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Application of 1D stagnation solutions to 3D wire array simulation in absence of radiation

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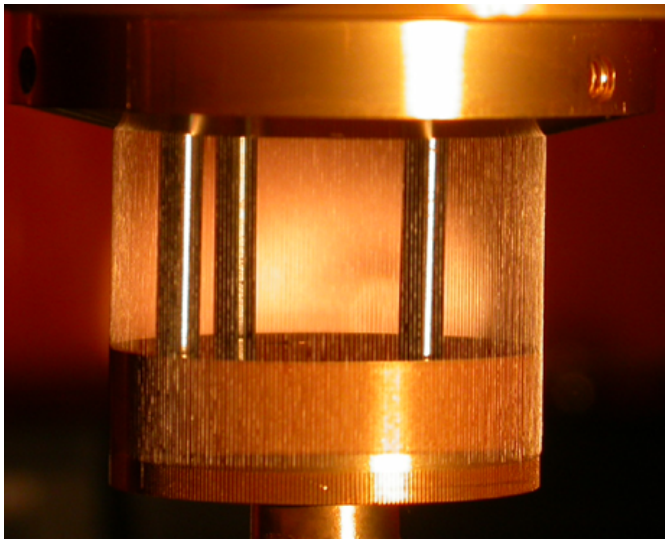
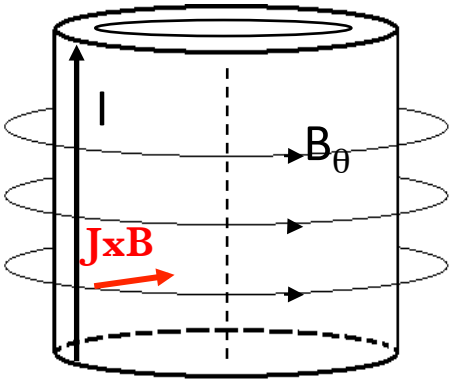
APS DPP 2013

Denver, CO

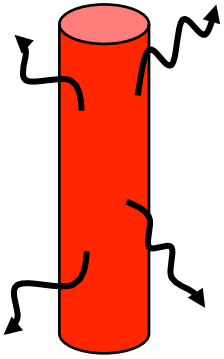
A wire-array Z pinch is a powerful radiation source



Wires blow up and form a plasma shell



Plasma stagnates on axis, converting kinetic energy to internal energy



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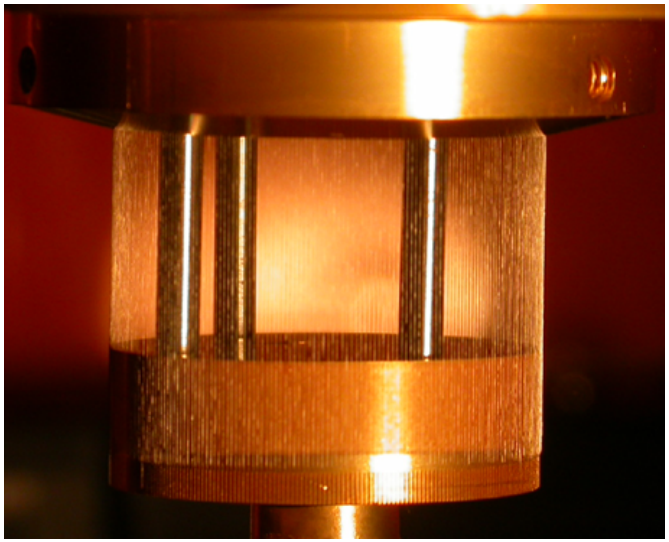
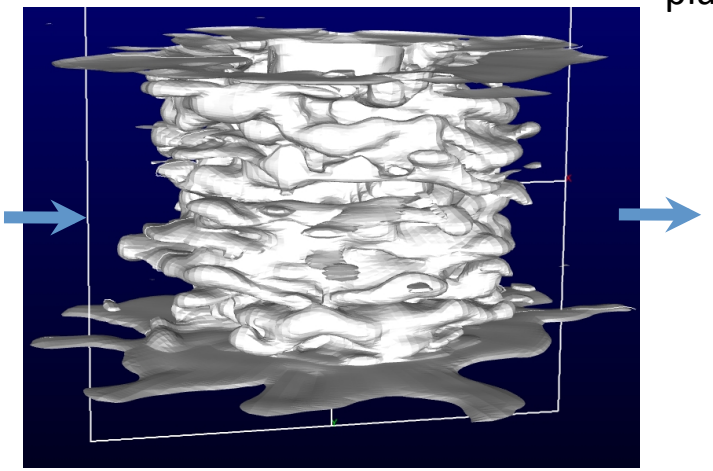
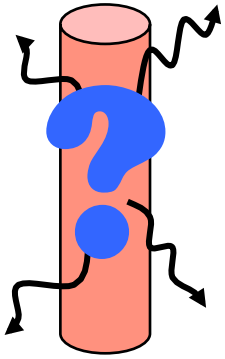
A wire-array Z pinch is a powerful radiation source





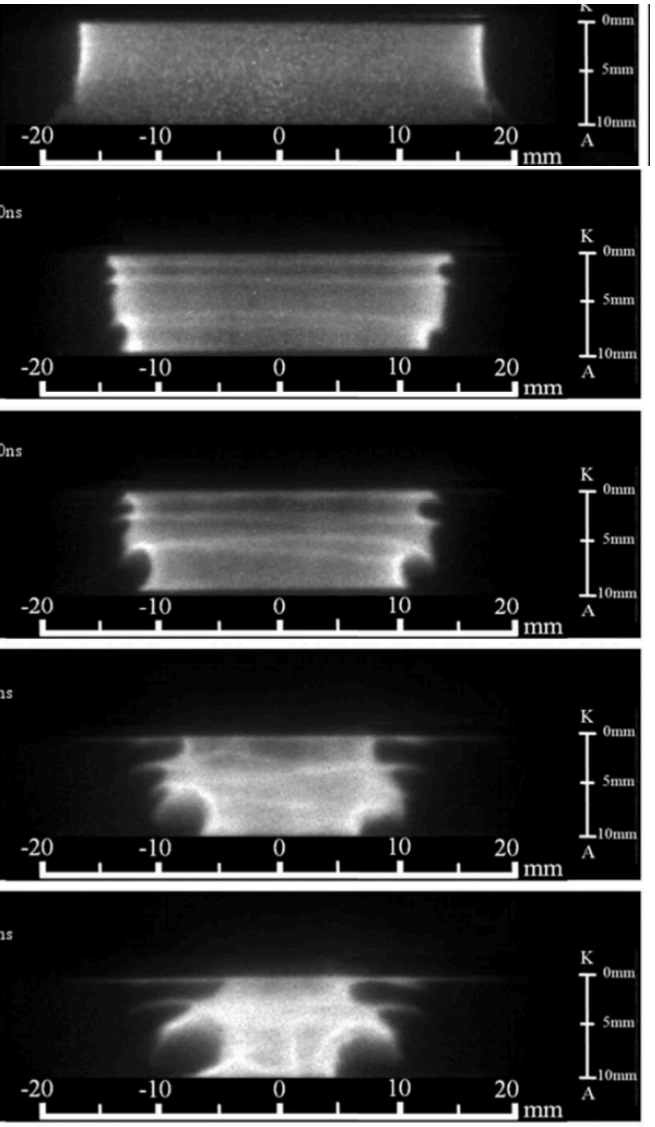
How does this 3D imploding plasma stagnate?



3D stagnation may be relevant to other HED configurations

Visible light images of gas puff

D. Osin et al., IEEE Trans. Plasma Sci. 39, 2392 (2011)



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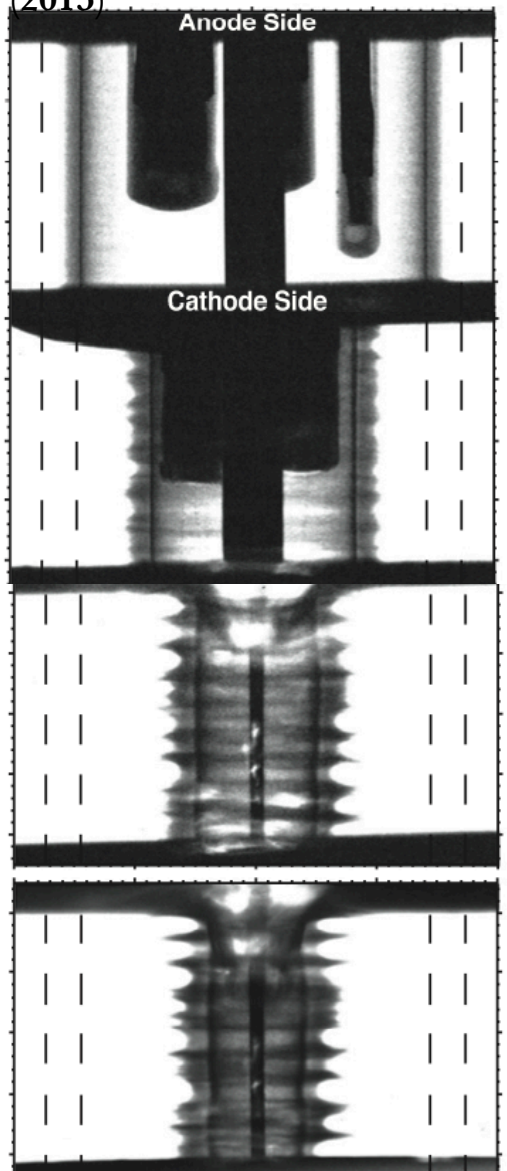
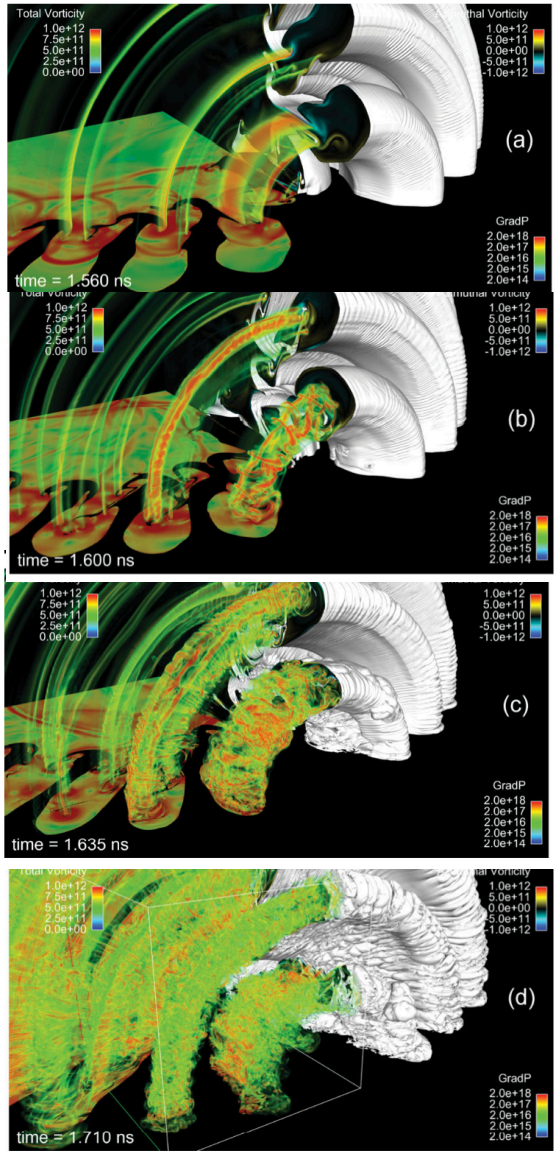
Radiographs of Be liner

McBride et al., PoP 20, 056309 (2013)

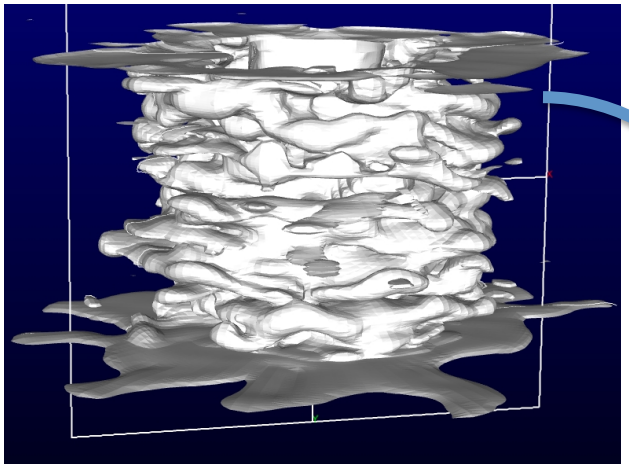


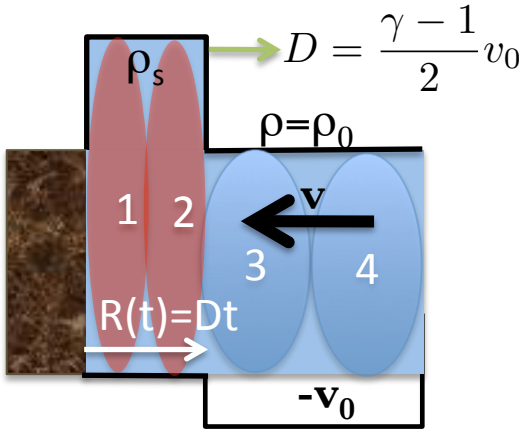
3D simulation of ICF capsule

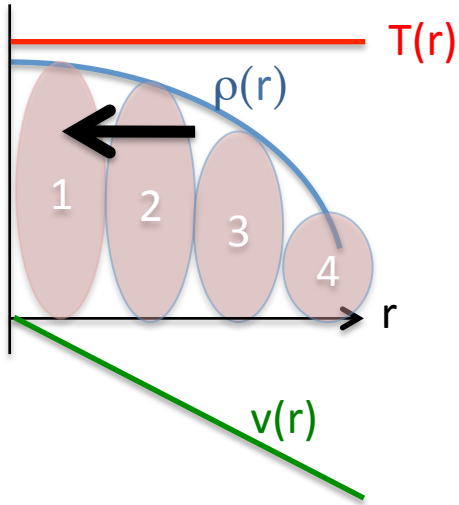
Thomas and Kares, PRL 109, 075004 (2011)



Can a 1D theory describe how 3D objects stagnate?







