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ON THE NUMERICAL APPROACHES TO MATERIAL BIFURCATION

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Summary This work presents numerical approaches to study the problem of material bifurcation, where the mechanical state of the material is given numerically in terms of its fourth-order tangent moduli. The detection of material bifurcation is then posed as a minimization problem, i.e., the determinant of the acoustic tensor is minimized with respect to directions. In this work, five different parametrization techniques to obtain direction vectors are proposed. Of particular interest are the efficiency and robustness of different approaches. The approaches described in this work are applicable to any general constitutive law as long as its fourth-order tangent moduli are given. A representative numerical example will be given to demonstrate the performance of different approaches.

INTRODUCTION

Understanding and detecting material bifurcation is a subject of considerable research interest since it impacts many problems in mechanics of materials. For instance, in metals, material bifurcation often signals localized deformations which may eventually lead to fracture. For some material models, closed form solution for material bifurcation might be tractable. In general, however, one has to rely on numerical methods. In this work, we are interested in numerically determining whether or not a material bifurcates given its mechanical state in terms of its fourth-order tangent moduli. Within the scope of this work, the material bifurcation condition refers to the existence of a non-zero vector $\boldsymbol{v} \in S^2$ such that

$$f(\boldsymbol{v}) = \det \boldsymbol{A}(\boldsymbol{v}) = 0, \quad \boldsymbol{A} = \boldsymbol{v} \cdot \mathbb{C} \cdot \boldsymbol{v} \tag{1}$$

where $\mathbb{C}$ is a fourth-order tangent modulus, which may have different specific forms or meanings in various contexts. When develop numerical procedures, we consider a general fourth-order tensor $\mathbb{C}$ and do not impose any restrictions.

MINIMIZATION PROBLEM

To numerically detect if the material bifurcates, condition (1) is tested at the minima of the bifurcation function $f(\boldsymbol{v})$ throughout the deformation process. Therefore, it is appropriate to consider the following minimization problem

$$\min \ f(\boldsymbol{v}) = \det \boldsymbol{A}(\boldsymbol{v}), \ \ \boldsymbol{v} \in S^2 \tag{2}$$

If the bifurcation function $f(\boldsymbol{v})$ is differentiable, the minimization problem can be rewritten equivalently as

$$\frac{\partial f}{\partial \boldsymbol{v}} = 0, \ \ \boldsymbol{v} \in S^2 \tag{3}$$

The problem is assumed to be solvable, i.e., there exists at least one $\boldsymbol{v}$ that minimizes $f(\boldsymbol{v})$. Due to the highly nonlinear nature of the minimization condition (3), a Newton-type iterative procedure is needed to numerically solve the problem.

PARAMETRIZATIONS

While the bifurcation condition (1) should be independent of how the minimization is performed, the minimization process itself will be significantly influenced by the specific form of the direction vectors chosen. To this end, this work proposes techniques to parametrize the unknown vector $\boldsymbol{v}$, and evaluates the performance of different parametrizations. Five different parametrizations for the vector $\boldsymbol{v}$ are explored in this work:

(i). Spherical parametrization, where spherical coordinates are used. This is the classical and most commonly used approach. However, it will be shown in the numerical examples, it may result in complicated landscape of the minimization function, and infinite number of minimal points.

(ii). Cartesian parametrization with constraint enforced by Lagrange multiplier [1].

(iii). Cubic parametrization, where $\det \boldsymbol{A}$ is assembled on three planes of a unit cube. The condition that $\boldsymbol{v} \in S^2$ has been relaxed to $\boldsymbol{v} \in \mathbb{R}^3$. Nevertheless, the location of the minimum is the same provided that $\det \boldsymbol{A} = 0$.

(iv). Inverse stereographic projection, which projects a vector from equatorial plane (z=0) onto unit-radius sphere.

(v). Exponential mapping, which projects a vector from tangent plane at the north pole (z = 1) onto unit-radius sphere.

