SAND2012-1679 C

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Quantum Error Correction: Optimal, Robust, Adaptive? Or

Where is The Quantum Flyball Governor?

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@ March 2012 Meeting of The American Physical Society

Research partially supported by the IARPA-QCS Program, an ARO MURI grant, and by the Laboratory Directed Research and Development program at Sandia National Laboratories.

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Where is the controller?

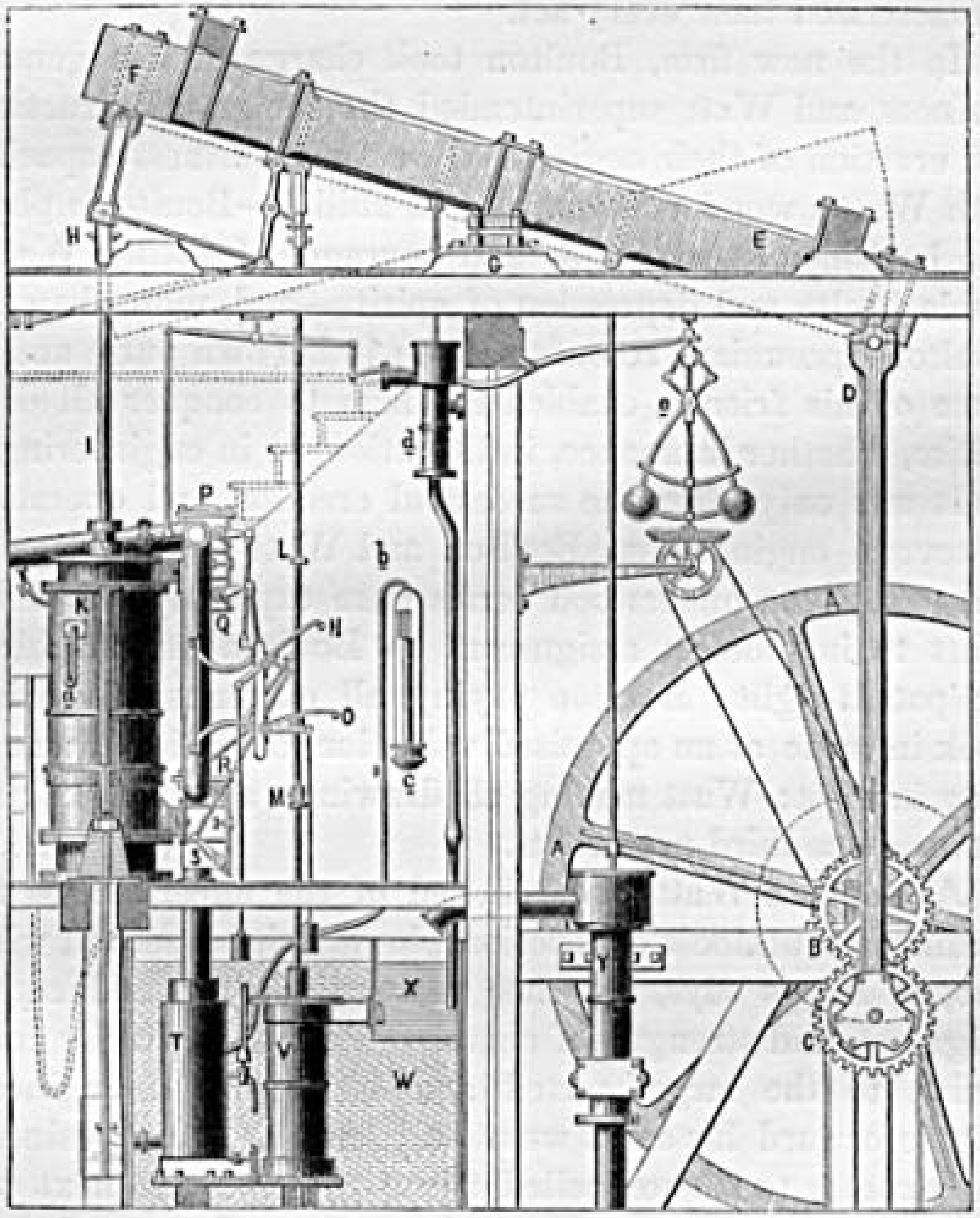
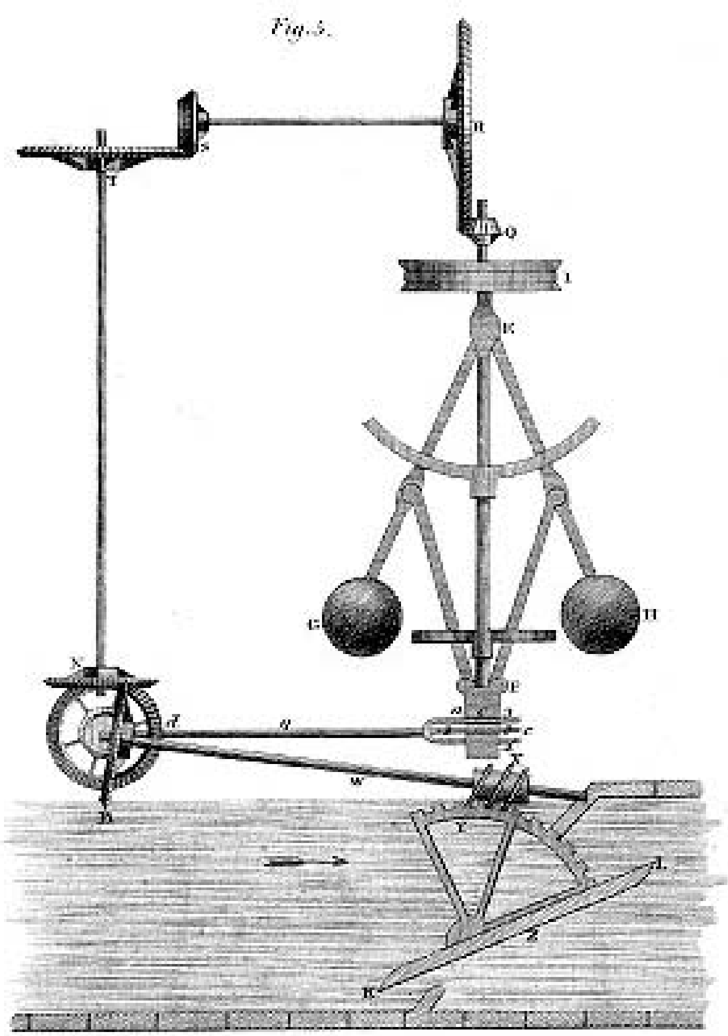