Maron et al. (PRL **111**, 035001 (2013)): found good agreement between 1D shock solution and experimental data for wire array and gas puff.

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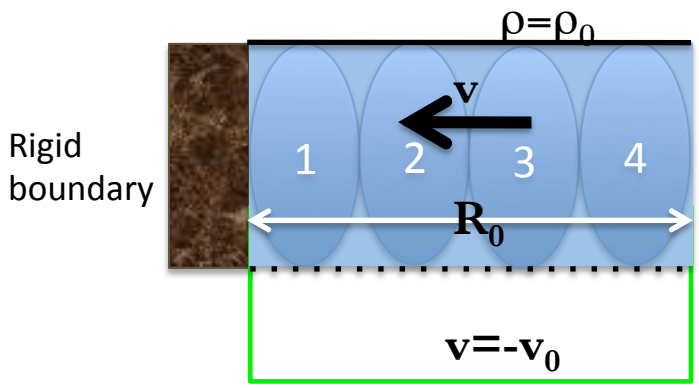
1D shock solution

1D planar geometry



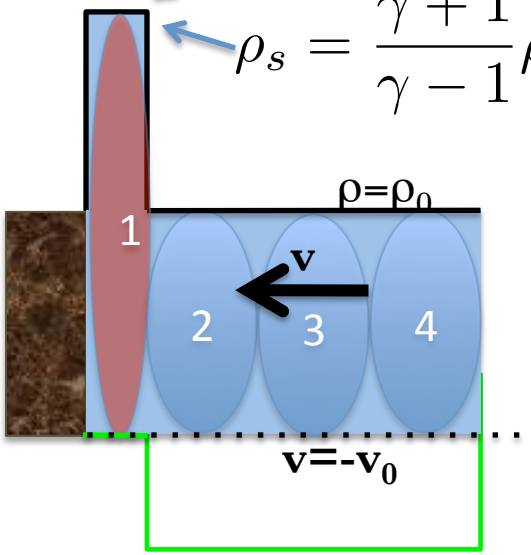
Consider cold fluid particles imploding towards a rigid boundary. Density and velocity profiles are flat.

Problem we will consider is purely hydrodynamic; we ignore thermal conduction, radiation, magnetic fields.

1D shock solution

$$k_b T_s = \frac{\gamma - 1}{2} m_i v_0^2 \overset{\gamma=5/3}{\Rightarrow} \frac{1}{3} m_i v_0^2$$

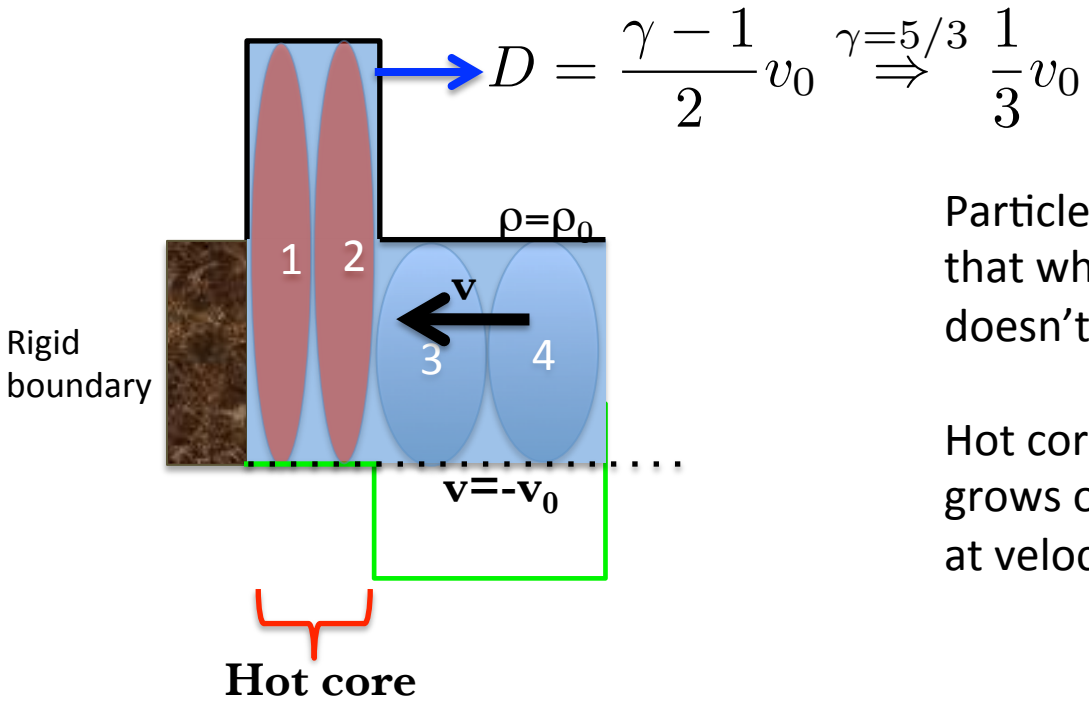
$$\rho_s = \frac{\gamma + 1}{\gamma - 1} \rho_0 \overset{\gamma=5/3}{\Rightarrow} 4\rho_0$$



Rigid boundary

Particle 1 collides into the boundary and converts all its kinetic energy into internal energy.

1D shock solution

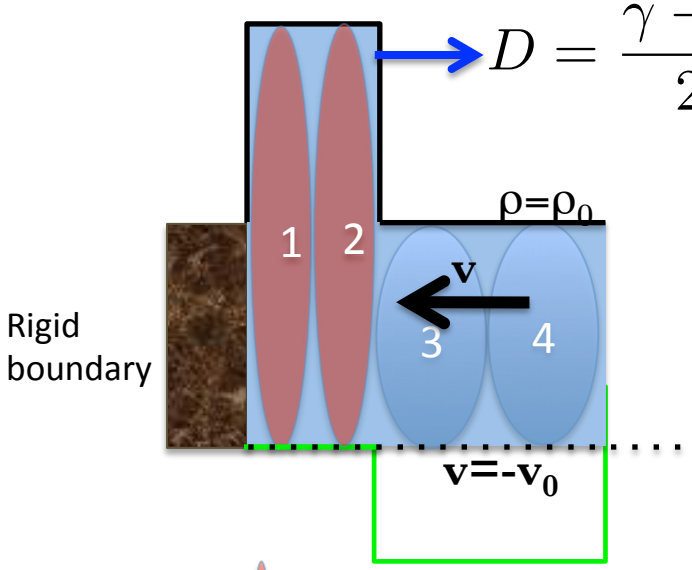


Particle 1 has sufficient thermal pressure that when particle 2 strikes it, particle 1 doesn't compress at all.

Hot core (i.e. stagnated "fluid particles") grows outward through a shock traveling at velocity D

1D shock solution

$$D = \frac{\gamma - 1}{2} v_0 \overset{\gamma=5/3}{\Rightarrow} \frac{1}{3} v_0$$



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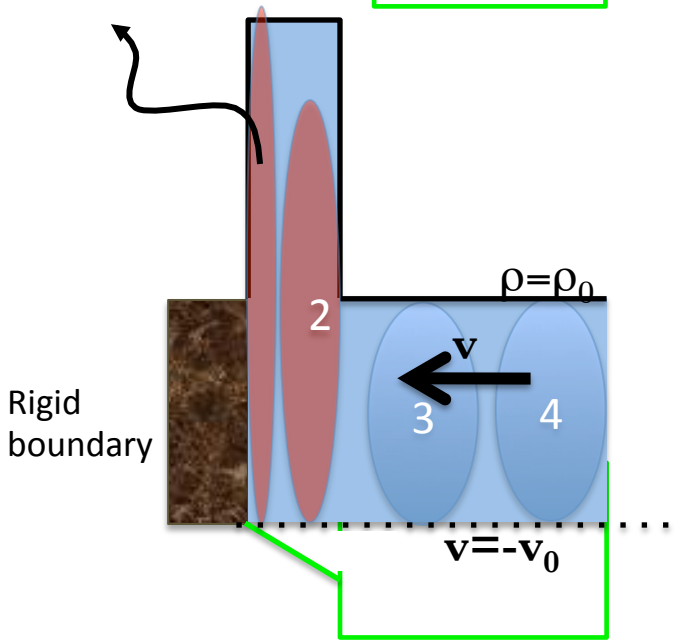

In presence of radiation*, particle 1 will radiate away its thermal pressure, and compress when particle 2 hits it, thus reducing D.

*M.M. Basko, P.V. Sasorov, M. Murakami, et al., Plasma Phys. Control. Fus. 54 (2012) 055003

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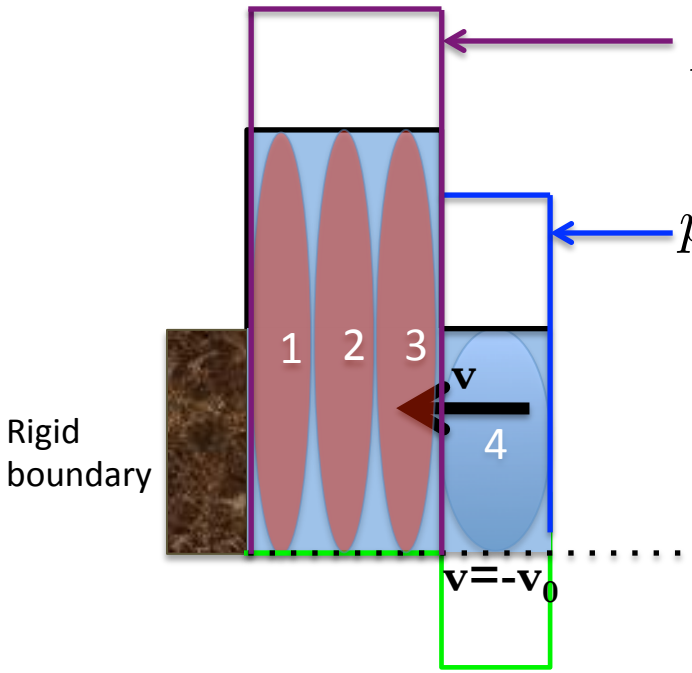
1D shock solution



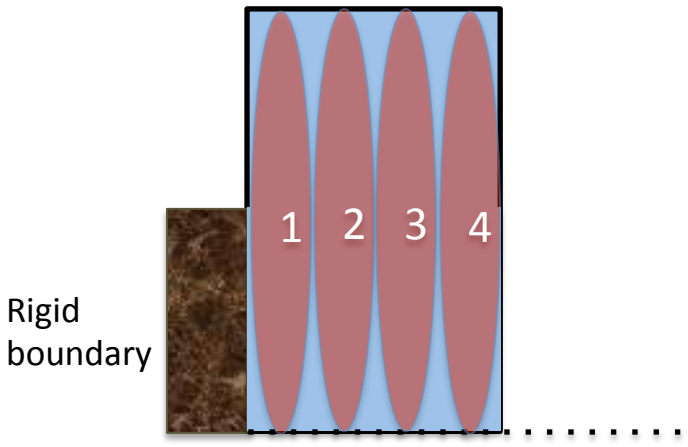
$$p_s = \frac{\gamma+1}{2}\rho_0 v_0^2 \overset{\gamma=5/3}{\Rightarrow} \frac{4}{3}\rho_0 v_0^2$$

$$p_{ram} = \rho_0 v_0^2$$

The hot core is confined in the sense that v=0 there.

This confinement is due solely to the incoming ram pressure (i.e. there is no magnetic field here).

