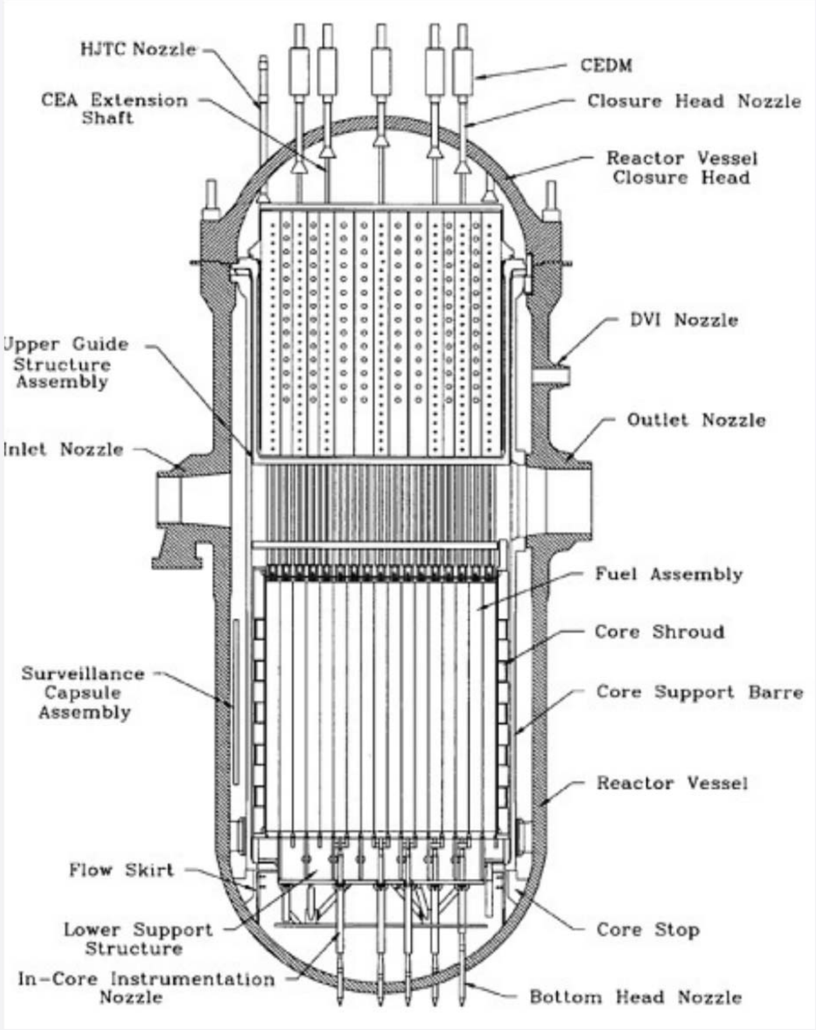
APR1400

Nuclear Steam Supply System in Containment

NSSS – Reactor Vessel

- 4 Inlet nozzles, 2 Outlet nozzles and 4 DVI nozzles
- Part of fission product containment system
- Part of coolant pressure boundary



NSSS – Reactor Vessel Internals

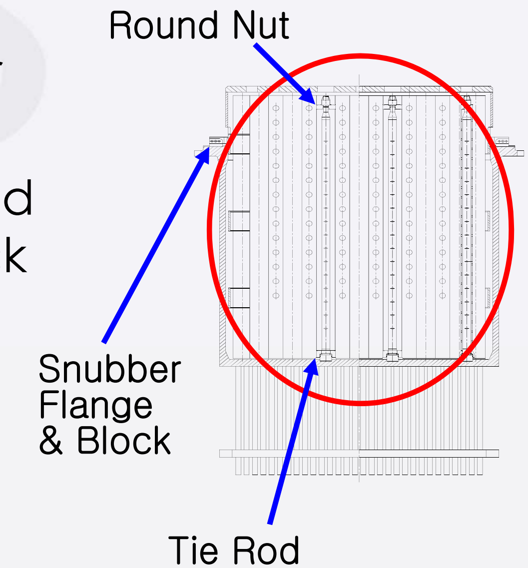
Integrated Inner Barrel Assembly (IBA)

- IBA is welded to UGS Upper Flange
- Elimination of Tie Rod, Round Nut, Snubber Flange & Block

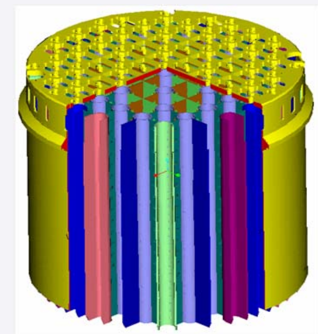
Integrated Lower Internal Assembly

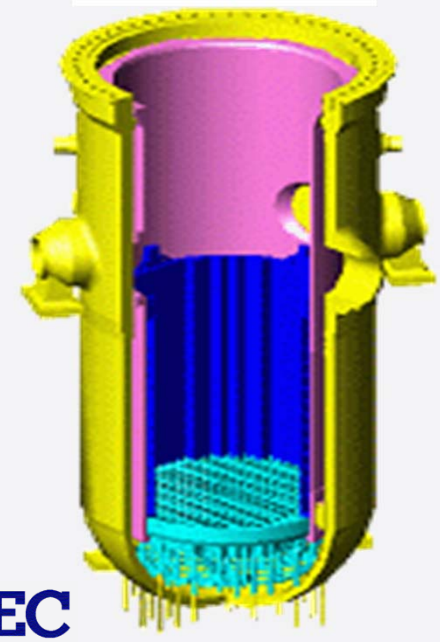
- Core support barrel + Core shroud + Lower support structure

Conventional UGS Ass'y



APR1400 UGS Ass'y





* UGS : Upper Guide Structure

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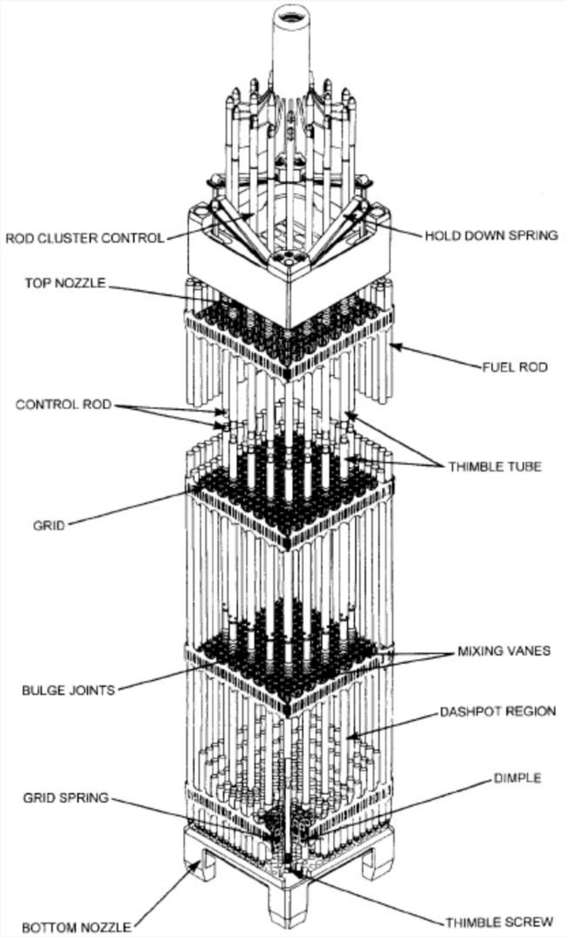
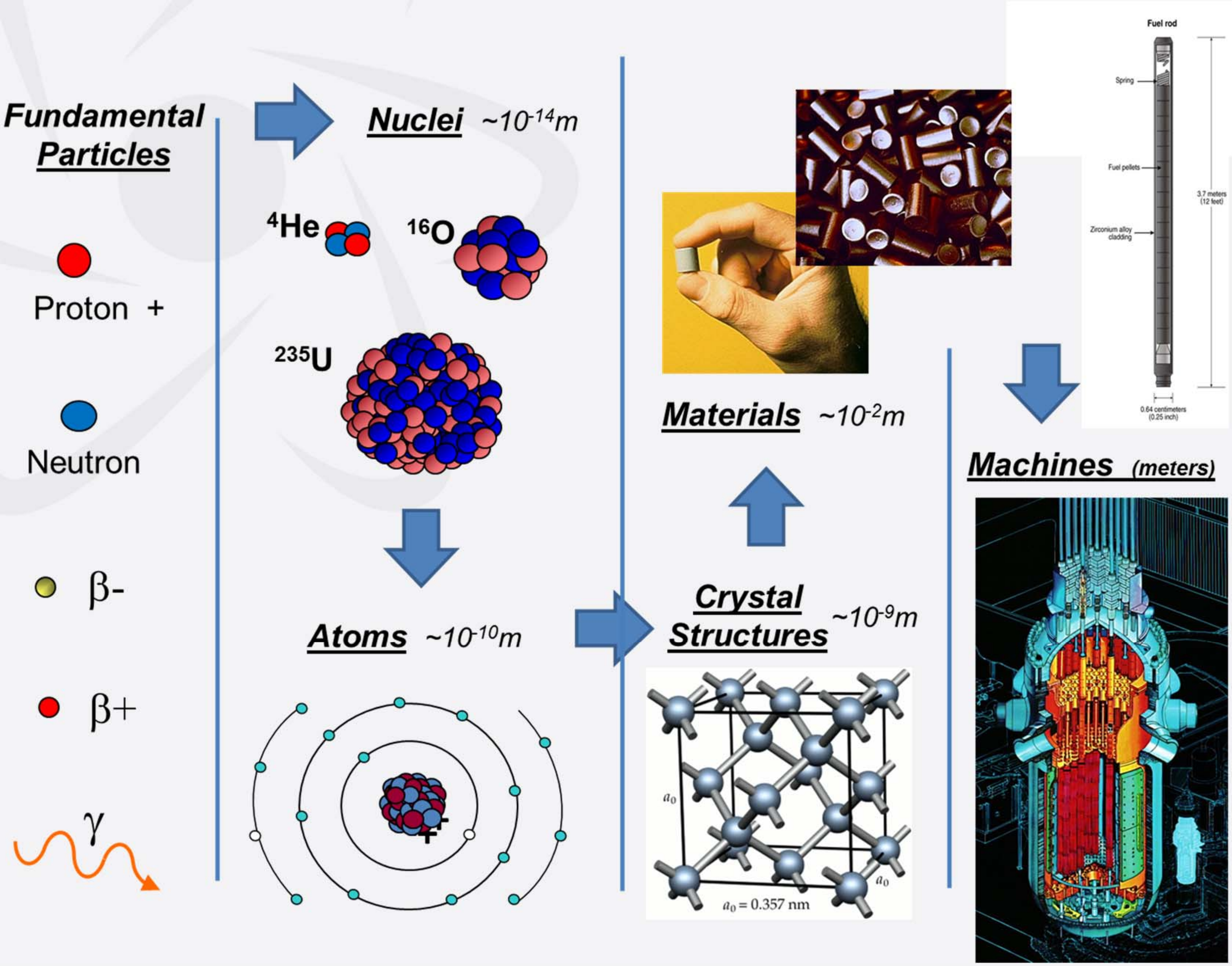
PWR: Fuel Assembly

- The fuel in PWR is enriched uranium dioxide (UO_2); the enrichment varies from 2 to 5%
- Small cylindrical pellets of UO_2 (1 cm diameter, 2 cm long)

are loaded into sealed stainless steel or Zircaloy tubes about 4 m long
  - These tubes are called fuel pins or fuel rods
  - Cladding is the outer layer of the fuel rods; it prevents radioactive fission fragments from escaping the fuel into the coolant



Source: "Nuclear Reactor Physics" by Stacey, W.M., second edition


Fundamental Particles
Proton +
Neutron
β-
β+
γ
Nuclei ~10^-14m
4He
16O
235U
Atoms ~10^-10m
Crystal Structures ~10^-9m
a0
a0
a0 = 0.357 nm
Materials ~10^-2m
Fuel rod
Spring
Fuel pellets
Zirconium alloy cladding
3.7 meters (12 feet)
0.64 centimeters (0.25 inch)
Machines (meters)

Importance of Basic Nuclear Physics for Understanding of Reactor Physics

- Reactor Physics
- Nuclear Reaction Rates
- Reactor Physics Elements

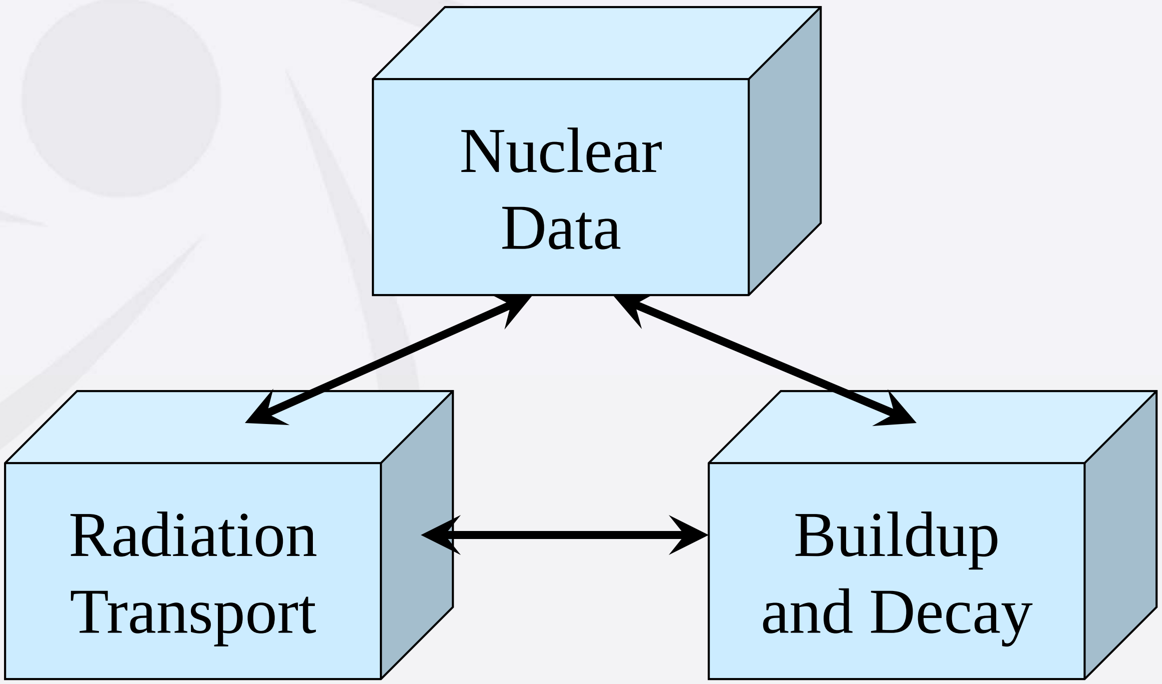
Reactor Physics

- Reactor physics is the study of the behavior and characteristics of neutron fission chain reacting systems
- In most cases, our primary interest is in determining
  - The rate at which different nuclear reactions are occurring in the system
- These reactions allow us to determine:
  - Reactor criticality and reactor power
  - Doses to humans and radiation damage to materials
  - Nuclear waste production

Nuclear Reaction Rates

- Nuclear reactions are generally induced by either radioactive decay or radiation interactions
- To know the rate at which reactions are occurring, we need to know:
  - Rate at which radiation travels through a material
  - Probability that radiation will cause a reaction
  - Probability that any radioactive products in the material will decay as a function of time
- Thus, we need to have nuclear data and have the ability to predict the transport of radiation

Reactor Physics Elements



The Importance of Basic Nuclear Physics

- To be able to understand any of the three basic building blocks of reactor physics
  - We need to have some basic nuclear physics knowledge
- We need to understand the constituents of a
  - Nucleus, mass and energy, radiation, radioactive decay, nuclear reaction physics, etc…

Fundamental Particles

- The world is composed of various subatomic particles
  - Also referred to as fundamental particles
- The fundamental particles are divided into:
  - Leptons (electrons, positrons, and neutrinos)
  - Hadrons (a subset of which includes protons and neutrons, also called baryons)
- To understand nuclear reactions
  - We need to have a basic understanding of some of these fundamental particles

Fundamental Particles (continued)

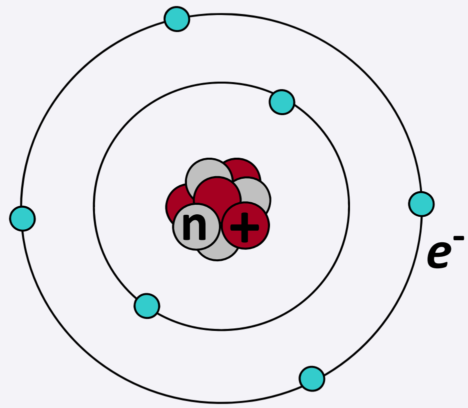
- For the purposes of reactor physics, we will concern ourselves with only the following fundamental particles:

Type	Particle	Symbol	Charge	Mass	Half-life
baryons	proton	p	+e	1.673×10^{-27} kg	stable?
	neutron	n	0	1.675×10^{-27} kg	10.4 min
leptons	electron	β^-, e^-	-e	9.109×10^{-31} kg	stable
	positron	β^+	+e	9.109×10^{-31} kg	stable
	neutrino	ν	0	?	stable
gluons	photon	γ	0	0	stable

Electron charge: -1.6022×10^{-19} C

Atomic Structure

- Atoms consists of a small, dense, positively charged nucleus surrounded by a cloud of negatively charged ***electrons***.
- Niels Bohr: *Electrons rapidly travel around the nucleus in discrete orbits. An electron in one of these orbits (or shells) has a discrete amount of energy.*
- Atomic nucleus contains
  - Positively charged ***protons*** and
  - Electrically neutral ***neutrons***.
    - Except for the atomic nucleus of hydrogen-1 which contains only one proton and no neutrons.
- Protons and neutrons are called ***nucleons***.



Simplified representation of an atom. NOT to scale.

Atomic Structure (continued)

- Chemical properties of an atom are determined by the number of electrons and their distribution around the nucleus.
- Composition of an atomic nucleus is responsible for all nuclear effects: radioactive transformation, nuclear energy, etc.
- Nucleus contains nearly all the mass of the atom.

Subatomic particle	Mass
Proton	1.67262×10^{-27} kg
Neutron	1.674929×10^{-27} kg
Electron	9.10939×10^{-31} kg

Terminology and Notation

- Atoms and nuclei are denoted as $^{A}_{Z}X$
- ***X* – symbol representing a chemical element.**
- ***Z* – atomic number**: number of protons in the atomic nucleus (also, number of electrons in an electrically neutral atom).
- ***A* – atomic mass number**: total number of nucleons in the nucleus.
- Atomic nucleus consists of *Z* protons and

  $N = A - Z$ neutrons. ***N* – neutron number.**
- ***<u>Nuclide</u>*** is a species of atom characterized by its atomic number *Z* and mass number *A* and the energy state of the nucleus.

Isotopes, Isotones, Isobars

- The atomic number Z (number of protons in the nucleus) uniquely identifies a chemical element.
- **Isotopes** – nuclides with the same number of protons but different number of neutrons.
  - Example: isotopes of carbon ^{12}C, ^{13}C, ^{14}C.
- **Isotones** – nuclides of different chemical elements that have the same number of neutrons.
  - Example: ^{12}B and ^{13}C both have 7 neutrons.
- **Isobars** – nuclides with the same atomic mass number. Example: ^{3}H and ^{3}He.

Atomic Mass Unit (amu)

- ***Atomic Mass Unit*** – one twelfth the mass of the neutral carbon-12 atom.

$$1\ amu = \frac{1}{12} \times m(^{12}C)$$

1 amu = 1.66057 × 10^{-24} g.

- The mass of any atom in **amu** is numerically equal to the atomic weight of the atom.
  - Atomic weight of a neutral atom ^{A}X, $M(^{A}X)$, is

$$M(^{A}X) = 12 \times \frac{m(^{A}X)}{m(^{12}C)} \rightarrow m(^{A}X) = M(^{A}X) \times 1\ amu$$

Atomic and Nuclear Dimensions

- Neither atom nor nucleus have a sharp outer boundary.
- Atomic radius represents the average distance between the nucleus and the boundary of the electron cloud. It depends on the atomic number Z.
- The average radii are approximately the same for all atoms: about 200 picometers (1 pm = 10^{-12} m).
  - Except for a few of the lightest atoms: H, B, C, N, etc.
- Nucleus ~ sphere with radius

$$R = 1.25\ fm \times A^{1/3}$$

where A is the atomic mass number, 1 fm = 10^{-15} m.

- An atom is mostly empty space.

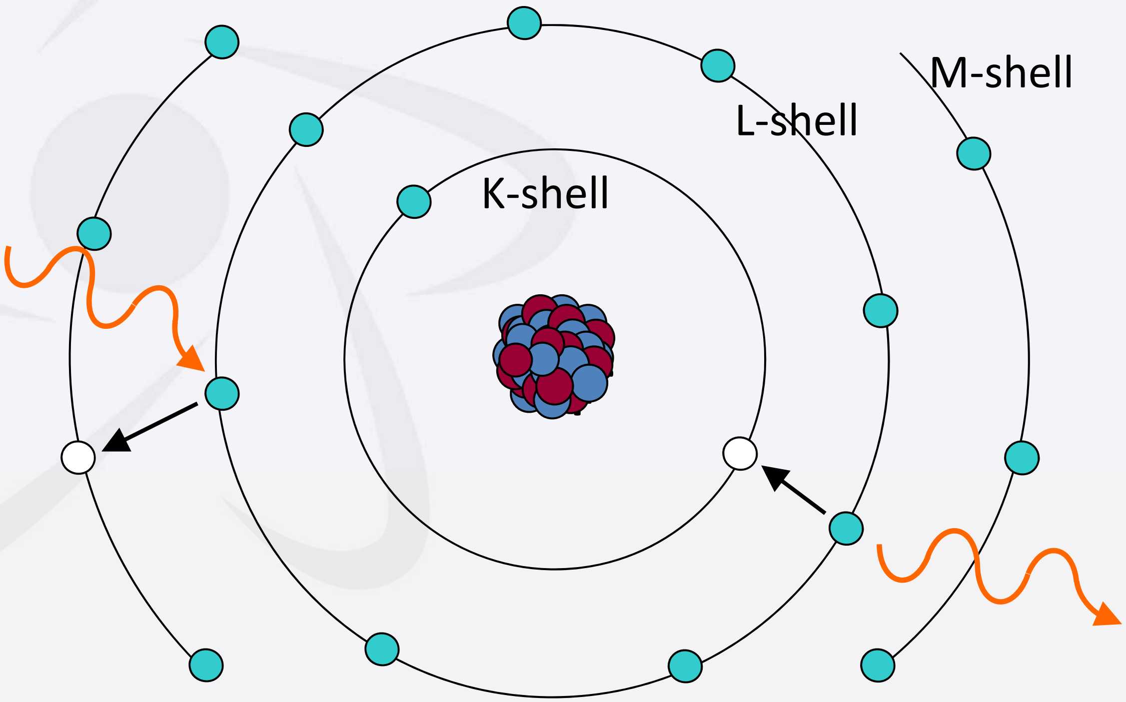
Energy States in Atoms

- In atoms, the electrons can only have certain discrete values of energy, called ***energy levels***.
- The position of an electron in an atom depends on electron's energy. Electrons exist more frequently in certain regions around the nucleus than others; such regions of space are referred to as ***atomic orbitals*** or ***shells***.
- The higher the energy of an electron, the further away from the nucleus the electron is likely to be found.
- If all the electrons in an atom are at their lowest possible energy level, the atom is in the ***ground state*** (the lowest energy state).
- An ***excited state*** is any state with energy greater than the ground state. An atom has many excited states.

Atomic Excitation

- Excitation occurs when a ground state electron absorbs a photon and jumps up to a higher energy state.
- An atom cannot remain in an excited state too long. It will return to a lower energy state (a lower excited state or the ground state) by emitting a photon.
  - ***A photon* is** a quantum of electromagnetic **energy**.
    - Photons have no mass, no charge; travel at the speed of light (in vacuum).

Atomic Excitation (continued)



To jump to a higher energy level (from an inner shell to an outer shell), an electron must absorb energy.

When an outer shell electron falls into an inner shell, it emits energy.

Four Fundamental Forces

- **Gravity** – attractive force between masses.
- **Weak force** – associated with some radioactive decay modes.
- **Electromagnetic force** – exists between charged particles.
- **Nuclear force (Strong force)** - strong attractive force between all nucleons (protons and neutrons).
- The strength of these four fundamental forces range over ~40 orders of magnitude:

Nuclear	Electromagnetic	Weak	Gravity
1	$1 \cdot 10^{-2}$	$1 \cdot 10^{-15}$	$1 \cdot 10^{-39}$

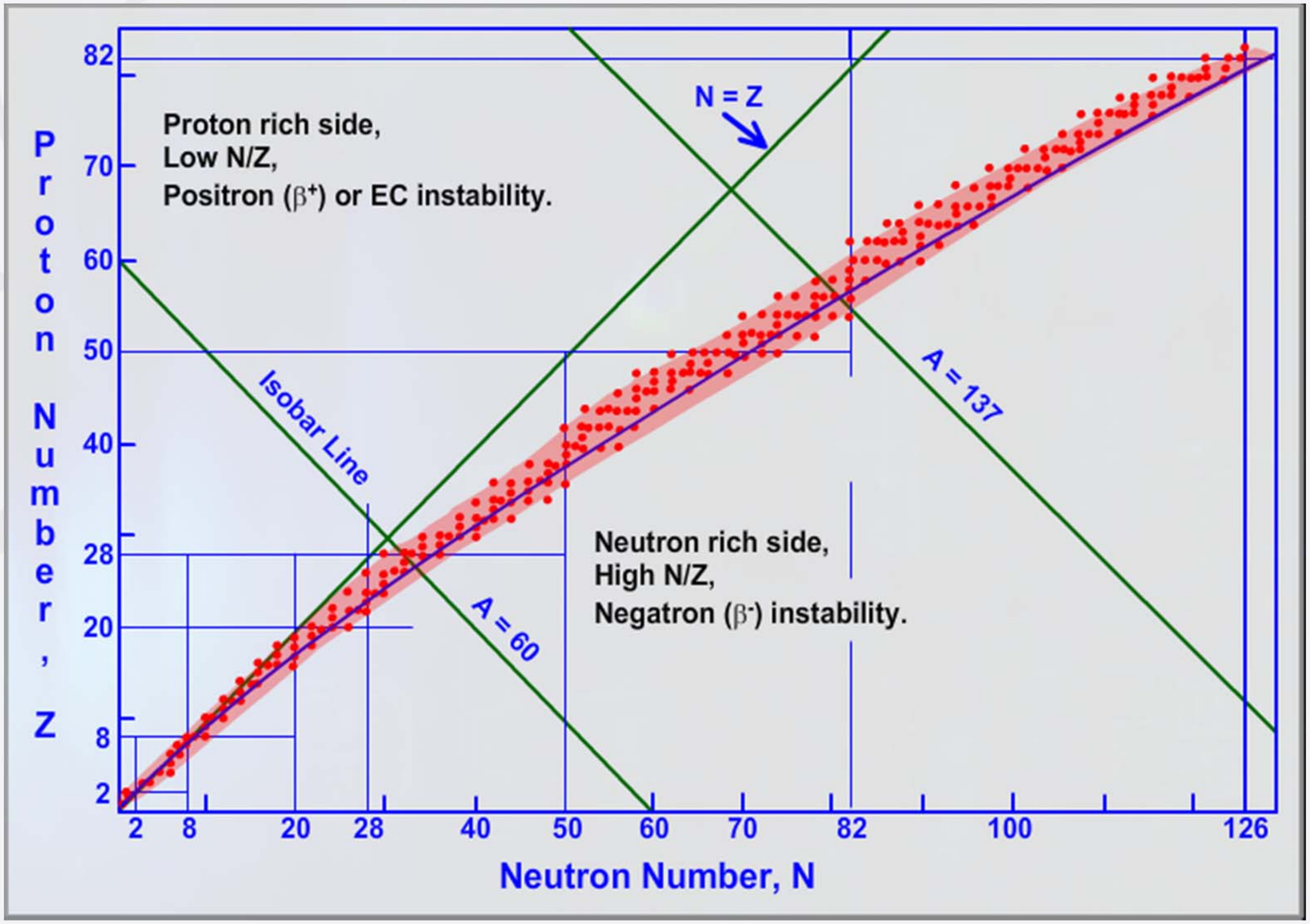
Forces Inside Atoms

- **Electrostatic force** described by Coulomb's law: like charges repel each other, opposite charges attract. This force holds atoms together: negatively charged electrons travel around the positively charged nucleus.
- In the nucleus, protons and neutrons are held together by a much stronger ***nuclear force*** (or ***strong force***).
  - Nuclear force has a very short range: ~10^{-15} meters.
  - Nuclear force overcomes the repulsive (electrostatic) force acting between protons.
  - Nuclear force is responsible for release of energy when a nucleus breaks apart.
- The gravitational force in atoms is negligible compared to the electrostatic force and the nuclear force.