Fig. 27.—Watt's Engine, 1781.

The Feedback Control design paradigm



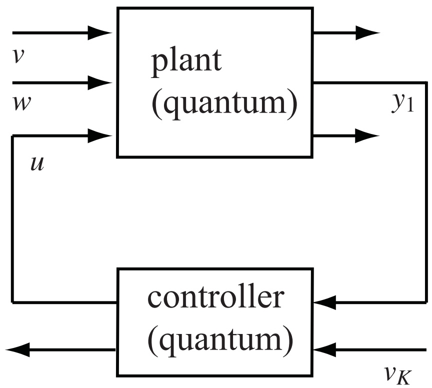
“Flyball Governor”
for steam engine control
James Watt (ca. 1780)

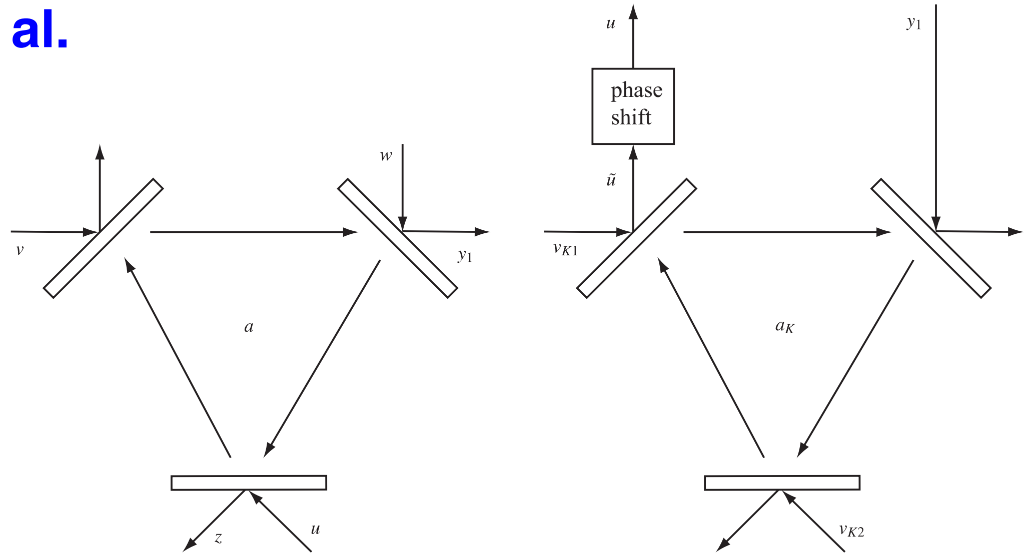
completely mechanical
“natural” speed regulation

Is there a
Quantum Flyball Governor?