1D shock solution



After a time

$$t_f = \frac{2}{\gamma+1}\frac{R_0}{v_0} \overset{\gamma=5/3}{\Rightarrow} \frac{3}{4}\frac{R_0}{v_0}$$

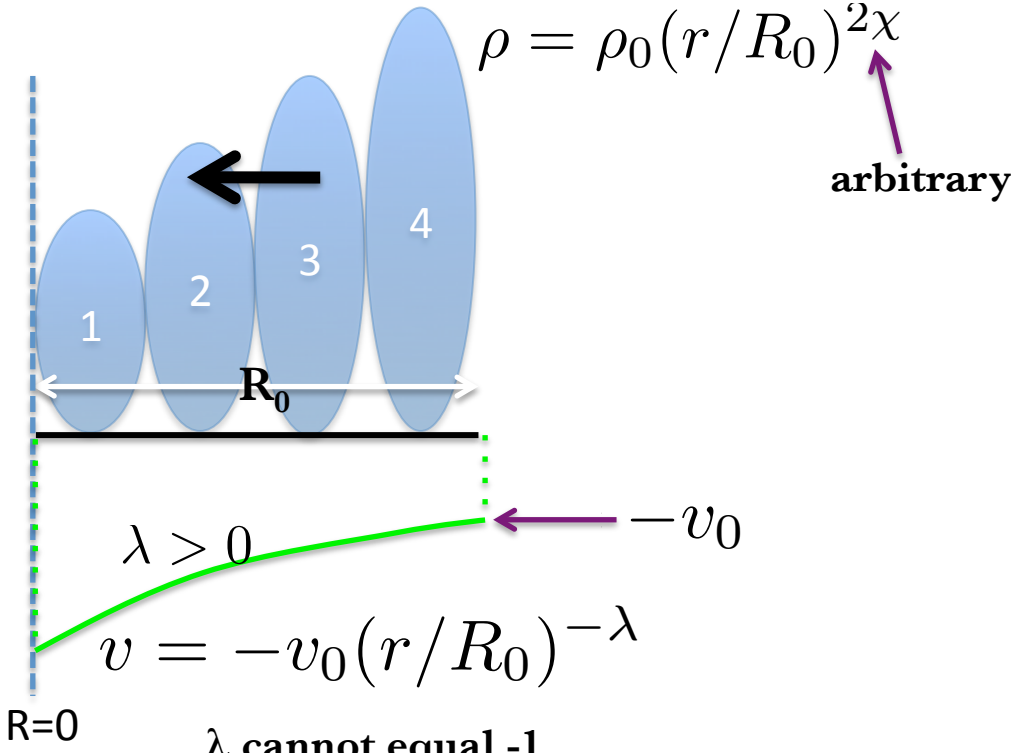
stagnation is COMPLETE.

This is the "Noh problem"*

*W.F. Noh, J. Comp. Phys. 72, 78 (1987)

1D generalized Noh solution

A generalization to the Noh solution has been developed by A. Velikovich, allowing non-uniform initial ρ(r), v(r).



λ cannot equal -1 (homogeneous solution)

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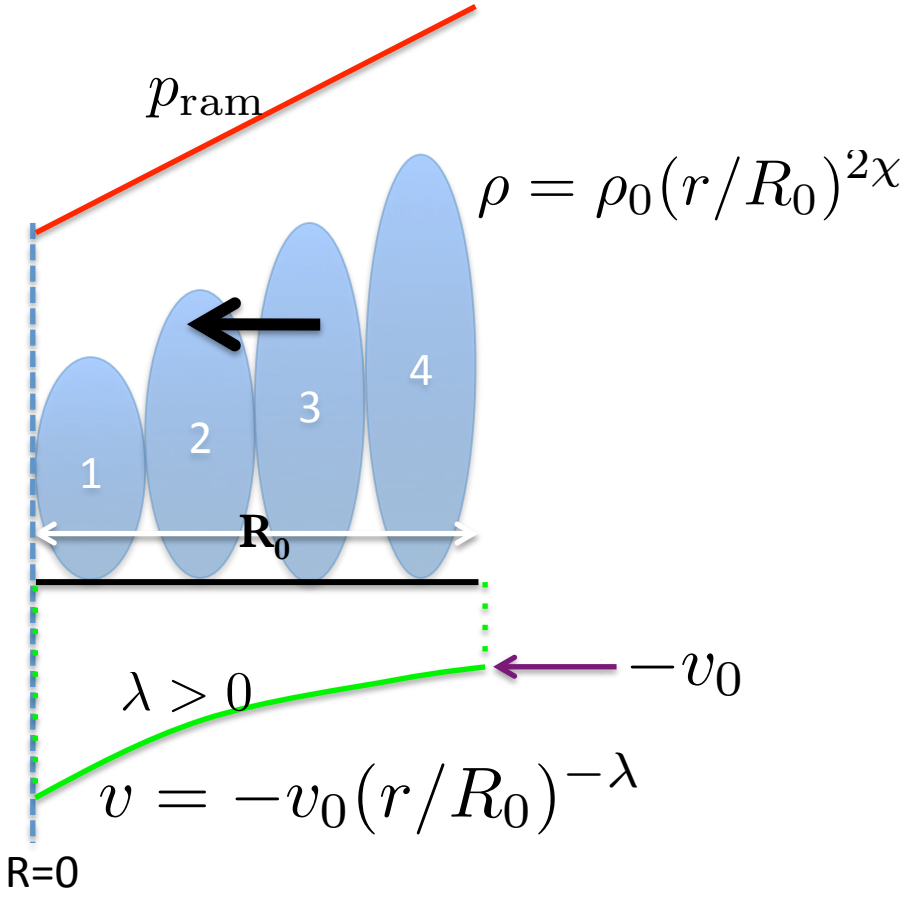
1D generalized Noh solution



A key parameter is the ram pressure:

$$p_{ram} = \rho v^2 = \rho_0 v_0^2 (r/R_0)^{2(\chi-\lambda)}$$

χ-λ>0: increasing ram pressure

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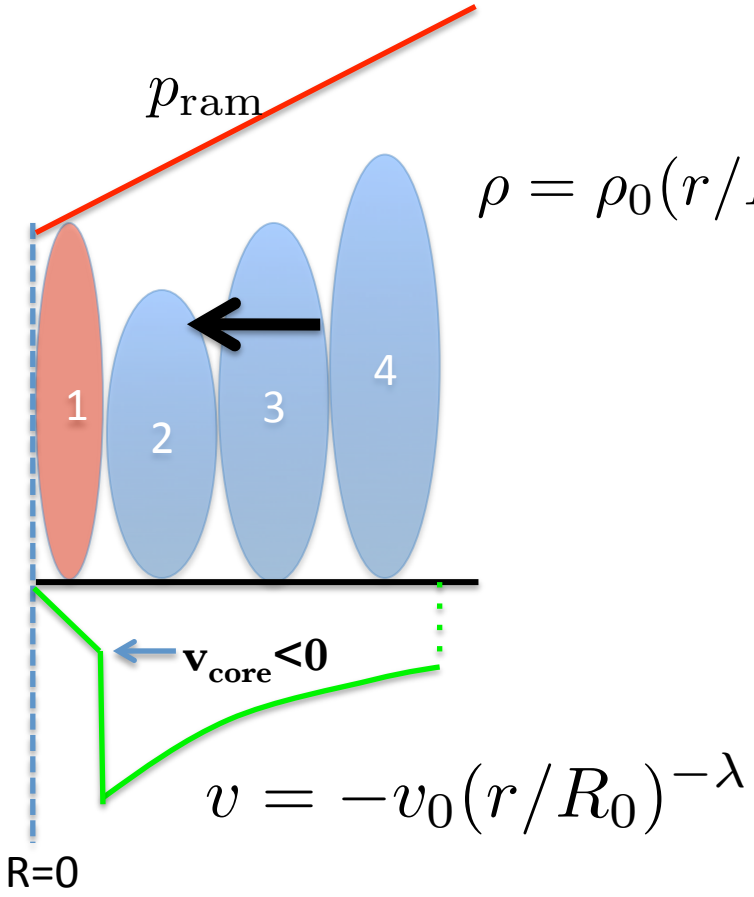
1D generalized Noh solution



$$\rho = \rho_0 (r/R_0)^{2\chi}$$

$$v = -v_0 (r/R_0)^{-\lambda}$$

A key parameter is the ram pressure:

$$p_{ram} = \rho v^2 = \rho_0 v_0^2 (r/R_0)^{2(\chi-\lambda)}$$

χ-λ>0: increasing ram pressure
v_{core}<0, continual compression in core

$$p_s(t) \propto t^{\frac{2(\chi-\lambda)}{1+\lambda}}$$

$$\rho_s(t) \propto t^{\frac{2\chi}{1+\lambda}}$$

$$R_s(t) \propto t^{\frac{1}{\lambda+1}}$$

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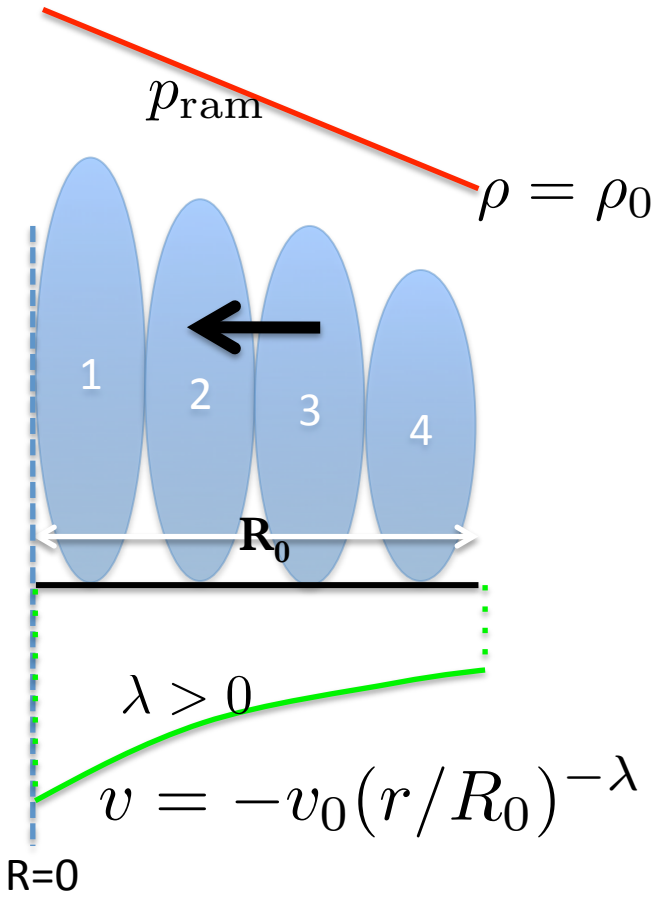
1D generalized Noh solution



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$$p_{ram} = \rho v^2 = \rho_0 v_0^2 (r/R_0)^{2(\chi-\lambda)}$$

χ-λ<0: decreasing ram pressure

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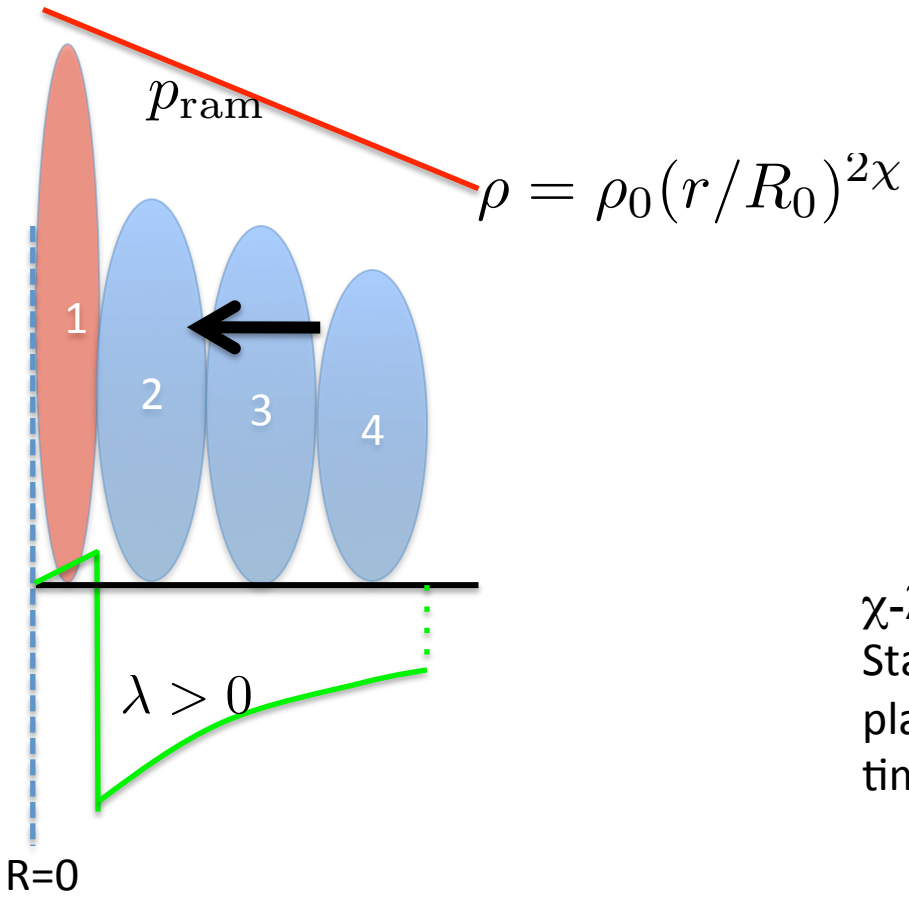
1D generalized Noh solution



A key parameter is the ram pressure:

$$p_{ram} = \rho v^2 = \rho_0 v_0^2 (r/R_0)^{2(\chi-\lambda)}$$

χ-λ<0: decreasing ram pressure
Stagnated plasma expands into the imploding plasma, so that core pressure decreases with time.

