SAND2012-3338 C

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Modeling and Simulation of Parameter Vector Random Processes in Mechanical Structures

53rd AIAA/ASME/ASCE/AHS/ASC

Structures, Structural Dynamics, and Materials Conference

14th AIAA Non-Deterministic Approaches Conference

Waikiki, HI

April 23-26, 2012

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Talk Outline

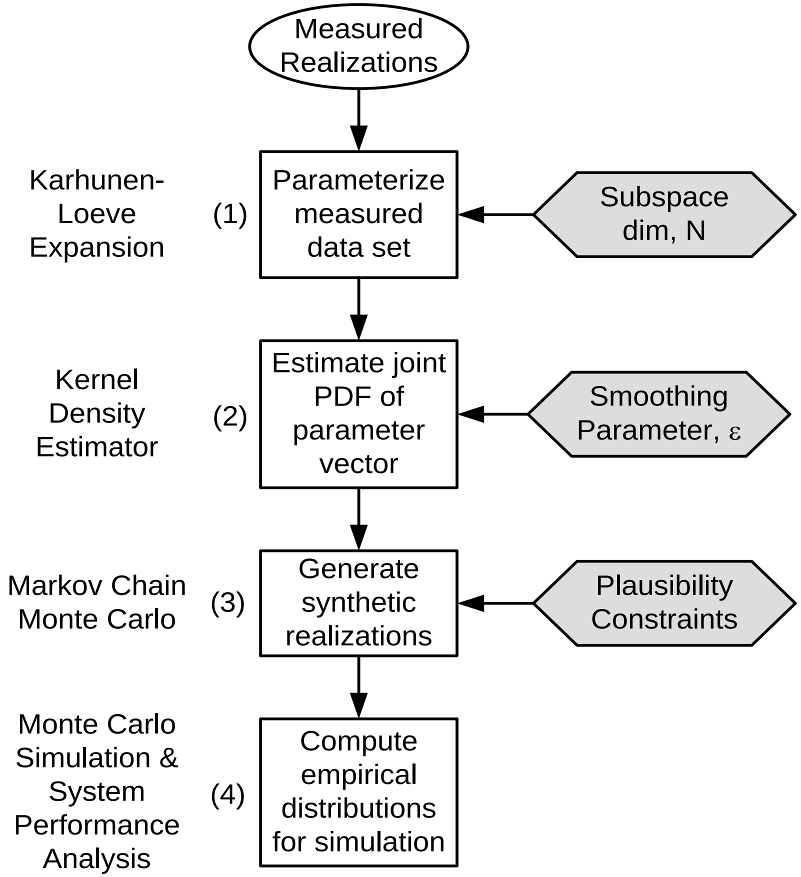
- Motivation
- Introduction – Objective, Control of Stochastic Systems
- Review of Karhunen-Loeve Expansion with Markov Chain Monte Carlo
- Shortcomings of Modeling Methods
- Modification of Markov Chain Monte Carlo
- Applications to Stochastic Frequency Response Function Modeling

Motivation

- Real structures are random – Practically any measure of behavior reflects randomness
  - Among nominally identical structures
  - From one assembly of a structure to the next
- When we develop a stable controller that is optimized for one structure in a stochastic ensemble of nominally identical structures
  - It may not have closed-loop stability for all members of the ensemble
  - It will not prove optimal for all members of the ensemble

Introduction – Objectives

- **Develop framework for modeling stochastic structures in terms of frequency response functions (FRF)**
  - **including some knowledge of the physics**
- Estimate probability of stability of random, controlled structures for a given controller
- Estimate probability of satisfying a closed-loop performance objective for random structures



This paper focuses on the modeling framework
The 2nd & 3rd objectives are addressed in a separate paper

Modeling and Generation of FRF Random Process – KLE

- **Framework for modeling frequency response function (FRF) needs to accommodate random processes that are**
  - **Nonstationary**
  - **Gaussian/non-Gaussian**
- **The Karhunen-Loeve expansion (KLE) satisfies these requirements**
- **The matrix form of the KLE is**

$$\mathrm{H} \cong \mathrm{v}\mathrm{w}^{1/2}\mathrm{U} + \mu_{\mathrm{H}}$$

- **This is a straightforward SVD or eigenvalue decomposition**
- **For MIMO FRFs, we vectorize the FRF matrix and stack the real and imaginary parts**