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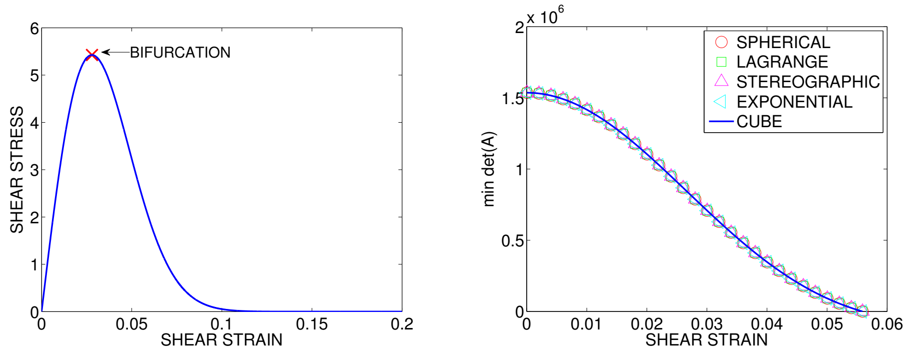
REPRESENTATIVE NUMERICAL EXAMPLE

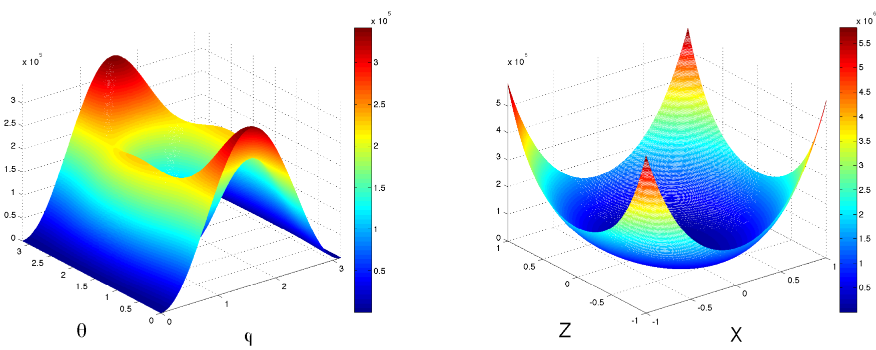
In this section, results of a simple shear test performed on the isotropic hyperelastic damage model are shown [2]. Material properties are $\lambda = 80$, $\mu = 80$, $\xi_\infty = 1.0$ and $\tau = 1.0$. The shear stress-strain behavior as well as the degradation of $\det \boldsymbol{A}$ are shown in Fig. 1.



(a) (b)

Figure 1. Simple shear test on isotropic damage model: (a) shear stress-strain behavior, with red cross indicating bifurcation, and (b) degradation of $\det \boldsymbol{A}$ for different parametrizations.

To illustrate the influence of parametrizations on minimization function, we plot the landscape of $\det \boldsymbol{A}$ for two parametrizations in Fig. 2, i.e., the spherical and cube. It can be seen that spherical parametrization yields a complicated landscape and an infinite number of minima. Cube parametrization, on the other hand, yields a smooth convex landscape and a unique minimum on this plane. It is clear that the Newton-type based optimization process is more likely to converge to the true global minimum if the cube parametrization is used.



(a) Spherical (b) Cube

Figure 2. Simple shear test on isotropic damage model: landscape of $\det \boldsymbol{A}$ at bifurcation.

CONCLUSIONS

Numerical approaches for detecting material bifurcation have been presented in this paper. Five different parametrizations on the direction vectors are explored. A hyperelastic damage model under simple shear loading is analyzed. While five parametrizations detect bifurcation at the same time, the resulting minimization functions may vary, which can be seen from the landscape of $\det \boldsymbol{A}$. To illustrate this point, the landscapes of spherical and cube parametrizations are plotted and compared. A smooth convex landscape from cube parametrization is clearly favored for Newton-type based optimization.

References

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[2] Holzapfel G.A.: Nonlinear Solid Mechanics: A Continuum Approach for Engineering Wiley, Chichester 2000.

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