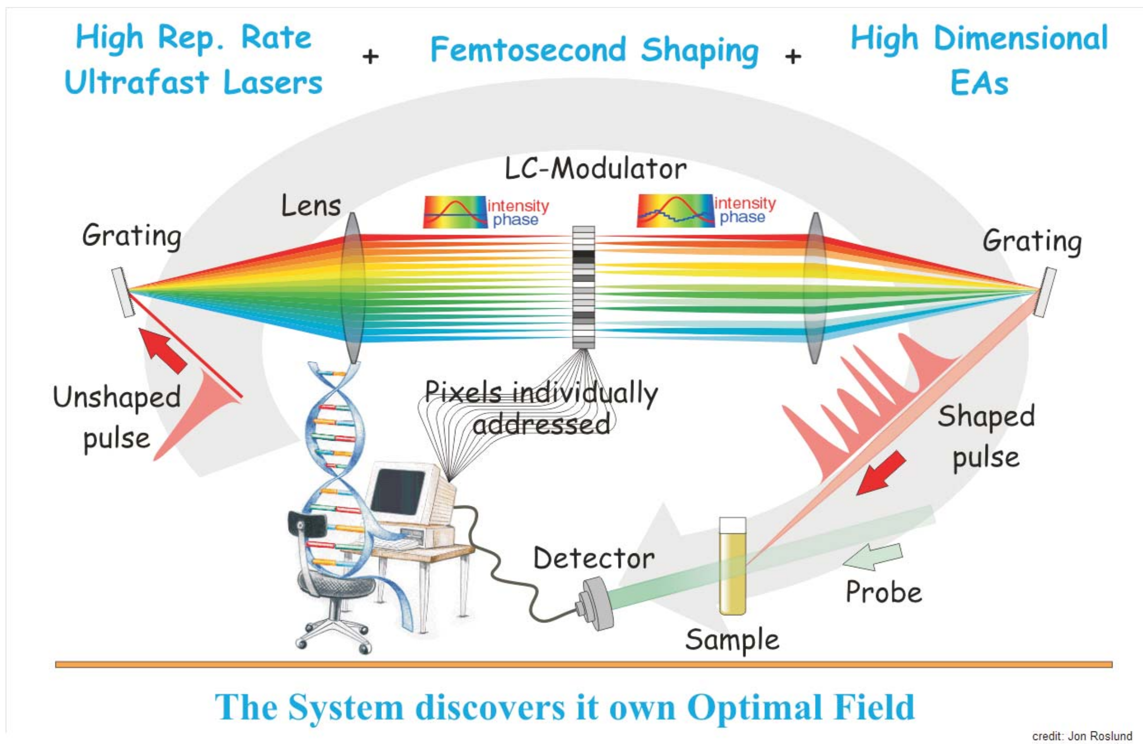
completely quantum mechanical
“natural” quantum error correction

Matt James, Ian Petersen, et al.

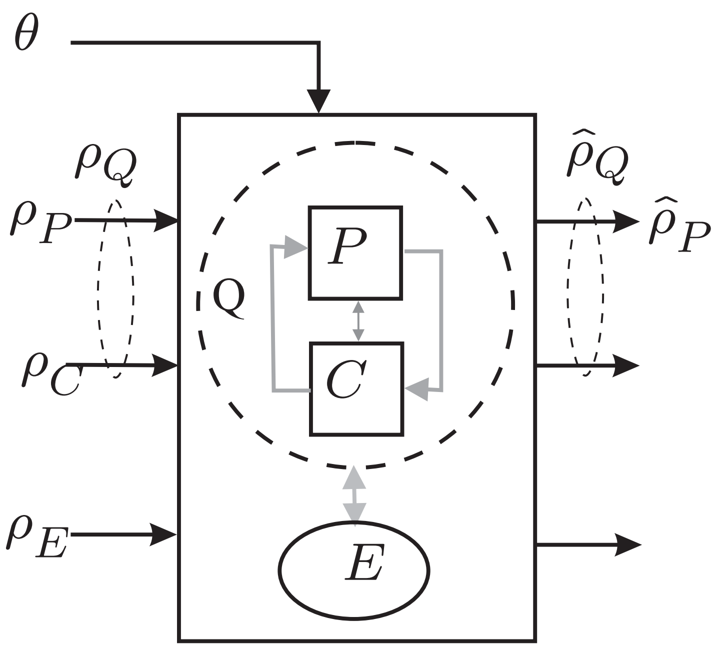




Hersch Rabitz et al.



Robust optimization with physical model



$P =$ "plant" or "system"
$C =$ "controller" or "ancilla"
$E =$ "environment" or "bath"
$\triangle =$ "uncertainty modeling constraints"

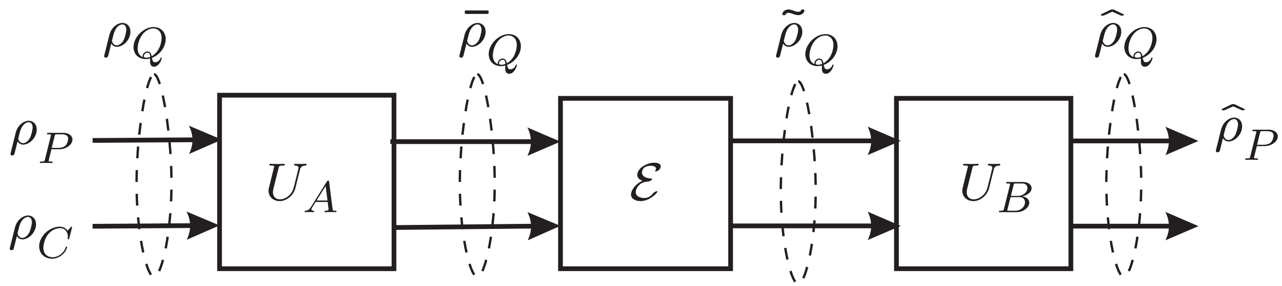
$$\begin{array}{ll} \text{maximize} & \min_{\triangle} F(\theta) \\ \text{subject to} & \theta \in \Theta \\ & (P, C, E) \in \triangle \end{array}$$