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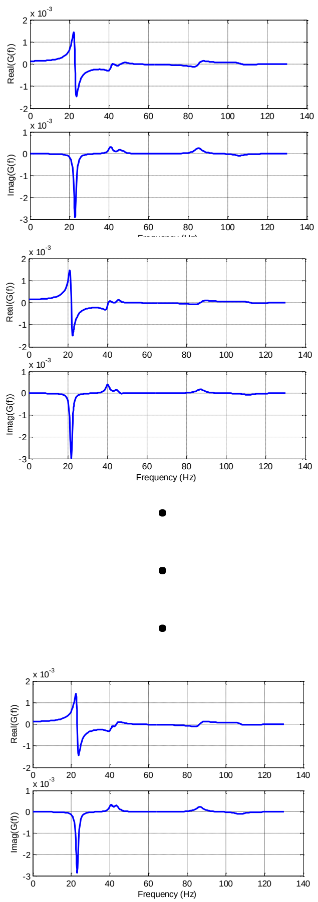
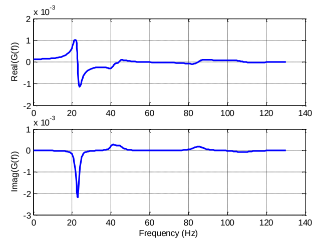
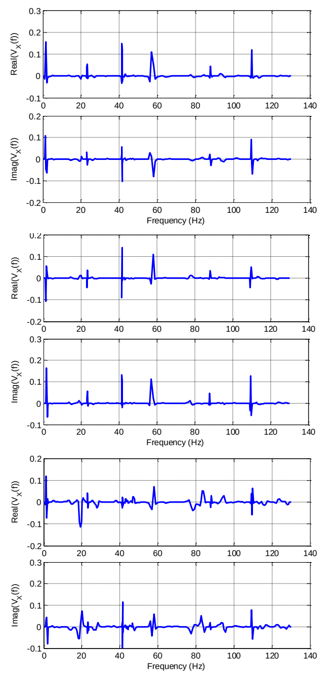
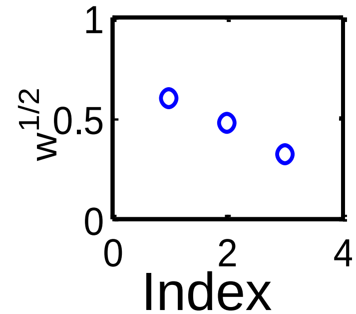
Karhunen-Loeve Expansion

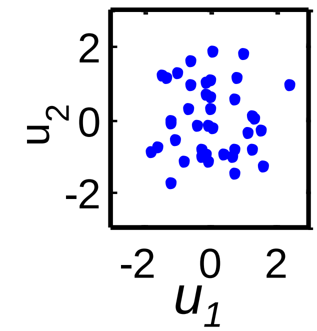
$$x(f) = \mu_X(f) + \sum_k v_k(f) \times w_k^{1/2} \times u_k$$











The first three can be approximated using measured realizations.

The u_k are zero-mean, unit variance, uncorrelated.

The KL expansion models a random process and its realizations as a mean function plus a product of shape functions, amplitude functions, and randomizing factors.

Modeling and Generation of FRF Random Process – KLE

- **Definitions**

$$\mathrm{H} \cong \mathrm{v}\mathrm{w}^{1/2}\mathrm{U} + \mu_{\mathrm{H}}$$

- H **is an nx1, discrete parameter (vector) RP**
- **v is an n x M, orthogonal matrix of the principle eigenvectors in the autocovariance matrix,** C_{HH}**, of** H

$$\mathrm{C}_{\mathrm{HH}} = E\left[(\mathrm{H}\text{-}\mu_{\mathrm{H}})(\mathrm{H}\text{-}\mu_{\mathrm{H}})^{T}\right]$$

- $\mathrm{w}^{1/2}$ **is an M x M, diagonal matrix of square roots of principle eigenvalues of the autocovariance matrix of H (components nonnegative)**

Modeling and Generation of FRF Random Process – KLE

- **Definitions**

$$H \cong v w^{1/2} U + \mu_H$$

- μ_H **is the n x 1, vector mean of the RP H**
- U **is an M x 1, vector of random variables (RV) – randomizing factors of the model.**
  - **Each RV is zero-mean and unit variance**
  - **RV pairs are uncorrelated**

Modeling and Generation of FRF Random Process – KLE

- **When measured realizations of the RP are available, then KLE parameters, $\left(\mathrm{v}, \mathrm{w}^{1/2}, \mu_{\mathrm{H}}\right)$, can be estimated.**

  Denote estimates $\left(\hat{\mathrm{v}}, \hat{\mathrm{w}}^{1/2}, \bar{\mathrm{h}}\right)$
- **Approximate KLE model**

$$\mathrm{H} \cong \hat{\mathrm{v}}\hat{\mathrm{w}}^{1/2}\mathrm{U} + \bar{\mathrm{h}}$$

- **KLE model can be inverted to evaluate realizations,** u**, of** U **corresponding to realizations,** h**, of** H

$$\mathrm{u}_j \cong \mathrm{w}^{-1/2}\mathrm{v}^T\left(\mathrm{h}_j - \bar{\mathrm{h}}\right) \qquad j = 1, \ldots, N$$

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Modeling and Generation of FRF Random Process – KLE

- **The realizations $\mathrm{u}_j,\ j=1,\ldots,N$, of the random vector (RVec) U are used to establish whether the source is Gaussian or non-Gaussian**
- **If Gaussian, realizations of RP, H, can be generated by generating Gaussian random sequences, U_G, and using them in**

$$\mathrm{H} \cong \hat{\mathrm{v}}\hat{\mathrm{w}}^{1/2}\mathrm{U}_G + \bar{\mathrm{h}}$$

- **If non-Gaussian, realizations of RP, H, can be generated by generating non-Gaussian random sequences, U_{nG}, and using them in**

$$\mathrm{H} \cong \hat{\mathrm{v}}\hat{\mathrm{w}}^{1/2}\mathrm{U}_{nG} + \bar{\mathrm{h}}$$