$$p_s(t) \propto t^{\frac{2(\chi-\lambda)}{1+\lambda}}$$

$$\rho_s(t) \propto t^{\frac{2\chi}{1+\lambda}}$$

$$R_s(t) \propto t^{\frac{1}{\lambda+1}}$$

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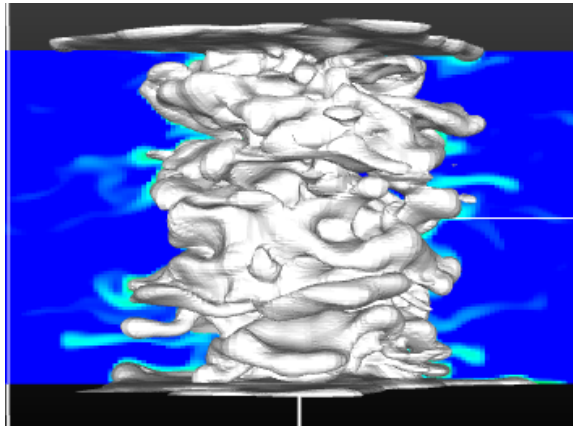
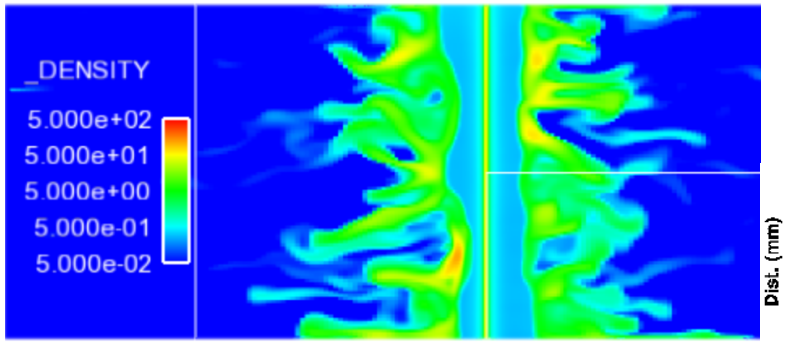
3D wire-array Z pinch simulation

t=2518 ns

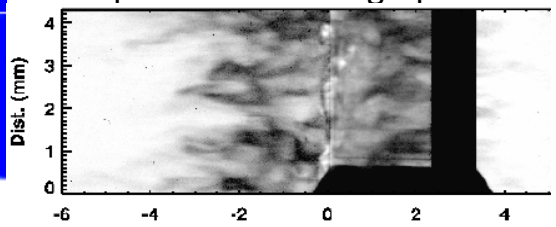
ρ=1 kg/m^3 contour



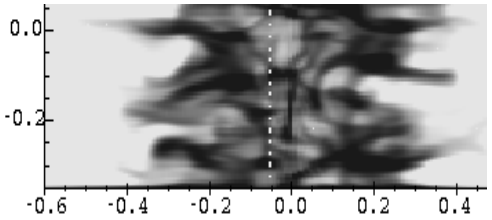


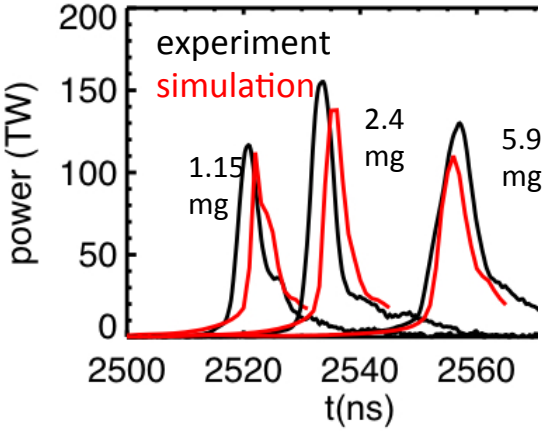
Alegra: 3D radiation MHD + thermal conduction
Voltage drive
Lee-More-Desjarlais conductivity
1.7 million elements (dz~60 μm, dr~20 μm)
Mass injection scheme*

Experimental radiograph



Simulation radiograph





*E.P. Yu, M.E. Cuneo, M. P. Desjarlais, R.W. Lemke, et al., Phys. Plasmas **15**, 056301 (2008)

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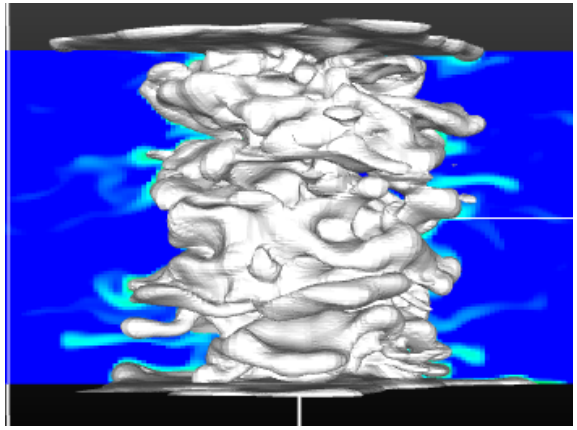
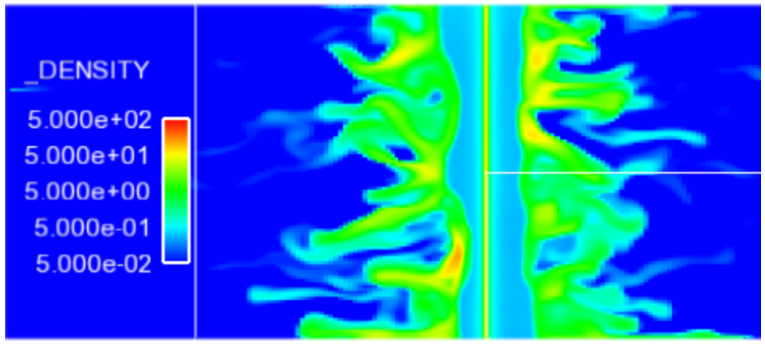
3D wire-array Z pinch simulation

t=2518 ns

ρ=1 kg/m^3 contour





Simulate 1.15 mg, W compact array (R_o=1cm)
At t=2518 ns, radiation is "turned off" (σ_r/1e4)

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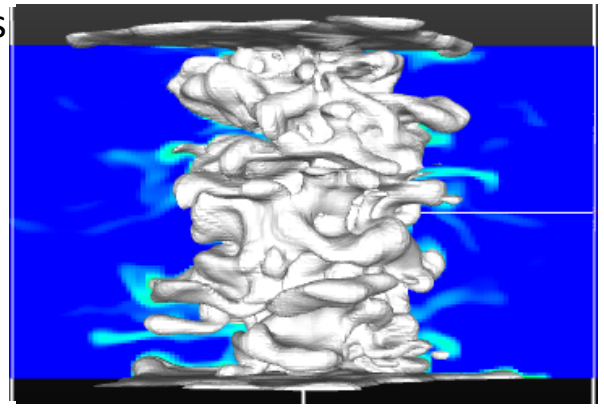
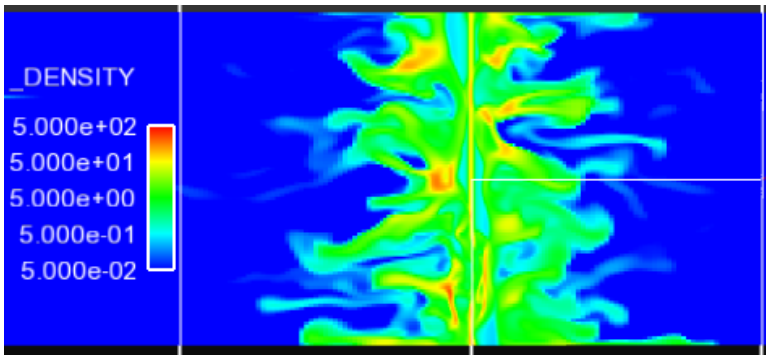
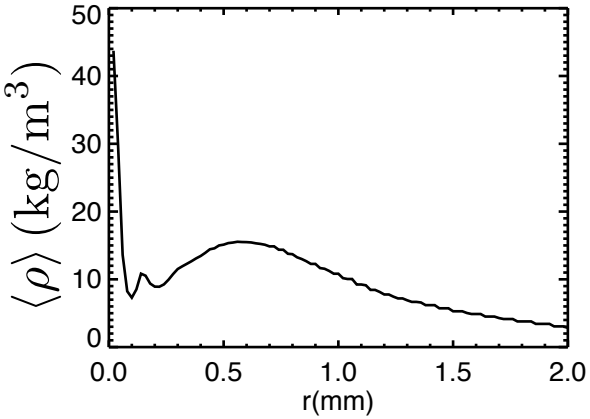
3D wire-array Z pinch simulation

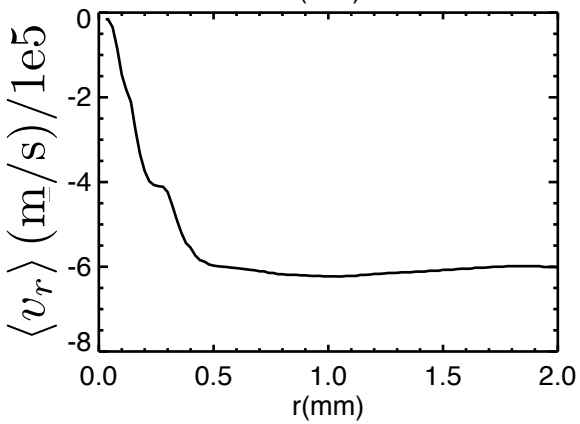
t=2518.8 ns

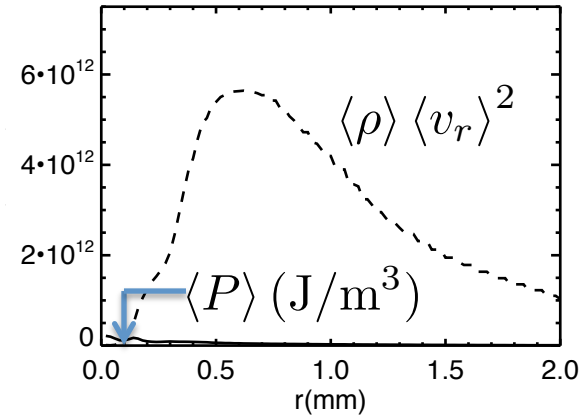
ρ=1 kg/m^3 contour











All quantities axially and azimuthally averaged

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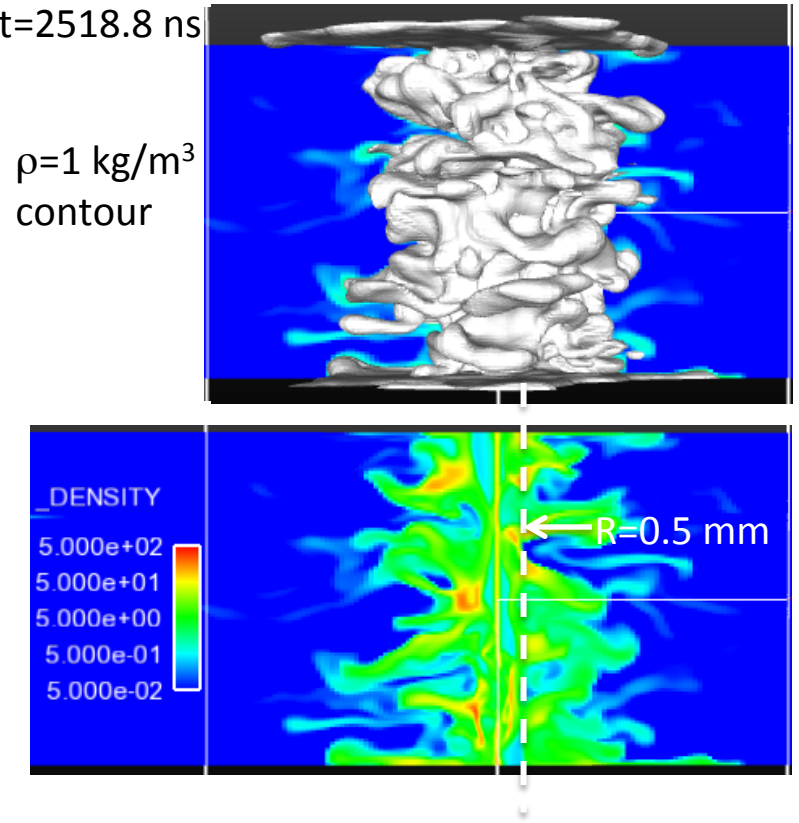
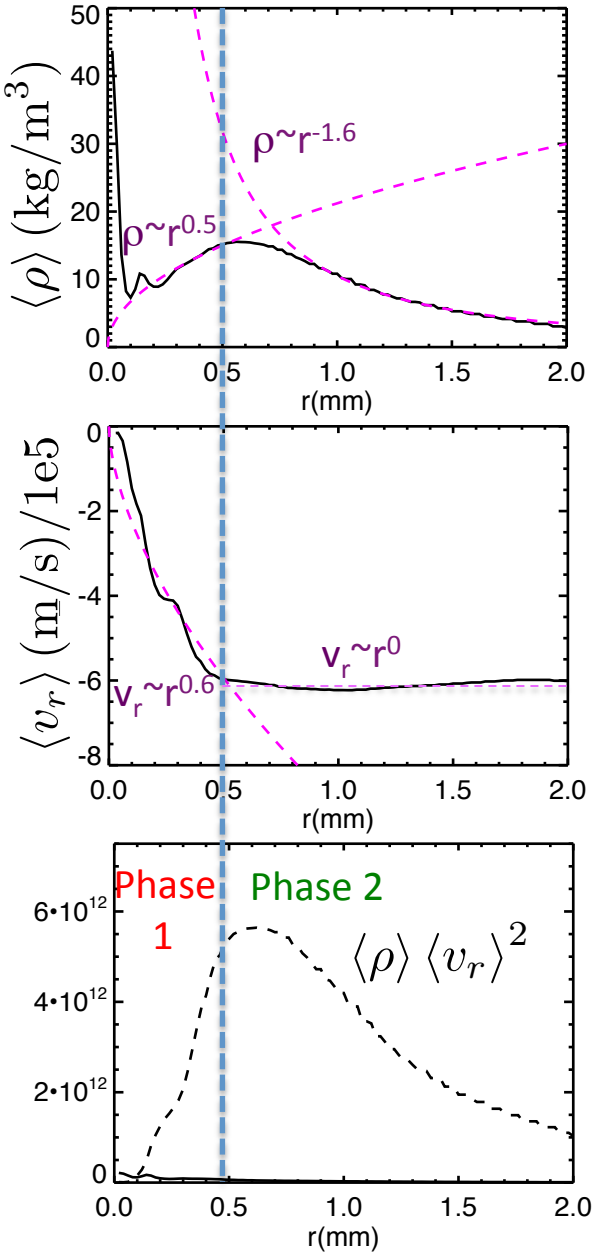
Pre-stagnation profiles exhibit "2 phase" profile