Stability of the Nucleus

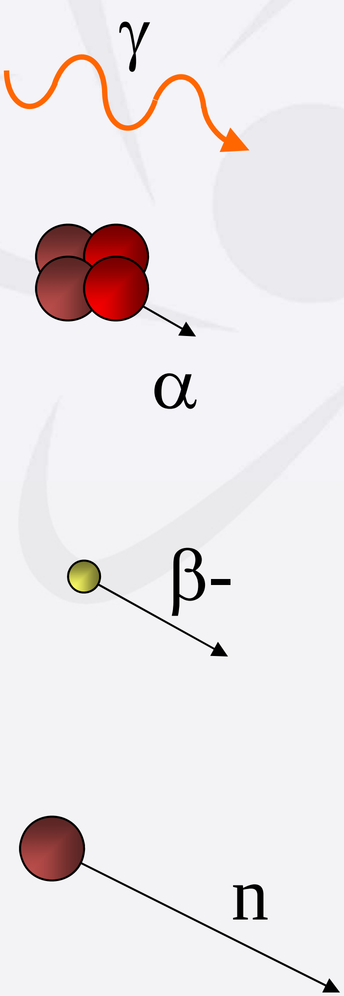
- The balance between the attractive and repulsive forces determines the stability of a nucleus.
- The most important factor is the **neutron-to-proton ratio**.
- For light nuclei, $Z \sim N$ and their neutron-to-proton ratio is about one ($N : Z$ is about 1:1).
- For nuclei with $A > 40$, more neutrons are needed to balance the repulsive forces between the protons. For stable atoms, the neutron-to-proton ratio approaches 1.5.
- If the nucleus has more neutrons than needed for stability or doesn't have enough neutrons, the atom is unstable.
- An unstable nucleus will attempt to become stable by emitting particles or energy.

Band of Stability



Source: Dr. Reece, NUEN , TAMU

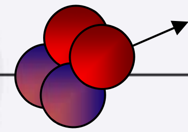
Radiation



- Radiation is energy traveling through space or matter in the form of particles or electromagnetic waves.
- Radiation includes:
  - Gamma-rays
  - Alpha particles
  - Beta-rays
  - Neutrons
  - Neutrinos
  - Cosmic rays

- Radiation is produced
  - via decay
  - as a byproduct of nuclear and atomic reactions
- Atoms with unstable nuclei are ***radioactive***. They can spontaneously transform into other nuclides. As they do so, they emit radiation of one type or another.
- Terminology:
  - ***Radionuclide*** – a radioactive nuclide.
  - ***Radioisotope*** – a radioactive isotope.

Alpha Particles α



- An a particle consists of two protons and two neutrons
  - Identical to a helium nucleus, ^4_2He
  - Carries positive charge +2e
  - Has a large size and mass (compared to other radiations)
    - 6.645×10^{-27} kg
  - Low penetrating power
    - Travel a few centimeters in air
    - Can be stopped by a sheet of paper
    - Does not penetrated through skin

Beta Rays β-

- Beta rays are energetic particles emitted from a nucleus
  - Electrons
    - β^-, e^-, charge –e
  - Positrons
    - β^+, charge +e
  - Mass 9.109×10^{-31} kg
- Beta rays have medium penetrating power
    - Travel about 20 cm in air
    - Can penetrate through the dead layer of skin cells and damage the living skin tissue
    - Can be stopped by a thin sheet of aluminum

Gamma Rays γ

- Gamma rays are electromagnetic waves
  - High frequency (short wavelength)
  - Carry no charge
  - Have no mass
  - Travel large distances in air, pass through human body
    - Better stopped by high density materials (concrete, lead, uranium)

Neutrons n

- Neutrons are
  - Electrically neutral, carry no charge
  - Mass 1.675×10^{-27} kg
  - Can travel large distances in air (hundreds of meters)
  - Can easily penetrate the human body
  - Better stopped by materials that contain hydrogen atoms
    - Water, polyethylene, and concrete
      - The nucleus of a hydrogen atom (proton) and a neutron have almost identical masses
      - When colliding with a hydrogen nucleus, a neutron loses a great amount of its energy

Module 1: Nuclear Energy Fundamentals (Week 2/Day 2)

Lecture #2: Nuclear Physics Part II

Dr. David Boyle

Lecture-2 Outline

1. Chart of the Nuclides

2. Nuclear Reactions

3. Nuclear Models

Chart of Nuclides

- A tabulated chart that lists the stable and unstable nuclides.
- The chart is a two-dimensional graph in which the nuclides are arranged with their atomic number *Z* along the vertical axis and their neutron number *N* along the horizontal axis.
- Each point (square) plotted on the graph represents the nuclide of a real or hypothetical chemical element.
- The chart offers a greater insight into the characteristics of isotopes than the periodic table, which shows only elements instead of each of their isotopes.
- http://atom.kaeri.re.kr/ton/nuc1.html

Isotopic Abundance

- Stable and unstable isotopes of various chemical elements are found in nature.
- Some isotopes of a given element are more abundant than other isotopes of the element.
- ***Isotopic abundances*** are given in atom percent. Abbreviation: *a/o*.
- In one million uranium atoms, how many atoms of each isotope are present?

Isotopic Abundance Example

- Uranium has three naturally occurring isotopes: ^{234}U, ^{235}U and ^{238}U.
- Their abundances 0.0055%, 0.72% and 99.2745% respectively.
- In one million uranium atoms, how many atoms of each isotope are present?
- Expected numbers of atoms:
  - approximately 55 ^{234}U atoms
  - 7200 ^{235}U atoms, and
  - 992745 ^{238}U atoms.

Enriched and Depleted Uranium

- All isotopes of uranium have
  - similar chemical properties
  - different nuclear properties.
- ^{235}U is used in most reactors. Natural uranium ore is only 0.72% ^{235}U.
- ***Enrichment*** is the process of increasing of the proportion of a particular isotope.
- ***Enriched uranium*** is uranium in which the concentration of ^{235}U is greater than in natural uranium ore.
- ***Depleted uranium*** is uranium in which the concentration of ^{235}U is less than its natural value of 0.72% .

- ***Atomic weight*** – the ratio of the average mass of neutral atoms of an element to 1/12 of the mass of a neutral atom of carbon-12.
- Atomic weight of a neutral atom ^{A}X, $M(^{A}X)$, is

$$M(^{A}X) = 12 \times \frac{m(^{A}X)}{m(^{12}C)}$$

Atomic Weight (continued)

- Atomic weight of an element

$$M = \sum_i \gamma_i M_i / 100$$

where γ_i where is the isotopic abundance in atom percent of the *i-th* isotope of atomic weight M_i.

- Note: atomic weight has no units!

Atomic Weight Example

- Using the data below, compute the atomic weight of naturally occurring uranium.
- Uranium has three naturally occurring isotopes:
  - ^{234}U, $\gamma(^{234}U) = 0.0055\%$, $M(^{234}U) = 234.0409456$
  - ^{235}U, $\gamma(^{235}U) = 0.72\%$, $M(^{235}U) = 235.0439231$
  - ^{238}U, $\gamma(^{238}U) = 99.2745\%$, $M(^{238}U) = 238.0507826$

$$M(U) = (0.0055 * 234.0409456 + 0.72 * 235.0439231 + 99.2745 * 238.0507826) / 100 = 238.0289127$$

Gram Atomic Weight. Mole

- ***Gram atomic weight*** is the amount of substance having a mass (in grams) equal to the atomic weight of the substance. Measured in moles.
- ***Mole*** – amount of material having a mass numerically equal to its atomic weight.
- ***Avogadro's number*** – the number of atoms or molecules in one mole of any substance.

$$N_A = 6.02214179(30) \times 10^{23}\ \text{mol}^{-1}$$

- Example: One mole of carbon-12 has a mass of exactly 12 g. Hence, we can calculate the mass of one atom of ^{12}C:

$$12 / 6.02214179(30) \times 10^{23} = 1.99 \times 10^{-23}\ \text{g}$$

Mass and Energy

- The energy produced in nuclear systems is derived from Einstein's famous formula:

$$E = mc^2$$

where $c = 2.9979 \times 10^8$ m/s is the speed of light.

- **Mass and energy are equivalent, or convertible into one another.**

Rest-mass Energy

- Rest-mass energy is energy of a body at rest
- Using Einstein's formula:

$$E_0 = m_0 c^2$$

  - Where c = 2.9979 × 10^8 m/s is the speed of light
  - E_0 is the rest energy
  - m_0 is the rest mass

Electron Rest-mass Energy

- Rest-mass energy is energy of a body at rest: $E_0 = m_0c^2$
- Rest-mass energy of an electron:
  - Rest-mass of an electron $m_e = 9.10954 \times 10^{-31}$ kg
  - Speed of light $c = 2.9979 \times 10^8$ m/s.
  - Rest-mass energy of an electron

$$m_e c^2 = 81.871 \times 10^{-15}\ \mathrm{kg \cdot m^2/s^2} = 81.871 \times 10^{-15}\ \mathrm{J}$$

where J denotes joule: $1\mathrm{J} = 1\ \mathrm{kg \cdot m^2/s^2}$

Total Energy

- Total energy of a body is the sum of the kinetic and potential energy.
- Total energy for a free particle:

$$E_{total} = mc^2 = E_{kinetic} + m_0 c^2$$

where m_0 is its rest mass.

- This equation is valid only for bodies (particles) with nonzero rest mass.
- It does not apply to photons.

Total Energy of Photon

- Total energy of a photon

$$E = h\nu = h\frac{c}{\lambda}$$

where *h* is Plank's constant and v is photon's frequency; λ is its wavelength and c is the speed of light.

h = 4.135667×10^{-15} eV·s = 6.626069×10^{-34} J·s

Electron Volt

- The electron volt is another useful practical unit.
- **One electron volt** is the energy gained by an electron when it passes through a potential difference of one volt

$$1\ \text{eV} = (1.602177 \times 10^{-19}\ \text{C})(1\ \text{Volt})$$

$$\mathbf{1\ eV = 1.602177 \times 10^{-19}\ J}$$

- Rest-mass energy of the electron

$m_e c^2 = 81.871 \times 10^{-15}$ J $= 51.0999 \times 10^4$ eV $= 0.511$ MeV

Energy Equivalent of 1 amu

- **1 amu** = 1.66057 × 10^{-24} g
- Using Einstein's formula:

$$E = mc^2$$

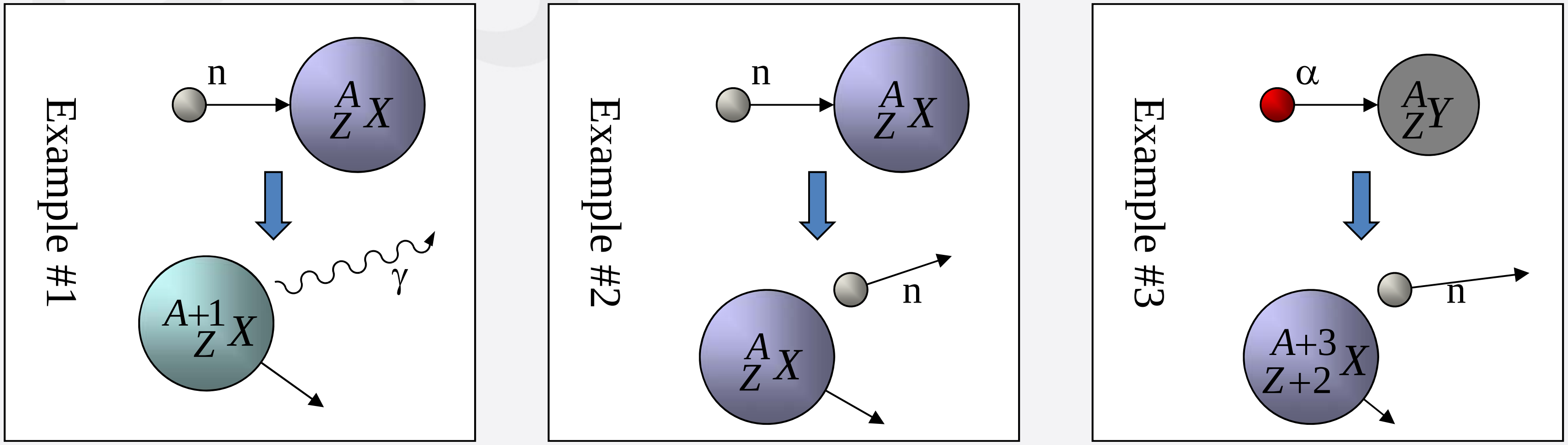
$$= \left(1.66057 \times 10^{-27} \, kg\right)\left(2.9979 \times 10^{8} \, \frac{m}{s}\right)^2$$

$$= 931.5 \; MeV$$

Nuclear Reactions

- Nuclear reactions occur when
  - two nuclear particles interact to produce one or more nuclear particles at the same or different energies
- We can represent a nuclear reaction with two interacting particles and two product particles by the equation

$$A + B \rightarrow C + D$$



- Fundamental laws governing nuclear reactions:
  - ***Conservation of nucleons***
  - ***Conservation of charge***
  - ***Conservation of momentum***
  - ***Conservation of energy***
- Important to note: conservation of nucleons and conservation of charge do not imply conservation of protons and neutrons separately.

Conservation of Nucleons

- Example 1

  Oxygen-16 is bombarded by energetic neutrons

  – $^{16}O(n,p)^{16}N$

    $^{16}O + n \rightarrow {}^{16}N + {}^{1}H$

    - nucleus of hydrogen-1 contains only one proton, no neutrons

- Example 2

  Tritium (^{3}H) is bombarded by deuterons (d, ^{2}H nuclei)

  – $^{3}H(d,n)^{4}He$

    $^{3}H + {}^{2}H \rightarrow {}^{4}He + n$

Conservation of Energy

- Can be used to predict whether a nuclear reaction is energetically favored.

$$A + B \rightarrow C + D$$

$$E_{before} = E_{after}$$

$$T_A + T_B + M_A c^2 + M_B c^2 = T_C + T_D + M_C c^2 + M_D c^2$$

where T_i denotes kinetic energy, $M_i c^2$ – rest mass energy.

- Rearrange terms:

$$(T_C + T_D) - (T_A + T_B) = [(M_A + M_B) - (M_C + M_D)]c^2$$

$$Q = T_{after} - T_{before} = [(M_A + M_B) - (M_C + M_D)]c^2$$

Q-value of the Reaction

$$Q = \text{Net K.E. after} - \text{Net K.E. before}$$

$$= T_{after} - T_{before}$$

$$= [(M_A + M_B) - (M_C + M_D)]c^2$$

- Q is always expressed in MeV.

$$Q = [(M_A + M_B) - (M_C + M_D)] \cdot 931 \text{ MeV}$$

Exothermic vs. Endothermic Reactions

- Exothermic reactions: $Q > 0$
  - there is a net increase in the kinetic energies of the particles
  - mass is converted into kinetic energy
- Endothermic reactions: $Q < 0$
  - there is a net decrease in the kinetic energies of the particles
  - kinetic energy is converted to mass

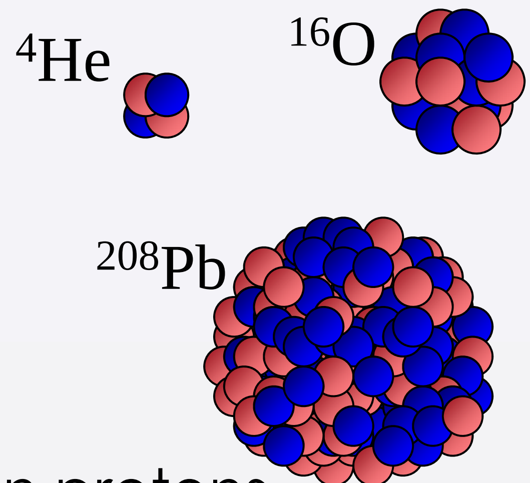
Exothermic vs. Endothermic Reactions. Example

- Exothermic reactions:
  - $^{10}B(n, \alpha)^{7}Li$
  - $^{6}Li(n, \alpha)^{3}H$
    - this reaction is used for the production of tritium
- Endothermic reactions:
  - $^{9}Be(\gamma,n)^{7}Be$
  - $^{7}Li(p, \alpha)^{3}H$

- We use nuclear models to help explain various nuclear phenomena
  - and to allow us to have a better understanding of the structure of the nucleus
- Nuclear models are based on theory and experimental observation
- The two models of interest are:
  - The shell model
  - The liquid drop model
- Neither of these models are exact.

The Liquid Drop Model

- A nucleus is much like a spherical drop of liquid with nearly uniform density.
- Model assumptions:
  - Nuclear matter is incompressible
  - All nuclei have the same density
  - Two forces exist between nucleons:
    - Coulomb repulsion force between protons
    - Strong nuclear force with same attractive magnitude for all nucleons
  - The nuclear force exists only between a nucleon and its nearest neighbors (saturation)



The Shell Model

- The shell model describes the structure of the nucleus in terms of energy levels.
- The shell model is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom:
  - The nucleus, like the atom, has discrete energy levels that correspond to different configurations of nucleons in the nucleus.
  - Nucleons exist in certain energy levels within the nucleus and move independently of each other.

References

1. Lamarsh, J. R., and Baratta, A. J., "Introduction to Nuclear Engineering", third edition, Prentice Hall, 2001
2. Turner, J. E., "Atoms, radiation, and radiation protection", third edition, Wiley-VCH, 2007
3. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf

List of Required Texts

1. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf

Module 1: Nuclear Energy Fundamentals (Week 2/Day 3)

Radioactive Decay
Nuclear Fission

Gulf Nuclear Energy Infrastructure Institute – 2012 Fundamentals Course

Dr. David Boyle
Texas A&M University

Module 1/Week 2:

- **Nuclear and Reactor Physics**

Week 2 Learning Objectives:

- Be able to explain how nuclear power works and why it is needed
- Be able to explain why nuclear power works
- Be able to explain why fission products and radiation are major issues for nuclear power
- Be able to explain the role of neutron interactions in nuclear power

Today's Primary Learning Objective

Primary Day 3 Learning Objective:

- Be able to explain how and why nuclear power works

Take away from this lecture:

- Fission releases energy because of the difference in binding energy per nucleon between heavy and medium mass nuclei

Module 1: Nuclear Energy Fundamentals (Week 2/Day 3)

Lecture #1: Radioactive Decay

Dr. David Boyle

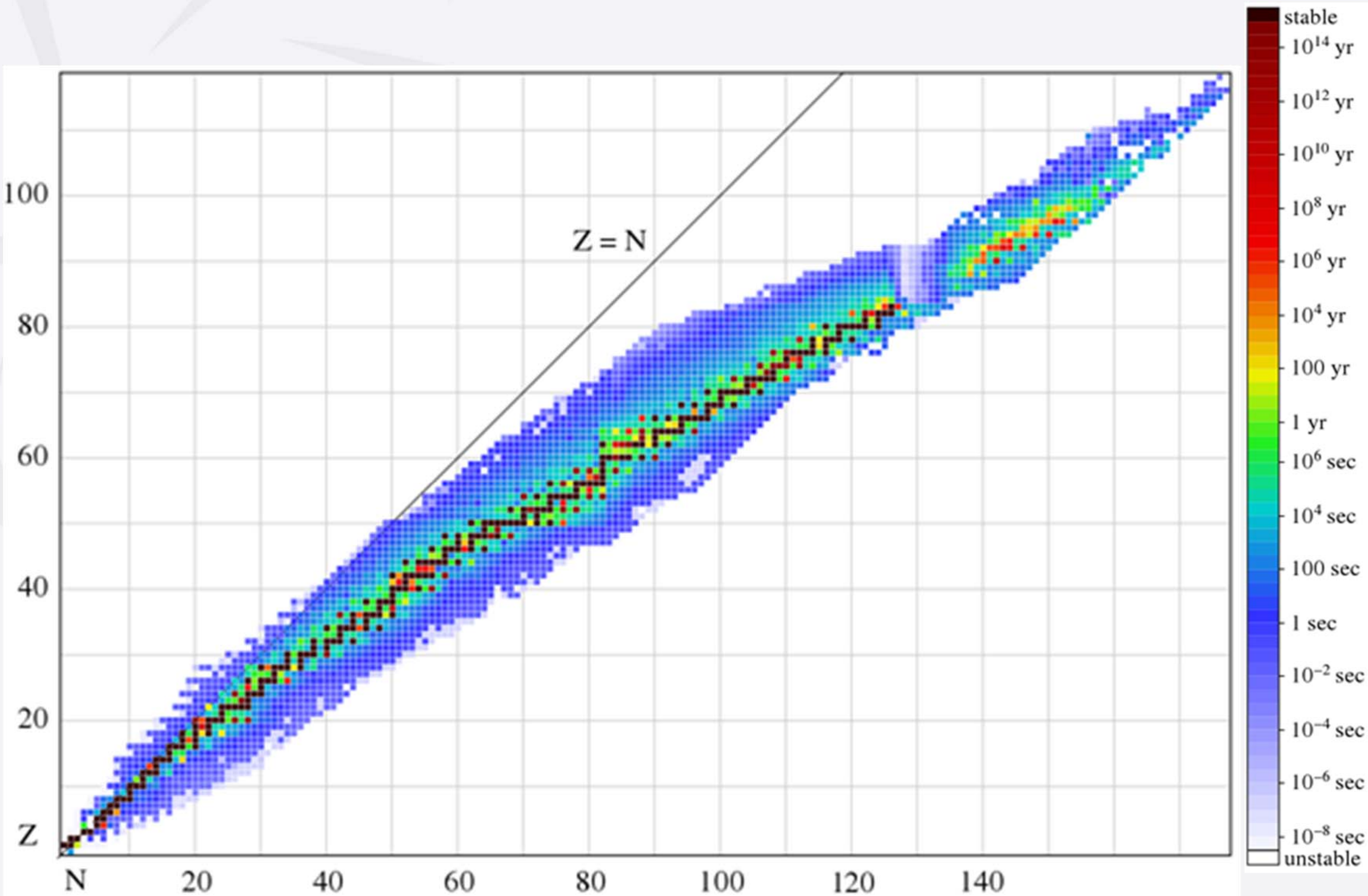
Lecture Outline

1. Nuclear Stability (review)
2. Modes of Radioactive Decay
3. Activity
4. Decay Constant and Half-life
5. Decay Chain Calculations
6. Radioactive Equilibrium

Stability of the Nucleus

- The balance between the attractive and repulsive forces determines the stability of a nucleus.
- The most important factor is the neutron-to-proton ratio.
- For light nuclei, $Z \sim N$ and their neutron-to-proton ratio is about one ($N : Z$ is about 1:1).
- For nuclei with $A > 40$, more neutrons are needed to balance the repulsive forces between the protons. For stable atoms, the neutron-to-proton ratio approaches 1.5.
- If the nucleus has more neutrons than needed for stability or doesn't have enough neutrons, the atom is unstable.
- An unstable atom will attempt to become stable by emitting radiation (particles or energy).

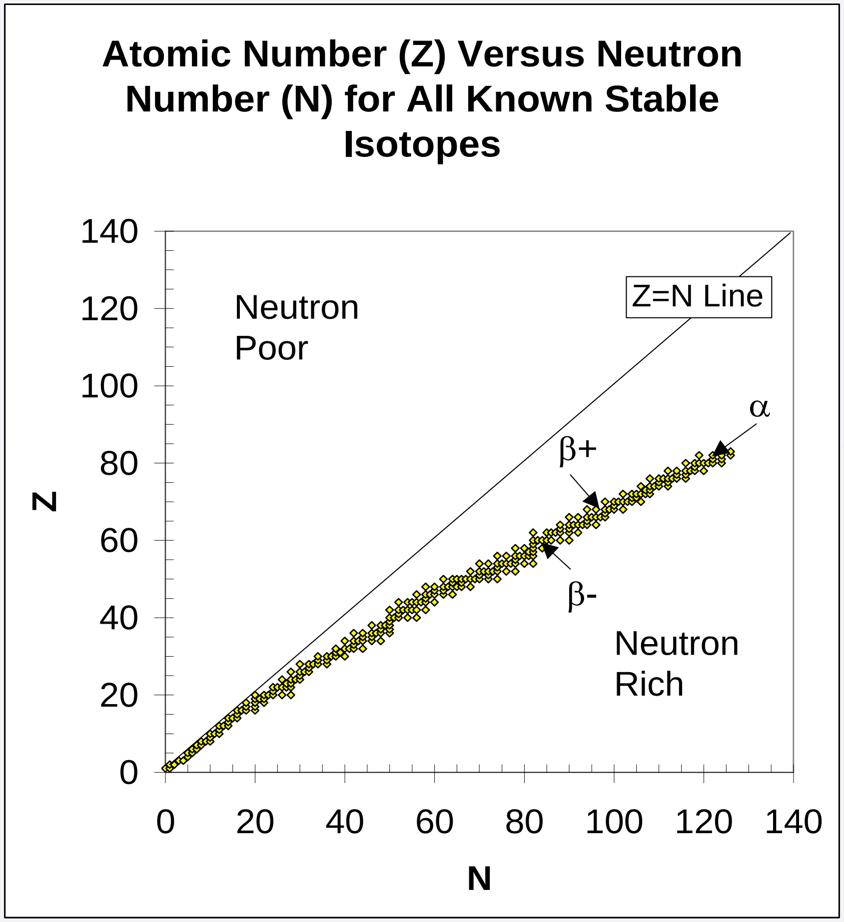
Band of Stability



Source: adapted from http://en.wikipedia.org/wiki/File:Isotopes_and_half-life.svg

Radioactive Decay

- The figure shows all of the stable nuclides as a function of Z (vertical axis) and N (horizontal axis)
  - Note that there are more neutrons than protons in most nuclei
  - The extra neutrons provide nuclear stability
- There are only certain combinations of N and Z that produce stable nuclei
  - The rest are radioactive



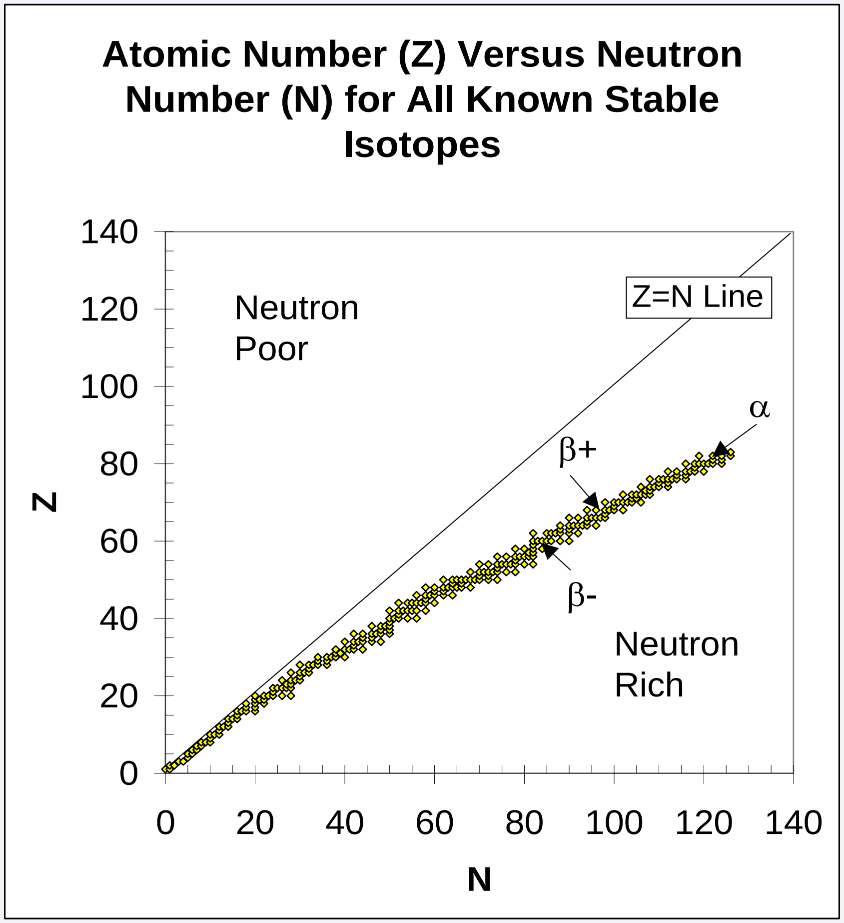
Author: Dr. Charlton, NUEN, TAMU

Alpha Decay

- The mode of decay for any isotope is determined by its position on the chart of the nuclides.
- Heavy elements are typically a (alpha) emitters
  - An alpha particle is an energetic helium nucleus (2 protons and 2 neutrons)

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha $$

- Heavy elements might also spontaneously fission



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Radioactive Decay (continued)

- Neutron rich isotopes are typically β - emitters

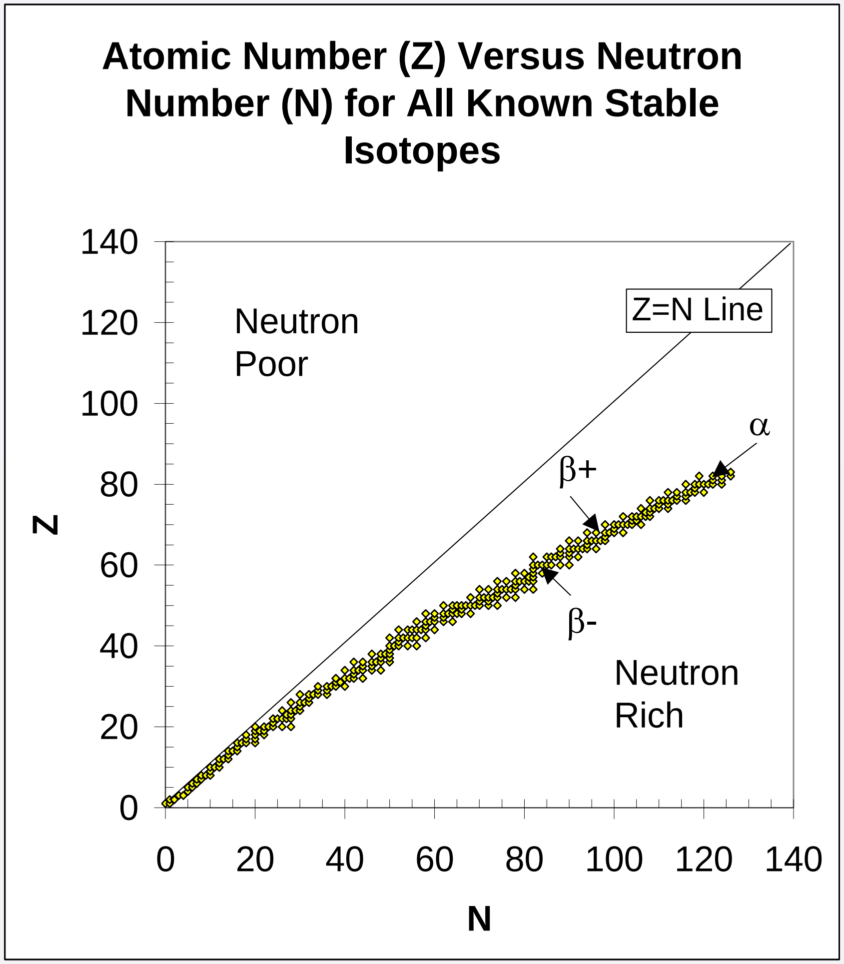
$$ {}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}\beta + {}_{0}^{0}\overline{\nu}_{e} $$

- Neutron poor isotopes are generally β+ emitters (or electron capture, E.C.)