- local solution via sequential convex optimization [a]

- θ can be parametrization of external control signals and/or internal system variables, *e.g.*, physical parameters, system-ancilla couplings, geometries, *etc.*

[a] A. Mutapcic *et al.*, *Engr. Opt.*, (2009), J. Zhang, R. Kosut, *ECC*, (2007), S. Boyd, *EE364b Lecture Notes*, M. Grace et al.,*J.Phys.B*,(2007), A. Levi, S. Haas (editors), *Optimal Device Design*, Cambridge, (2009)

Robust optmization with operator model



$$
\begin{array}{ll}
\text{maximize} & \min_{\triangle} F(U_A, U_B) \\
\text{subject to} & U_A,\ U_B \text{ unitary} \\
& \mathcal{E} \in \triangle
\end{array}
$$

- local solution via equivalent bi-convex iterations, *e.g.*, solve for U_A given U_B, solve for U_B given U_A, repeat ... [a]
- provides limit of performance
- physical implementation is not accounted
- useful for optimization intialization

[a] M. Reimpell, R. F. Werner, *Phys. Rev. Lett.* (2005), N. Yamamoto, S. Hara, K. Tsumura, *Phys. Rev. A*, (2005), A. S. Fletcher, P. W. Shor, M. Z. Win, *Phys. Rev. A* (2007), R.L. Kosut, A. Shabani, D.A. Lidar, *Phys. Rev. Lett.* (2008), S. Taghavi, R.L. Kosut, D.A. Lidar, *IEEE Trans. Information Theory* (2009), R.L.Kosut, D.A.Lidar, *Quant. Inf. Proc.*, (2007)

Solution via Sequential Convex Optimization (SCO)

Initialize

select an initial control $\theta \in \Theta$
select a sample of uncertain parameters $\delta_i \in \Delta,\ i = 1, \ldots, L$
set "step-size" β

Repeat

1. *Calculate fidelities, gradients & Hessians with respect to* θ

$$F(\theta, \delta_i),\ g(\theta, \delta_i) = \nabla_\theta F(\theta, \delta_i),\ R(\theta, \delta_i) = -\nabla_\theta^2 F(\theta, \delta_i),\ i = 1, \ldots, L$$

2. *Solve convex optimization for* $\tilde{\theta}$ *using the linearized fidelity.*

$$\begin{array}{ll} \text{maximize} & \min_{i=1,\ldots,L} F(\theta, w_i) + g^T(\theta, \delta_i)\tilde{\theta} - \tilde{\theta}^T [R(\theta, \delta_i)]_+ \tilde{\theta}/2 \\ \text{subject to} & \theta + \tilde{\theta} \in \Theta,\ -\beta \le \tilde{\theta} \le \beta \end{array}$$