Phase 1:

$\rho_s \sim t^{1.25}$

$p_s \sim t^{4.25}$

$R_s \sim t^{5/2}$

Phase 2:

$\rho_s \sim t^{-1.6}$

$p_s \sim t^{-1.6}$

$R_s \sim t$

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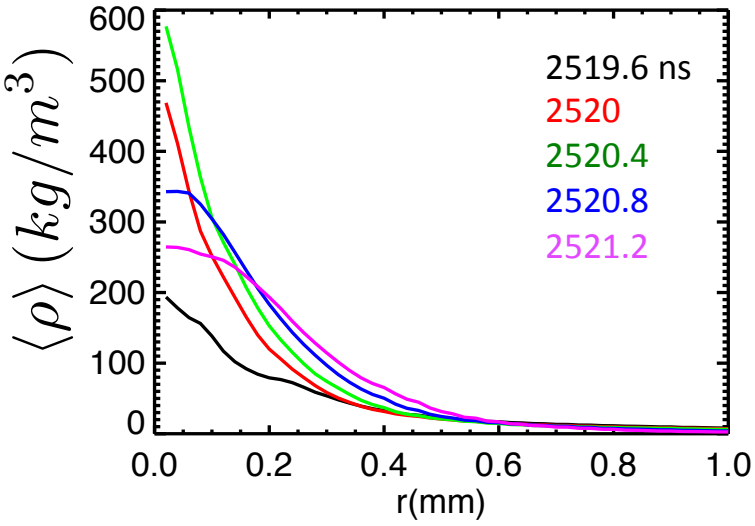
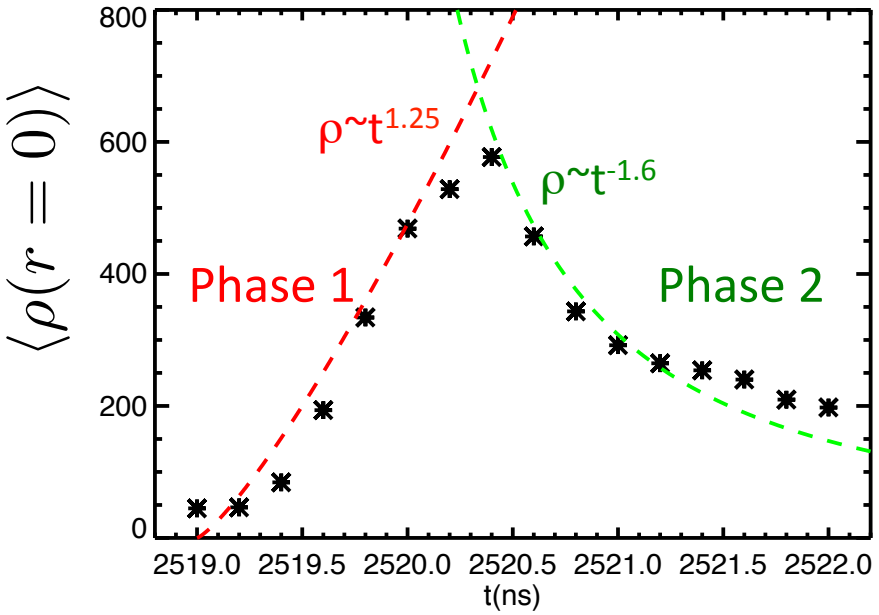
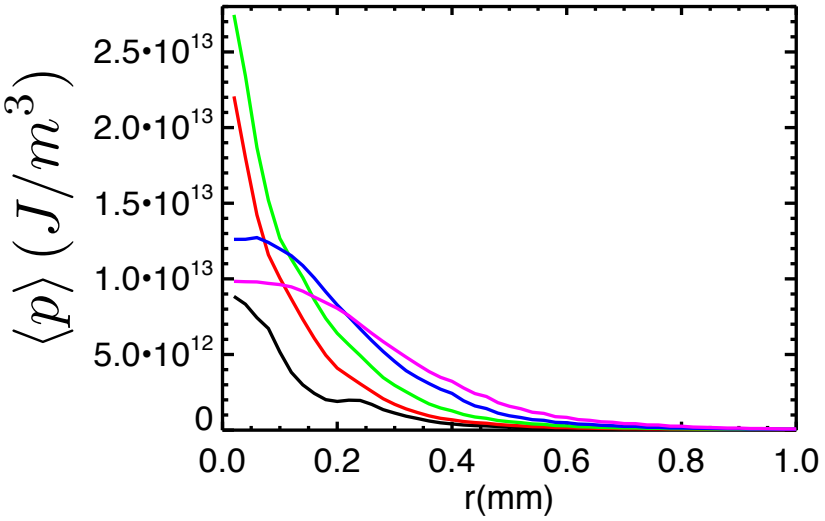
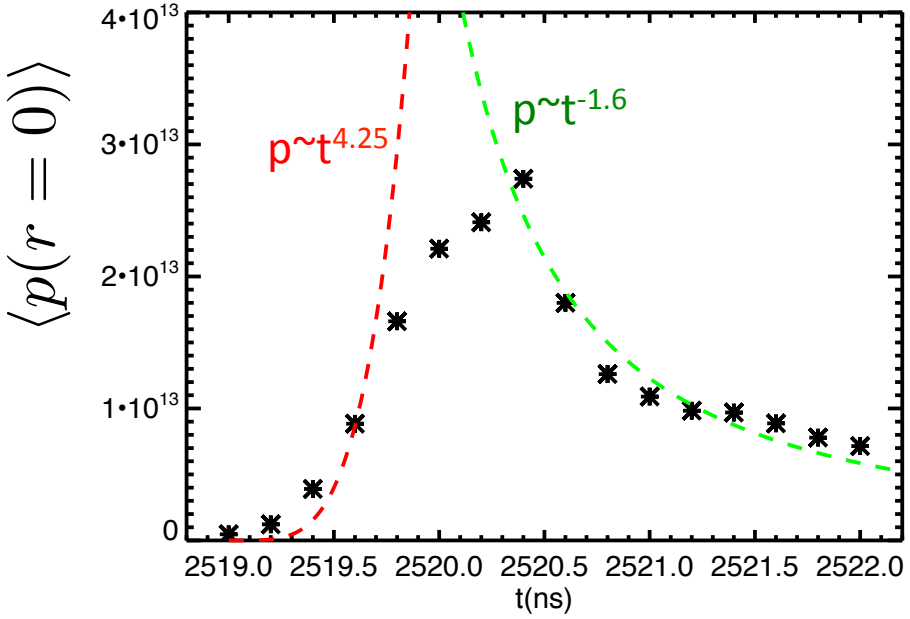
Comparison of generalized Noh with 3D simulation: ρ(t), p(t)









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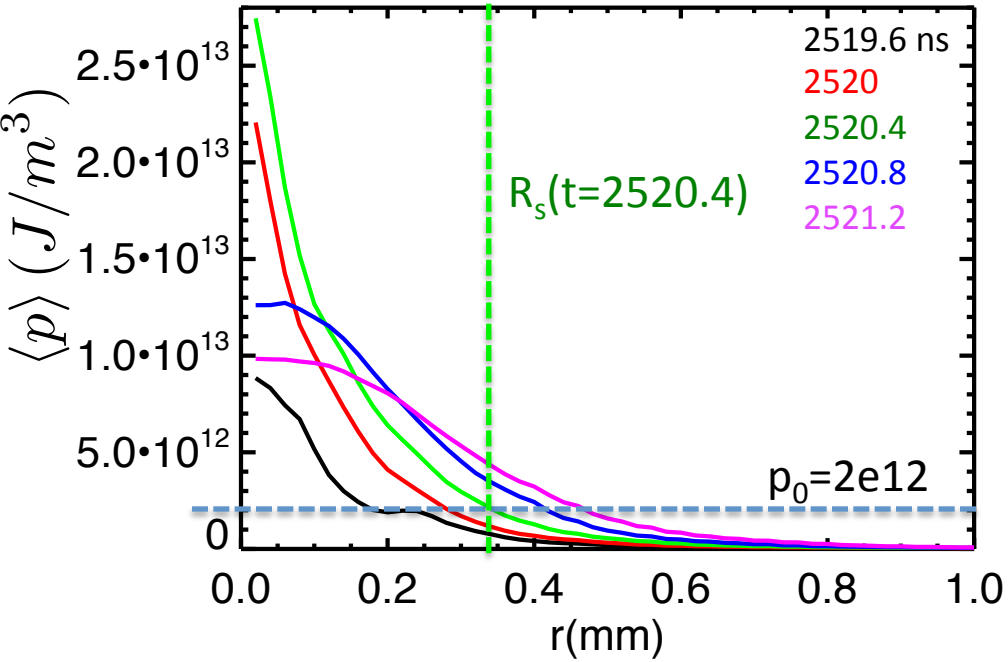
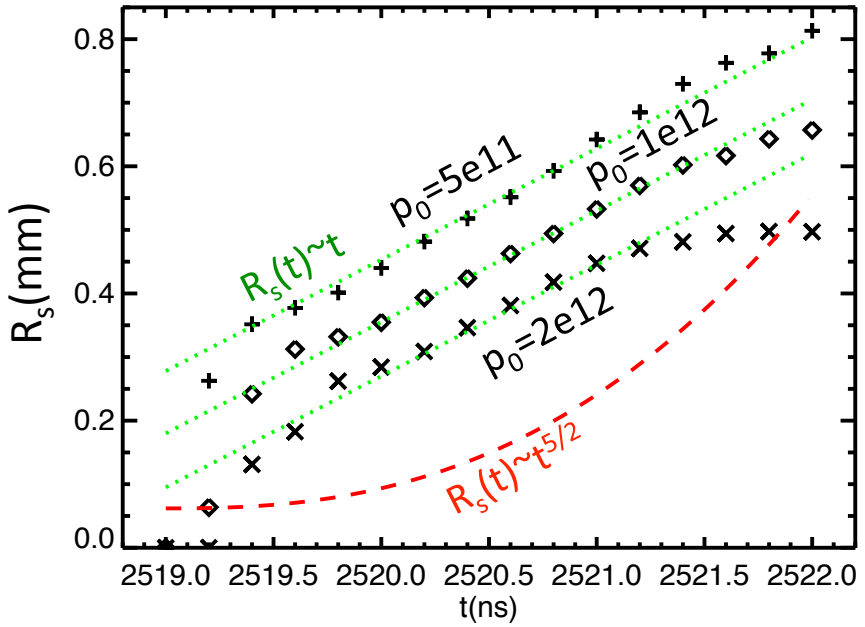
Comparison of generalized Noh with 3D simulation: $R_s(t)$





Once again, phase 2 agrees better with 3D simulation than phase 1. Why?