Modeling and Generation of FRF Random Process – KLE

- **Consider non-Gaussian case – RP H normally non-Gaussian**
- **Data, $\boldsymbol{u}_j, j = 1, \ldots, N$, from the non-Gaussian source, $\boldsymbol{U}$, can be used to generate more data from the same source via Markov Chain Monte Carlo (MCMC)**
- **How?**
  - **Compute the *kernel density estimator* of $\mathrm{u}_j, j = 1, \ldots, N$ the approximation to the probability density function of the RV U**
  - **MCMC uses the likelihood (PDF) of U to generate more realizations of U**

Modeling and Generation of FRF Random Process – KLE

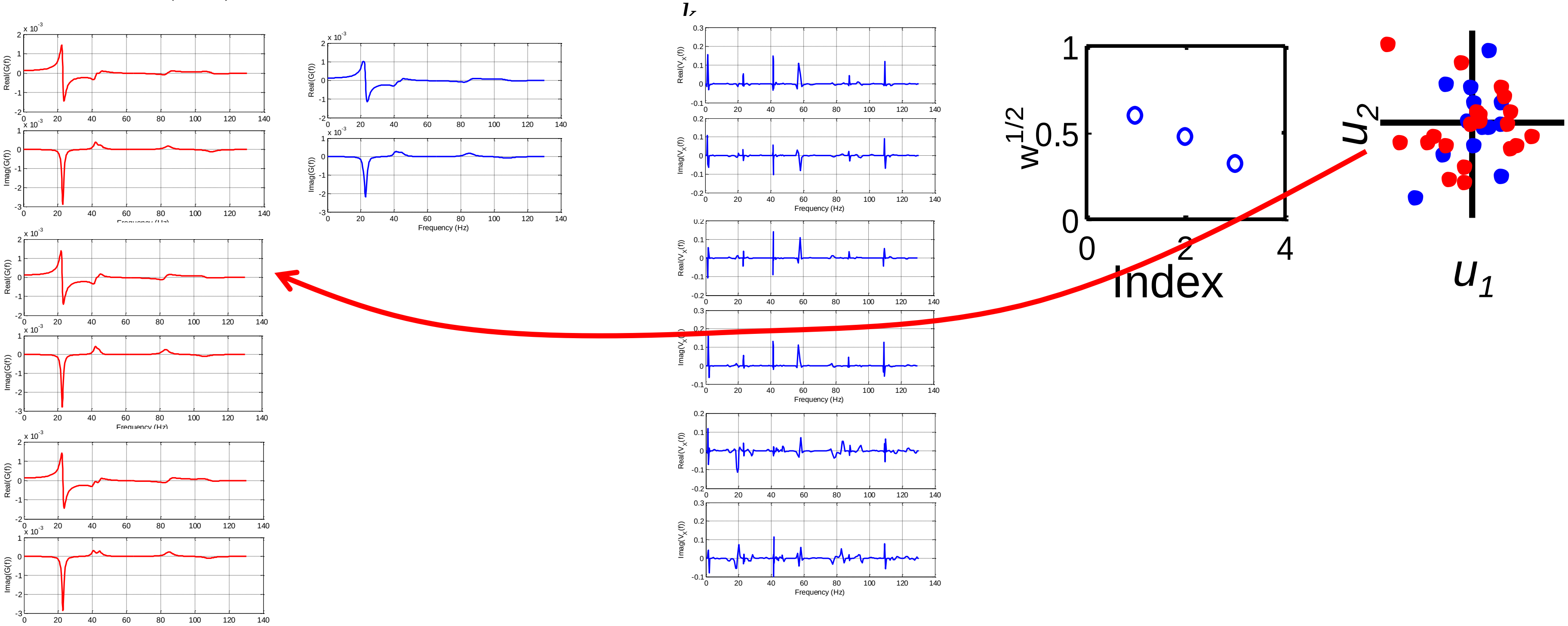
- **Denote the generated data** $\mathrm{u}_j^{(gen)}, j = 1,\ldots, N_{gen}$
- **These data lead to generated realizations of the FRF**

$$\mathrm{h}_j^{(gen)} \cong \hat{\mathrm{v}}\hat{\mathrm{w}}^{1/2}\mathrm{u}_j^{(gen)} + \overline{\mathrm{h}} \qquad j = 1\ldots, N_{gen}$$

- **The generated realizations are linear combinations of the measured realizations**
  - **The span of the generated realizations is limited by the measured data from which we start**
  - **We need a "good" measured ensemble**
- **There is no knowledge of the physics (yet)**

Modeling and Generation of FRF Random Process – KLE

$$h^{(gen)}(f) = \bar{h}(f) + \sum_k \hat{v}_k(f) \times \hat{w}_k^{1/2} \times u^{(gen)}{}_k$$



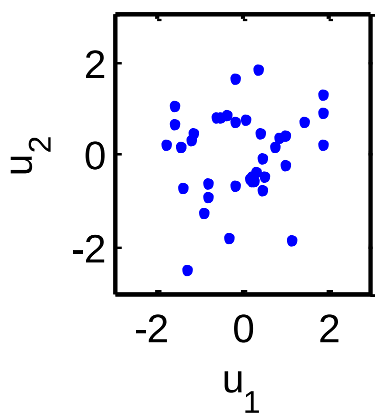
Synthetic FRFs = Mean FRF + Shape functions x Amplitudes x Generated Realizations