3. *Update*

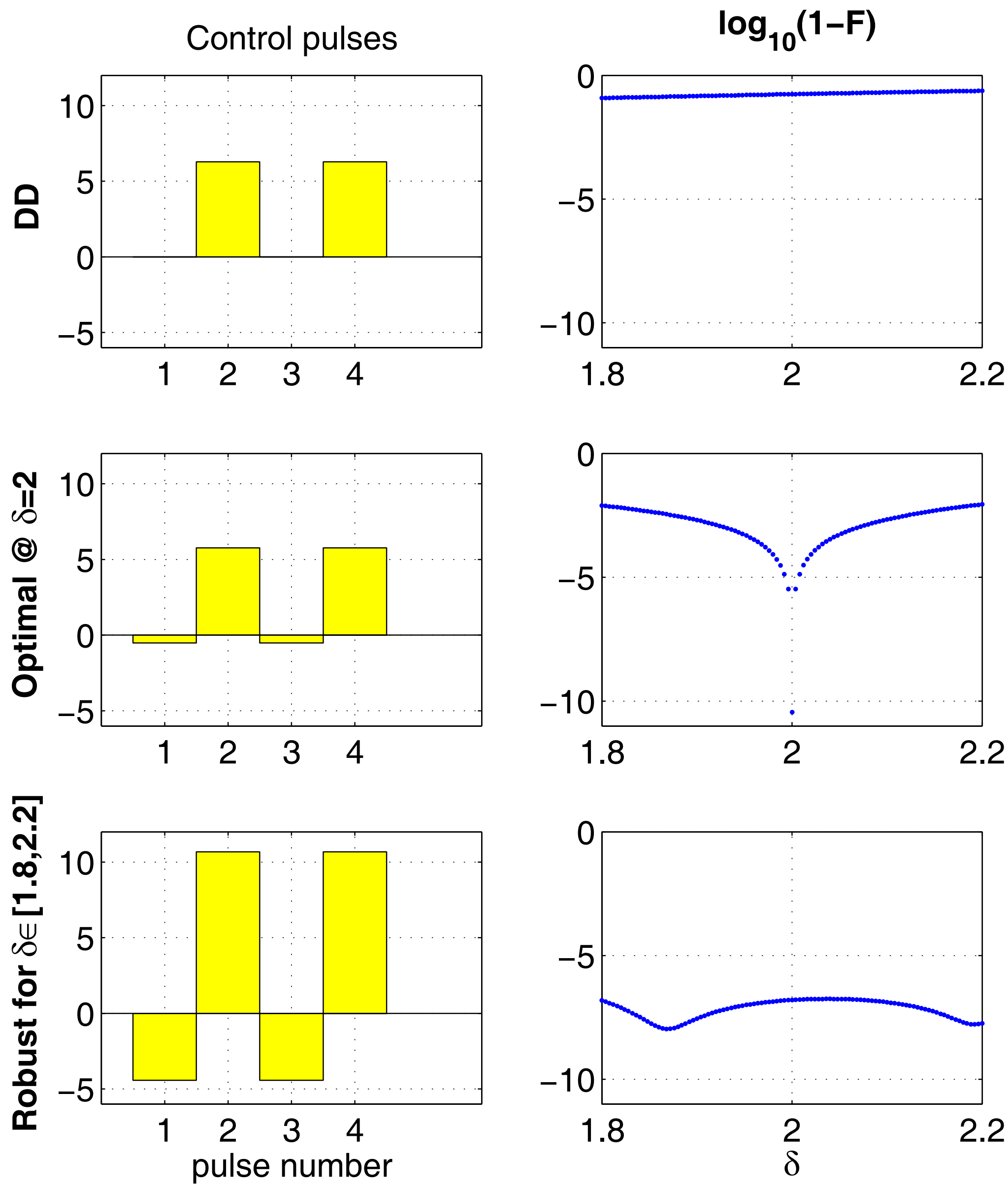
$$\begin{array}{ll} \text{IF} & \min_{i=1,\ldots,L} F(\theta + \tilde{\theta}, \delta_i) > \min_{i=1,\ldots,L} F(\theta, \delta_i) \\ \text{THEN} & \theta \leftarrow \theta + \tilde{\theta},\ \text{increase step-size } \beta \\ \text{ELSE} & \text{decrease step-size } \beta \end{array}$$