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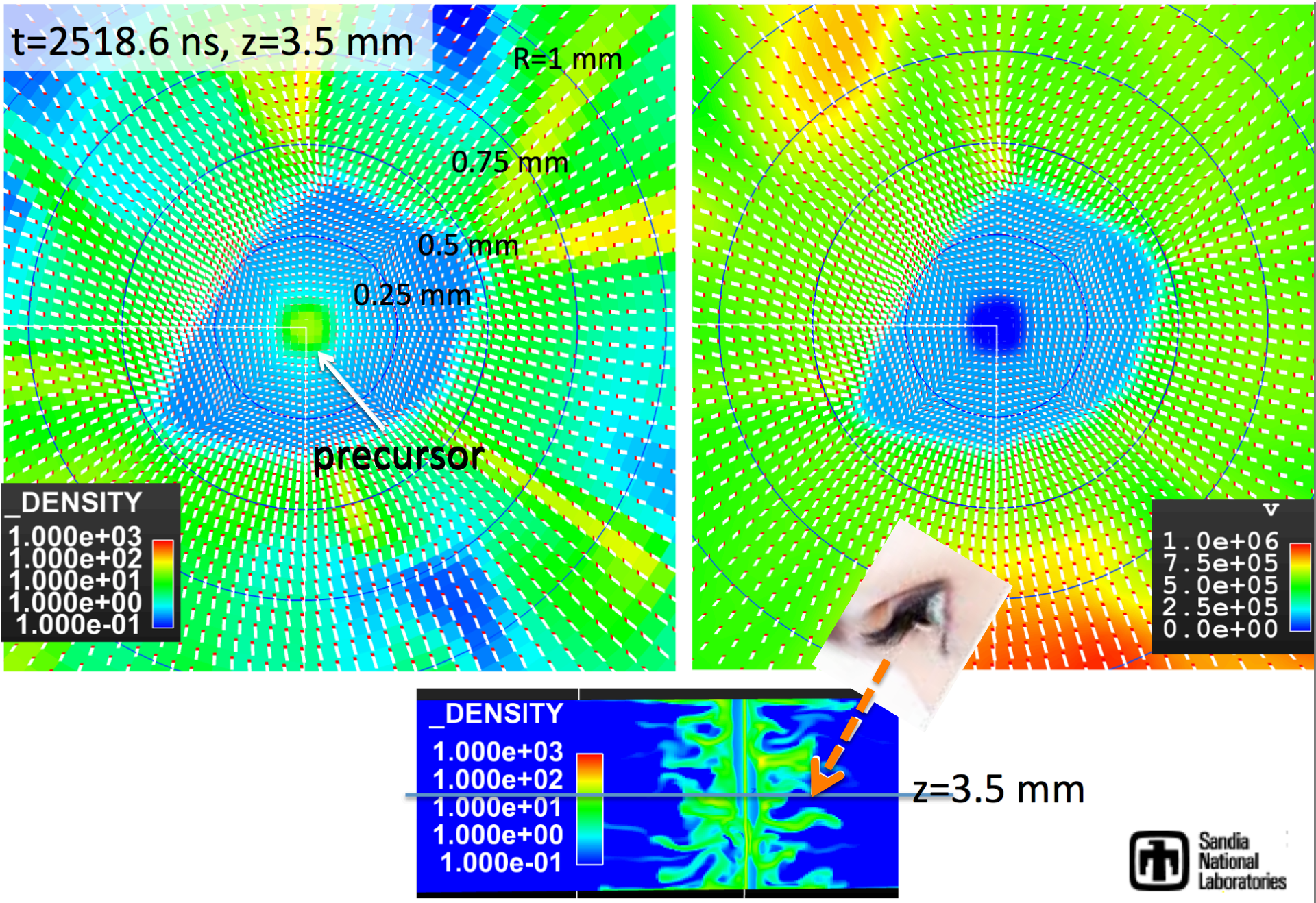
3D simulation fluid flows



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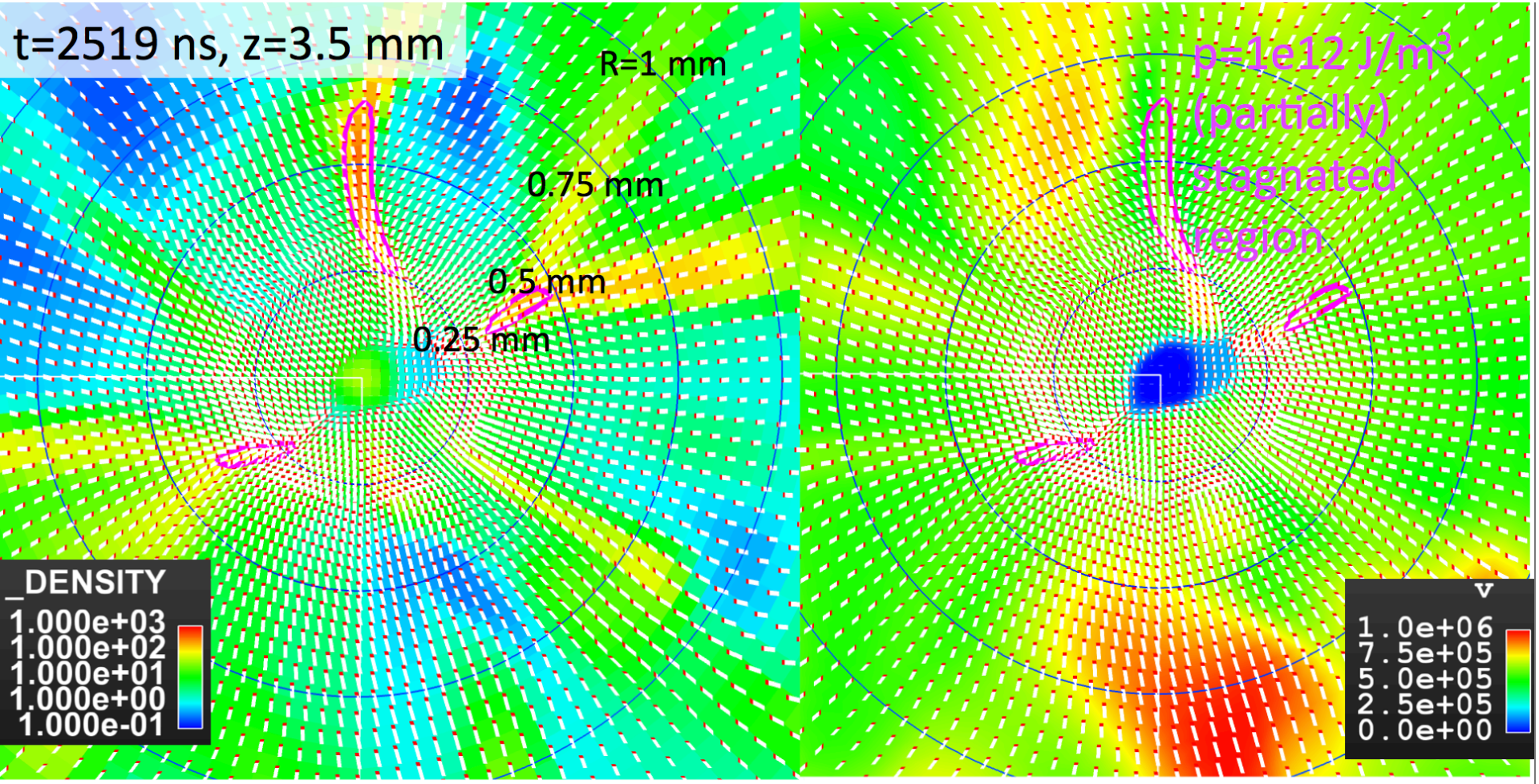
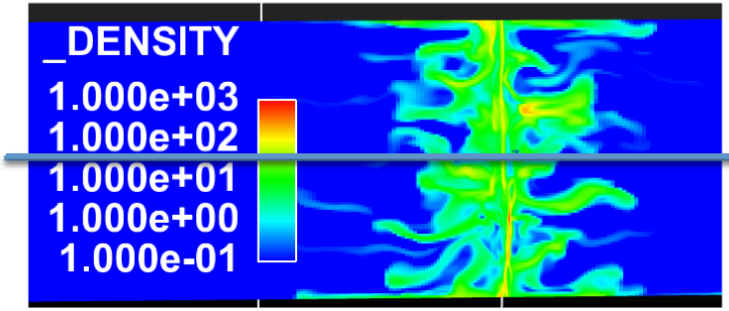
3D simulation fluid flows



In general, plasma collides obliquely and off-axis.



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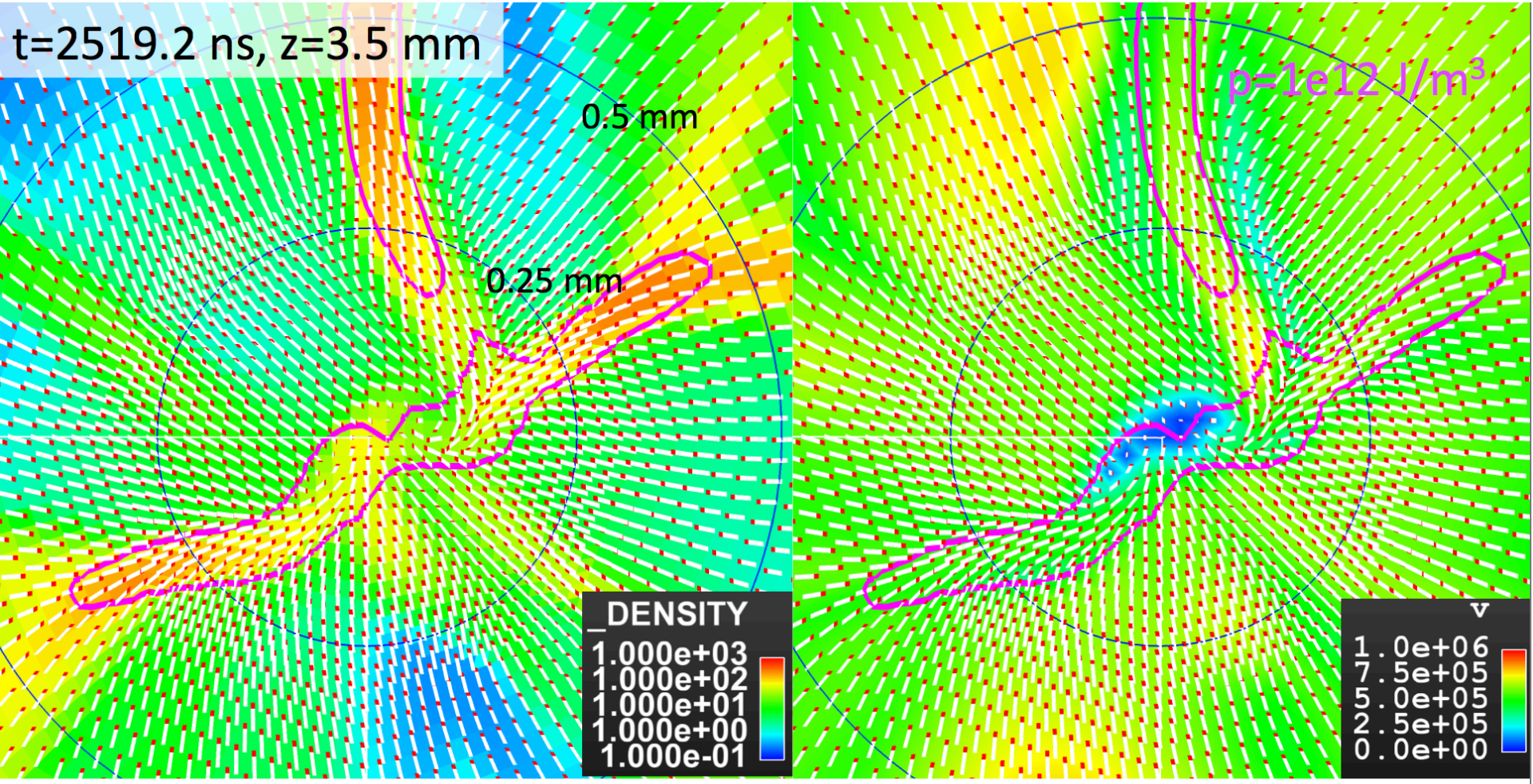
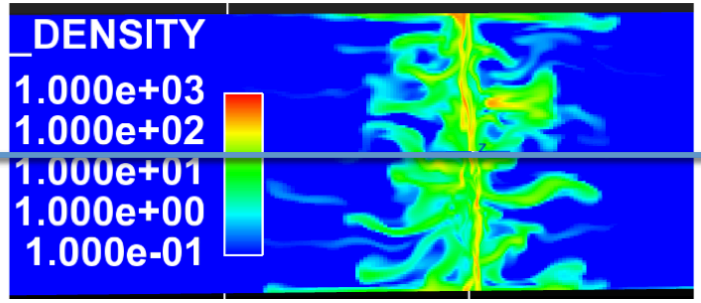
3D simulation fluid flows