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{1}^{0}\beta + {}_{0}^{0}\nu_{e} $$

$$ {}_{Z}^{A}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}^{A}Y + {}_{0}^{0}\nu_{e} $$

- Fission products are typically neutron rich



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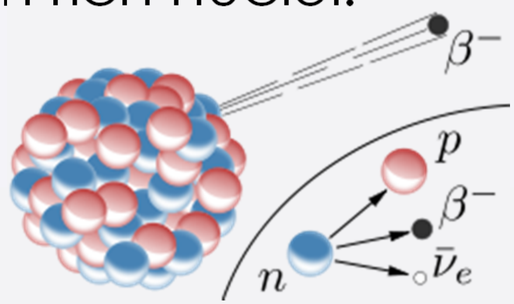
Beta Decay

- In β− decay, the weak interaction converts a neutron into a proton while emitting an electron and an electron antineutrino.
  - β− decay generally occurs in neutron rich nuclei.

$$n \xrightarrow{\beta^-} p + e^- + \bar{\nu}_e$$



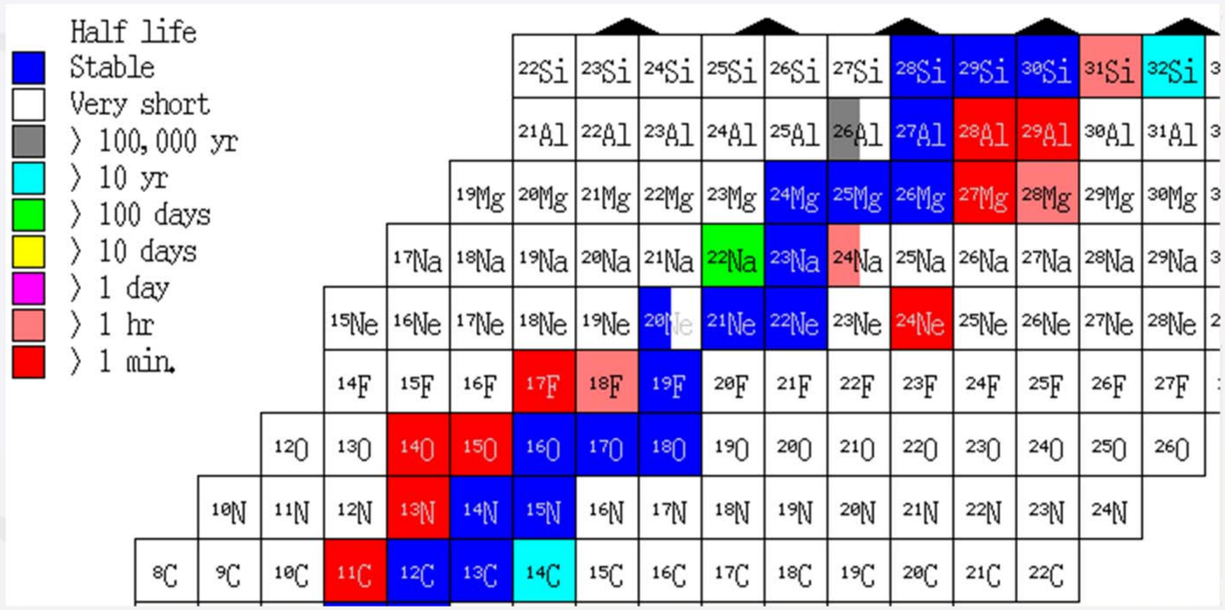
Source: http://en.wikipedia.org/wiki/Beta_decay

- In β+ decay, energy is used to convert a proton into a neutron, a positron (e+) and an electron neutrino (ν_e):

$$n \xrightarrow{\beta^+} p + e^+ + \nu_e$$

Radioactive Decay (continued)

- Consider the excerpt from the Chart of the Nuclides



Source: http://atom.kaeri.re.kr/ton/nuc1.html

- The element sodium (Z=11) has various isotopes
  - (N=11, N=12, N=13, …)
  - only Na-23 (Z=11, N=12) is stable
  - Na-22 decays by b^{+} emission
  - Na-24 decays by b^{-} emission

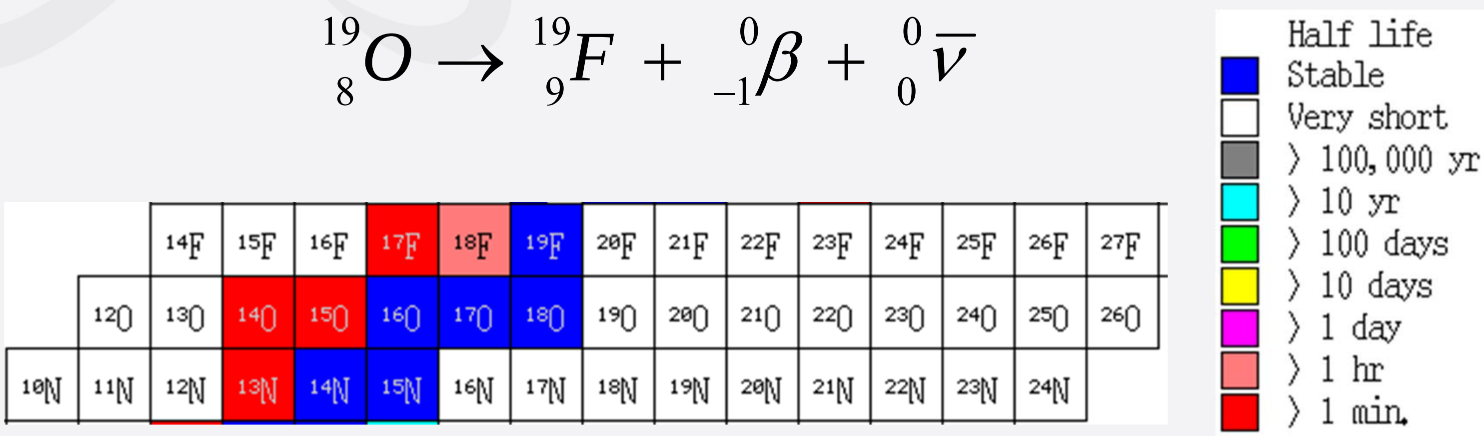
Radioactive Decay (continued)

- Nuclei (such as O-15) which are lacking in neutrons undergo b$^+$ decay:

$$^{15}_{8}O \rightarrow \, ^{15}_{7}N + \, ^{0}_{+1}\beta + \, ^{0}_{0}\nu$$

- On the other hand, nuclei (such as O-19) that are excessively neutron rich undergo b$^-$ decay:

$$^{19}_{8}O \rightarrow \, ^{19}_{9}F + \, ^{0}_{-1}\beta + \, ^{0}_{0}\overline{\nu}$$



Source:http://atom.kaeri.re.kr/ton/nuc1.html

Radioactive Decay (continued)

- Often, the daughter nucleus is also unstable and will undergo an additional b- decay
- This leads to a decay chain:

$$^{20}_{8}O \rightarrow \, ^{20}_{9}F + \, ^{0}_{-1}\beta + \, ^{0}_{0}\overline{\nu}$$

$$^{20}_{9}F \rightarrow \, ^{20}_{10}Ne + \, ^{0}_{-1}\beta + \, ^{0}_{0}\overline{\nu}$$

  – Where Ne-20 is stable

Source:http://atom.kaeri.re.kr/ton/nuc1.html

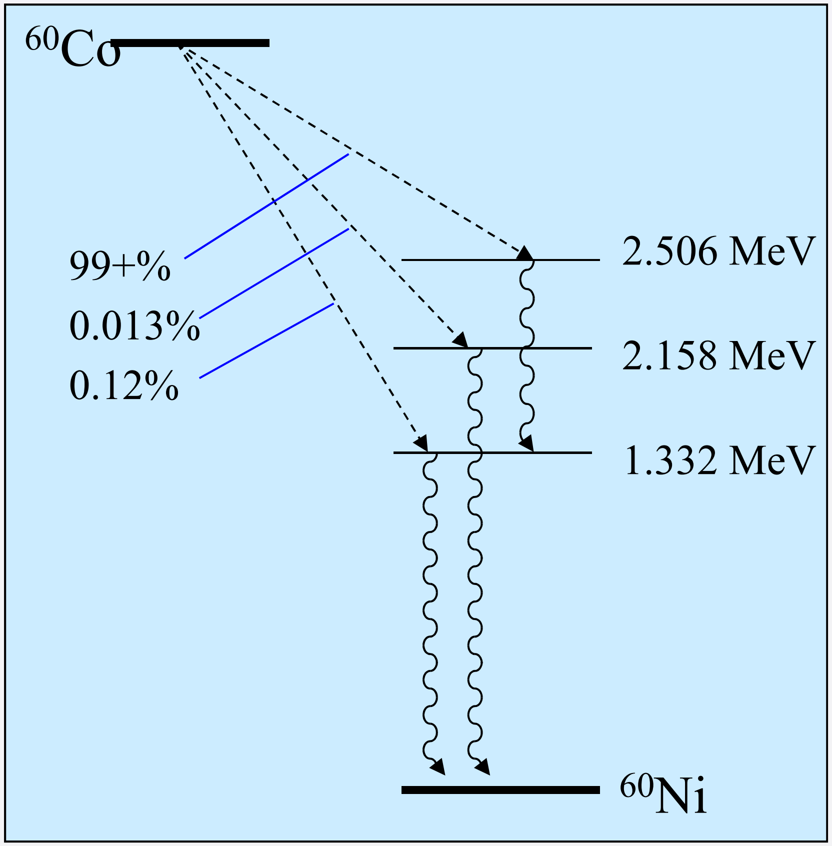
Gamma Decay Example

Nuclides often emit gamma rays in addition to other radiations; these emissions usually serve to bring the product nucleus to its ground state

- ^{60}Co decays as follows:

$$^{60}_{27}Co \rightarrow \, ^{60}_{28}Ni + \, ^{0}_{-1}\beta + \, ^{0}_{0}\overline{\nu} + \gamma$$

- Thus, most of the time ^{60}Co emits two gamma rays of 1173 and 1332 keV



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^{60}Co has a half-life of 5.27 years or

$$t_{1/2} = 5.27 \ yrs$$

Modes of Decay

Decay Mode	Product Nucleus	Principle Radiations	Other Radiations
α Alpha	A-4, Z-2	α-lines	γ-lines (often)
β^- Beta-	Z+1, isobaric	β^- –continuous	γ-lines (often)
β^+ Beta+	Z-1, isobaric	β^+ –continuous	γ-lines (often), annihilation γ (511 keV)
ε Electron Capture	Z-1, isobaric	ν	x-rays, auger electrons, lines of daughter
I.C. Internal Conversion	Z, A	e^- -lines	x-rays, auger electrons, lines of parent
I.T. Isomeric Transition	Z, A	γ-lines	n/a
S.F. Spontaneous Fission	distribution	fission fragments	n, β^-, γ, etc...

Activity

- Source activity (*A*)
  - The rate of radioactive decay (decays/sec)
  - Common units
    - Becquerel (Bq) – one Bq is one decay per second
    - Curie (Ci) – one curie is equal to 3.7×10^{10} decays per second
- Calculating activity:

$$A = \lambda \cdot N$$

N is the number of radioactive atoms in the sample

  - λ is the decay constant
    - Probability of decay per unit time per radioactive atom

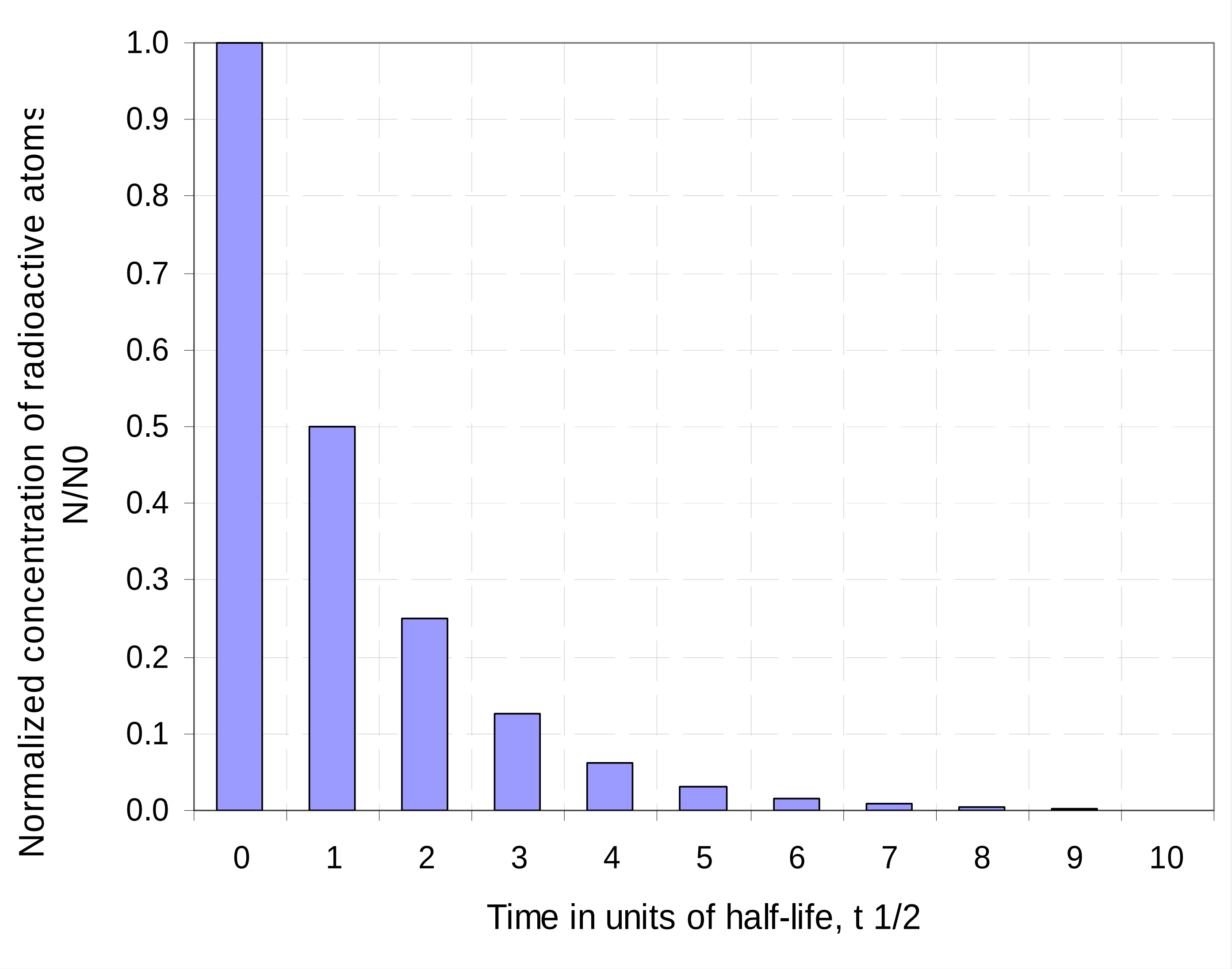
Decay Constant and Half-life

- Decay constant (λ)
  - Probability of decay per unit time per radioactive atom
  - λ is constant for all time and for all atoms of the same species

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

  - Half-life ($t_{1/2}$) of the radioactive atoms in a sample is
    - The time required for half of the radioactive atoms in the sample to decay: $N(t_{1/2}) = ½\, N(t=0)$
    - The time during which the activity of the sample decreases by a factor of two: $A(t_{1/2}) = ½\, A(t=0)$

Decay of a Radionuclide Sample



Decay Calculations

- We can derive most activity and decay relationships from the simple balance equation:

$$\frac{dN}{dt} = \left[\begin{matrix}\text{rate of}\\ \text{gain}\end{matrix}\right] - \left[\begin{matrix}\text{rate of}\\ \text{loss}\end{matrix}\right]$$

- Assume we

  Have a sample which contains only N_0 radioactive atoms at time $t=0$

  – Interested in the number of atoms [$N(t)$] at new time t

$$\frac{dN}{dt} = 0 - \lambda N(t) = -\lambda N(t)$$

$$\frac{1}{N(t)} dN = -\lambda\, dt$$

$$\int_{N_0}^{N(t)} \frac{1}{N'} dN' = \int_0^t -\lambda\, dt'$$

$$\ln[N(t)] - \ln[N_0] = -\lambda t - 0$$

$$\ln\left[\frac{N(t)}{N_0}\right] = -\lambda t$$

$$\frac{N(t)}{N_0} = e^{-\lambda t}$$

Decay Calculations (continued)

- Thus, number of atoms at time *t* is

$$N(t) = N_0 e^{-\lambda t}$$

- This also means the activity of our sample is

$$A = \lambda N$$

$$A(t) = \lambda N_0 e^{-\lambda t}$$

$$A(t) = A_0 e^{-\lambda t}$$

  – Where A_0 is the initial activity of the sample

- If the $N(t_{1/2})$ is half of the number of atoms we began with:

$$\frac{N(t_{1/2})}{N_0} = \frac{1}{2}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln 1 - \ln 2 = -\lambda t_{1/2}$$

$$0 - \ln 2 = -\lambda t_{1/2}$$

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Production of Radioisotopes

- If we had radioactive atoms being produced at some constant rate R
  - Then our balance equation would become:

$$\frac{dN}{dt} = R - \lambda N$$

  - Let's assume that at t=0 there are no radioactive atoms in the sample:

$$N(t=0) = 0$$

- Solution to this system is

$$N(t) = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

  - Where *t* is the irradiation time

  - We will assume we reach saturation after 4-5 $T_{1/2}$

Production of Radioisotopes Example

- Suppose we begin with a sample of ^{27}Al

$$^{27}Al + n \rightarrow {}^{28}Al + \gamma$$

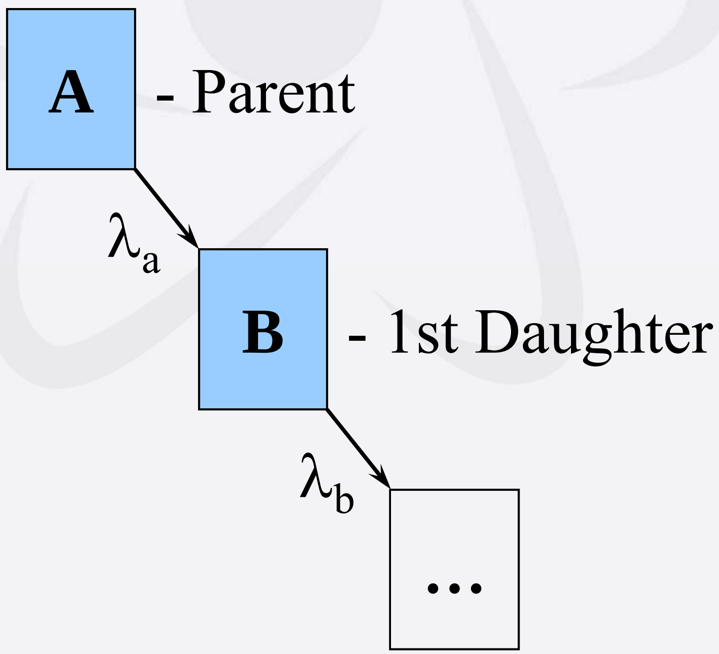
  - ^{28}Al has a half-life of 2.25 minutes
  - Thus, if we allow the sample to irradiate for ~10 minutes
    - We will have saturated the sample with ^{28}Al
- For an activated product

$$A(t) = R\left(1-e^{-\lambda t}\right)$$

  - As we saturate the sample, the activity will approach

$$A(t=\infty) = R \quad \leftarrow \text{saturation activity}$$

Decay Chains



- Many nuclides decay through complicated parent-daughter chains
- The concentrations of these nuclides can be determined analytically
- We will assume the following initial conditions:
  - $N_A(t=0)=N_0$
  - $N_B(t=0)=0$

Decay Chain Solution for A

- The governing equations for this system are

$$\frac{dN_A}{dt} = -\lambda_a N_A(t)$$

$$\frac{dN_B}{dt} = \lambda_a N_A(t) - \lambda_b N_B(t)$$

- The solution for A is

$$\frac{1}{N_A(t)} dN_A = -\lambda_a \, dt$$

$$\int_{N_0}^{N_A(t)} \frac{1}{N_A'} dN' = \int_0^t -\lambda_a \, dt'$$

$$\ln\left[\frac{N_A(t)}{N_0}\right] = -\lambda_a t$$

$$N_A(t) = N_0 e^{-\lambda_a t}$$

Decay Chain Solution for *B*

- Thus, the equation for *B* becomes

$$\frac{dN_B}{dt} = \lambda_a N_0 e^{-\lambda_a t} - \lambda_b N_B$$

- The solution for *B* is

$$\frac{dN_B}{dt} + \lambda_b N_B = \lambda_a N_0 e^{-\lambda_a t}$$

$$\frac{dN_B}{dt} e^{\lambda_b t} + \lambda_b N_B e^{\lambda_b t} = \lambda_a N_0 e^{-\lambda_a t} e^{\lambda_b t}$$

$$\frac{d}{dt}\left[N_B e^{\lambda_b t}\right] = \lambda_a N_0 e^{(\lambda_b - \lambda_a)t}$$

Decay Chain Solution for *B* (continued)

- Solving the ODE yields:

$$N_B e^{\lambda_b t} = \int \lambda_a N_0 e^{(\lambda_b - \lambda_a)t}\, dt$$

$$N_B e^{\lambda_b t} = \frac{\lambda_a N_0}{\lambda_b - \lambda_a} e^{(\lambda_b - \lambda_a)t} + D$$

$$N_B(t) = \frac{\lambda_a N_0}{\lambda_b - \lambda_a} e^{-\lambda_a t} + De^{-\lambda_b t}$$

- Our initial condition is:

$$N_B(t=0) = 0$$

Decay Chain Solution for *B* (continued)

- Substituting *t=0* into our equation above:

$$N_B(t=0) = \frac{\lambda_a N_0}{\lambda_b - \lambda_a} + D = 0$$

$$D = -\frac{\lambda_a N_0}{\lambda_b - \lambda_a}$$

$$N_B(t) = \frac{\lambda_a N_0}{\lambda_b - \lambda_a} e^{-\lambda_a t} - \frac{\lambda_a N_0}{\lambda_b - \lambda_a} e^{-\lambda_b t}$$

- Thus, the solution for *B* is

$$N_B(t) = \frac{\lambda_a N_0}{\lambda_b - \lambda_a}\left(e^{-\lambda_a t} - e^{-\lambda_b t}\right)$$

Radioactive Equilibrium

- Radioactive equilibrium exists when the production rate and a decay rate for a given radioisotope are equal.
- Transient equilibrium
  - The parent isotope must have a longer half-life then the daughter
    - $t_{1/2\ parent} > t_{1/2\ daughter}$ ($\lambda_{parent} < \lambda_{daughter}$)

Example: $^{140}Ba \rightarrow {}^{140}La$.

- Secular equilibrium
  - The parent isotope has a long half-life compared to the daughter
    - $t_{1/2\ parent} >> t_{1/2\ daughter}$ ($\lambda_{parent} << \lambda_{daughter}$)

Example: $^{247}Pu \rightarrow {}^{247}Am$.