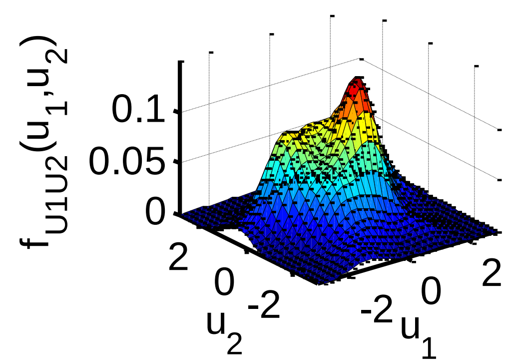
Kernel Density Estimator

- **The joint PDF of a collection of data can be expressed with the Kernel Density Estimator (KDE)**

$$\hat{f}_{\boldsymbol{u}}(\boldsymbol{\alpha}) = \frac{1}{M}\sum_{j=1}^{M} \frac{1}{(2\pi\varepsilon^2)^{M/2}} \exp \left[\frac{1}{2\varepsilon^2}\left\|\boldsymbol{\alpha} - \boldsymbol{u}_j\right\|^2\right]$$

- **It is the superposition of Gaussian PDFs centered at the data points**
- **Finds the likelihood of a randomizing vector α occurring based on the original randomizing vectors $\boldsymbol{u}_j$**
- **The smoothing factor ε determines how tightly the distribution is grouped around the original data**

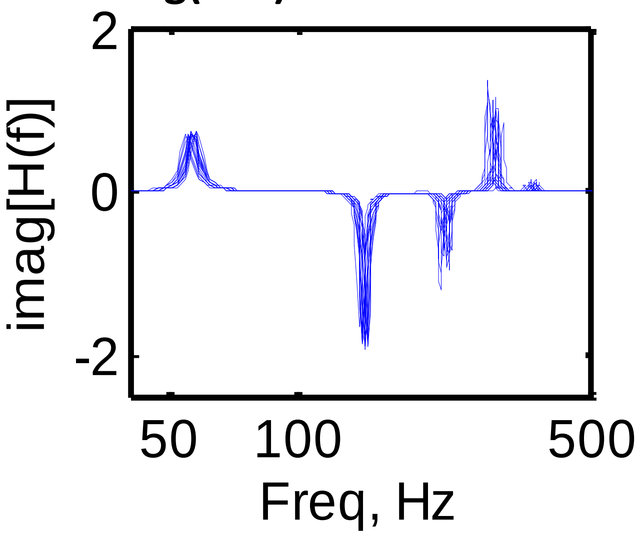




Example – Shortcomings of Existing Technique

- **The KLE doesn't know about the structure…**
  - **It simply combines basis vectors in random linear combinations which can give FRFs that aren't physical**

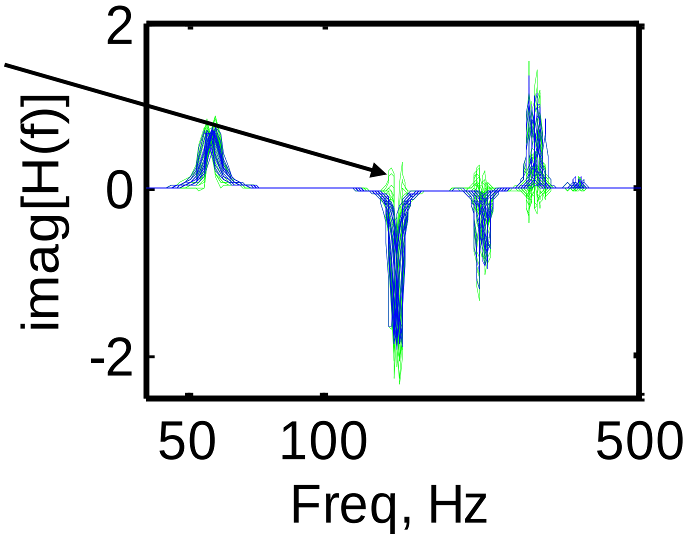
Imag(FRF)s of 20 structures



We use imaginary part of FRF, only, because real part is Hilbert transform

Some synthetic FRFs are "implausible."

Too many extrema
Therefore implausible



"Measured" Imag(FRF) (blue) and synthetic Imag(FRF) (green)

The Plausibility Constraint

- **Some identifiable $\mathrm{u}_j^{(gen)}, j = 1,.., N_{gen}$ result in implausible (not probable) $\mathrm{h}_j^{(gen)}, j = 1,..., N_{gen}$**
  - **They have some features that we can quantitatively identify**
  - **Denote these $\mathrm{u}_j^{(gen,NP)}, j = 1,..., N_{NP}$, and $\mathrm{h}_j^{(gen,NP)}, j = 1,..., N_{NP}$**
- **The remainder of the generated realizations are OK (i.e., probable) $\mathrm{u}_j^{(gen,P)}, j = 1,..., N_P$**
  - **Denote them $\mathrm{h}_j^{(gen,P)}, j = 1,..., N_P$, and**
- **If the constraint is a hard one, or there are no prior data, the plausible set of accepted synthetic realizations is $\mathrm{h}_j^{PL} = \mathrm{h}_j^{(gen,P)}, j = 1,..., N_P$**

The Plausibility Constraint

- **The plausibility constraint may be fuzzy or we may want to judge plausibility probabilistically**
  - **This requires prior knowledge of the plausibility likelihood**
- **To diminish chance of non-probable realizations, generate realizations $\mathrm{u}_j^{(gen)}$ "as usual"**
- **When a realization is accepted in MCMC analysis, test it to determine whether it is "probable" or "not probable"**
- **Compute plausibility probability**