Precursor has been crushed



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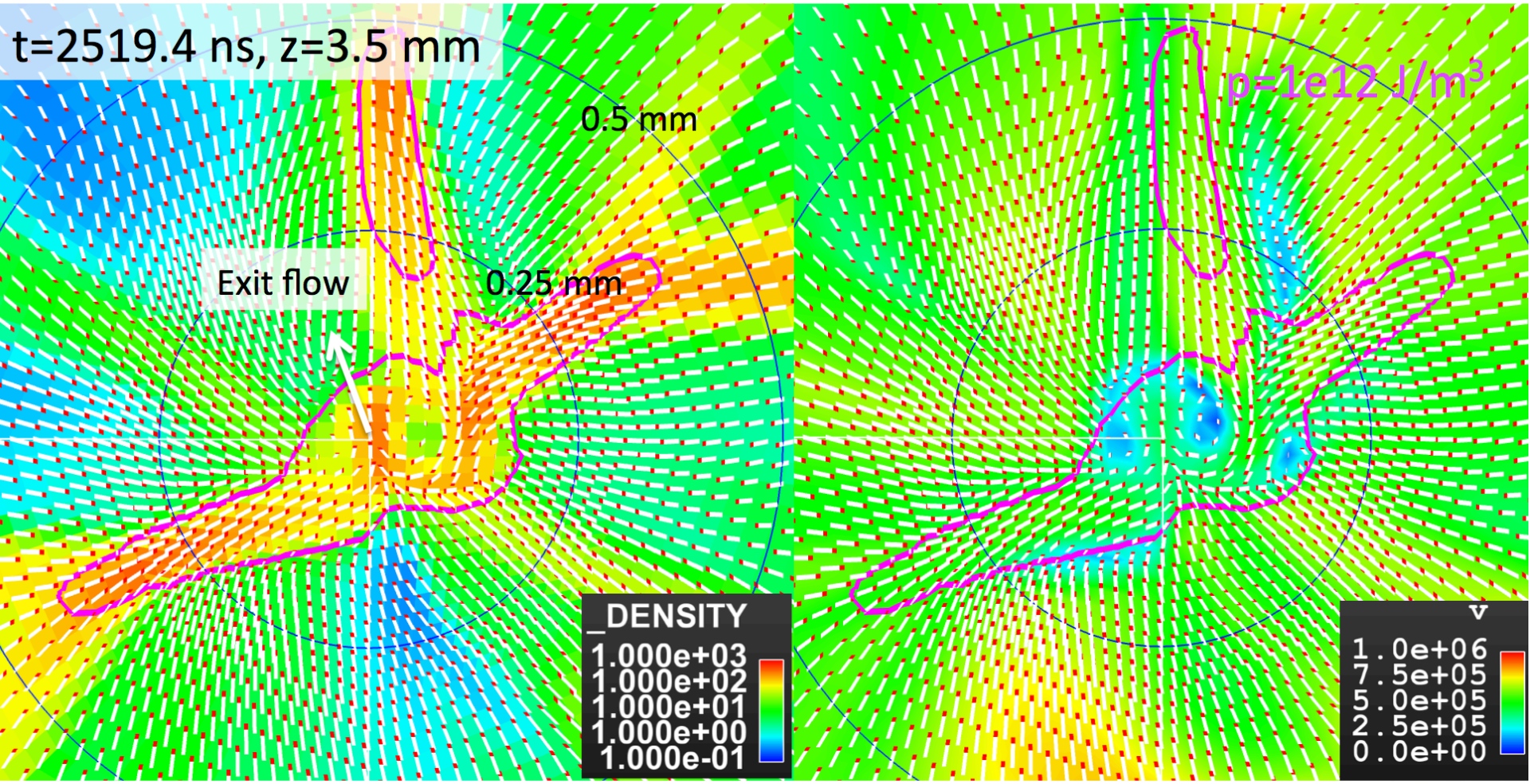
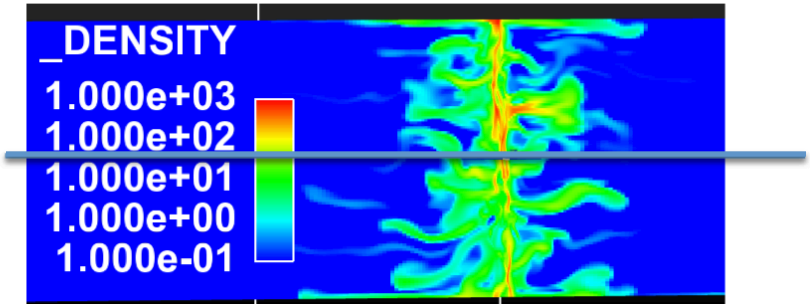
3D simulation fluid flows



Stagnated region (purple contour) is growing via accretion.

Exit flow is forming.



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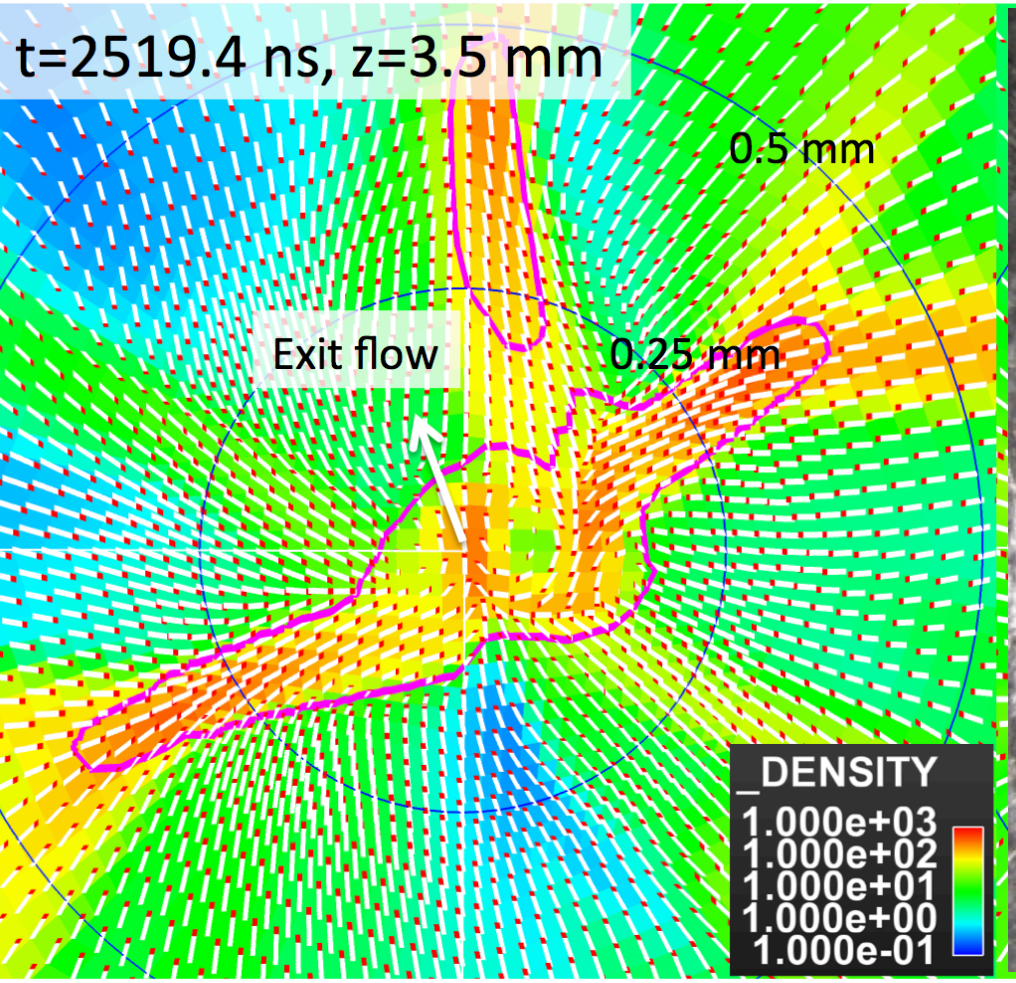
3D simulation fluid flows



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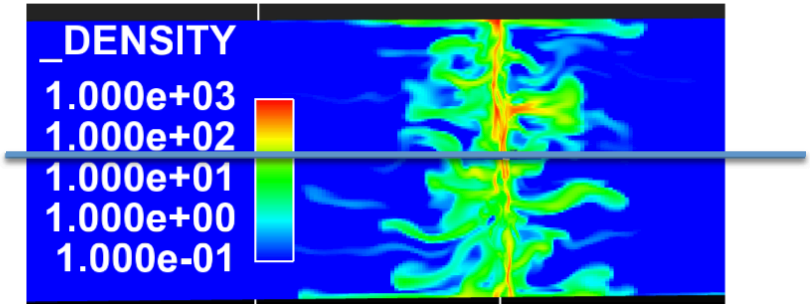

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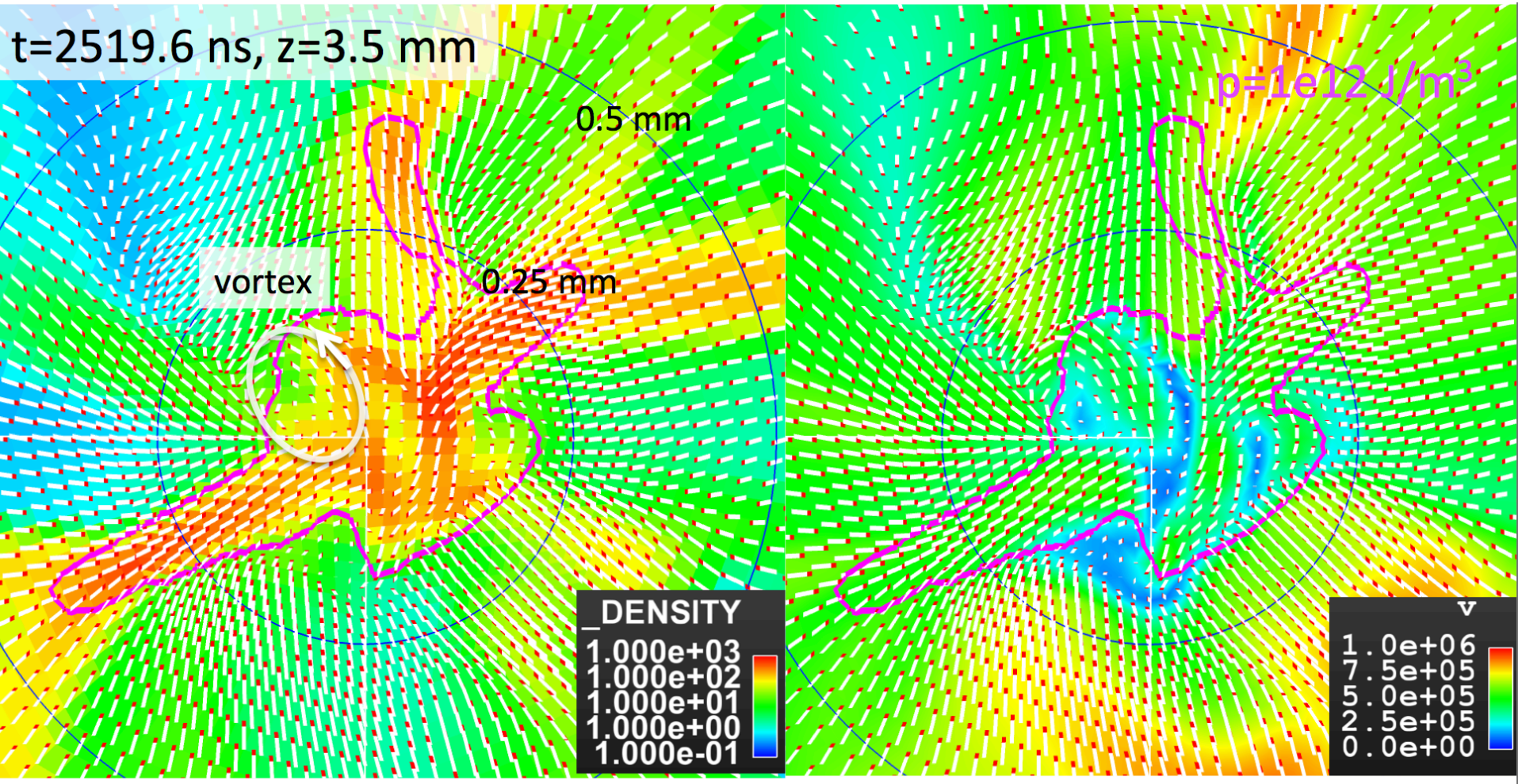
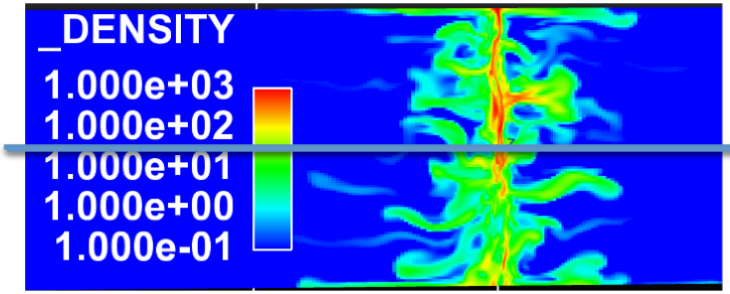
3D simulation fluid flows