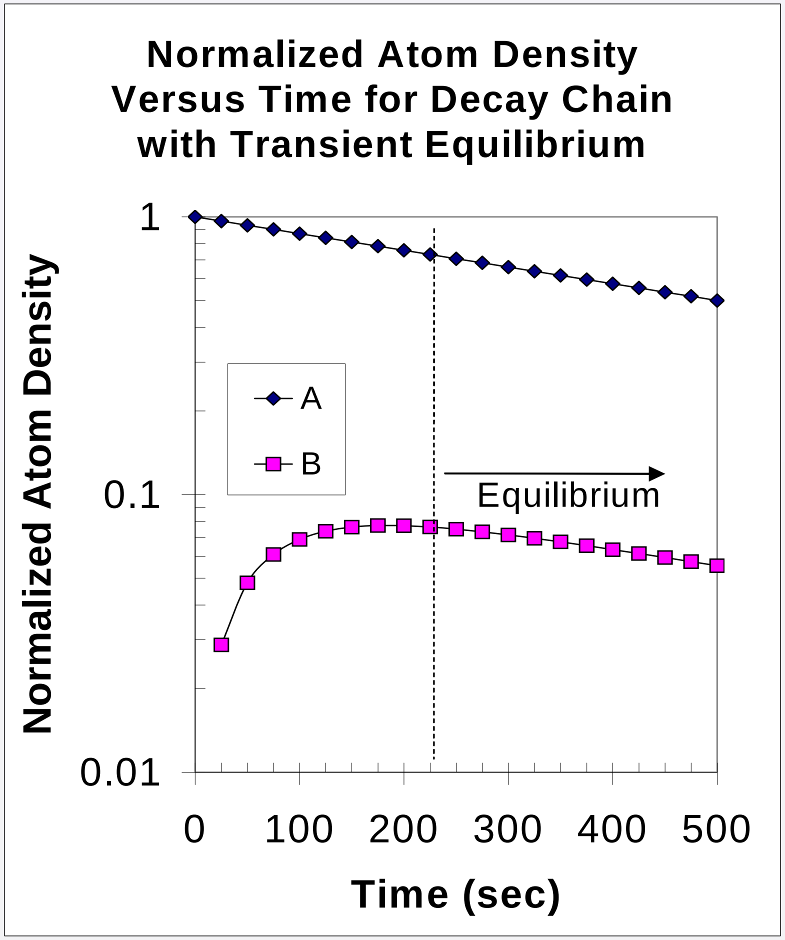
Transient Equilibrium

- If $\lambda_b > \lambda_a$, then when t gets large

$$N_B(t) = \frac{\lambda_a N_0}{\lambda_b - \lambda_a}\left(e^{-\lambda_a t} - e^{-\lambda_b t}\right)$$

$$N_B(t) = \frac{\lambda_a N_0}{\lambda_b - \lambda_a} e^{-\lambda_a t}$$

$$N_B(t) = \frac{\lambda_a}{\lambda_b - \lambda_a} N_A(t)$$



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Secular Equilibrium

- If parent has extremely long half-life (i.e., λ_a is very small):

$$N_B(t) \cong \frac{\lambda_a N_0}{\lambda_b - \lambda_a}\left(1 - e^{-\lambda_b t}\right)$$

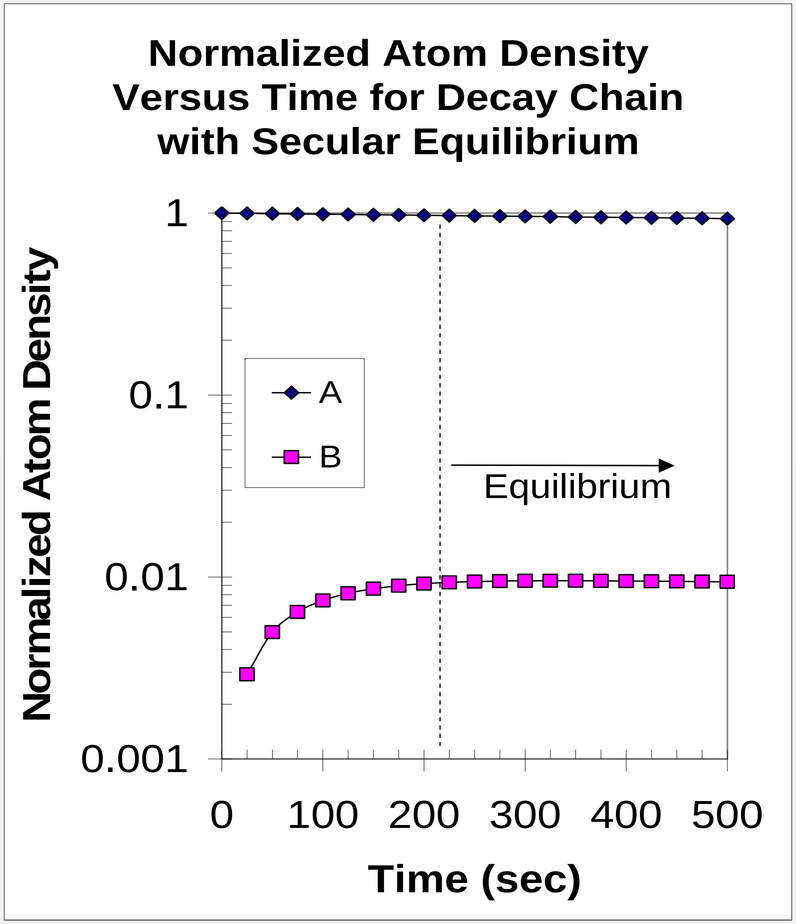
- This implies $\lambda_b >> \lambda_a$

$$N_B(t) \cong \frac{\lambda_a N_0}{\lambda_b}\left(1 - e^{-\lambda_b t}\right)$$

$$\lambda_b N_B(t) \cong \lambda_a N_0\left(1 - e^{-\lambda_b t}\right)$$

- After 4-5 half-lives, we reach secular equilibrium

$$\lambda_b N_B(t) \cong \lambda_a N_0$$



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Module 1: Nuclear Energy Fundamentals (Week 2/Day 3)

Lecture #2: Fission

Dr. David Boyle

Mass Defect

- The mass of a "heavy" atom is slightly less than the sum of the masses of the composite subatomic particles.
- **Mass defect,** ΔM**,** is the difference between the mass of the atom and the sum of the masses of the free protons, neutrons, and electrons comprising the atom.

$$\Delta M = Zm_p + (A - Z)m_n + Zm_e - M_{atom}$$

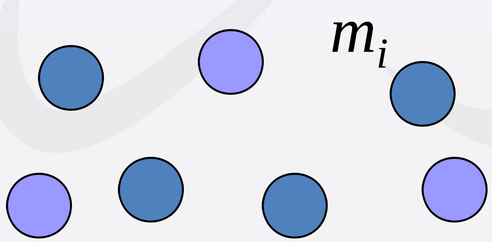
$$m_p = rest\ mass\ of\ a\ proton = 1.007277\ amu$$
$$m_n = rest\ mass\ of\ a\ neutron = 1.008665\ amu$$
$$m_e = rest\ mass\ of\ an\ electron = 0.000548597\ amu$$

- ΔM **= *Mass before assembly – Mass after assembly***

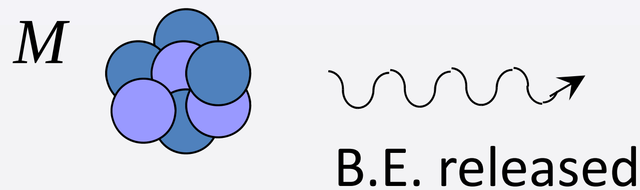
Binding Energy (B.E.)

- The **Binding Energy** (B.E.) of a group of particles which are assembled into a system is the energy released by combining the system of free particles.

Before assembly



After assembly



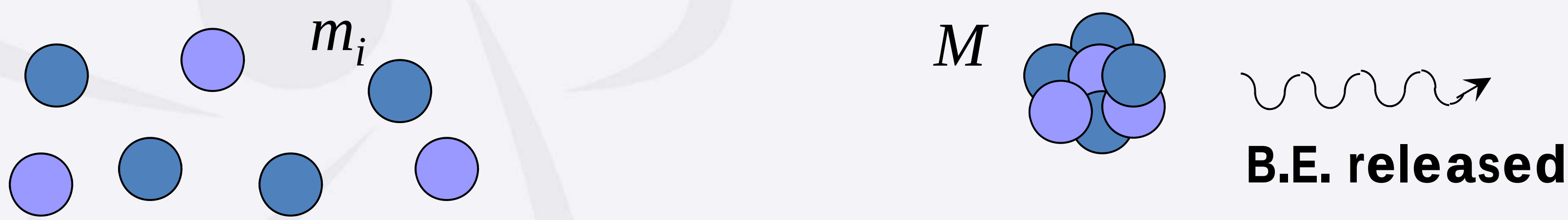
Free particles at rest; m_i – rest mass.

Bound particles are at rest. M – rest mass of assembly.

Binding Energy (continued)

- Conservation of energy: $E_{before} = E_{after}$

Before assembly | After assembly



$$\sum m_i c^2 = Mc^2 + B.E.$$

$$B.E. = \sum m_i c^2 - Mc^2 = \left(\sum m_i - M\right)c^2$$

$$B.E. = \Delta M c^2$$

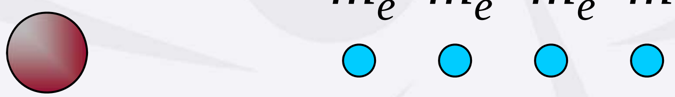
- If B.E. > 0, energy is produced during assembly and corresponding mass is consumed.

B.E. of Orbital Electrons to the Nucleus

Before assembly

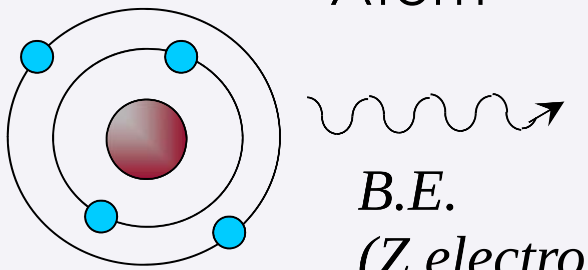
Nucleus — $M_n(Z,A)$

Z electrons — m_e m_e m_e m_e



After assembly

Atom



B.E. (Z electrons)

$M_a(Z,A)$

$$\Delta M = \left(M_n(Z,A) + Zm_e\right) - M_a(Z,A)$$

$$B.E.(Z\ electrons) = \Delta Mc^2$$

- Thomas-Fermi statistical model of the atom:

$$B.E.\ (Z\ electrons) \approx 15.7\ Z^{7/3}\ \text{eV}$$

For Hydrogen-1, B.E. = 13.6 eV.

Example

- Consider ^{238}U:

  92 protons, 238 – 92 = 146 neutrons, 92 electrons.

  B.E. (92 electrons) ≈ 15.7 $(92)^{7/3}$ eV ≈ 0.6 MeV

  ΔM = *(0.6 MeV) / (931.5 MeV/amu) = 6.4 $\times 10^{-4}$ amu*
- This is very small compared to the atomic mass of 238 amu.
- If we neglect the B.E. of the electrons, we obtain:

$$M_a(Z, A) = M_n(Z, A) + Zm_e$$

- We can use this unless otherwise stated.

B.E. of Nucleons in a Nucleus

Before assembly

Z protons ***(A-Z)*** neutrons

m_p m_n

After assembly

Nucleus

$M_n(Z,A)$

B.E. (Z electrons)

$$B.E. nucleus = (mass\ before - mass\ after)c^2$$

$$B.E. nucleus = \Delta M c^2 = \left([Z m_p + (A-Z) m_n] - M_n(Z,A)\right) c^2$$

The energies associated with nuclear transformations are usually in the MeV range. There is a huge difference in the energy released when an atom undergoes a nuclear transformation as compared with a chemical reaction (rearrangement of electrons).

B.E. of Nucleons in a Nucleus (continued)

- Nuclear masses are difficult to measure. Rather, atomic masses are measured and tabulated

$$B.E.nucleus = \Delta M c^2 = \left([Zm_p + (A-Z)m_n] - M_n(Z,A) \right) c^2$$

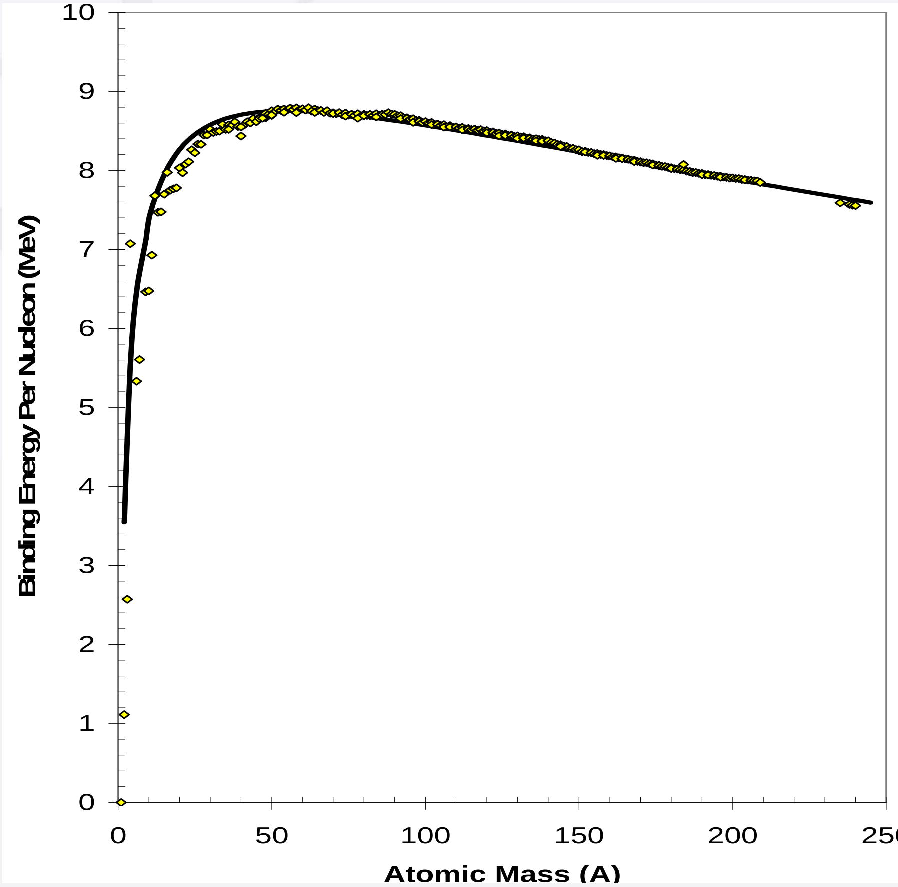
$$B.E.nucleus = \left([Zm_p + (A-Z)m_n] - [M_a(Z,A) - Zm_e] \right) c^2$$

$$= \left(Zm_p + (A-Z)m_n - M_a(Z,A) + Zm_e \right) c^2$$

$$= \left(Zm_p + Zm_e + (A-Z)m_n - M_a(Z,A) \right) c^2$$

$$B.E.nucleus = \left(ZM_a\left({}_1^1H\right) + (A-Z)m_n - M_a(Z,A) \right) c^2$$

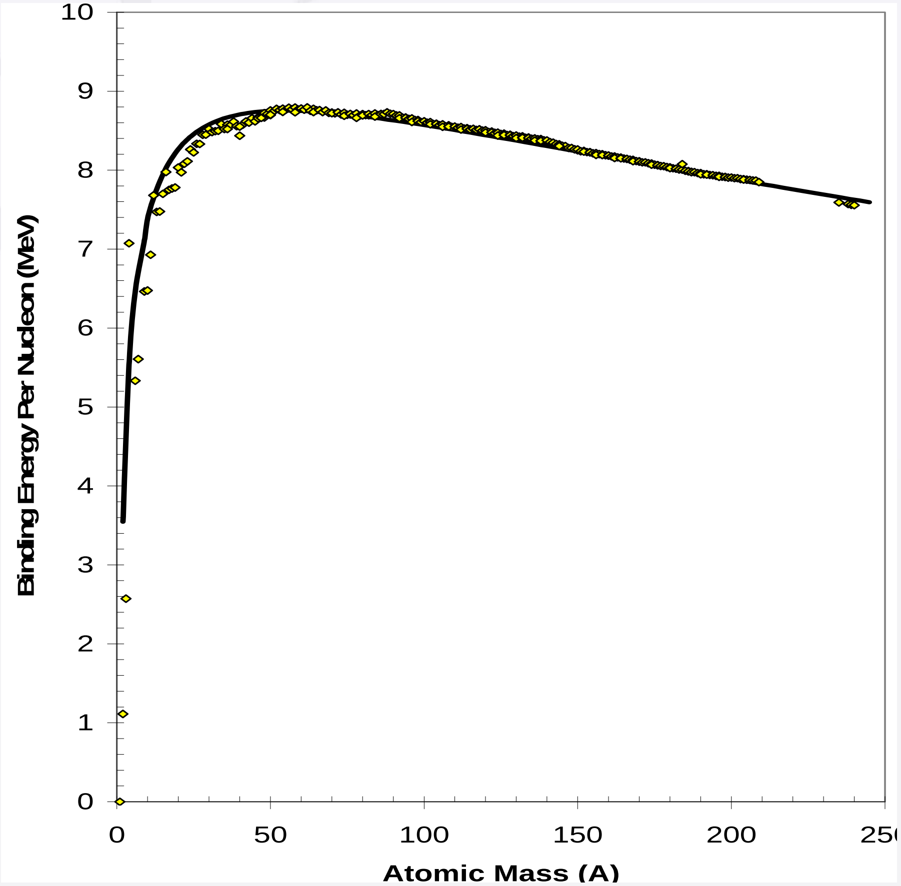
B.E. of Nucleons in a Nucleus (continued)



- BE is the energy released when forming a nucleus; it is also the energy required to break the nucleus into its constituent nucleons.
- The total BE of a nucleus is a function of ***A***.
- Nuclei where the BE per nucleon is high are very stable.

Author: Dr. Charlton, NUEN, TAMU

B.E. of Nucleons in a Nucleus (continued)



- If during a nuclear reaction the average BE/nucleon increases, energy is liberated and mass is consumed. The energy released is equal to the total change in BE of all the nucleons.
- Energy is released for fission of heavy nuclei and for fusion of light nuclei.

Author: Dr. Charlton, NUEN, TAMU

Binding Energy of the "Last Neutron"

- Referred to as the amount of energy released when adding the "last" neutron to the nucleus

$$ {}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A+1}_{Z}X $$

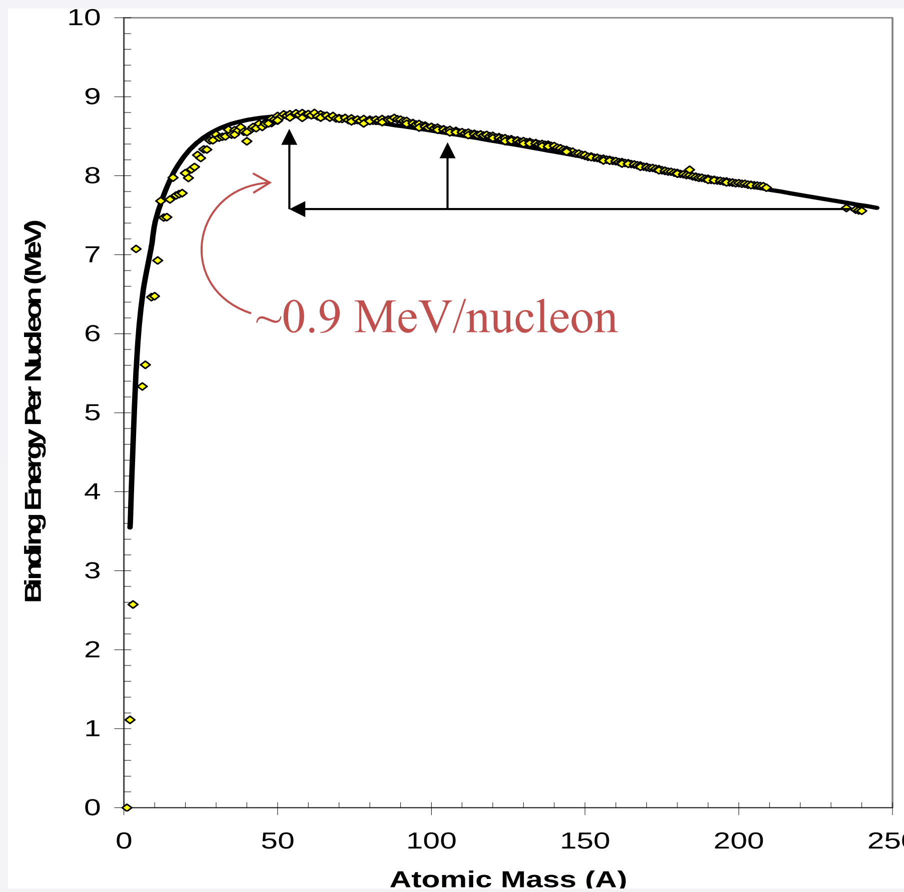
- This binding energy is also called the separation energy (E_s)

$$ E_s = \left[m_n + m\left({}^{A}_{Z}X\right) - m\left({}^{A+1}_{Z}X\right) \right] c^2 $$

- E_s is just sufficient to remove a neutron without providing it with any kinetic energy

Why Fission Occurs

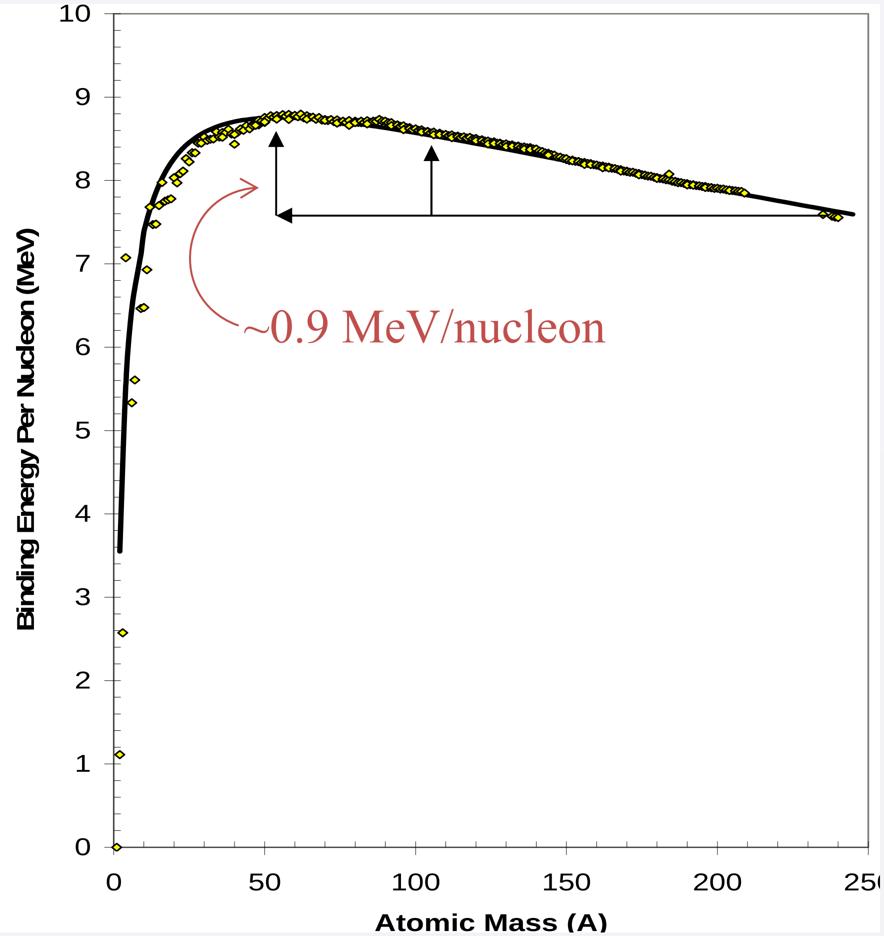
- The liquid drop model binding energy curve explains why fission occurs
  – the binding energy per nucleon of nuclei decreases with increasing atomic mass for *A > 50*
  – thus, a more stable configuration of nucleons is produced when a heavy nuclei is split into two or more parts



Author: Dr. Charlton, NUEN, TAMU

Fission Energy

- The initial estimate of energy released in fission also came from the liquid drop model binding energy curve.
- The average binding energy per nucleon is
  - About 7.6 MeV in the vicinity of U
  - About 8.5 MeV for *A* from 40 to 140



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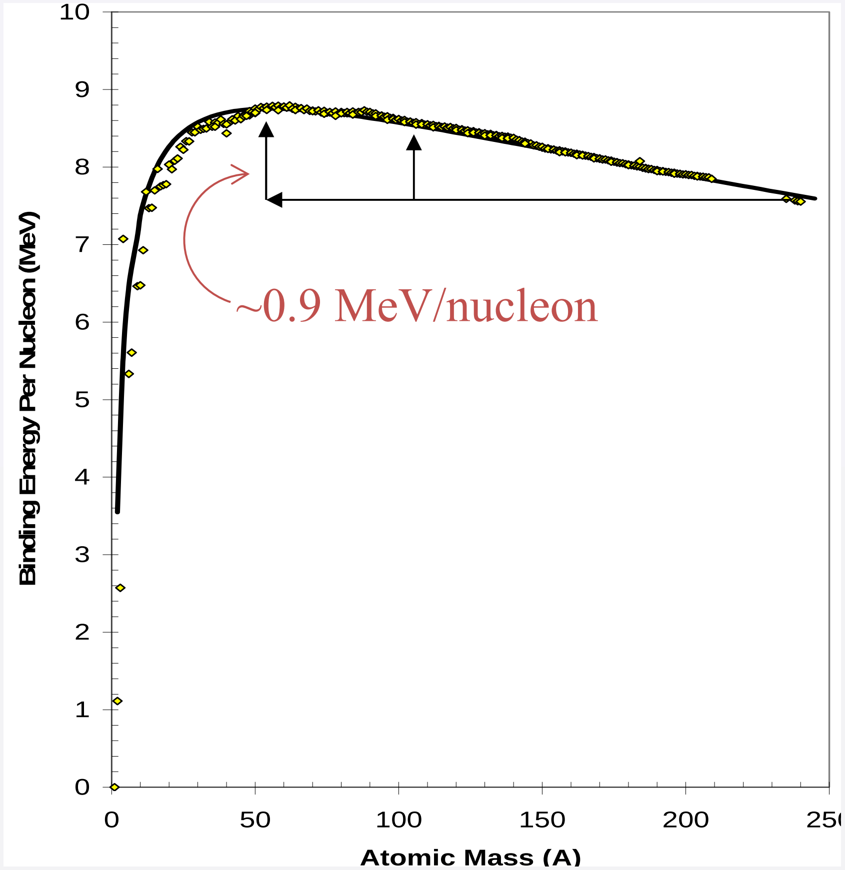
Fission Energy (continued)

- The average binding energy per particle is
  - About 0.9 MeV greater in the fission products than in the U
- Since there are about 235 nucleons in U
  - The total energy released is around 210 MeV



$$ {}_{0}^{1}n + {}_{92}^{235}U \rightarrow \left({}_{92}^{236}U\right)^{*} \rightarrow {}_{55}^{140}Cs + {}_{37}^{93}Rb + 3\left({}_{0}^{1}n\right) $$

Fissile Nuclides.
Fissionable but Nonfissile Nuclides

- Fissile nuclides
  - Undergo fission following the absorption of a zero-energy neutron (room temperature neutron).
    - Example: ^{233}U, ^{235}U, ^{239}Pu, and ^{241}Pu

- Fissionable but non-fissile nuclides
  - Undergo fission if the incident neutron has some threshold kinetic energy
    - Example: ^{232}Th, ^{238}U, ^{240}Pu

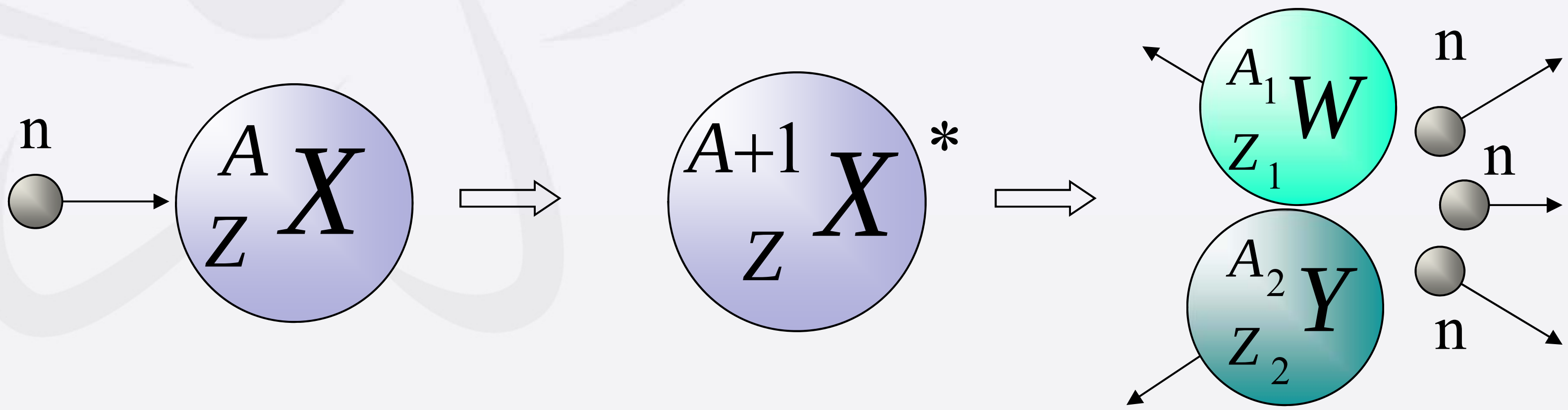
- It is possible to manufacture certain fissile nuclides from non-fissile nuclides
- The non-fissile nuclides from which fissile isotopes can be produced by neutron absorption are said to be *fertile*.
- Fissionable but non-fissile nuclides ^{232}Th and ^{238}U can be used to produce fissile nuclides: ^{233}U and ^{239}Pu, accordingly

$$^{232}\mathrm{Th}(n,\gamma)^{233}\mathrm{Th} \xrightarrow{\beta^-} {}^{233}\mathrm{Pa} \xrightarrow{\beta^-} {}^{233}\mathrm{U}$$

$$^{238}\mathrm{U}(n,\gamma)^{239}\mathrm{U} \xrightarrow{\beta^-} {}^{239}\mathrm{Np} \xrightarrow{\beta^-} {}^{239}\mathrm{Pu}$$

Neutron-induced Fission

- A neutron is absorbed by a target nucleus: a compound nucleus is formed in an excited state.