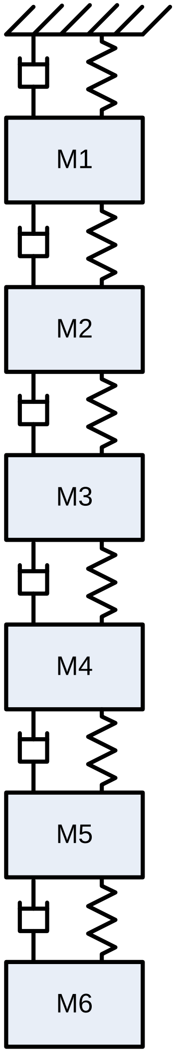
$$p_{PL} = \frac{\gamma L_{U^{(PR)}}(u)}{\gamma L_{U^{(PR)}}(u) + \beta L_{U^{(NP)}}(u)}$$

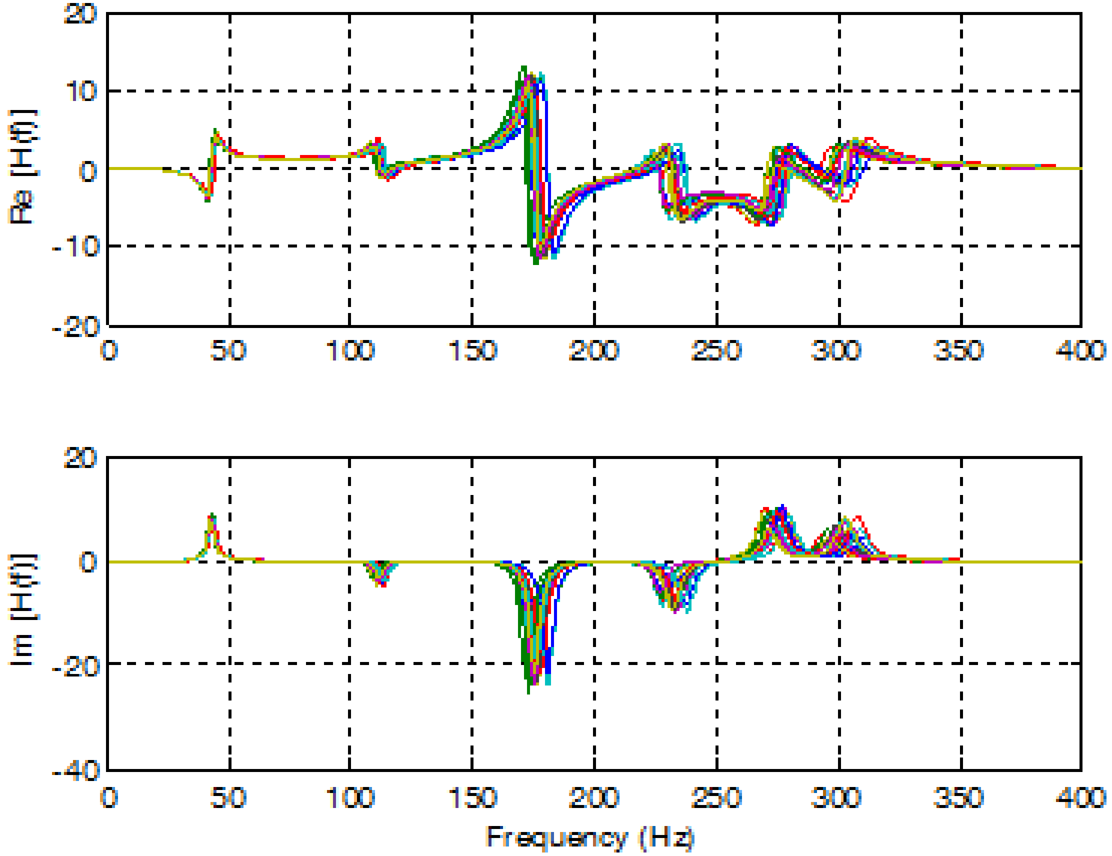
and accept the generated realization with this probability

The Plausibility Constraint

- **If no prior likelihoods are available, use a (first) round of MCMC analysis to construct them**
- **Generate the synthetic realizations from a 2^{nd} MCMC analysis that incorporates the probabilistic plausibility constraint**
- **If we use a reduced order KLE, there is no guarantee that the approximations of the measured ensemble satisfy the plausibility constraints**
  - **This provides another criterion for selecting the KLE order**