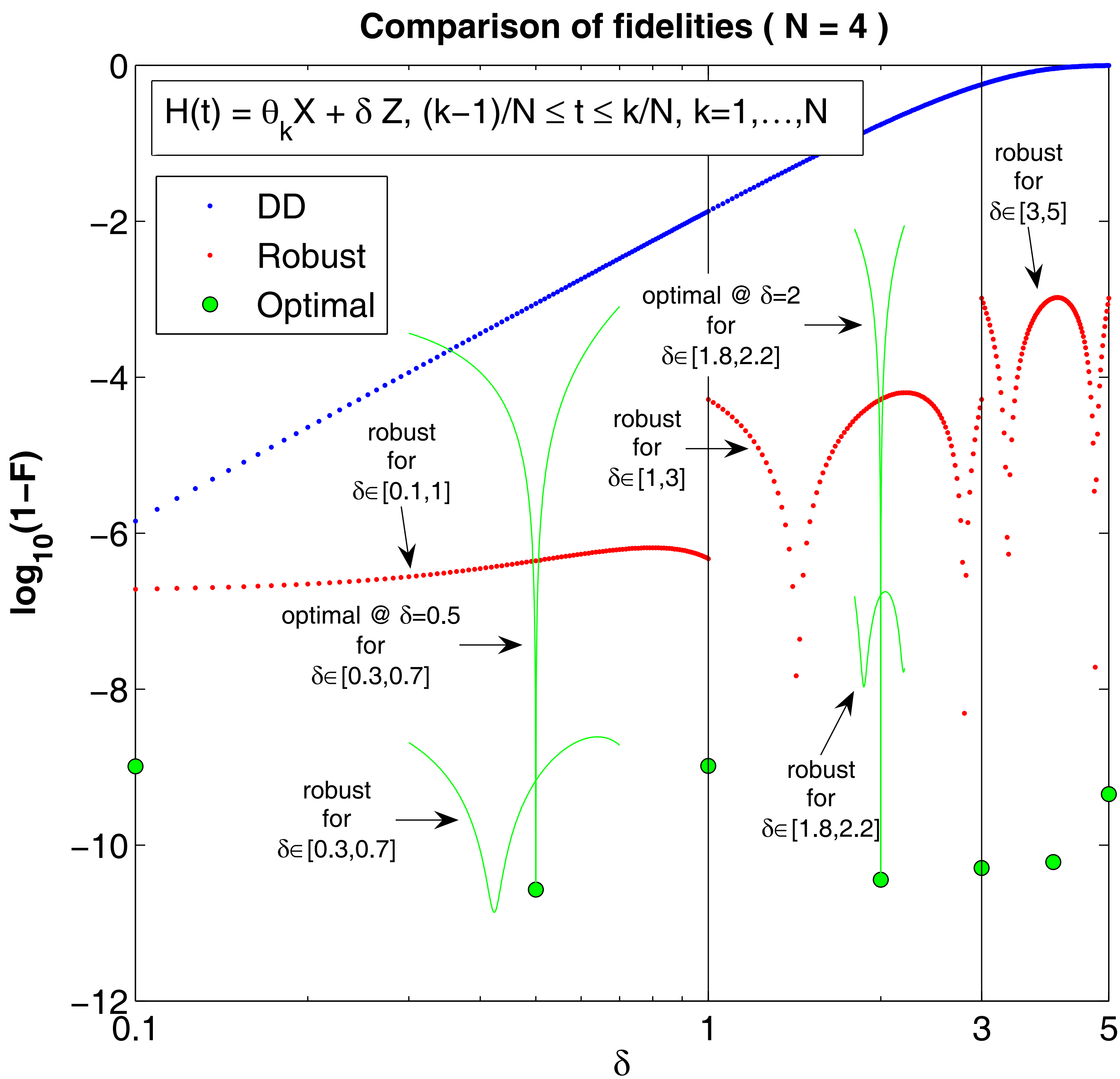
Until *Stopping criteria satisfied*

Validate *evaluate* $F(\theta, \delta_{\text{val}}),\ \delta_{\text{val}} \in \Delta \Rightarrow$ repeat SCO with worst-case samples from δ_{val}.