Kinematics of Fission

- The fission reaction consists initially of
  - The absorption of a neutron by a target nucleus to form a compound nucleus and the subsequent scission of this nucleus into two fragments

$$ {}^{A}_{Z}X + {}^{1}_{0}n \rightarrow \left[{}^{A+1}_{Z}X^{*} \right] \rightarrow {}^{A1}_{Z1}V + {}^{A2}_{Z2}W $$

- The Q-value for fission is then

$$ Q = 931.5 \frac{MeV}{amu} \left[M\left({}^{A}_{Z}X\right) + m_n - M\left({}^{A1}_{Z1}V\right) - M\left({}^{A2}_{Z2}W\right) \right] $$

- For the ^{236}U compound nucleus, we get

  Q = 170 MeV

  – this is less than 200 MeV because the fission fragments are unstable and will release the remaining energy by β^- decays

- For symmetric fission, we would have a Q-value of approximately

$$Q = 931.5\frac{MeV}{amu}\left[\, M\left({}^{A+1}_{Z}X\right) - 2M\left({}^{(A+1)/2}_{Z/2}V\right)\right]$$

Kinematics of Fission (continued)

- Fission occurs in a variety of ways
  - therefore the masses of the products vary from fission event to fission event
  - thus, the Q-value for fission is not a single well-defined quantity
    - it is an average over the probability distributions of the fragment masses
    - regardless of mode it is found to lie between 185 MeV and 220 MeV
      - average for ^{235}U of 206 MeV
- The production of equal mass fragments is rare
  - frequency of about 1 in 100,000 fissions

Kinematics of Fission (continued)

- Scission of the compound nucleus into three or more fragments also occurs, though it is rare
- Thermal fission of ^{235}U:
  - frequency of ternary fission is 1 in 400
  - frequency of quaternary fission is less than 1 in 3000 fissions
- Ternary fission almost always occurs with the emission of an alpha particle; a triton is emitted in 1 in 10,000 fissions

Kinematics of Fission (continued)

- Based on the liquid drop model binding energy curve, fission is energetically possible
  - In fact, it is an exothermic reaction for nuclei with masses greater than about 90 amu
- This would suggest that nuclei with *A > 90* would undergo spontaneous fission
  - Actually spontaneous fission has never been observed for any nuclide lighter than Th-230
    - Whose half-life is 10^{17} years
  - The half-life for fission decreases rapidly with increasing atomic number
    - Until at Z=102 it is only of the order of seconds

Kinematics of Fission (continued)

- The half-life for the spontaneous fission of ^{236}U from its ground state is about 2×10^{16} years
- How is it then that the absorption of a slow neutron by ^{235}U may result in the essentially instantaneous fission of the ^{236}U compound nucleus?
  - the nucleus requires an excitation energy to allow it to overcome a barrier presented by the short-ranged nuclear forces
  - this excitation energy is supplied by the binding energy of the neutron absorbed in ^{235}U

Apparent Thresholds

- The "critical energy," E_{crit} , is the minimum amount of energy added to the nucleus necessary to cause fission
- With most heavy nuclei, the binding energy of the last neutron is insufficient to induce fission
  - in these cases, the incident neutron must have enough kinetic energy to supply E_{crit}-E_s
  - otherwise the probability of fission is very small
    - but not zero, thus these thresholds are "apparent thresholds"

Fission Process Examples

- ^{235}U (Z=92, A=235)
  - Compound nucleus ^{236}U (Z=92, A=236)
  - ^{236}U critical energy $E_{crit} \approx 5.3$ Mev
  - ^{236}U separation energy $E_s \approx 6.4$ MeV
  - $E_{crit} < E_s$

  Thus, ^{235}U is fissile.

- ^{208}Pb (Z=82, A=208)
  - Critical energy $E_{crit} \approx 20$ Mev
  - Separation energy $E_s \approx 6$ MeV
  - $E_{crit} > E_s$

  Thus, ^{208}Pb is fissionable (but it's expensive).

Separation Energy of the Last Neutron and Critical Energies*

Target Nucleus	Fissioning Nucleus (Compound Nucleus)	E_{crit} (MeV)	E_s (MeV)	Apparent Threshold (Mev)
^{232}Th	^{233}Th	6.5	5.1	1.4
^{233}U	^{234}U	4.6	6.6	0
^{235}U	^{236}U	5.3	6.4	0
^{238}U	^{239}U	5.5	4.9	0.6
^{239}Pu	^{240}Pu	4.0	6.4	0

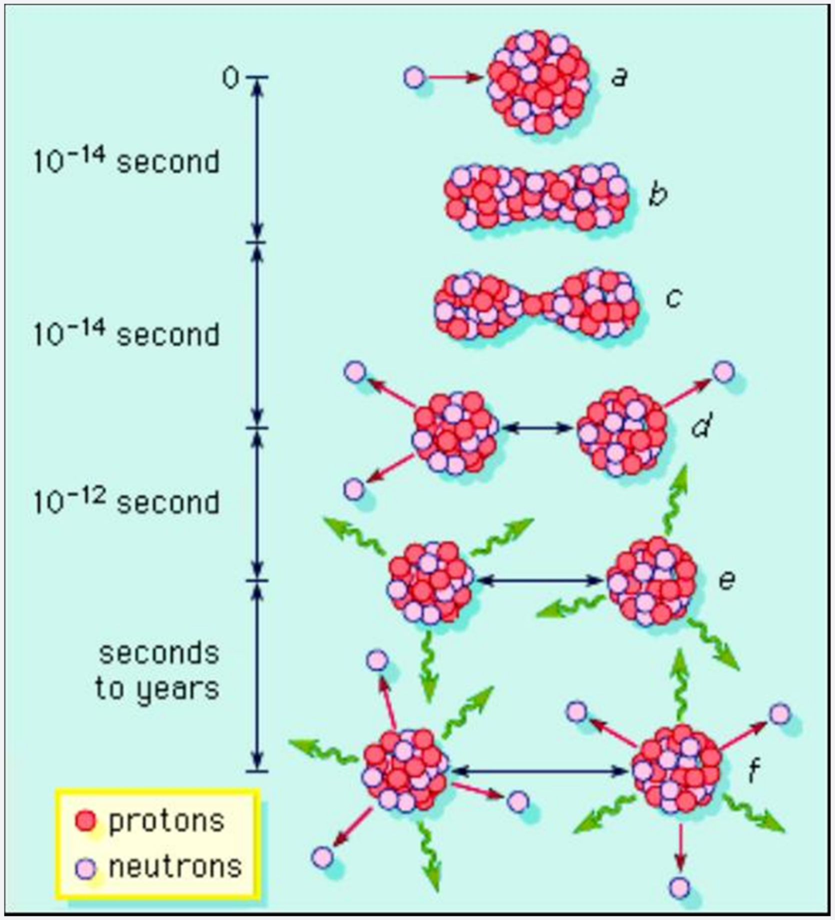
*** Data from “Introduction to Nuclear Engineering” by Lamarsh and Baratta, third edition, 2001**

Characteristics of Fission Reactions

- The fission reaction has characteristics different from those of our 5 other general types of neutron-nucleus reactions:
  - Large energy release (~200 MeV per reaction) which is not single valued for all reactions
  - Not completed in one or two stages
    - The actual splitting (or scission) is only the first stage in a complex chain of events
    - 40 MeV of the fission energy resides in excitation of the fission fragments
  - Produces dozens of emitted particles
    - Including neutrons (usually 2-3 neutrons)

Characteristics of Fission Reactions (continued)

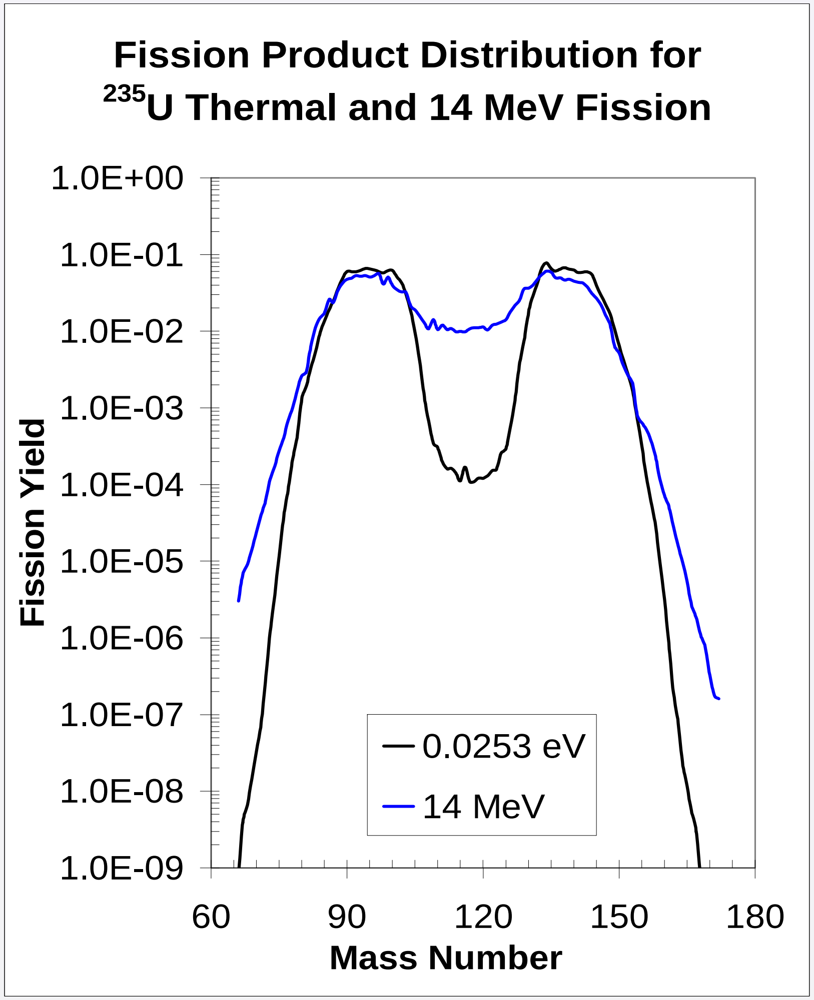
- Fission fragments give up their internal energy in a series of steps
  - Prompt neutrons and gamma-rays within 10^{-14} to 10^{-12} sec
  - Around 10^{-11} sec, fission fragments come to rest
  - Above 10^{-3} sec, fission products begin chains of beta decays; some decay by neutron emission (delayed neutrons)



Source: unknown

Fission Products

- Fission product distributions as a function of FP mass number tend to show a binomial distribution
  - This behavior is reflected in the liquid drop model
- On average, a fissioning nuclide splits in half (i.e., two equal mass, equal charge products)
  - However, symmetric fission is in fact a rare event



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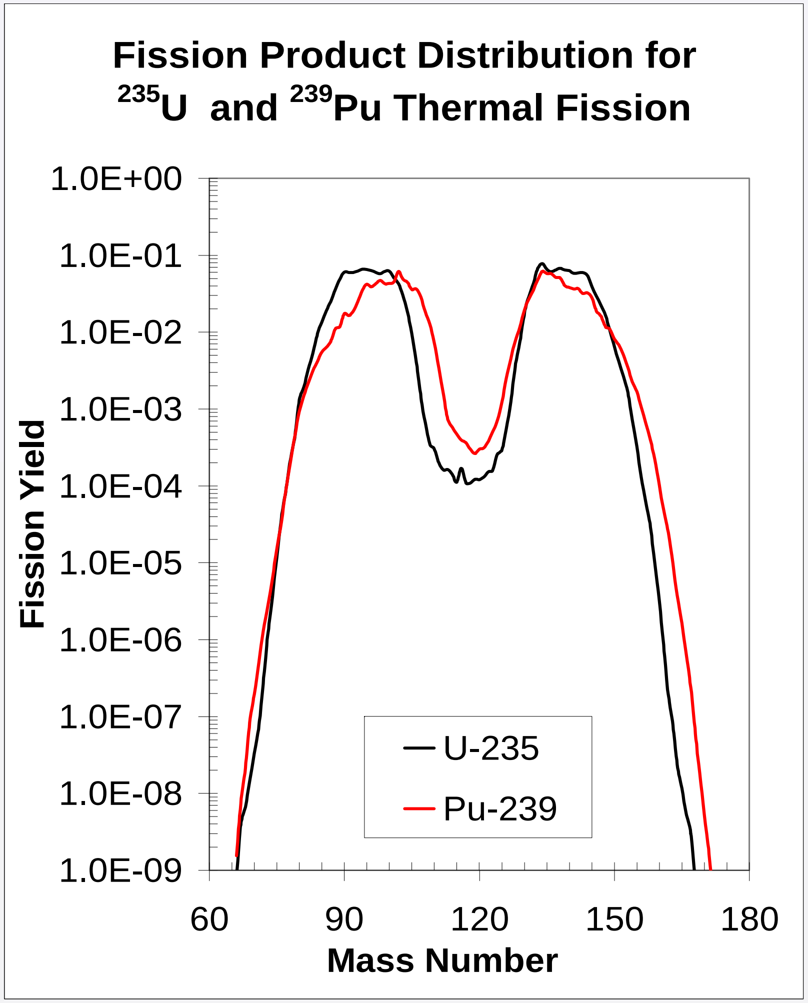
Fission Products (continued)

- The fission product distributions are a function of
  - Fissioning nuclide
  - Incident neutron energy
- Fission products tend to be neutron rich (b- and g emission)
- Fission products:
  - Are biological hazards
  - Produce decay heat
  - Are neutron poisons



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Energy from Fission

Particles	Energy Emitted (MeV)	Energy Recovered (MeV)	Where Deposited
Fission fragments	168	168	fuel
Prompt neutrons	5	5	fuel, clad, coolant
Prompt γ-rays	7	7	fuel, clad, coolant, shield
Decay β's	8	8	fuel
Decay γ-rays	7	7	fuel, clad, coolant, shield
Decay neutrinos	12	0	universe
Capture γ-rays	0	3-12	fuel, clad, coolant, shield
Total	*207*	*198-207*	

References

1. Lamarsh, J. R., and Baratta, A. J., "Introduction to Nuclear Engineering", third edition, Prentice Hall, 2001
2. Turner, J. E., "Atoms, radiation, and radiation protection", third edition, Wiley-VCH, 2007
3. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf

List of Required Texts

1. DOE Fundamentals Handbook “Nuclear Physics and Reactor Theory”, http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf

Module 1: Nuclear Energy Fundamentals (Week 2/Day 4)

Neutron Interactions

Gulf Nuclear Energy Infrastructure Institute – 2012 Fundamentals Course

Dr. David Boyle
Sandia National Laboratories

Module 1/Week 2:

- **Nuclear and Reactor Physics**

Week 2 Learning Objectives:

- Be able to explain how nuclear power works and why it is needed
- Be able to explain why nuclear power works
- Be able to explain why fission products and radiation are major issues for nuclear power
- Be able to explain the role of neutron interactions in nuclear power

Today's Primary Learning Objective

Primary Day 4 Learning Objective:

- Types and characteristics of neutron interactions with matter

Take away from this lecture:

- Main ways neutrons interact with matter

Module 1: Nuclear Energy Fundamentals (Week 2/Day 4)

Lecture #1: Neutron Interactions

Dr. David Boyle

Lecture Outline

1. Neutron Interactions
   - Scattering
   - Absorption
   - Fission

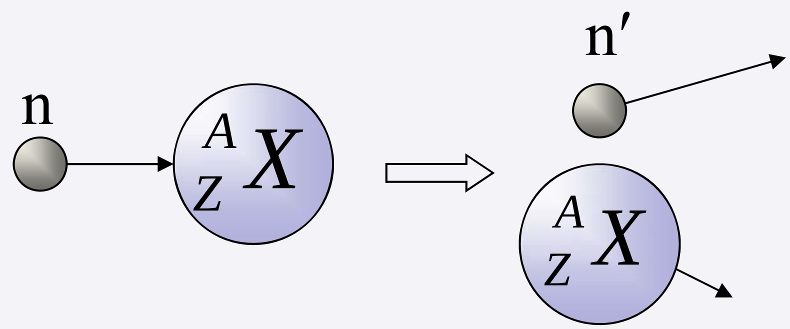
2. Cross Sections

Neutron Interactions

- Neutrons have zero net electrical charge. They pass through electron clouds and interact with atomic nuclei.
- Types of neutron interactions:
  - Scattering
    - Elastic Scattering – the most important process of slowing down neutrons in nuclear reactors
    - Inelastic Scattering
  - Absorption
    - Radiative Capture
    - Charged-Particle Reactions
    - Neutron-Producing Reactions
    - Fission

Elastic Scattering (n, n')

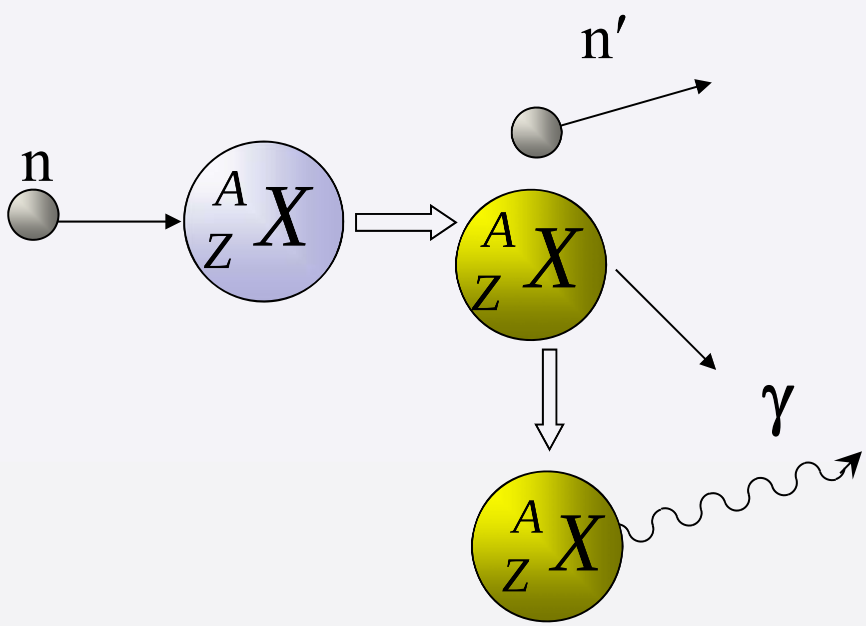
- A neutron strikes a nucleus; the target nucleus is almost always in its ground state.



- Some kinetic energy is transferred from the neutron to the target nucleus.
- The neutron reappears (the initial and final neutrons are not necessarily the same). The target nucleus is left in its ground state – there is no energy transferred into nuclear excitation.
- **Total kinetic energy is conserved**!

Inelastic Scattering (n, n')

- The target nucleus absorbs the incident neutron: a compound nucleus is formed.
- The compound nucleus emits a neutron of lower kinetic energy; the target nucleus is left in an excited state.



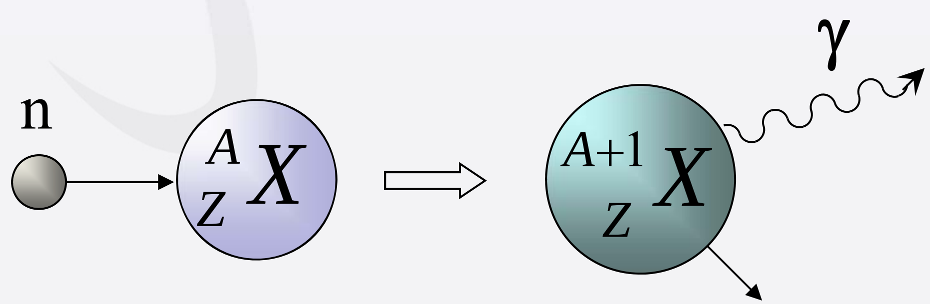
The target nucleus will reach its ground state by emitting one or more gamma rays.

Total energy is conserved (but not the kinetic energy!):

Initial kinetic energy of incident neutron	**=**	***Kinetic energy of the emitted neutron + kinetic energy of the target nucleus + energy of all emitted gamma rays***

Radiative Capture (n, γ)

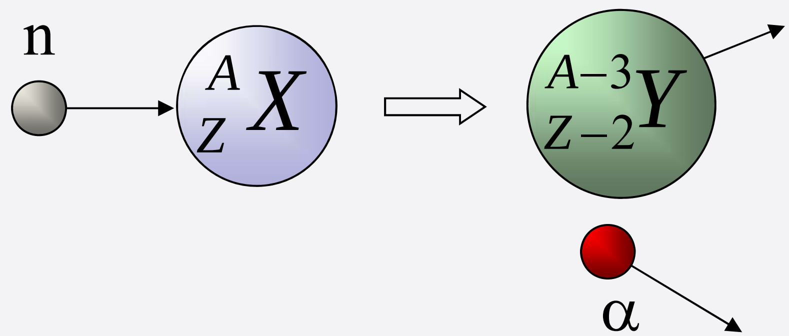
- The target nucleus absorbs the incident neutron: a compound nucleus is formed.
- The compound nucleus *immediately* emits a gamma ray.



Charged-Particle Reactions

(n,a), (n, p), (n, d), (n, t)

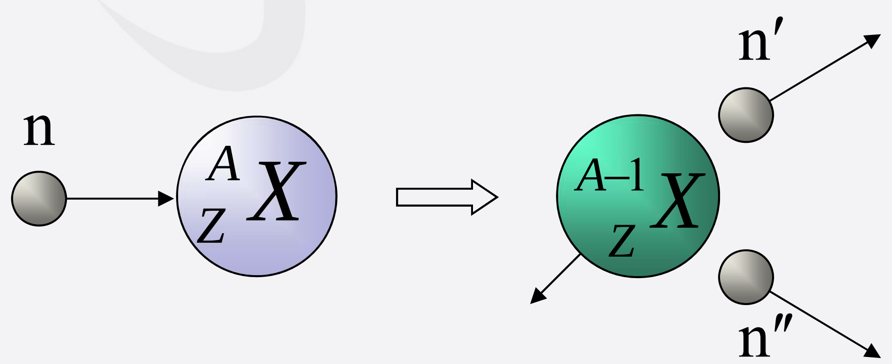
- The target nucleus absorbs the incident neutron: a compound nucleus is formed.
- The compound nucleus possess relatively high excitation energy: it ejects a charged particle (e.g. proton, alpha particle, etc.)



Neutron-Producing Reactions

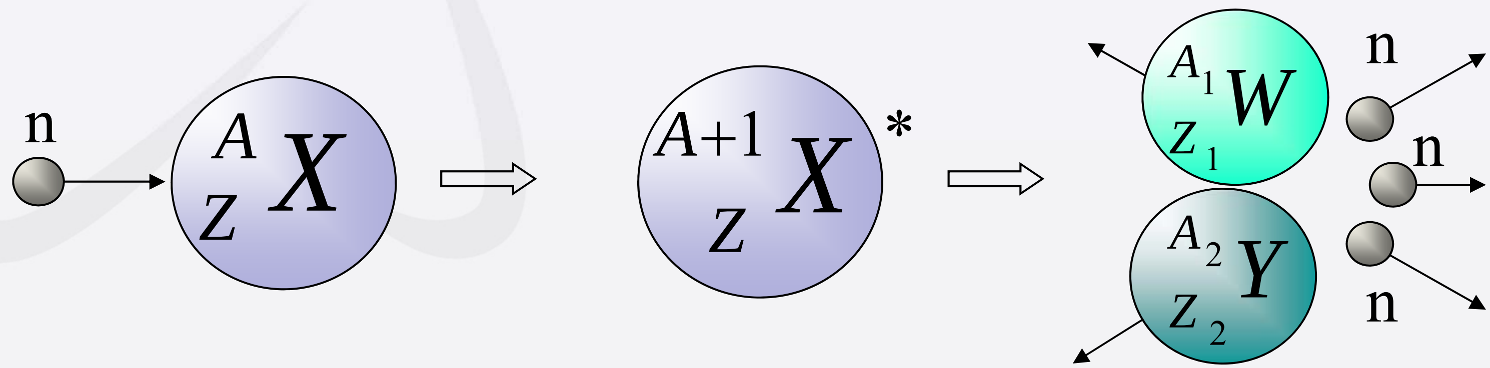
(n,2n), (n,3n), (n,xn)

- These reactions occur with energetic neutrons.
- The (n, 2n) reaction is important in reactors containing heavy water or beryllium.
- Be-9 undergoes a (n,2n) neutron reaction to Be-8.



Nuclear Fission

- Nuclear Fission is a process in which an atomic nucleus splits and a large amount of energy is released in the form of radiation and energetic fission products.



- Typical nuclear fission reaction:

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U\right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n\right) $$

Nomenclature

- Spontaneous Fission:
  - when a nucleus spontaneously decays by fission.
- Neutron-Induced Fission:
  - when a fission reaction is induced by the absorption of a neutron into a nucleus.
- Thermal Fission:
  - when a fission reaction is induced by the absorption of a room-temperature thermal neutron into a nucleus.
- Fast Fission:
  - when a fission reaction is induced by the absorption of a fast (around 1-MeV) neutron into a nucleus.

Cross Sections

- Neutron Interactions
- Neutron Beam Experiments. Collision Density
- Nuclear Cross Sections. Mean Free Path

Neutron Interactions

- To safely and efficiently operate a nuclear reactor we need to be able to predict changes in neutron population over time and space
- Our principle interest is in calculating nuclear reaction rates:

The number of incident neutrons interacting with the target nuclei per unit time.

Types of Neutron Interactions

- Scattering: neutron collides with an atomic nucleus.
  - Elastic Scattering (n, n)
  - Inelastic Scattering (n, n')
- Absorption
  - Radiative Capture (n, γ)
  - Charged-Particle Reactions (n,a), (n, p), (n, d), (n, t)
  - Neutron-Producing Reactions (n,2n), (n,3n), (n,xn)
  - Fission

Neutron Beam Experiment

- A beam of monoenergetic and monodirectional neutrons strikes a thin target.
- The neutrons in the beam have the following characteristics:

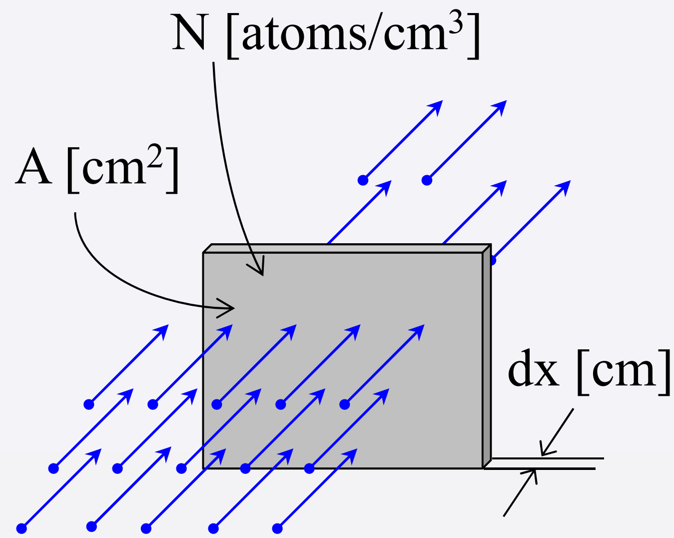
  -- speed v [cm/s]

  -- neutron density n [neutrons/cm^3]

- The intensity of the neutron beam (I) is equal to the number of neutrons striking the target per unit area per unit time

$$I = nv$$

- Units of beam intensity = neutrons/cm^2-sec

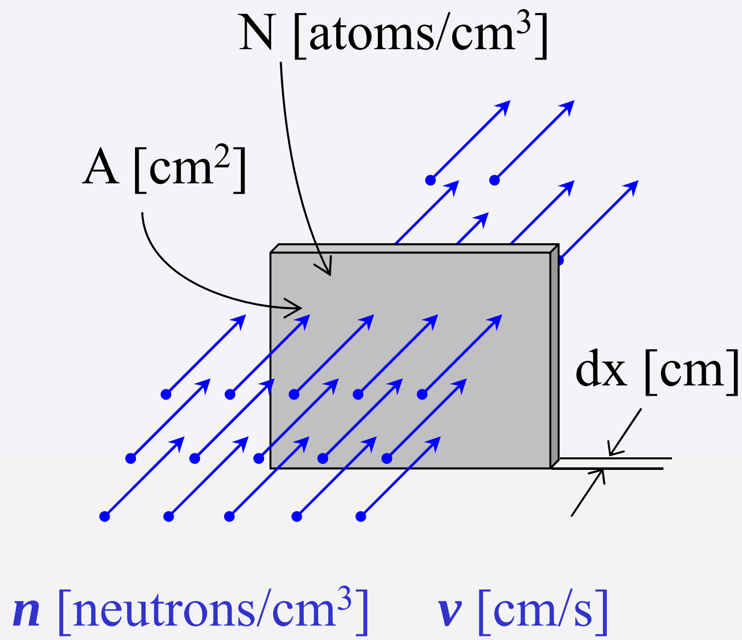


n [neutrons/cm^3]

v [cm/s]

Number of Interactions in the Target

- The number of interactions (collisions) between the beam neutrons and the target's nuclei is proportional to the
  - Atom density N of the target
  - Area A of the target
  - Thickness dx of the target
  - Intensity of the beam ($I = nv$)



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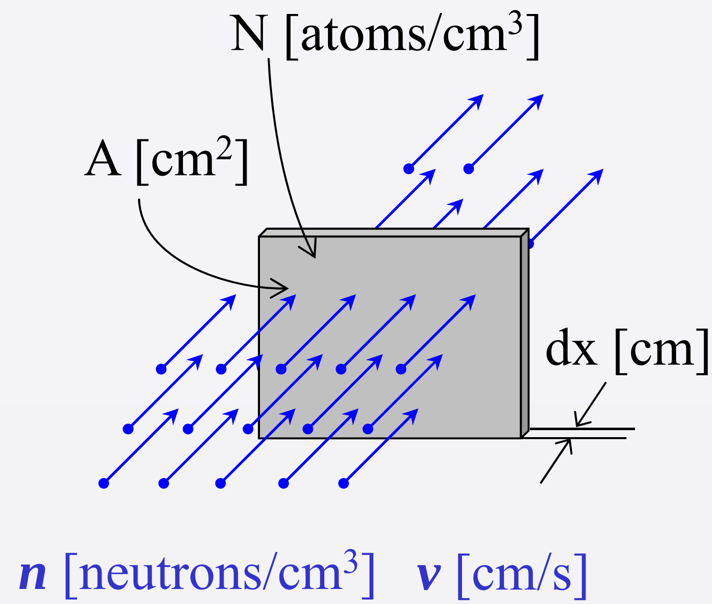
No. of Collisions ?? N A dx I

Number of Interactions (continued)

- In entire target,

Number of interactions per second = $\sigma N A \, dx \, I$



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- σ is called microscopic cross-section.
- σ is the proportionality constant that depends on the speed (energy) of the incident particles.