Example – Modified Technique

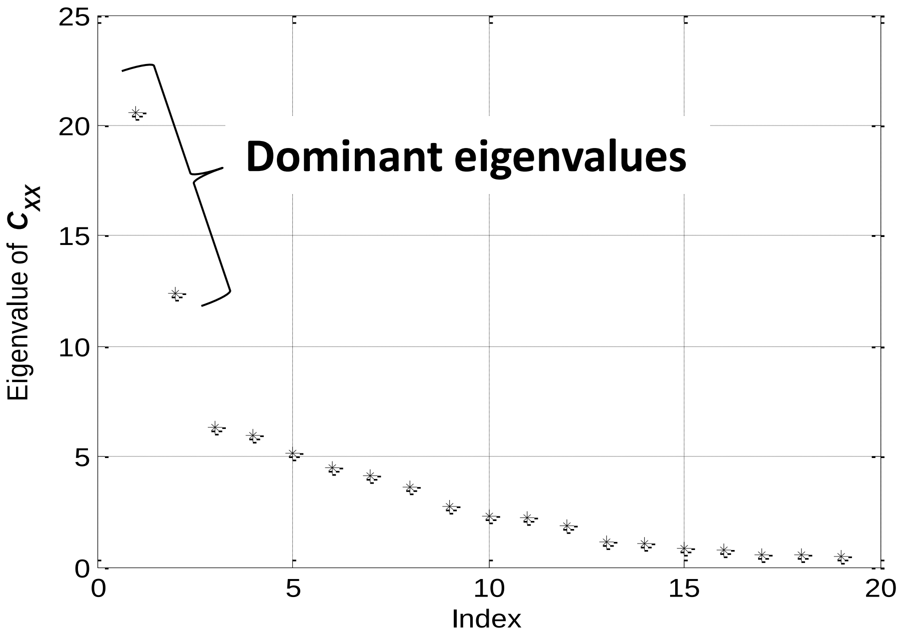




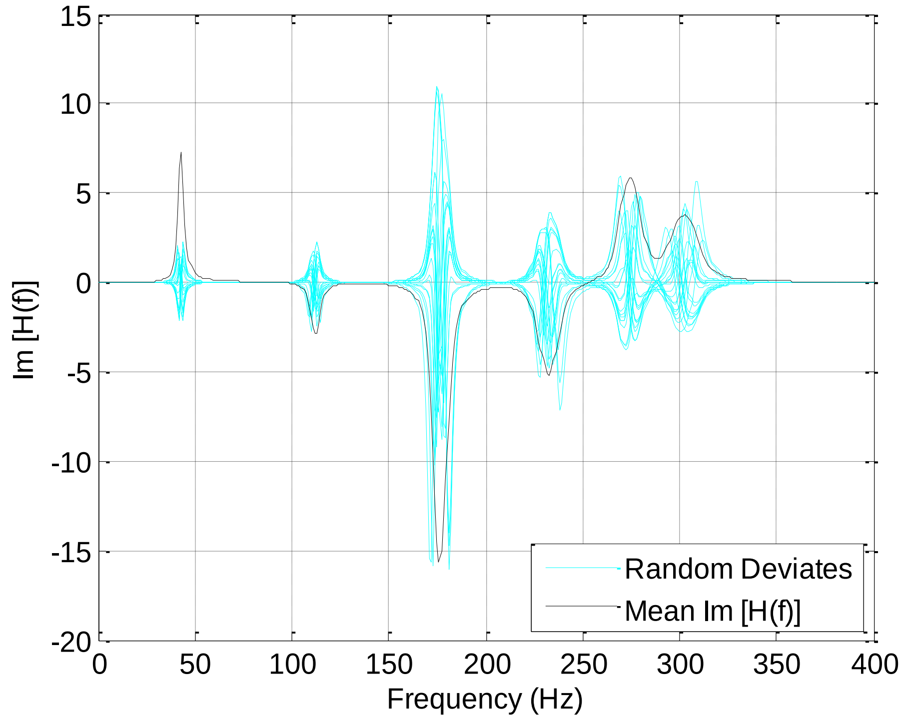
Real and Imaginary Parts from the FRFs of the 20 Structure Test Ensemble

Example – FRF KLE Expansion

- We consider only the imaginary part of the FRFs
  - The real part is the Hilbert transform
- Constraint – Number of peaks = 6



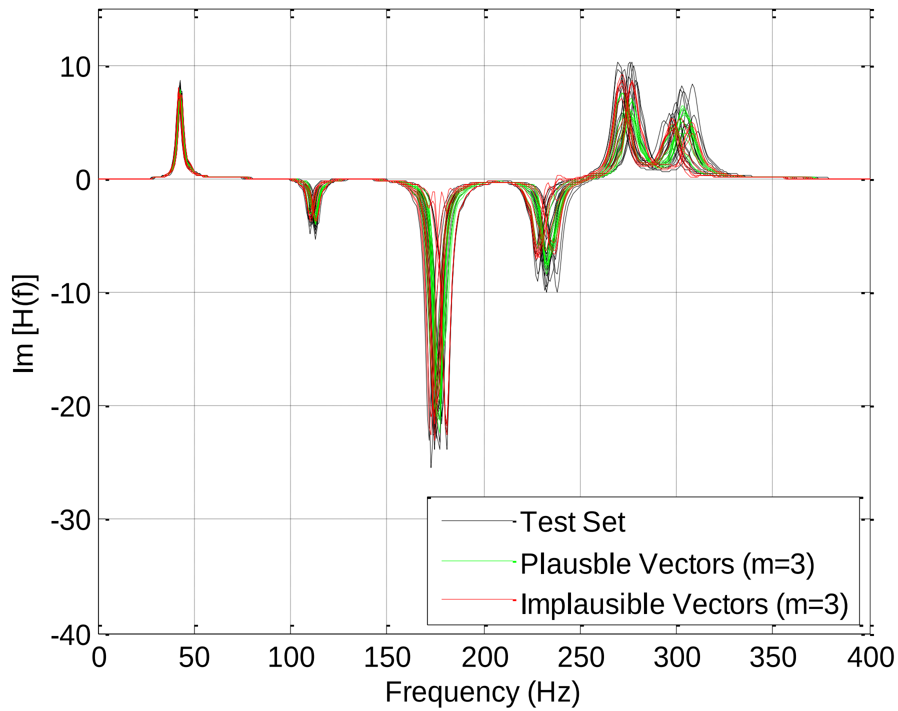
Eigenvalues of C_{XX}



Mean and Random Deviates of the Imaginary Part of the 20 Structure Test Ensemble

Example – FRF KLE Expansion

- We considered KLE expansions with m=19 and m=3
- m=3
  - Only 11 approximations of the original 20 met the order criterion
- m > 15 was required to ensure the all approximations are plausible