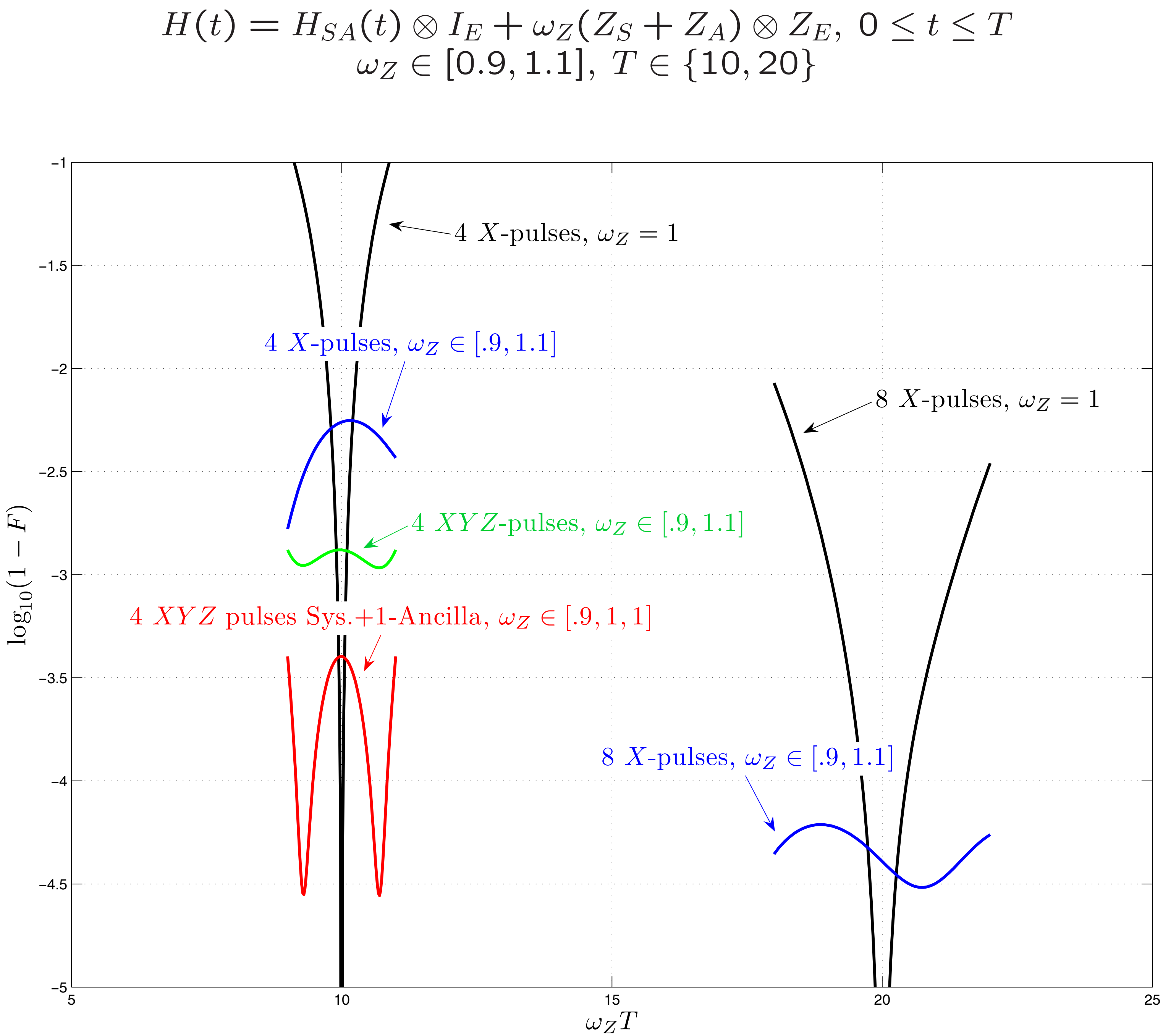
Example

$$H(t) = \theta_k X + \delta Z,\ (k-1)/N \leq t \leq k/N,\ k = 1, \ldots, N$$





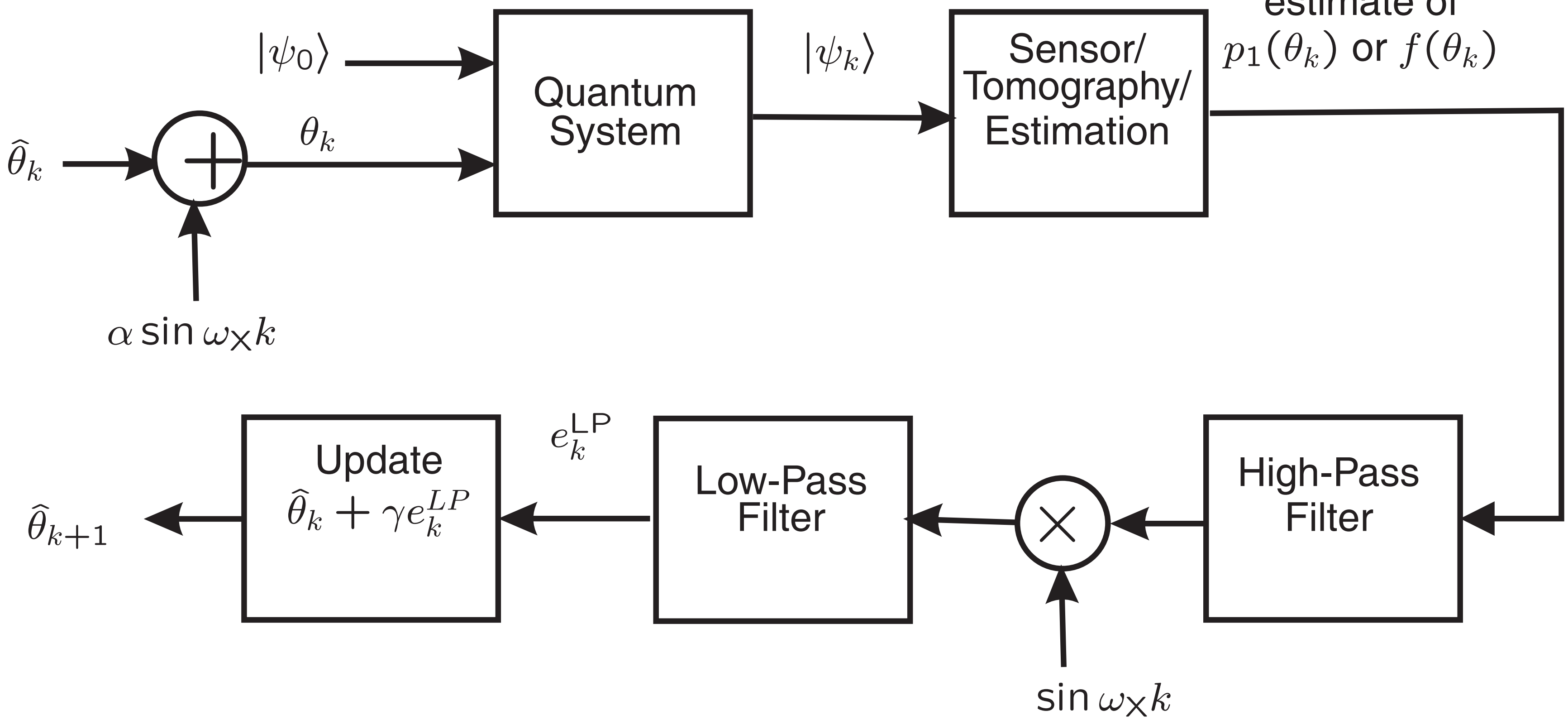
Comparison of fidelities (N = 4)
H(t) = θkX + δ Z, (k−1)/N ≤ t ≤ k/N, k=1,…,N
DD
Robust
Optimal
robust for δ∈[3,5]
optimal @ δ=2 for δ∈[1.8,2.2]
robust for δ∈[1,3]
robust for δ∈[0.1,1]
optimal @ δ=0.5 for δ∈[0.3,0.7]
robust for δ∈[0.3,0.7]
robust for δ∈[1.8,2.2]
log10(1−F)
0
−2
−4
−6
−8
−10
−12
0.1
1
3
5
δ