Microscopic Cross Section Symbols

- **Microscopic cross section** (σ) is the probability of a particular interaction occurring between a neutron and a target nucleus.
- Microscopic cross sections are expressed in units of **barns**, where 1 barn = 10^{-24} cm^2.
- Each of the types of interactions discussed previously has a characteristic microscopic cross section
  - σ_e: Elastic scattering cross section
  - σ_i: Inelastic scattering cross section
  - σ_g: Radiative capture cross section
  - σ_f: Fission cross section
  - σ_a: (n, a) cross section
  - σ_{2n}: (n,2n) cross section

Microscopic Cross Section Symbols (continued)

- The sum of all cross sections is known as the total cross section (s_t)

$$\sigma_t = \sigma_e + \sigma_i + \sigma_\gamma + \sigma_f + \sigma_\alpha + \ldots$$

- The sum of all absorption reaction cross sections is known as the absorption cross section (s_a)

$$\sigma_a = \sigma_\gamma + \sigma_f + \sigma_p + \sigma_d + \sigma_\alpha + \ldots$$

- The sum of all scattering reaction cross sections is known as the total scattering cross section (s_s)

$$\sigma_s = \sigma_e + \sigma_i$$

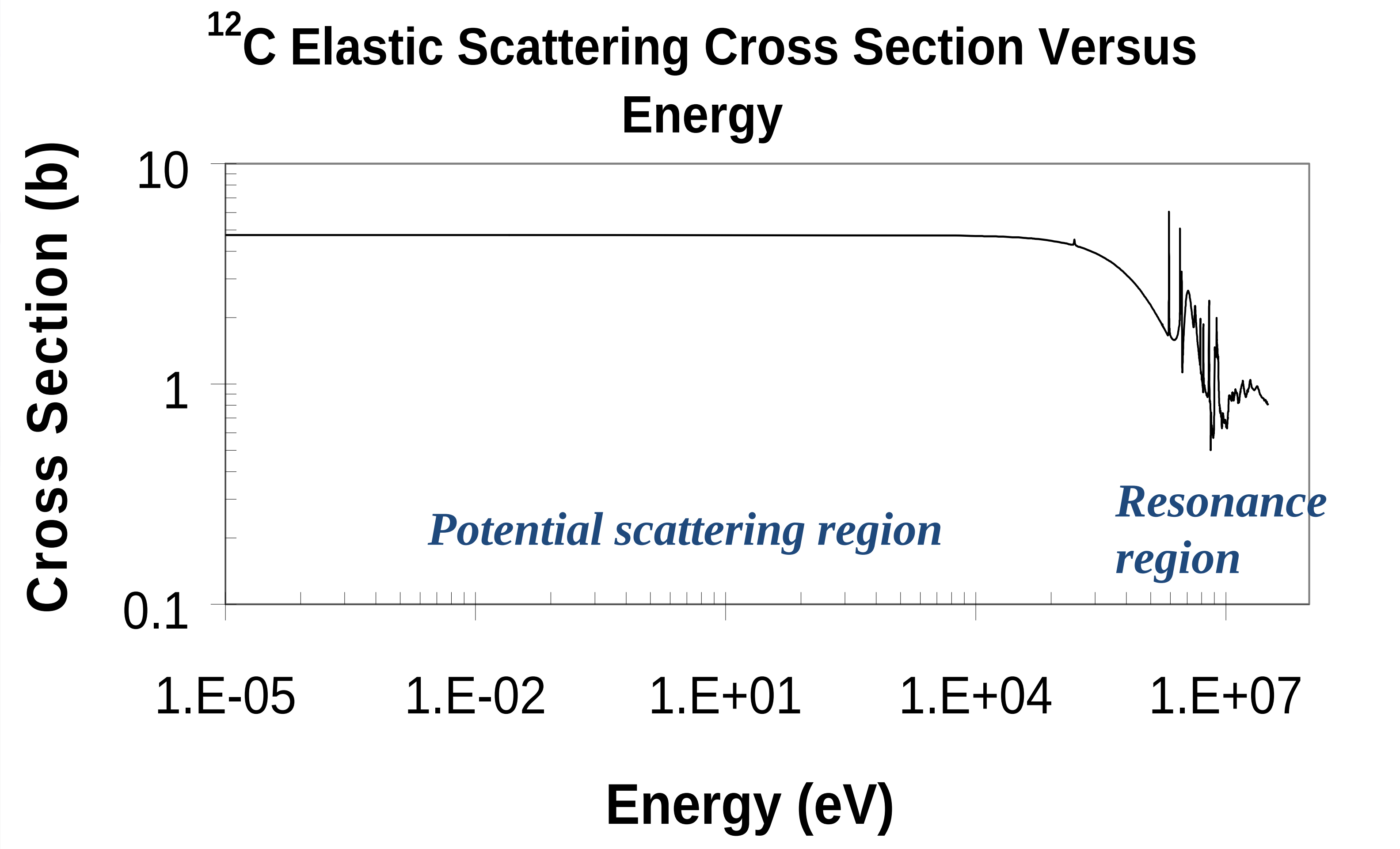
Microscopic Cross Sections Dependencies

- The microscopic cross section is a function of
  - The target nuclide
  - The incident particle
  - The relative speed between the incident particle and the target nuclide
    - We will often assume the target nuclide is at rest;
    - Then the microscopic cross section is a function of
      - The target nuclide
      - The incident particle
      - The energy of incident particle.

Where to get cross section data?

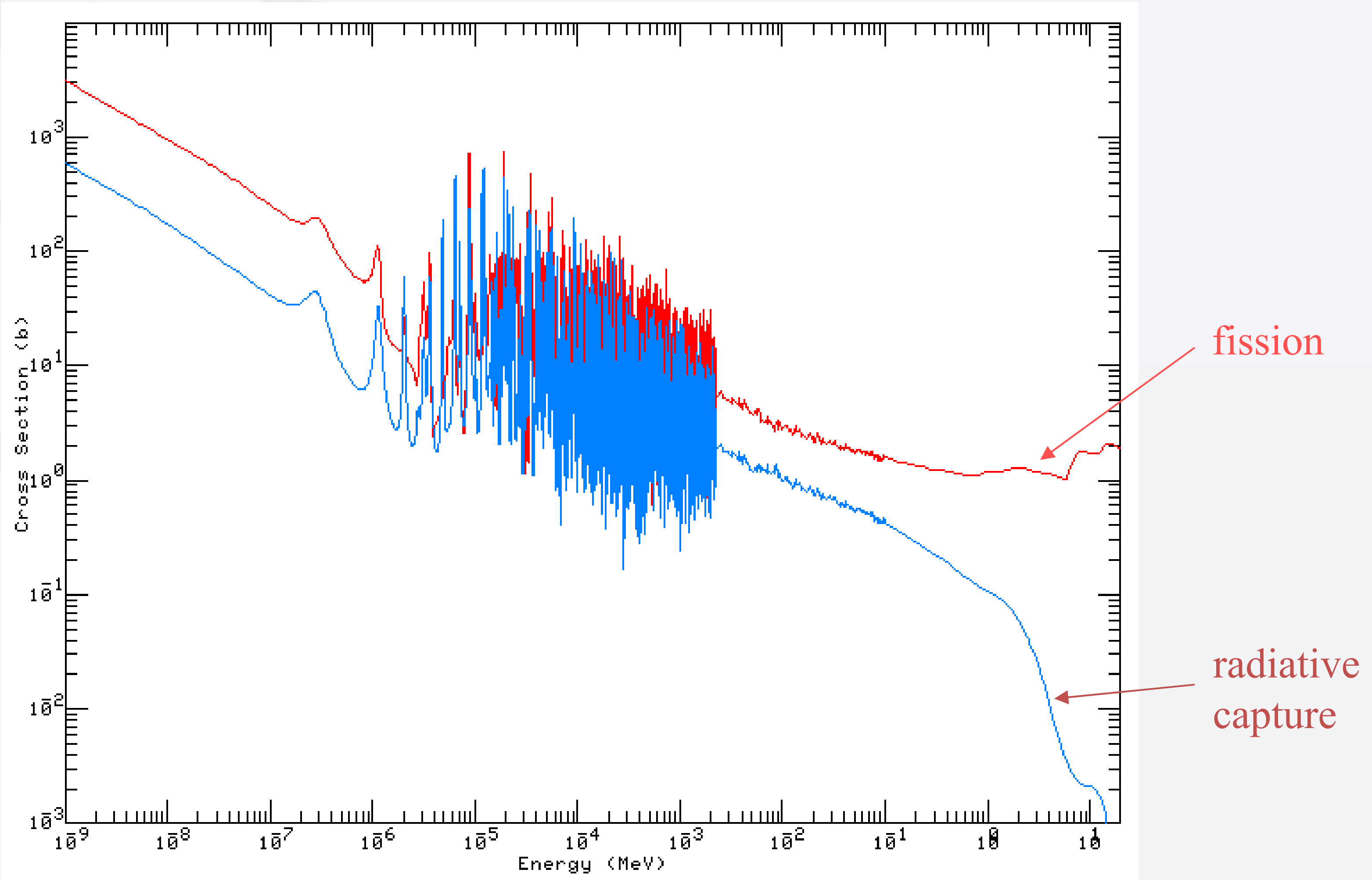
- Someone else has already measured/calculated most of the cross sections for most isotopes:
  - Online Chart of the Nuclides
    - http://www.nndc.bnl.gov/nudat2/index.jsp
    - http://sutekh.nd.rl.ac.uk/CoN/
    - http://wwwndc.tokai.jaeri.go.jp/CN03/index.html
    - http://atom.kaeri.re.kr/ton/
- The microscopic cross sections provided on most charts and tables are measured for a standard neutron velocity of 2200 meters/second (ambient temperature of 68F).
- Cross sections must be corrected for the temperature of the target material.

Elastic Scattering Cross Section Versus Energy of Incident Neutron



Author: Dr. Charlton, NUEN, TAMU

^{235}U Fission and Radiative Capture Cross Sections



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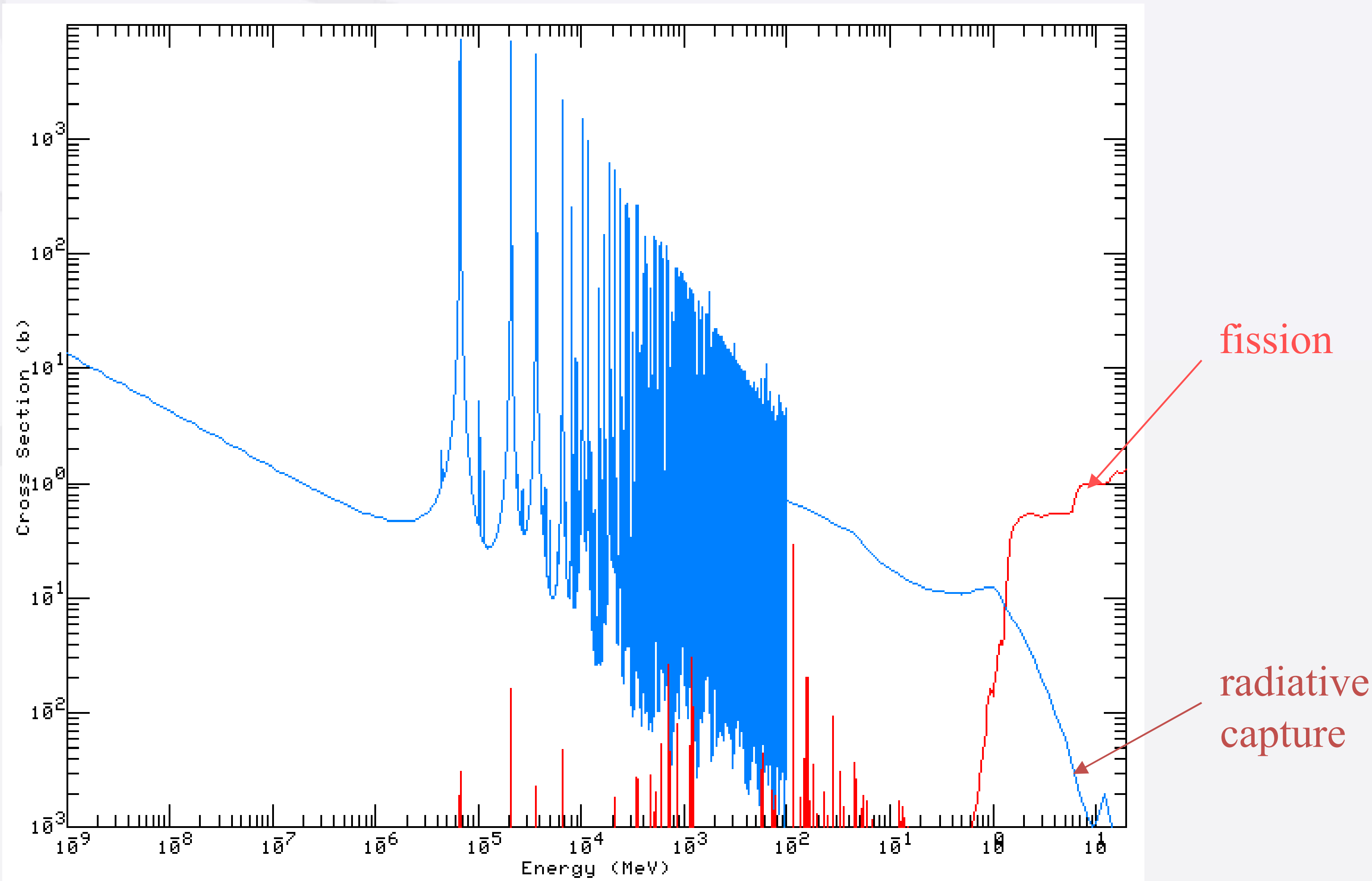
^{238}U Fission and Radiative Capture Cross Sections



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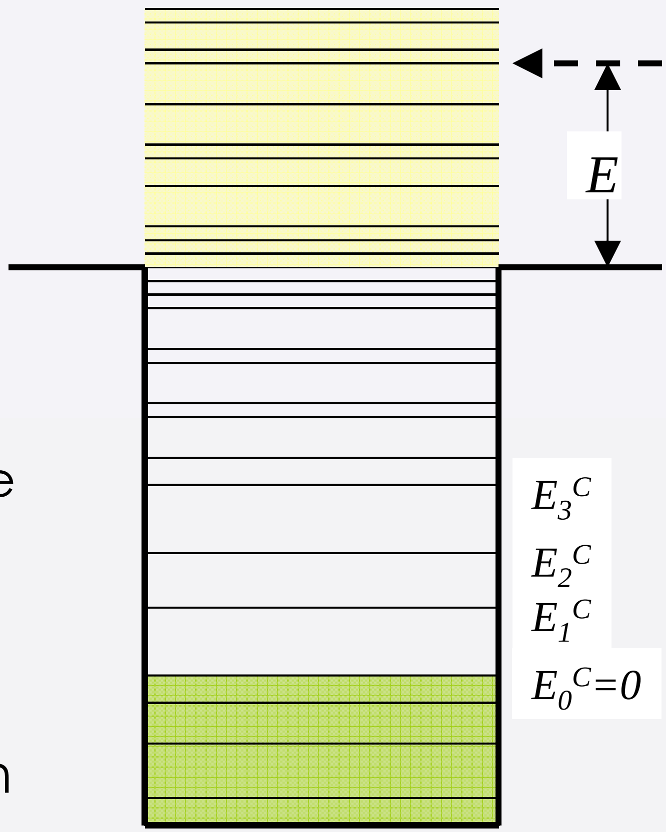
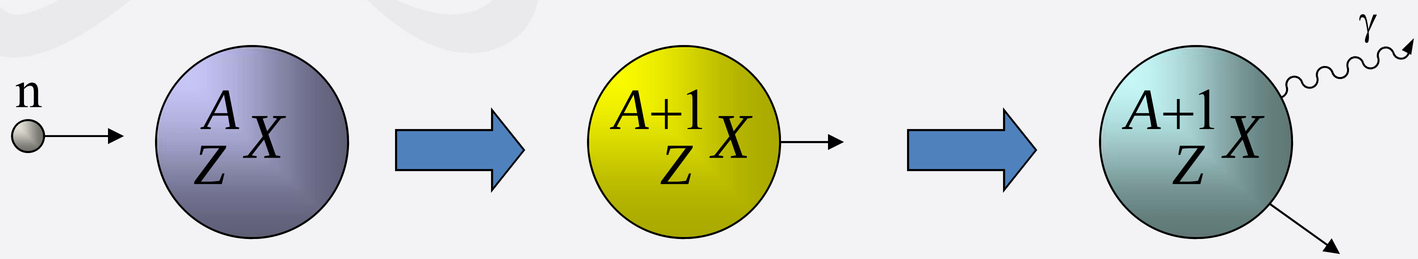
Resonance

- All nuclear cross sections vary with energy of the incident particle
  - strong fluctuations are called resonances.
- In most reactions, a neutron
  - interacts with a nucleus,
  - gets absorbed, and
  - forms a compound nucleus prior to the final reaction products being formed
- The target nucleus may have various excited states corresponding to different configurations of nucleons in the nucleus



Author: Dr. Charlton, NUEN, TAMU

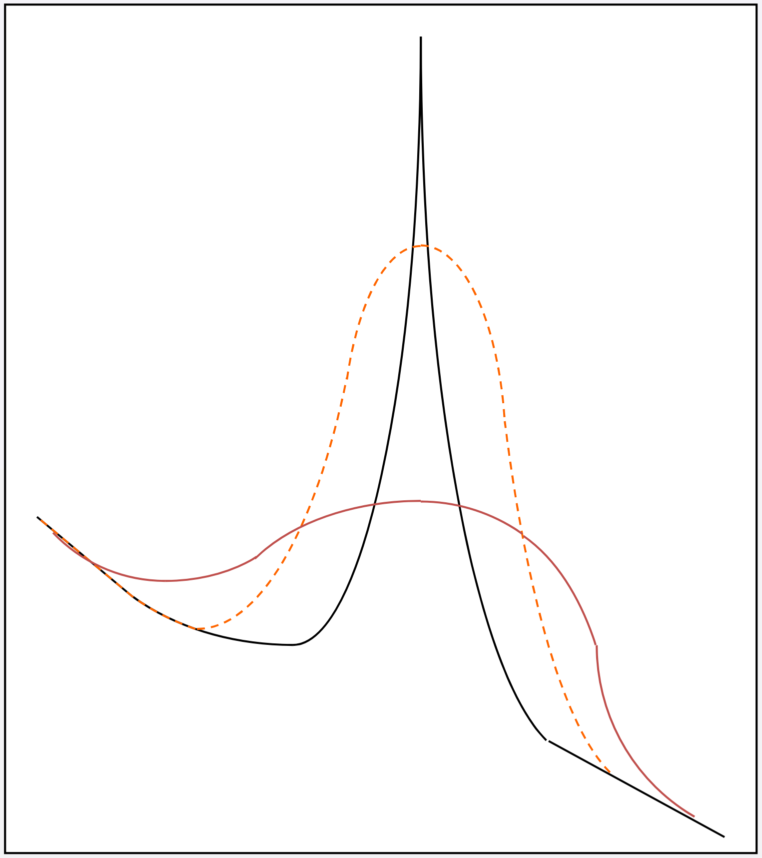
- The incident neutron and target nucleus are more likely to form a compound nucleus if
  - The sum of the binding and kinetic energies of the incident neutron corresponds to an excited state of the compound nucleus



  - The resonances arise in the cross-section because the compound nucleus must be formed before the reaction can proceed.

Cross Sections Versus Temperature

- Cross sections can vary with material temperature
- Temperature is related to the relative motion of the nuclei within a material
- These temperature changes mainly affect the resonances
  - Resonances broaden at higher temperatures
- The temperature effect on resonances is called doppler broadening



Author: Dr. Charlton, NUEN, TAMU

Module 1: Nuclear Energy Fundamentals (Week 2/Day 4)

Lecture #2: Neutron Flux

Dr. David Boyle

Lecture Outline

1. Neutron Attenuation
2. Neutron Flux and Reactor Power
3. Moderation

Collision Density

- The number of interactions per unit time (in the target):

$$\frac{\text{interactions}}{\text{second}} = \sigma_t \, N \, A \, dx \, I$$

- **Collision density (*F*)** is the number of interactions per unit volume per unit time:

$$F = \frac{\text{interactions}}{\text{s} \cdot \text{cm}^3} = \sigma_t \, N \, I$$

- The product of the atom density *N* and a cross-section is called the **macroscopic cross-section: $\Sigma = N\sigma$**.

Macroscopic Cross-section

- **Macroscopic cross-section (Σ)** is the probability of an interaction per unit length of travel of an incident particle.
- **Macroscopic cross-section Σ has units of cm-1.**
- **The macroscopic cross-section depends on**
  - The target atom density *N* (a macroscopic quantity)
  - Interaction type
  - Projectile energy
- **Macroscopic total cross section $\Sigma_t = Ns_t$**
- **Macroscopic scattering cross section $\Sigma_s = Ns_s$**
- **Macroscopic absorption cross section $\Sigma_a = Ns_a$, etc.**

Mixture Cross-section

- The macroscopic cross-section of a mixture of materials can be calculated as

$$\Sigma = \sum_i N_i \sigma_i = \sum_i f_i N \sigma_i = N \sum_i f_i \sigma_i$$

where N_i is the atom density for isotope i and f_i is the fractional abundance.

- Example: Uranium has three naturally occurring isotopes: ^{234}U, ^{235}U and ^{238}U. Their fractional abundances are 0.000055, 0.0072 and 0.992745 respectively. What is the macroscopic absorption cross section for ^{235}U in natural uranium?

Example Solution

- The fractional abundance of ^{235}U is 0.0072.
- Atomic weight of U is 238.0289; density of U is 19.1 g/cm^3.
- The microscopic absorption cross-section for ^{235}U is 680.8 b.
- Avogadro number $N_A = 0.6022 \times 10^{24}$ mol^{-1}
- Recall: number of atoms per unit volume $N = \rho N_A / M$
- Then the macroscopic absorption cross-section for ^{235}U is

$$\Sigma_a\left({}^{235}U\right) = N \cdot f\left({}^{235}U\right) \cdot \sigma_a\left({}^{235}U\right)$$

$$= \frac{19.1(0.6022\times10^{24})}{238.0289}(0.0072)680.8\times10^{-24}$$

$$= 0.237\ cm^{-1}$$

Mean Free Path

- The average distance a neutron travels without an interaction is called a mean free path (l)

$$\lambda = \frac{1}{\Sigma_t}$$

- Example 1. Suppose S_t for thermal neutrons in a research reactor is 10 cm^{-1}. On average, how far do thermal neutrons travel between collisions?

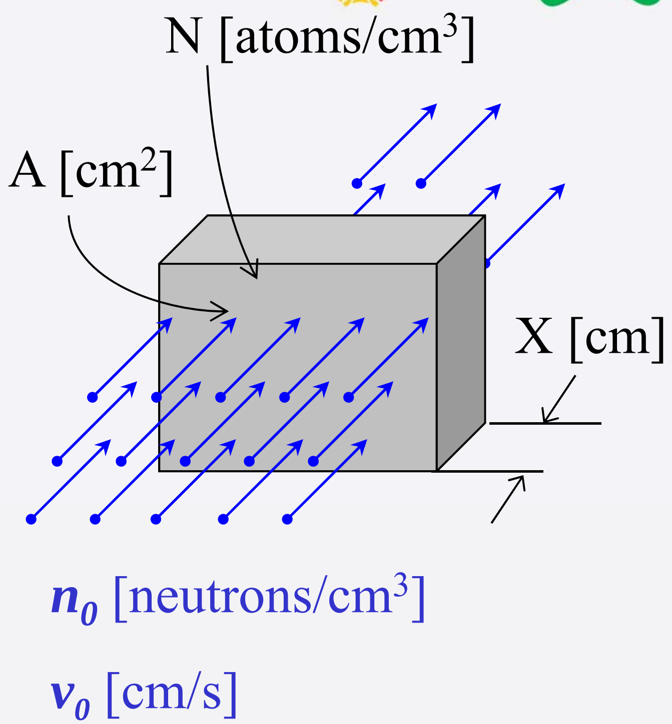
$$\lambda = \frac{1}{\Sigma_t} = 0.1 \cdot cm$$

- Example 2. Suppose S_a for thermal neutrons in a research reactor is 4 cm^{-1}. On average, how far do thermal neutrons travel before they are absorbed?

$$\lambda = \frac{1}{\Sigma_t} = 0.25 \cdot cm$$

Neutron Attenuation

- Let us revisit our mono-beam experiment.
- This time, we will let
  - The target be of some thickness *X* and
  - The beam will be some intensity $I_0 = n_0 v_0$
- The beam on the other side of the target will be composed of only uncollided neutrons



Neutron Attenuation (continued)

- As the beam passes through the target some portion of the neutrons collide and are removed from the beam
- We will refer to the beam intensity at a distance x into the target as *I(x)*
- To determine *I(x)* we will use the simple balance equation

$$[Rate\ \ of\ \ Change] = [Production\ \ Rate] - [Loss\ \ Rate]$$

Neutron Attenuation (continued)

- We know:
  - The rate of change at x into the target is *dI(x)/dx*
  - The production rate at x in the target is 0
  - The loss rate in the target is the collision density at x (number of interactions per unit volume per unit time)
- The collision density is $F = \dfrac{interactions}{cm^3\, s} = \sigma_t N I$
- Thus, our balance equation becomes

$$Rate \;\; of \;\; Change = 0 - Loss \;\; Rate$$

$$\frac{dI}{dx} = 0 - \Sigma_t I(x) \quad \Rightarrow \quad \frac{dI}{dx} = -\Sigma_t I(x)$$

Neutron Attenuation (continued)

- This balance equation can be solved:

$$\frac{dI}{dx} = -\Sigma_t I(x)$$

$$\frac{dI}{I} = -\Sigma_t dx \qquad \int \frac{dI}{I} = -\int \Sigma_t dx \qquad \ln(I) = -\Sigma_t x + C_1$$

$$I(x) = e^{-\Sigma_t x + C_1} = e^{-\Sigma_t x} e^{C_1}$$

$$I(x) = Ce^{-\Sigma_t x}$$

Neutron Attenuation (continued)

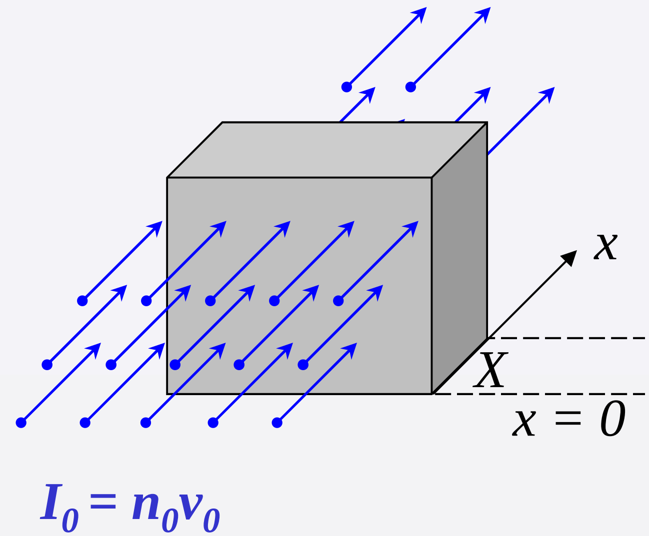
- At x = *0* (the front surface of the target),
  - the beam intensity is I_0 thus

$$I(x) = Ce^{-\Sigma_t x}$$

$$I(x=0) = I_0 = Ce^{-\Sigma_t 0}$$

$$C = I_0$$

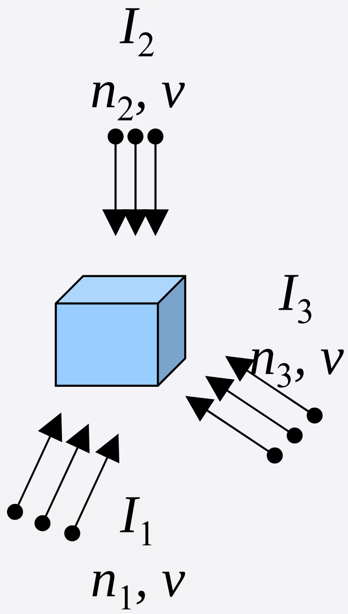
$$I(x) = I_0 e^{-\Sigma_t x}$$



- The intensity of the beam (uncollided neutrons) decreases exponentially and is a function of distance inside the target.