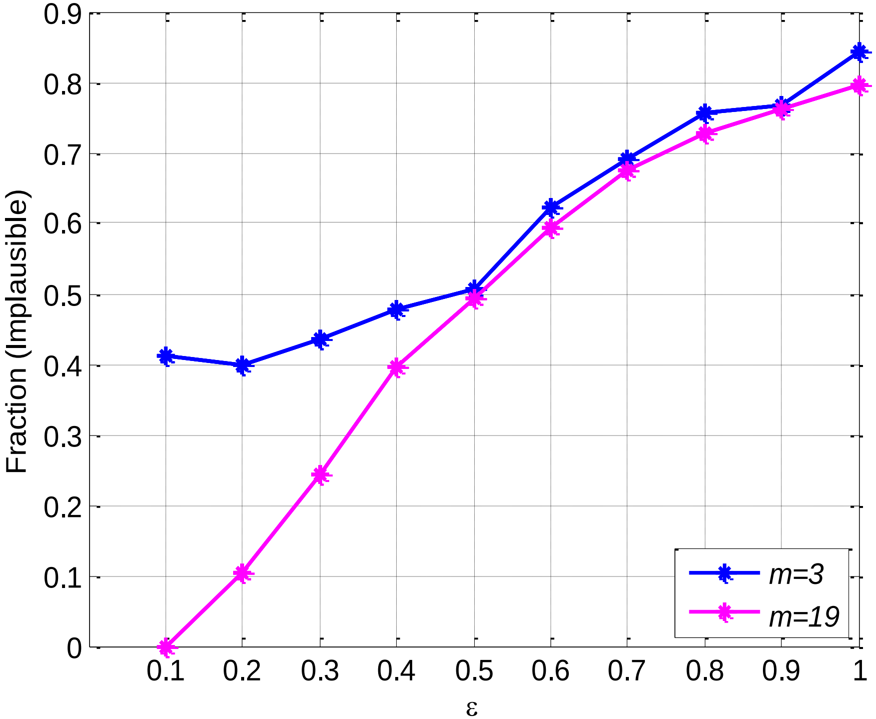
Approximation of the Test set Im[H] showing Plausible and Implausible Realizations

Example – KDE Space

- The fraction of accepted realizations was set to 0.43
- KLE smoothing factor

$$0.1 \leq \varepsilon \leq 1.0$$

  - This parameter controls how far away from the original space the synthetic FRFs can stray
  - ε small – stay close
- Upper bound on implausibility = 0.884
  - Original 20 are plausible
- Suggests $\varepsilon \leq 0.5$ for KDE smoothing parameter

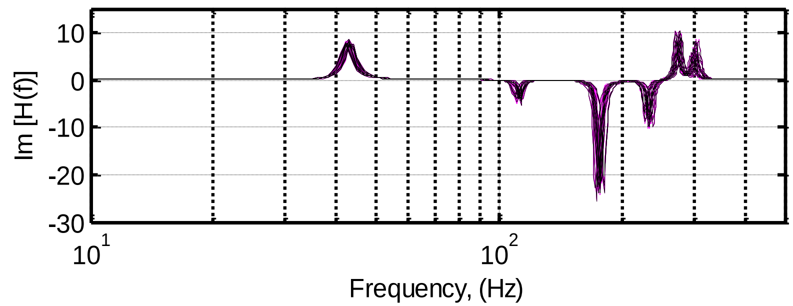


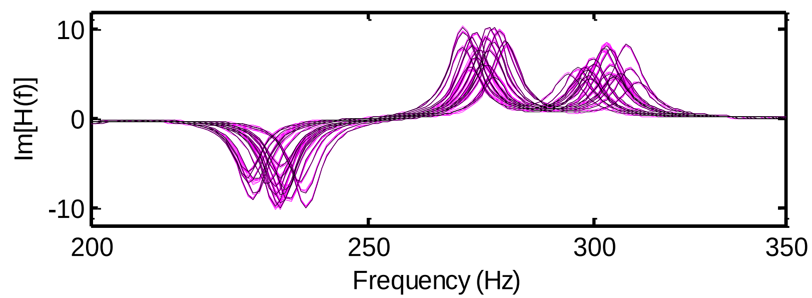
Implausibility Ratio
(Acceptance Ratio = 0.43)