$H(t) = H_{SA}(t) \otimes I_E + \omega_Z(Z_S + Z_A) \otimes Z_E,\ 0 \le t \le T$
$\omega_Z \in [0.9, 1.1],\ T \in \{10, 20\}$
4 X-pulses, $\omega_Z = 1$
4 X-pulses, $\omega_Z \in [.9, 1.1]$
8 X-pulses, $\omega_Z = 1$
4 XYZ-pulses, $\omega_Z \in [.9, 1.1]$
4 XYZ pulses Sys.+1-Ancilla, $\omega_Z \in [.9, 1, 1]$
8 X-pulses, $\omega_Z \in [.9, 1.1]$
$\log_{10}(1 - F)$
$\omega_Z T$
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
5
10
15
20
25

Extremum Seeking Feedback*

ESF system in the k-th iteration.



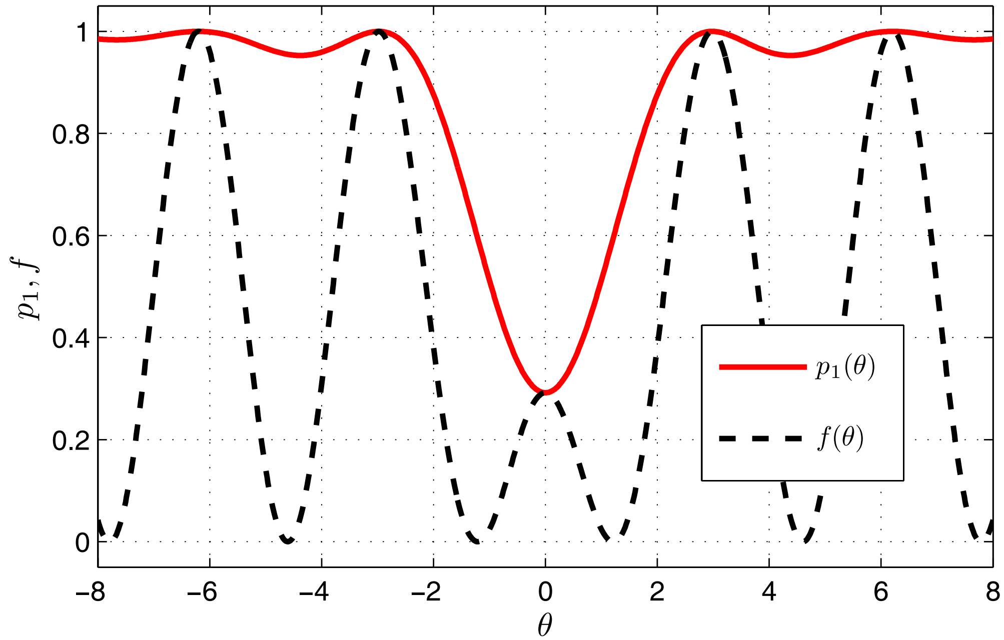
"Theory" IF:

- $f(\theta_k) \approx 1 + \left(\frac{\partial^2 f}{\partial \theta_k^2}(\theta_\star)/2\right)(\theta_k - \theta_\star)^2$ with $\frac{\partial^2 f}{\partial \theta_k^2}(\theta_\star) < 0$
- $\omega_{\mathsf{LP}} \leq \omega_{\mathsf{HP}} \leq \omega_{\mathsf{X}}$
- $\gamma\alpha\frac{\partial^2 f}{\partial \theta_k^2}(\theta_\star) < 0$ and "small"

THEN: $\widehat{\theta}_k \rightarrow \theta_\star$

* K.B. Ariyur & M. Krstic, *Real-Time Optimization by Extremum Seeking Feedback*, Wiley, 2003.

Example: outcome probability and fidelity vs. control



- both $p_1(\theta)$ and $f(\theta)$ vs. θ peak at the same periodic control values.
- $p_1(\theta)$ has multiple global maxima and no local maxima.
- fidelity $f(\theta)$ has both, specifically many global maxima and one local maxima.
- for quantum systems the outcomes of these functions are not instantaneously available, *e.g.*, typically we can record how many times an eigenvalue of an observable was obtained.

Summary remarks

- numerical optimization reveals robust control performance tradeoffs, *e.g.*, control magnitude, power, energy, bandwidth, temporal and spatial resources, *etc.*— requires “uncertainty modeling,” *e.g.*,
  - knowledge of range of coupling coefficients, bath/environment dynamics
  - “hardware” variables, *e.g.*, bias, scaling effects, classical noise, time constants, *etc.*...
- Complementary approach is on-line control tuning
  - control protocol must be amenable to performance improvement.
  - need “just-right” minimal data for fast estimation
- choice of system and control design are interdependent
- finally,

still looking for ...