Several Neutron Beams Bombard a Small Target

- Consider an experiment in which several neutron beams bombard a small target
  - The intensities of the beams are different
  - The neutrons in all the beams have the same speed (same energy)



- Then the total collision density is given by

$$F_{total} = \Sigma_t \left(I_1 + I_2 + I_3 + \ldots \right) = \Sigma_t \left(n_1 v + n_2 v + n_3 v + \ldots \right)$$
$$= \Sigma_t \left(n_1 + n_2 + n_3 + \ldots \right) v = \Sigma_t n v$$

where *n* is the total density of neutrons striking the target, $n = n_1 + n_2 + n_3 + \ldots$

Neutron Flux

- The situation at any point in a reactor is a generalization of this experiment with a very small sample and beams from all directions; thus, our equation

  $F = \Sigma_t n v$ is valid for any point in a reactor.

- The quantity *nv* is called the ***neutron flux*** and denoted by the symbol φ

$$\phi = nv$$

- The units of neutron flux are the same as the units of beam intensity: *neutrons/cm²-s*
- Then the collision density is

$$F = \Sigma_t nv = \Sigma_t \phi$$

Neutron Flux (continued)

- Another way to define neutron flux is
  The total track length traveled by all neutrons in unit volume per unit time

$$\phi\left[\frac{track\ length}{volume-time}\right]=n\left[\frac{particles}{volume}\right]v\left[\frac{track\ length}{time}\right]$$

- We will usually write the units as *neutrons/cm*^{2}*-s*

Neutron Flux (continued)

- Neutron fluxes can be conceptually viewed as
  - Related to the overall population of neutrons in a volume
  - The number of neutron track lengths per unit volume per unit time
  - the number of neutrons passing through a unit area per unit time

Neutron Flux and Reactor Power

- Example: A research reactor core has a volume of 60 x 10^3 cm^3. Calculate the power of the reactor if the neutron flux is 1.2 x 10^{13} neutrons/cm^2-s and the fission cross-section is 0.1 cm^{-1}.
- Solution:
  - – Find fission rate in the reactor
  - – Energy released per fission = 200 MeV
  - – Power = Energy release per fission x fission rate
    - Units for power: 1 Watt = 6.242 x 10^{12} MeV/s

$$\text{Total Fission Rate} = V\,\Sigma_f\,\phi = 60\times10^3\ (0.1)\ 1.2\times10^{13} = 7.2\times10^{16}\ \text{fission/s}$$

$$\text{Power} = \frac{7.2\times10^{16}\ (200)}{6.242\times10^{12}} = 2.3\times10^{6}\ \text{W} = 2.3\ \text{MW}$$

Neutron Moderation

- In thermal reactors, the neutron-induced fission is more likely when the neutrons have low energy < 1 eV (0.025 eV at 20°C).
- Since neutrons from fission events are fast neutrons (born at high energies, up to several MeVs),
  - They must be slowed down to thermal energies to induce further fission reactions.
- The energy of a neutron is reduced through scattering.
- This process is known as thermalization or ***moderation***.
- The material used for the purpose of reducing neutron energies is called a ***moderator***.

Moderator

- We want a moderator that will slow the neutrons quickly but not absorb them
- Thus for an ideal moderator, we want
  - Large scattering cross section
    - Need to slow down neutrons in a small number of collisions
  - Small absorption cross section
    - Most neutrons are not absorbed by moderator
  - Large energy loss per collision

Neutron Moderation (continued)

Moderator	A	ξ	Average number of collisions to thermalize
H_2O	17	0.927	19
D_2O	18	0.510	35
He	4	0.427	42
Be	9	0.207	86
C	12	0.158	114

References

1. Lamarsh, J. R., and Baratta, A. J., "Introduction to Nuclear Engineering", third edition, Prentice Hall, 2001
2. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf
3. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v2.pdf
4. Stacey, W. M., "Nuclear Reactor Physics", second edition, Wiley-VCH, 2007

List of Required Texts

1. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf
2. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v2.pdf

GNEII at KHALIFA UNIVERSITY

Module 1: Nuclear Energy Fundamentals (Week 2/Day 5)

Reactor Physics:
Criticality and The Neutron Balance Equation

Gulf Nuclear Energy Infrastructure Institute – 2012 Fundamentals Course

Dr. David Boyle
Sandia National Laboratories

Module 1/Week 2:

- **Nuclear Physics**

Week 2 Learning Objectives:

- Be able to explain how nuclear power works and why it is needed
- Be able to explain why nuclear power works
- Be able to explain why fission products and radiation are major issues for nuclear power
- Be able to explain the role of neutron interactions in nuclear power

Today's Primary Learning Objective

Primary Day 5 Learning Objective:

- Neutron balance and criticality

Take away from this lecture:

- Criticality (stable operation) in a nuclear reactor results from balancing neutron creation, absorption, and leakage

Module 1: Nuclear Energy Fundamentals (Week 2/Day 5)

Lecture #1: Criticality and The Neutron Balance Equation

Dr. David Boyle

Lecture Outline

1. Neutron Balance
2. Leakage
3. Criticality
4. Reactivity

Neutron Balance Equation

- To safely and efficiently operate a nuclear reactor we should be able to predict changes in the neutron population over time and space.

- We will use a balance equation of the form

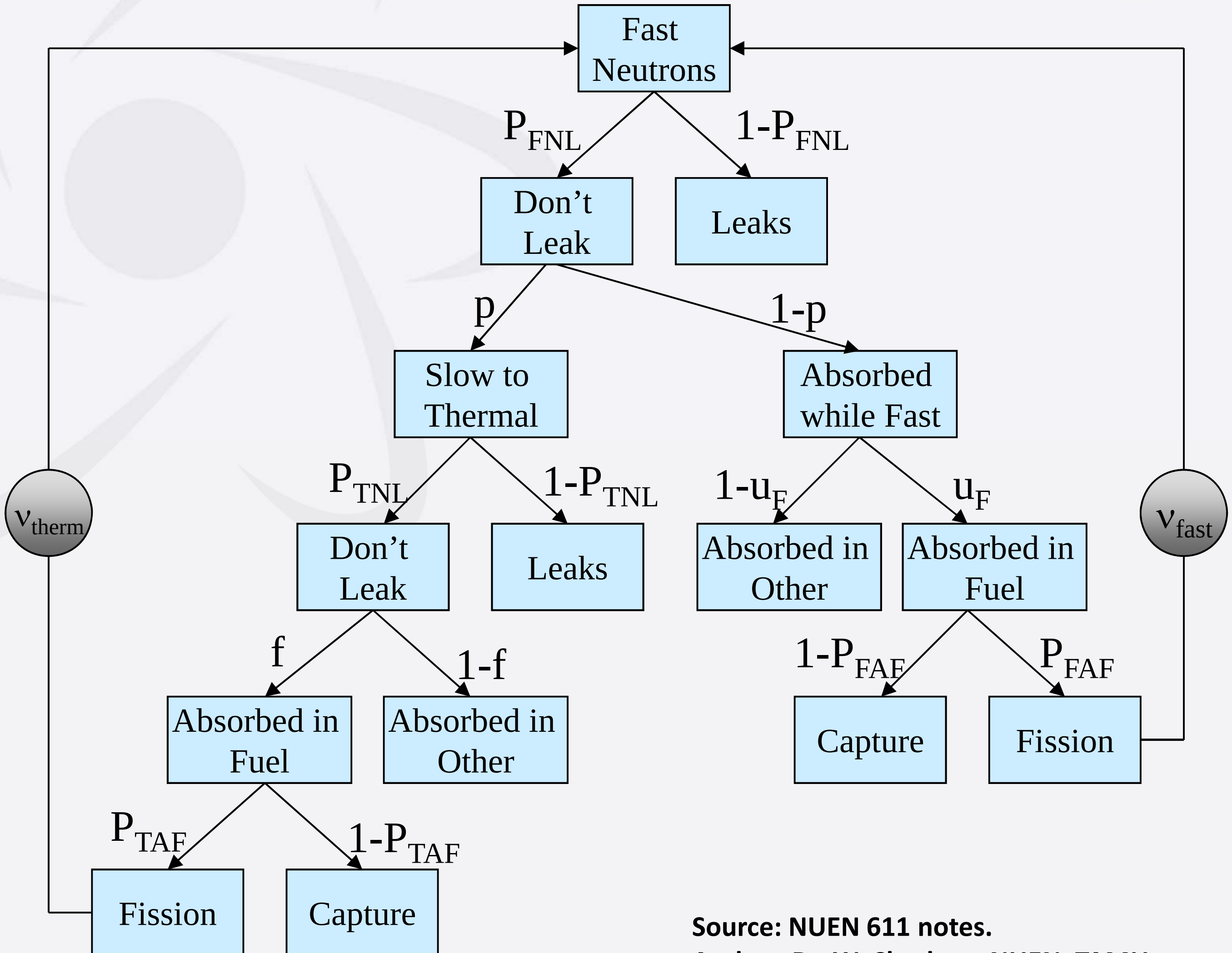
$$\begin{pmatrix} \text{change rate of} \\ \text{neutron population} \end{pmatrix} = \begin{pmatrix} \text{rate of} \\ \text{gain} \end{pmatrix} - \begin{pmatrix} \text{rate of} \\ \text{loss} \end{pmatrix}$$

Neutron Production and Loss Rates

- The production rate is given by
  - Production rate due to fixed neutron sources
  - Production rate due to neutron producing reactions [including fission and (n,2n)]

- The loss rate is given by
  - Loss rate due to absorption reactions
  - Loss rate due to "leakage"

Life Cycle of a Neutron



Source: NUEN 611 notes.
Author: Dr. W. Charlton, NUEN, TAMU

Criticality

- Criticality condition (k) for the system is defined as

$$k = \frac{\#\text{ neutrons in given generation}}{\#\text{ neutrons in previous generation}}$$

  - Without the source present

- The value of k immediately tells us what the time evolution of the system will be

Criticality (continued)

- With a source of fission neutrons present and without any externally injected neutrons (i.e., fission neutrons only)
  – $k < 1$, neutron population will decrease
    - called a subcritical reactor
  – $k = 1$, neutron population will stay constant
    - called a critical reactor
  – $k > 1$, neutron population will increase
    - called a supercritical reactor

Criticality (continued)

- We will traditionally use a more convenient definition for criticality
  - Every neutron in a single generation is lost by some mechanism and
  - The only neutrons in a given generation (when the external source is not present) are those produced from fission
  - Thus:

$$k = \frac{\text{rate of neutron production}}{\text{rate of neutron loss}}$$

Criticality (continued)

- For an infinitely large reactor, which has no neutron leakage, this is

$$k_\infty = \frac{\text{production rate}}{\text{absorption rate}}$$

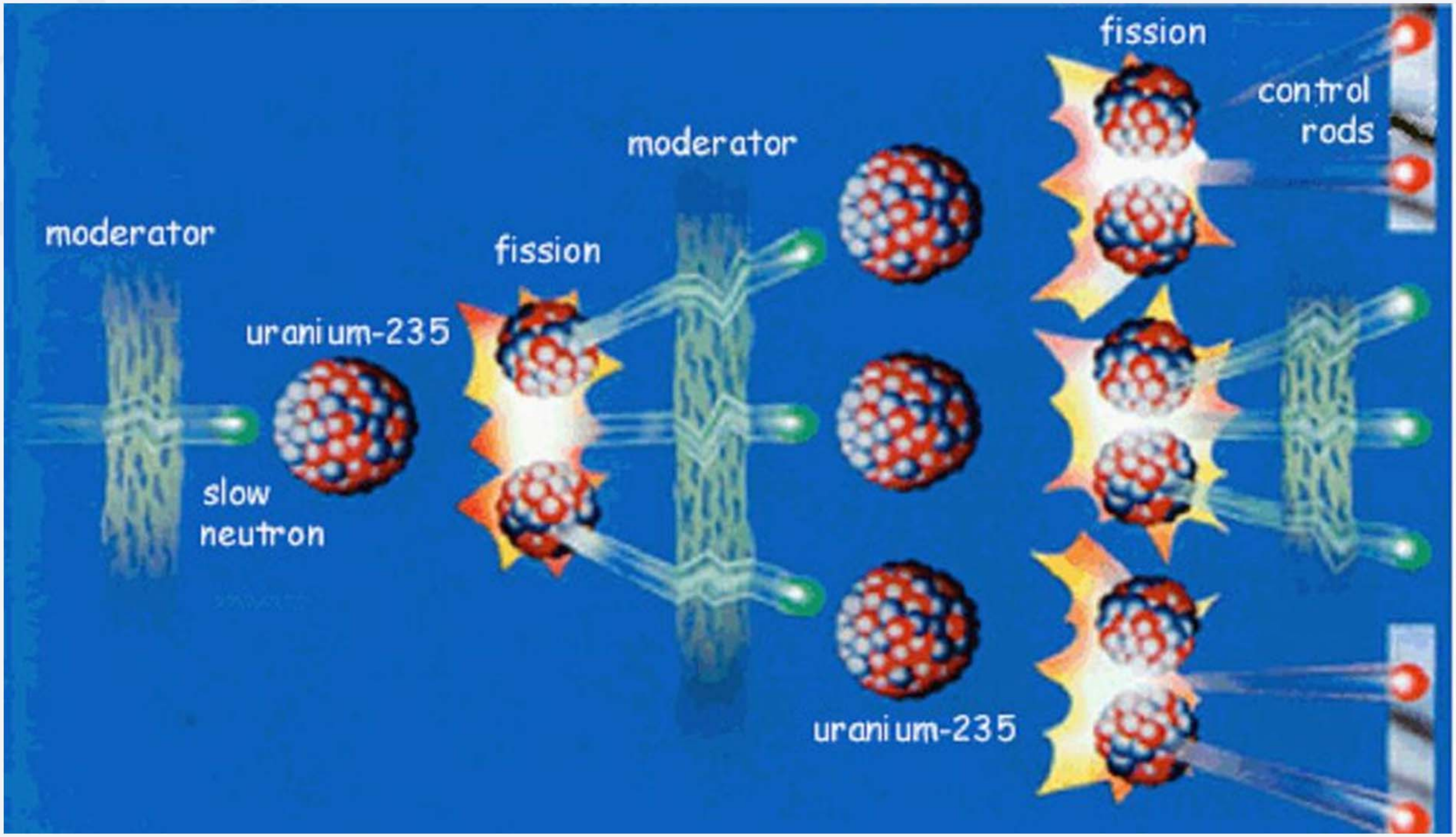
- For a more realistic system, this will be

$$k = \frac{\text{production rate}}{\text{absorption rate} + \text{leakage rate}}$$

Fission Chain Reaction

- Fission reactions not only generate an enormous amount of energy but also emit additional neutrons.
- Some number of the fast neutrons produced by fission in one generation will undergo moderation and eventually cause fission in the next generation.
- This leads to the possibility of a
  - Self-sustaining neutron-induced fission chain reaction
- Nuclear power plants operate by precisely controlling the rate at which nuclear reactions occur.

Fission Chain Reaction (continued)



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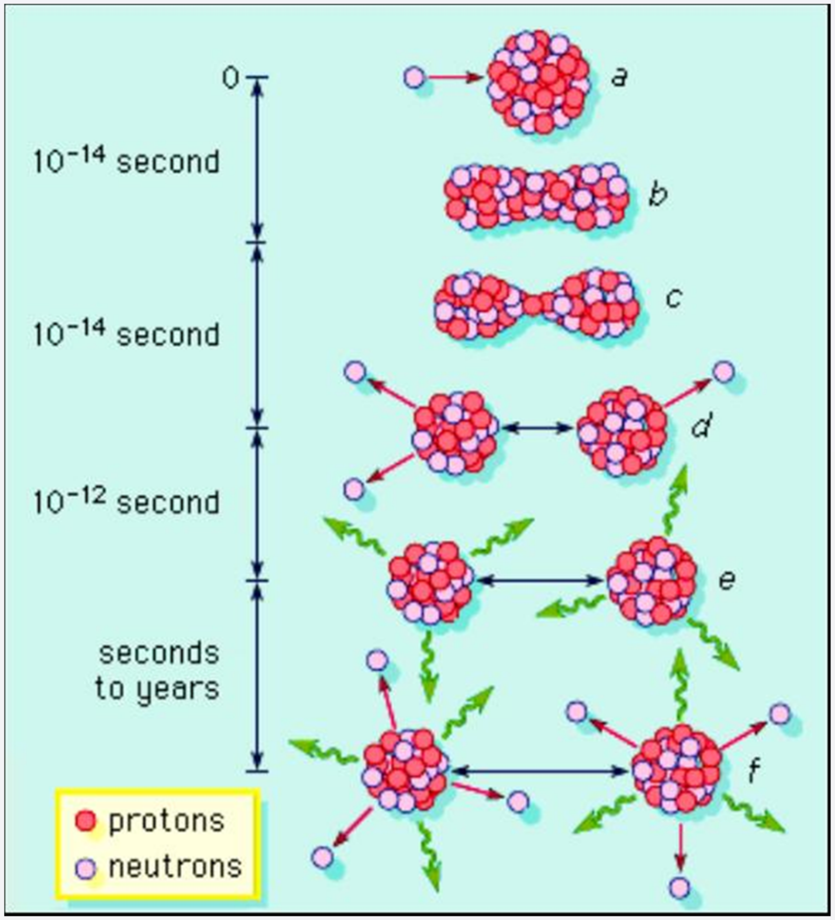
- For a nuclear chain reaction to continue at steady-state
  - The neutron production rate must be perfectly balanced with the neutron loss rate

- Any deviation from this balance will result in
  - A time-dependence of the neutron population and
  - A time-dependence of the power level of the reactor

Characteristics of Fission Reactions (continued)

- Fission fragments give up their internal energy in a series of steps
  - Prompt neutrons and gamma-rays within 10^{-14} to 10^{-12} sec
  - Around 10^{-11} sec, fission fragments come to rest
  - Above 10^{-3} sec, fission products begin chains of beta decays; some decay by neutron emission (delayed neutrons)



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Role of Delayed Neutrons

- Neutrons are emitted and fission products are created
  - From the decay of these fission products additional neutrons are generated at a very long time after the fission event
    - This will only be a small fraction of the total number of neutrons emitted by fission, but they are still very important
- Most neutrons are emitted directly from fission (*prompt neutrons*)
- The neutrons that are emitted from the decay of fission products are called *delayed neutrons*
  - Delayed neutrons affect the average neutron generation time and thus reactor control

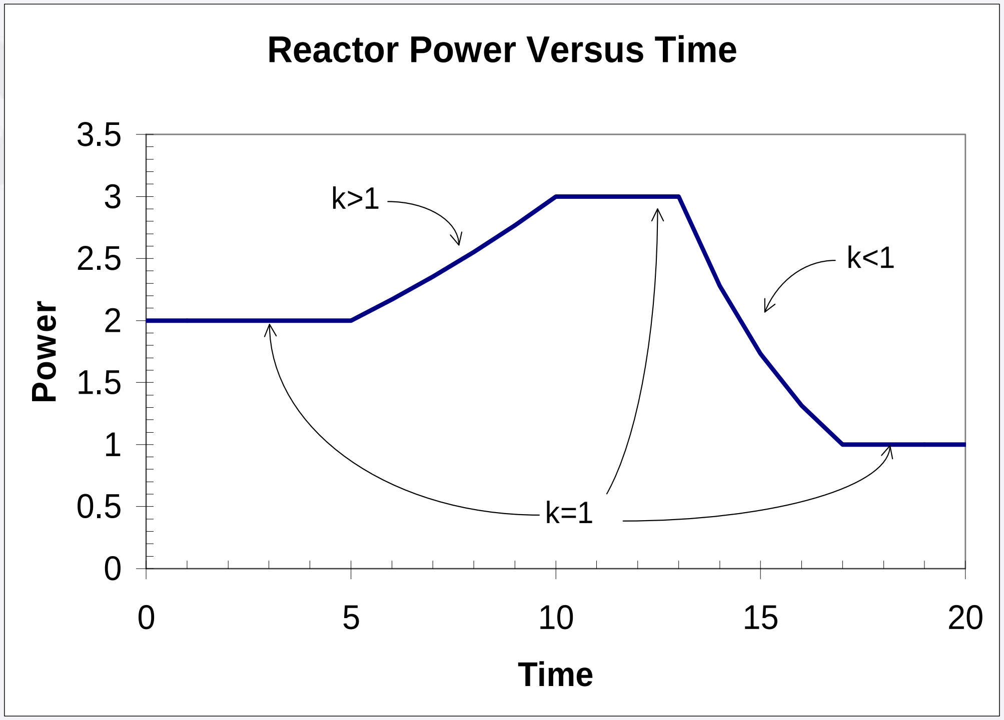
Role of Delayed Neutrons (continued)

- The prompt neutron lifetime is typically on the order of 10^{-4} seconds in a thermal reactor
  - 10^{-7} seconds in a fast reactor
- If only prompt neutrons were generated in the course of fission chain reaction, the neutron flux would increase by a factor of $e^{10}=22026.47$ in one second!
  - This means that we would have no chance of ever controlling this reactor
- Delayed neutrons are generated a “very long” time after the fission event
  - For uranium-235, the average delayed neutron generation time is about 12.5 seconds
- If we include the delayed neutrons, the power will increase in one second by a factor of $e^{1/57}=1.018$

Changing a Reactor's Power

- The reactor power is usually related to the value for *k*

- We will change the value for *k* by
  - adding/removing fuel
  - adding absorber
  - changing the number of neutrons that leak



Author: Dr. W. Charlton, NUEN, TAMU

Reactivity

- Reactivity (ρ) is the fractional change in neutron population per generation

$$\rho(t) = \frac{k(t) - 1}{k(t)}$$

- Reactivity is a measure of the deviation of the core from its critical value (i.e., $k=1$)
- It is used to predict how the neutron population of a reactor will change over time

Reactivity (continued)

- From the definition of reactivity, we can acquire:

$$\rho(t)=\frac{k(t)-1}{k(t)}$$

$$\rho(t)k(t)=k(t)-1$$

$$k(t)(\rho(t)-1)=-1$$

$$k(t)=\frac{1}{1-\rho(t)}$$

Units of Reactivity

- Reactivity in its simplest form has units of $\Delta k/k$:

$$\rho(t) = \frac{k(t) - 1}{k(t)}$$

- We can also express reactivity in units of
  - $\%\Delta k/k$
  - pcm (percent millirho)
  - Dollars ($1=1*b* $\Delta k/k$)

- Conversions:
  - $1\%\ \Delta k/k = 0.01\ \Delta k/k$
  - $1\ pcm = 0.00001\ \Delta k/k$

Changes from Power Increases

- As the reactor power increases many things might change in the reactor
  - Deplete ^{235}U
  - Temperature rises
  - Materials expand
  - Resonances in cross sections broaden
  - etc.

- All of these changes will effect the reactivity of the system

Reactivity Feedback

- These changes due to changes in neutron population (and power) generate a feedback effect in the reactivity
- Thus, if a core was originally critical and we add some reactivity to the system
  - The power level will begin to rise
  - This will increase the fission heating of the fuel
  - This will increase the temperature of the fuel
  - This increased temperature may change the reactivity in the system
  - This could either decrease or increase the rise in power

Reactivity Feedback (continued)

- We will express our feedback effect on reactivity in terms of a feedback coefficient

$$\rho(t) = \rho_0 + \alpha_n \left[n(t) - n_0 \right]$$

where

α_n is the neutron population coefficient of reactivity,

r_0 is the initial reactivity due to some external change in the system,

$n(t)$ is the neutron population at time t, and

n_0 is the initial neutron population at time $t=0$

Feedback Coefficients

- This feedback coefficient would rarely be expressed in terms of neutron population
- It would be more common to express in terms of
  - Reactor power

$$\rho(t) = \rho_0 + \alpha_P\left[P(t) - P_0\right]$$

  - Fuel temperature

$$\rho(t) = \rho_0 + \alpha_T\left[T(t) - T_0\right]$$

  - etc.