Shows the link between KLE order and KDE smoothing

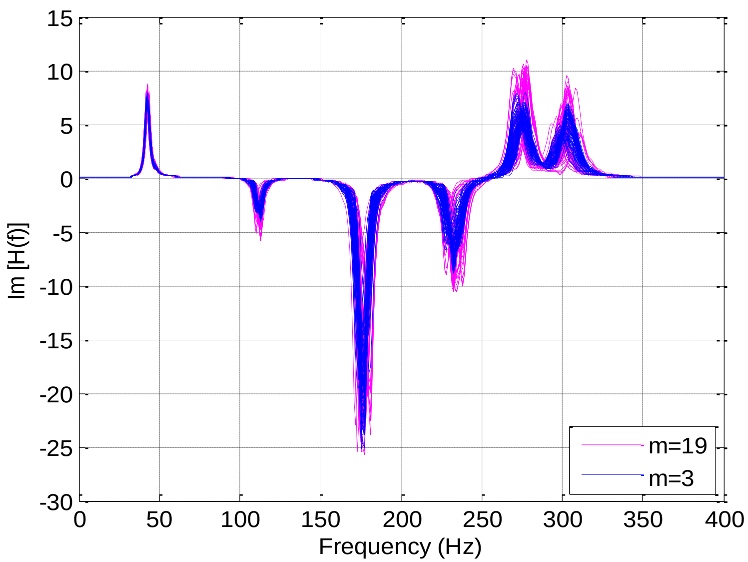
Example – KDE Space

- **For a very small KDE smoothing parameter, the synthetic FRFs are very close to the original set**
  - **Not very useful**
- **m = 3 synthetic FRFs seem to have more damping**





Comparison of Synthesized Realizations (Magenta) and Original Set (Black) [*m*=19, ε= 0.1]



Plausible Synthetic Realizations [ε= 0.5]

Example – KDE Space

- **Variability metric compares synthetic FRFs to original set**
- **Synthetic FRFs**

$$\sigma_{j,i} = \left\| h_j^{(PL)}(f) - h_i(f) \right\|_2$$

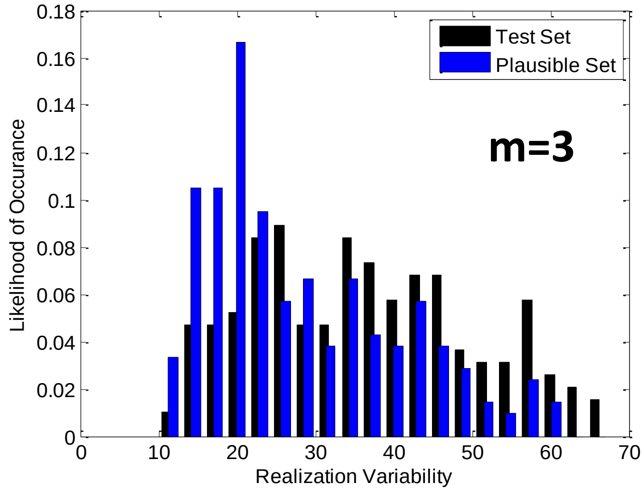
$$j = 1 \ldots n_{PL}, i = 1 \ldots M$$

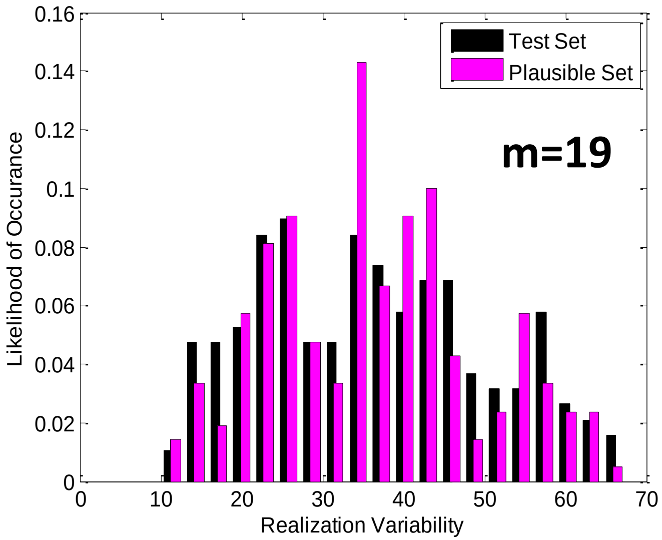
- **Original set**

$$\bar{\sigma}_{j,i} = \left\| h_j(f) - h_i(f) \right\|_2$$

$$i, j = 1 \ldots M, i \neq j$$

	Test Set (M = 20)	Plausible Realizations (m = 3)	Plausible Realizations (m = 19)
Mean	36.29	28.24	36.11
Std	13.85	12.48	13.02





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Conclusions

- **Original technique - Karhunen-Loeve expansion plus kernel density estimator and Markov chain Monte Carlo – developed to generate realizations of FRF**
  - **Direct simulation does not work well enough**
- **The technique was modified to permit exclusion of realizations with implausible characteristics**
  - **Accommodates inclusion of information on which generated realizations are improbable and which are probable**

Conclusions

- Showed various sensitivities of the KLE-KDE-MCMC process
  - Approximation quality is a function of KLE order
    - Ordering eigenvalues or summing them may not be sufficient to ensure a good approximation
  - KDE smoothing parameter is linked to KLE order
  - KDE smoothing parameter is linked to plausibility ratio
    - Plausibility provides another criterion for setting this parameter

Acknowledgements

- This work was a collaboration between AFRL, Sandia National Laboratories and MannaTech
- Dr. Tom Paez provided wrote most of the simulation code and provided the basis for the KLE approach
- Dr. Seth Lacy and Dr. Steven Lane contributed insights on the fundamental idea of adding physical constraints to the KLE process to produce plausible synthetic FRFs