Module 1: Nuclear Energy Fundamentals (Week 2/Day 5)

Lecture #2: Control Rods, Decay Heat, Burnup

Dr. David Boyle

Lecture Outline

1. Control Rods
2. Decay Heat Removal
3. Poisons
4. Absorbers
5. Fuel Burnup

Excess Reactivity

- Nuclear reactors are initially loaded with a significantly larger amount of fuel than is necessary merely to achieve criticality
  - this is often called *excess reactivity*
- This is done because the multiplication factor of the core will change during core operation
  - due to processes such as *fuel burnup* and *fission product poisoning*
- Thus, we design the core loading to provide enough excess reactivity to allow for full power operation for a specified time period

Excess Reactivity (continued)

- To compensate for this excess reactivity,
  - It is necessary to introduce an amount of negative reactivity which the operators (or designers) can adjust with time

- We generally use one of the following methods to account for reactivity changes with time:
  - Control rods
  - Burnable poisons
  - Chemical shims

Some Definitions

- Excess Reactivity (r_{ex}):
  - The core reactivity present with all control elements withdrawn from the core
- Shutdown Margin (r_{sm}):
  - The negative reactivity present when all control elements have been fully inserted to achieve minimum core multiplication
- Total Control Element Worth (Dr):
  - The difference between the Excess Reactivity and the Shutdown Margin

Some Definitions (continued)

- Control Element Worth ($D\rho_i$):
  - The reactivity change induced by the full insertion of a single control rod (control rod *i*)
- We can define several different types of control requirements:
  - Scram control
  - Power regulation
  - Shim control

Control Rods

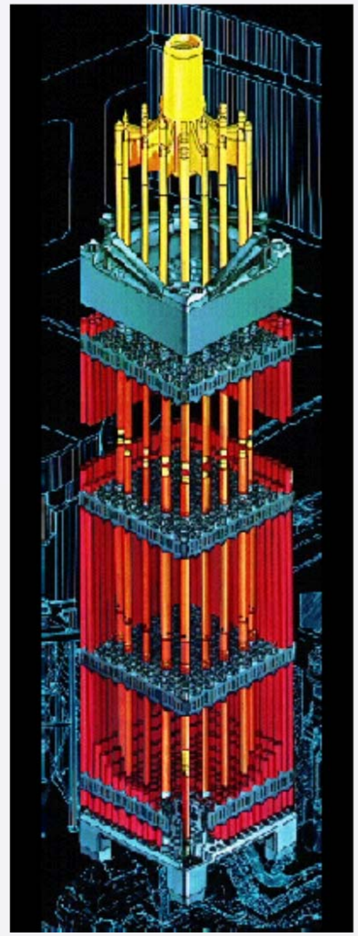
- Control rods are used in two ways:
  - To change the degree of reactor criticality for the purpose of raising and lowering the power level
  - To keep a reactor critical by compensating for the changes in the properties of the system that take place over its lifetime
- The worth of each control rod determines its ability to affect the core reactivity
- The worth can be a function of the rod position in the core
  - Both radially and from axial insertion

Control Rods in Power Reactors

- Most reactors however are controlled by a large number of rods
- Some research reactors are controlled with a small number of control rods
  - In practice, almost no reactors are controlled with one central rod
    - This is mainly because it would lead to large distortions in the flux
    - Also, from a safety standpoint this is bad since if the rod gets stuck we have no control capability

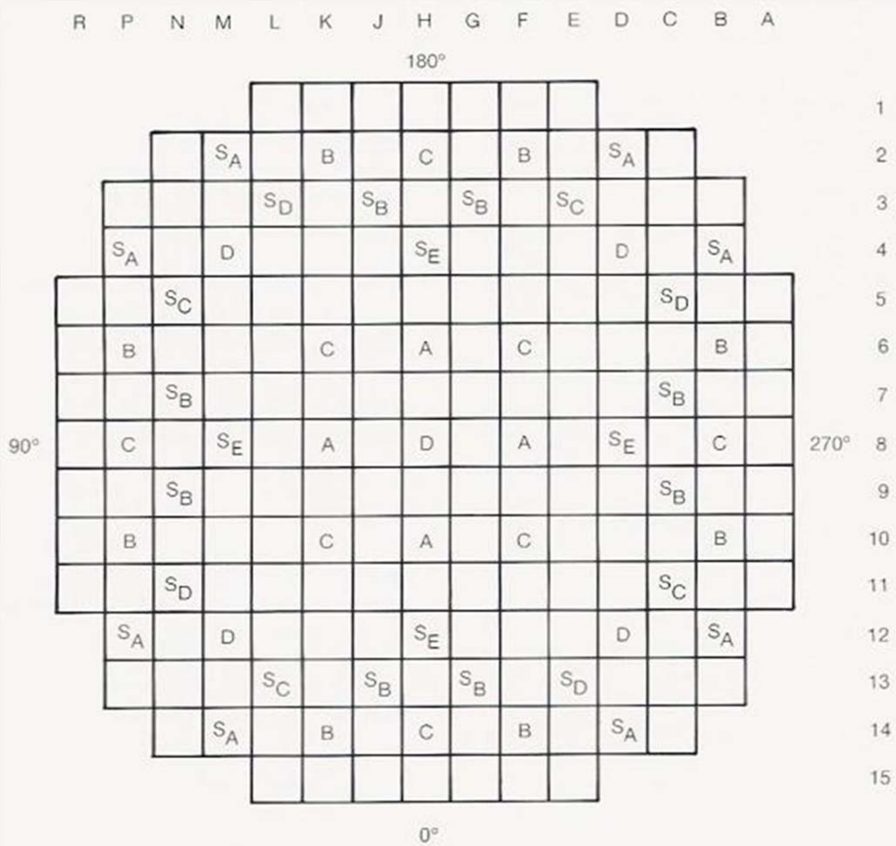
Control Rods in Power Reactors (continued)

- For power reactors there are two types of control rods generally in use today:
  - Cluster rods
  - Cruciform control rods
- In both cases, they are filled with a strong absorber like:
  - Boron carbide
  - Hafnium
  - Cadmium
  - Indium
  - Boron-loaded steel



Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC
http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

Control Rod Arrangement



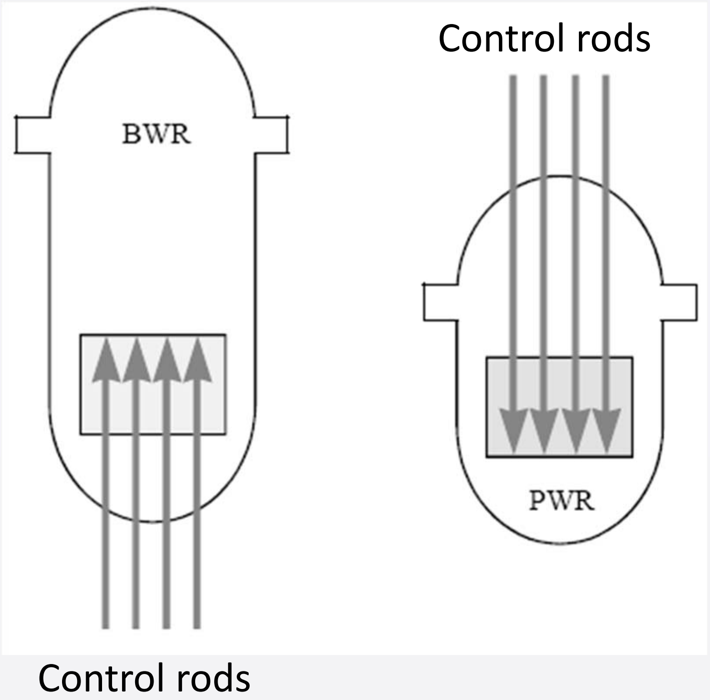
CONTROL BANK	NUMBER OF RODS
A	4
B	8
C	8
D	5
TOTAL	25

SHUTDOWN BANK	NUMBER OF RODS
S_A	8
S_B	8
S_C	4
S_D	4
S_E	4
TOTAL	28

Source: unknown

Reactor Scram (Trip)

- A reactor "scram" (or "trip") is the rapid insertion of the control rods into the core to shutdown the fission chain reaction
  - Employed for emergencies
  - The reactor power level decreases significantly in just a few seconds
    - But not all fissions are stopped



Source: USNRC "Reactor Concepts Manual: The Fission Process and Heat Production"

- The residual heat removal system is designed to remove decay heat

Decay Heat

- In reactor, decaying fission products emit beta and gamma radiation.
- Even after the reactor has been shutdown, the accumulated fission products continue to decay
  - Decay heat is the fission product decay energy released within the reactor.
- The amount of heat produced by decay heat is sufficient to cause fuel damage if not removed.
- The auxiliary feedwater system and the steam dump system (turbine bypass valves) work together to allow the operators to remove the decay heat from the reactor and reduce the temperature of the reactor coolant system to the desired level.

Source: "Nuclear Reactor Concepts" Workshop Manual, U.S. NRC
http://www.nrc.gov/reading-rm/basic-ref/teachers/unit3.html

Decay Heat

- Decay heat is always present following reactor operation due to energy resulting from the decay of fission products.
- The amount of decay heat present in the reactor is dependent on three factors
  - The pre-shutdown power level
  - How long the reactor operated
  - The amount of time since reactor shutdown
- Decay heat immediately after shutdown is approximately 5-6% of the preshutdown power level.
- Decay heat will decrease to approximately 1% of the pre-shutdown power level within one hour of reactor shutdown.

PWR: Emergency Core Cooling Systems (ECCS)

- The ECCS serves two purposes:
  - To provide core cooling to minimize fuel damage following a loss of coolant accident
    - Large amounts of cool, borated water are injected into the reactor coolant system
  - To provide extra neutron poisons to ensure the reactor remains shutdown following the cool down associated with a main steam line rupture
    - The same borated water source is used (this water source is called the refueling water storage tank)

Chemical Shims

- Most water-cooled or water-moderated reactors are controlled at least in part by varying the concentration of boric acid (H_3BO_3) in the water
- This boric acid is called a *chemical shim*
  - control rods are sometimes called shim safety rods
- Boric acid is soluble in water at 63.5 g/liter at room temperature (267 g/liter at 100° C)
  - usually dissolved to concentrations of about 500-2000 ppm in cooling water

Chemical Shims (continued)

- These chemical shims have several advantages:
  - Since the poison distribution is uniform and independent of the amount of reactivity being controlled, the fuel loading can be more easily distributed to yield a uniform power distribution
  - Chemical shims reduce the mechanical control rod requirements considerably
    - This saves in cost since mechanical control rods are expensive

Chemical Shims (continued)

- There are disadvantages to this control mechanism:
  - – the rate at which we can inject or withdraw poison from the coolant is small; the reactivity insertion rates are correspondingly small
  - – so chemical shims are used primarily to reduce control rod requirements and compensate for relatively slow reactivity changes
    - like fuel burnup, fission product poisoning, and moderator temperature changes

- The lifetime of a given core loading is generally determined by the amount of fuel initially loaded into the core
  - Or by damage to the clad

- In order to increase the allowable initial core fuel loading,
  - It is common to load into the core, materials with high absorption cross sections that compensate for all of the excess reactivity during the early stages of the core life

Burnable Absorbers (continued)

- These absorbers are chosen so that they "burn out" somewhat faster than the fuel burnup, so that as they are removed positive reactivity is added back to the core
  - This helps offset the negative reactivity added by the destruction of the ^{235}U

- We call these materials "burnable absorbers" or "burnable poisons"
  - Usually either gadolinium or borosilicate glass

Burnable Absorbers (continued)

- These poisons have several advantages:
  - They increase core lifetime without any decrease in control safety
  - Reduce the amount of mechanical control requirement
  - If distributed properly, can also flatten the core power distributions
- These poisons are usually fabricated into initial fuel as either fixed control blades (or curtains) or mixed into certain pins

Core Burnup

- As a reactor is operated, atoms of fuel are constantly consumed, resulting in the slow depletion of the fuel frequently referred to as core burnup.
- Major effects of this fuel depletion
  - The control rods must be withdrawn or chemical shim concentration reduced to compensate for the negative reactivity effect of this burnup
  - Supplemental burnable poisons (if used) burn out at a rate that approximates the burnout of the fuel
    - Thus, they reduce the amount of control rod movement necessary to compensate for fuel depletion early in core life
  - As control rods are withdrawn to compensate for fuel depletion, the effective size of the reactor is increased causing neutron leakage to decrease

Burnup: Changes in Fuel Composition

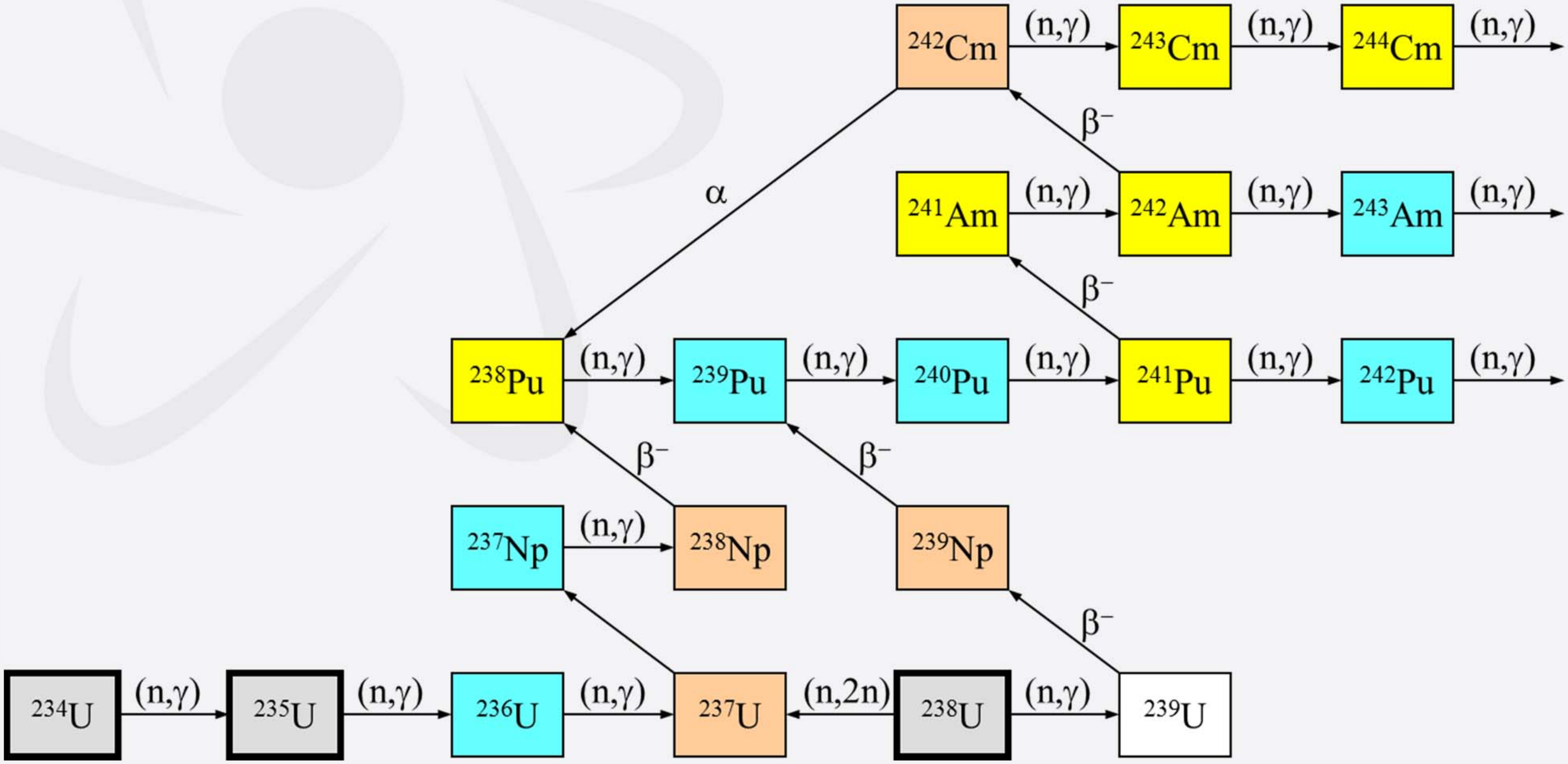
- The initial fuel in a reactor essentially consists of only ^{234}U, ^{235}U, and ^{238}U
- As the reactor is operated the fuel composition begins to change
  - Some neutrons cause fission reactions which result in the production of fission products
    - These fission products might then decay to other isotopes
  - Some neutrons are absorbed through (n,g) reactions in U isotopes and produces higher mass U isotopes
    - These higher mass U isotopes might then decay by b^- emission to higher actinide isotopes (Np, Pu, Am, Cm, etc.)

Reactions Resulting in Material Changes

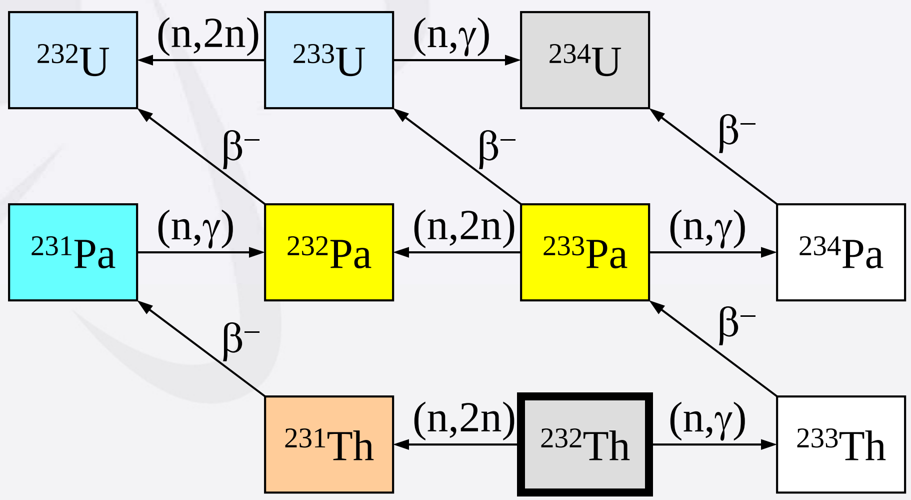
- Scattering reactions (elastic and inelastic) leave the materials unchanged

- All other reactions result in material changes:
  - radiative capture
  - fission
  - (n,2n), (n,3n), etc..
  - (n,p), (n,d), (n,t), (n,a), etc..
- Some of these isotopes can then decay to new isotopes or absorb neutrons and form new isotopes

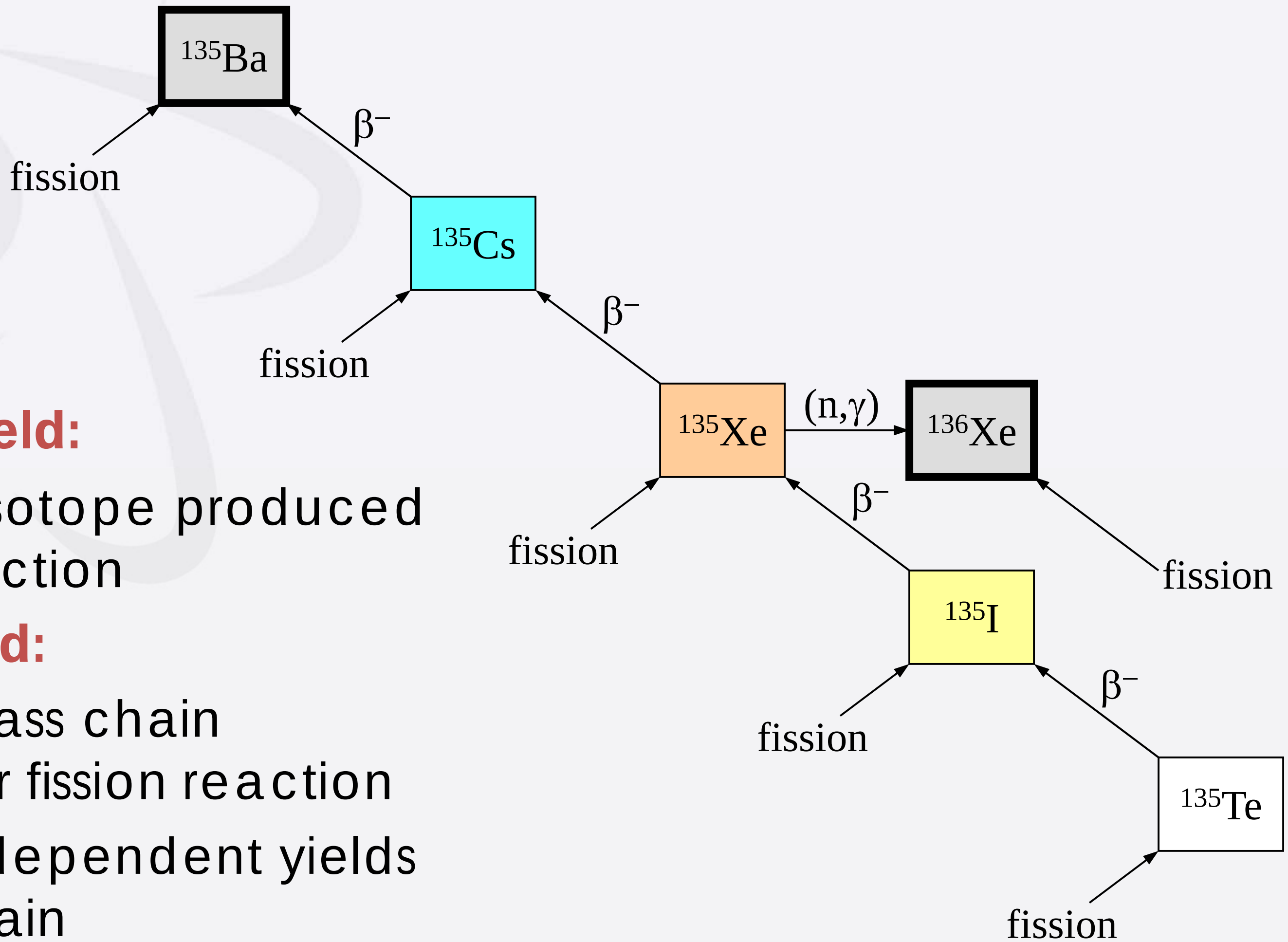
Uranium Buildup and Decay Chain



Thorium Buildup and Decay Chain



Fission Product Buildup and Decay Chain



- **Independent Yield:**
  - atoms of an isotope produced per fission reaction
- **Cumulative Yield:**
  - atoms of a mass chain produced per fission reaction
  - sum of the independent yields for a mass chain

Buildup and Depletion Calculations

- We refer to the calculations that determine these isotopic compositions as buildup and depletion calculations
- These calculations may be crucial for various reasons including
  - Computing the cross sections at different times in the reactor
  - Determine the production of isotopes for spent fuel management and reactor kinetics
  - Determine the heat produced by decays in the fuel
  - Calculating power production in different fuel rods

- In general, isotope *A* builds up and depletes due to:
  - Loss due to radioactive decay of *A*
  - Loss due to neutron absorption in *A*
  - Gain due to radioactive decay of *B* to *A*
  - Gain due to neutron absorption in *C* producing *A*

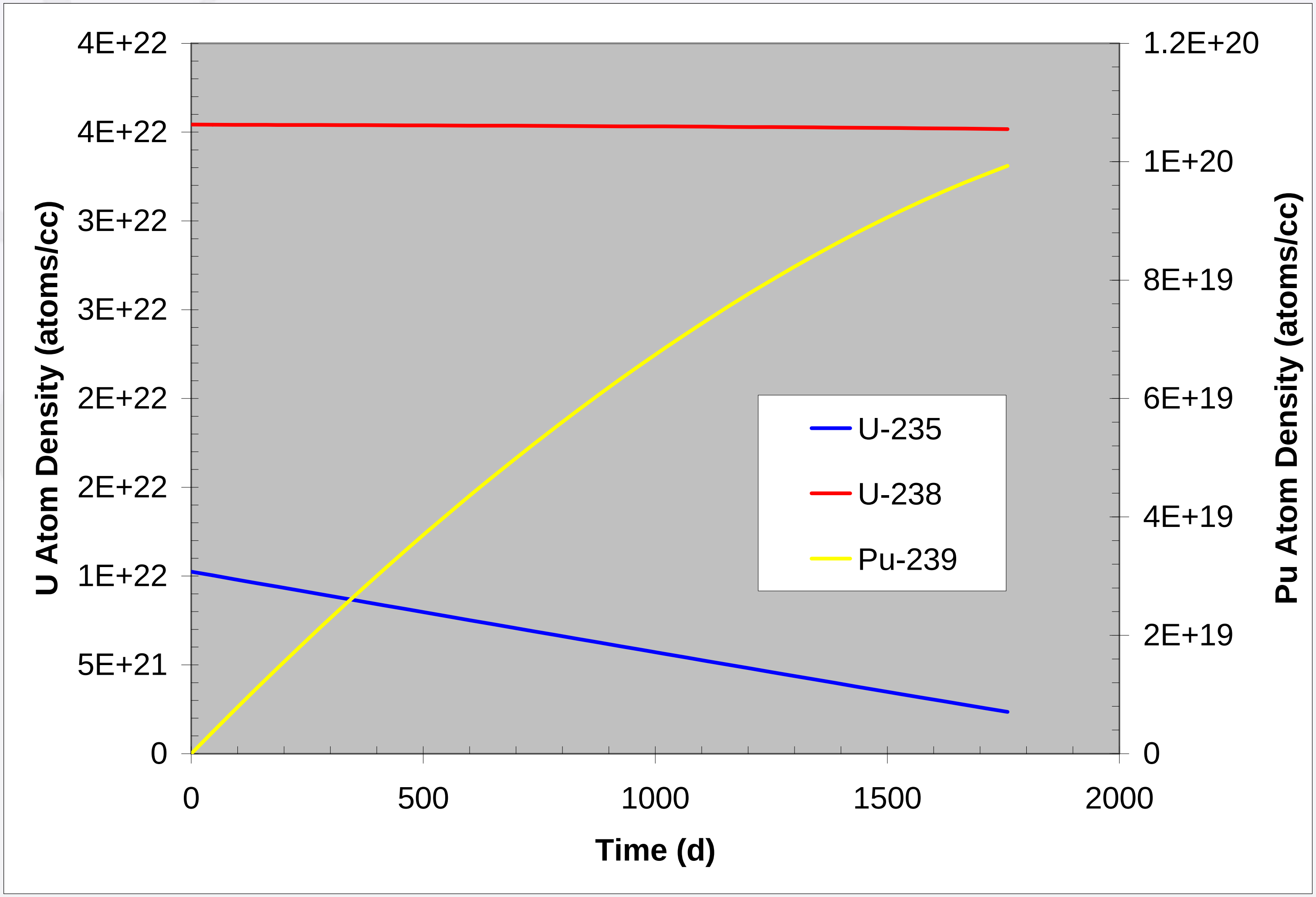
$$\frac{dN_A}{dt} = -\lambda_A N_A - \sigma_a^A \phi N_A + \lambda_B N_B + \sigma_a^C \phi N_C$$

Simple ^{239}Pu Production Example

- Let us consider a simple example of production of ^{239}Pu in a reactor originally fueled with only ^{235}U and ^{238}U

- We will consider the following:
  - Power production from ^{235}U and ^{239}Pu fission
  - Production of ^{239}Pu from absorption in ^{238}U and ignoring the decay chain leading to ^{239}Pu
  - Neglect all other isotopes

^{239}Pu Production Example



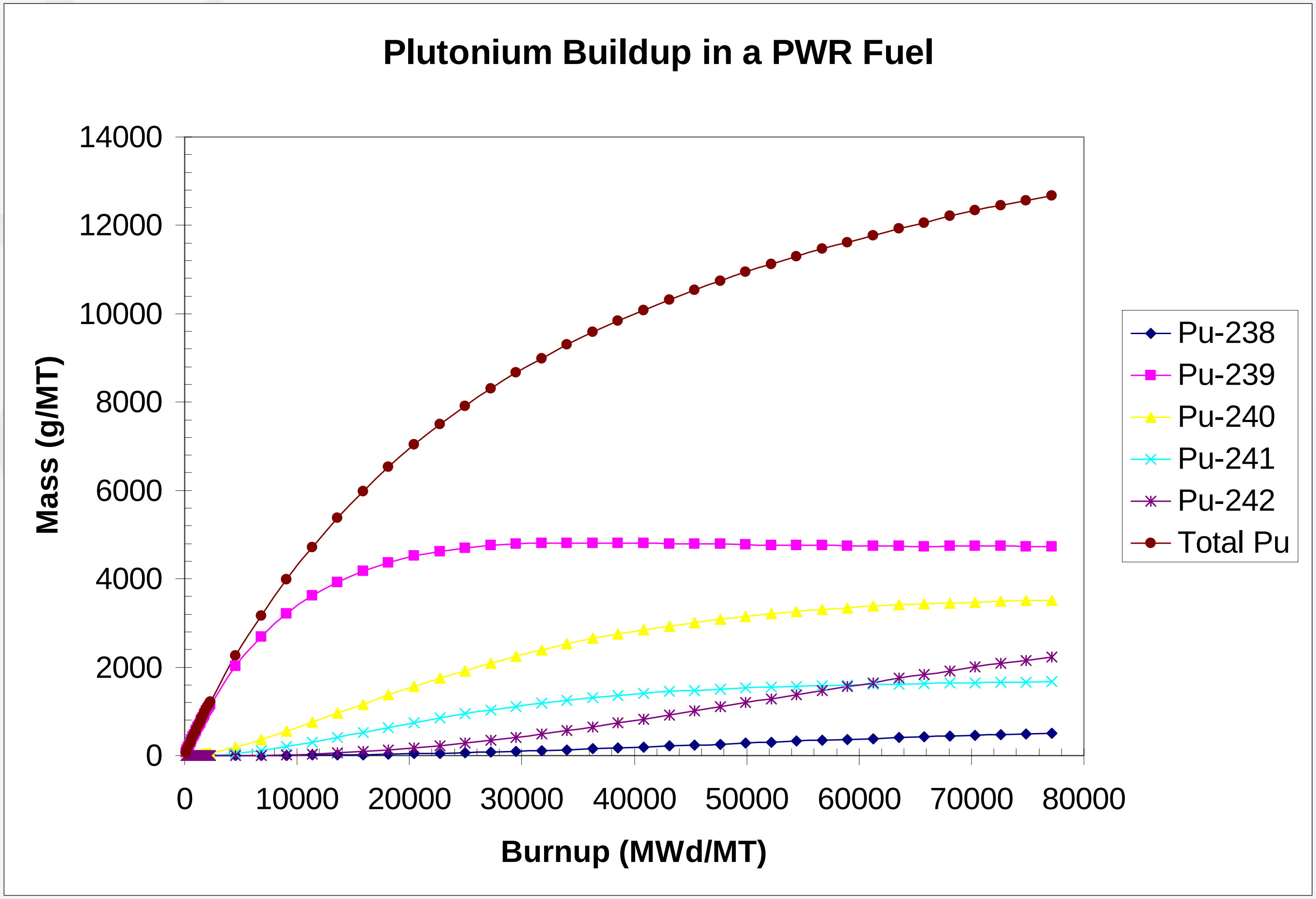
Author: Dr. Charlton, NUEN, TAMU.

Measure of Fuel Burnup

- The most commonly used measure of burnup is the fission energy released per unit mass of fuel
  - megawatt-days per ton of initial uranium (MWd/MTU)
  - for example: a reactor with 100,000 kg of uranium operating at 3000 MW for 1000 days would have a burnup of 30,000 MWd/MTU

- Typical burnups
  - PWRs or BWRs: 30,000-60,000
  - LMFBR: 100,000-200,000
  - CANDU: 10,000-20,000

Plutonium Production in PWRs



Author: Dr. Charlton, NUEN, TAMU.

Core Power Distribution

- It important to ensure predictable temperatures and uniform depletion of the fuel installed in a reactor
  - Need to provide an even distribution of neutron flux (power distribution)
  - Methods used to shape (flatten) the neutron flux profile across the core
    - Reflectors (reduce the ratio of peak flux to the flux at the edge of the core fuel area)
    - Poisons (suppress the neutron flux where desired)
    - Varying the fuel enrichment or fuel concentrations in the core radially, axially, or both

References

1. Lamarsh, J. R., and Baratta, A. J., "Introduction to Nuclear Engineering", third edition, Prentice Hall, 2001
2. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf
3. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v2.pdf
4. Stacey, W. M., "Nuclear Reactor Physics", second edition, Wiley-VCH, 2007

List of Required Texts

1. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v1.pdf
2. DOE Fundamentals Handbook "Nuclear Physics and Reactor Theory", http://www.hss.doe.gov/nuclearsafety/ns/techstds/standard/hdbk1019/h1019v2.pdf