Exit flow is turned around by ram pressure, forming a vortex



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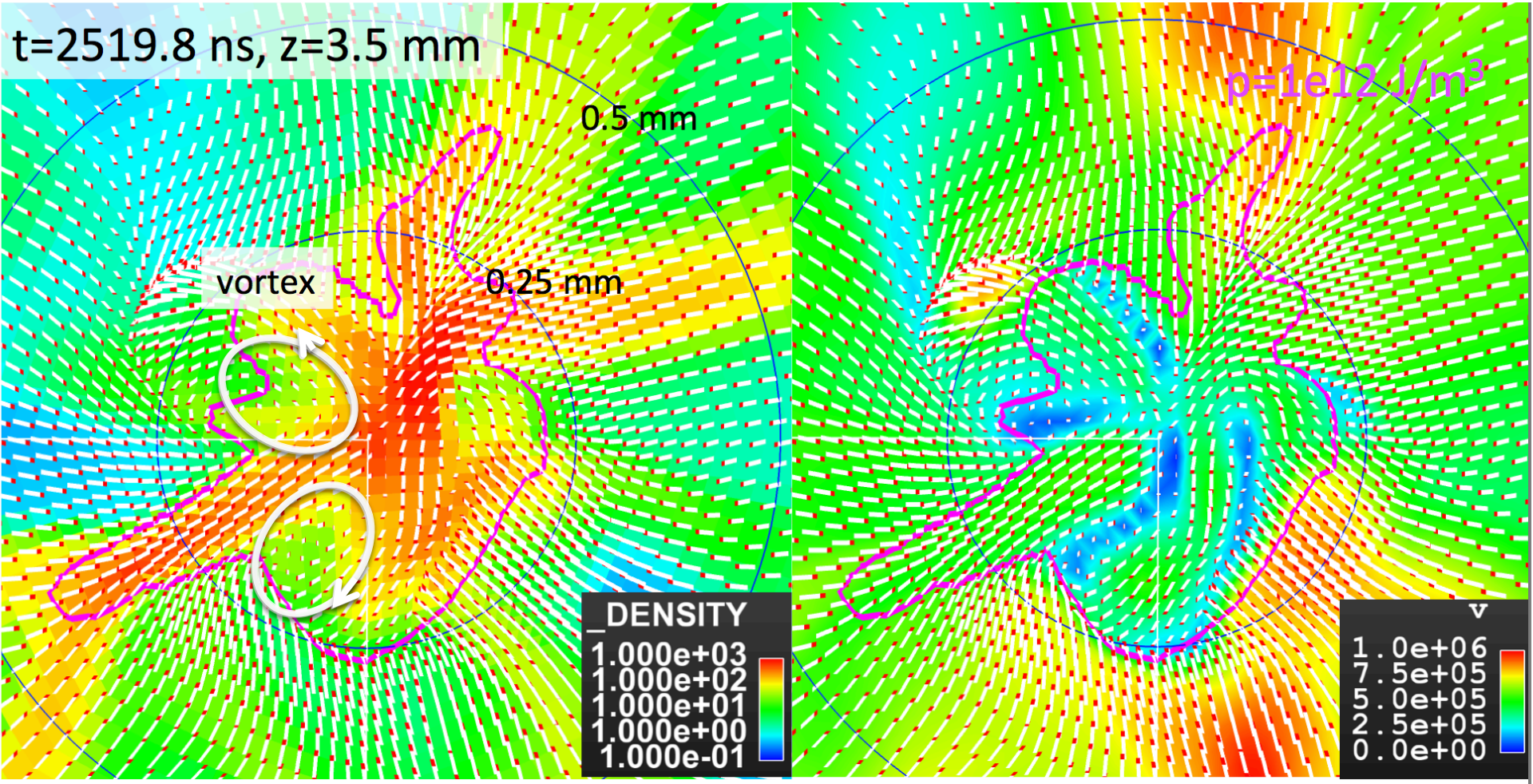
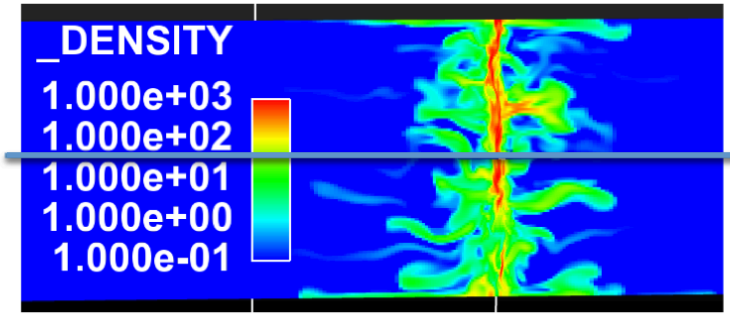
3D simulation fluid flows



"Stagnated" core is really not stagnant at all. Escaping flows form vortices, which generate centrifugal pressure, and enhanced thermal conduction



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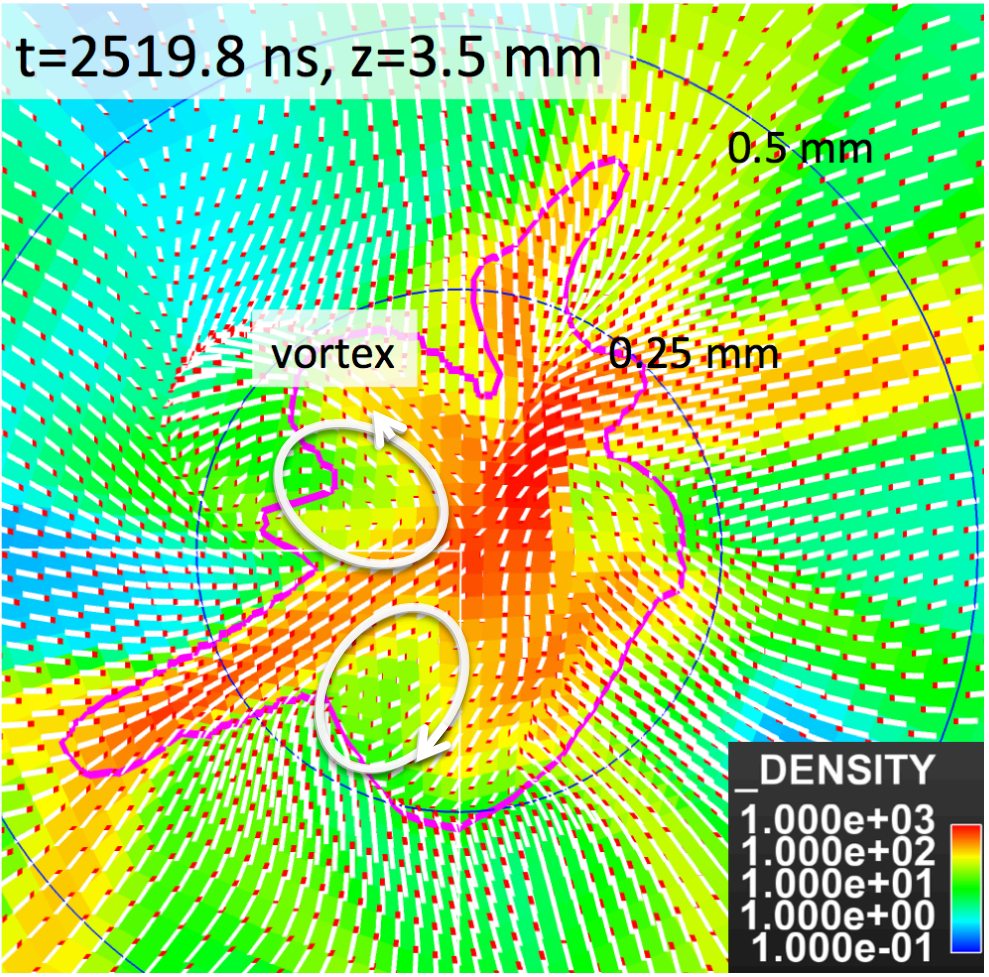
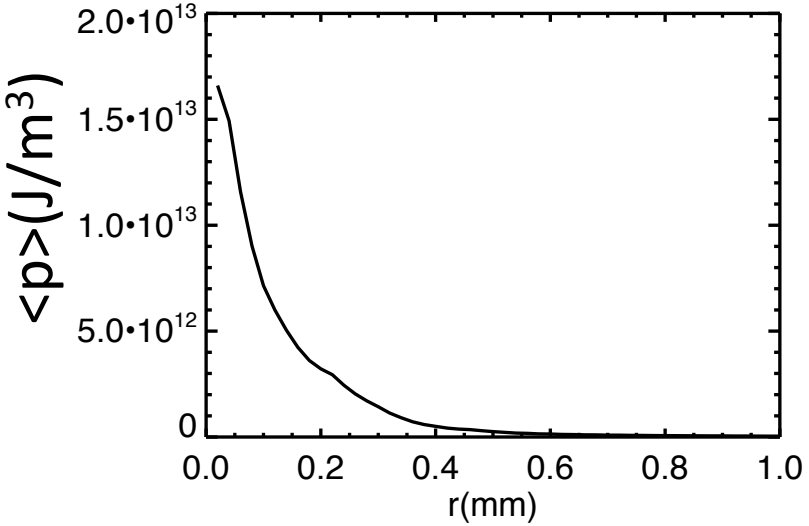
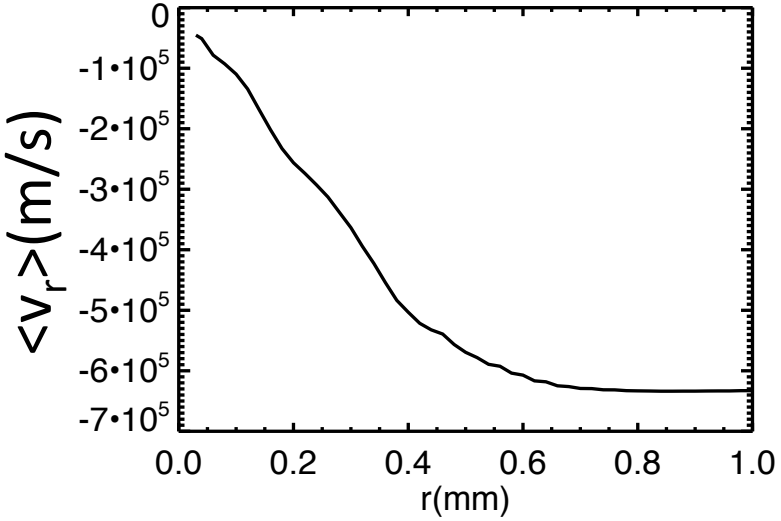
3D simulation fluid flows







We have a high pressure (partially stagnated) core, with significant (nearly linear) radial velocity. This suggests our final analytic solution.

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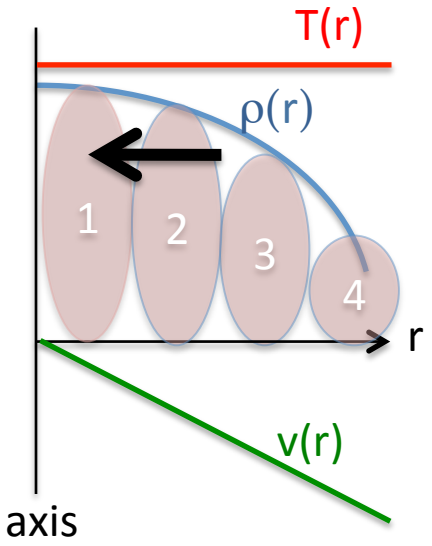
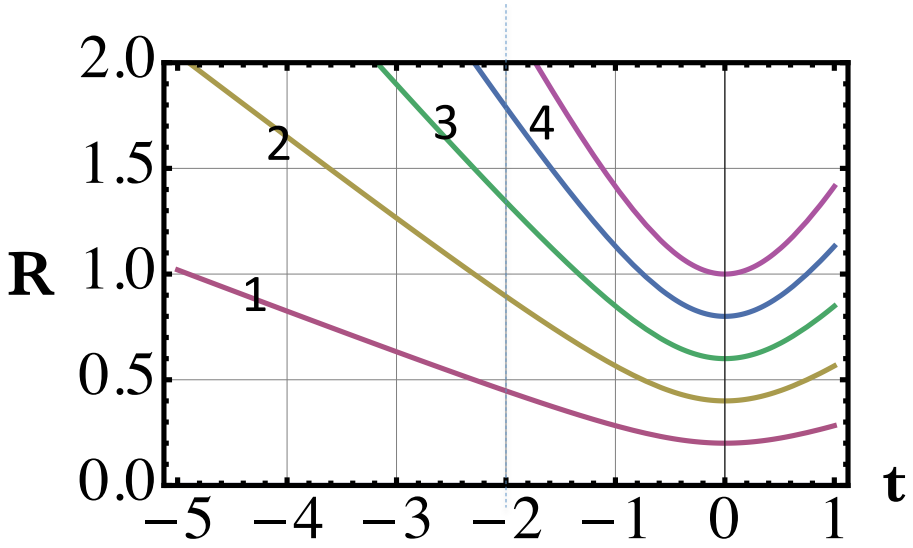
Homogeneous (shockless) stagnation





Whereas the shock solution had flat v(r), now it is linear. This allows particles to compress in unison, without shocks. There is no accretion.

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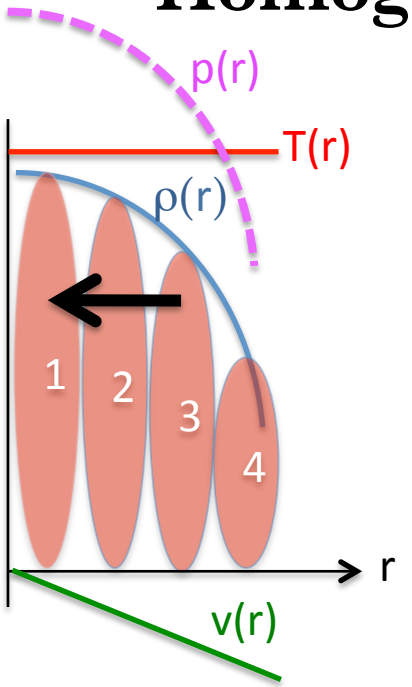
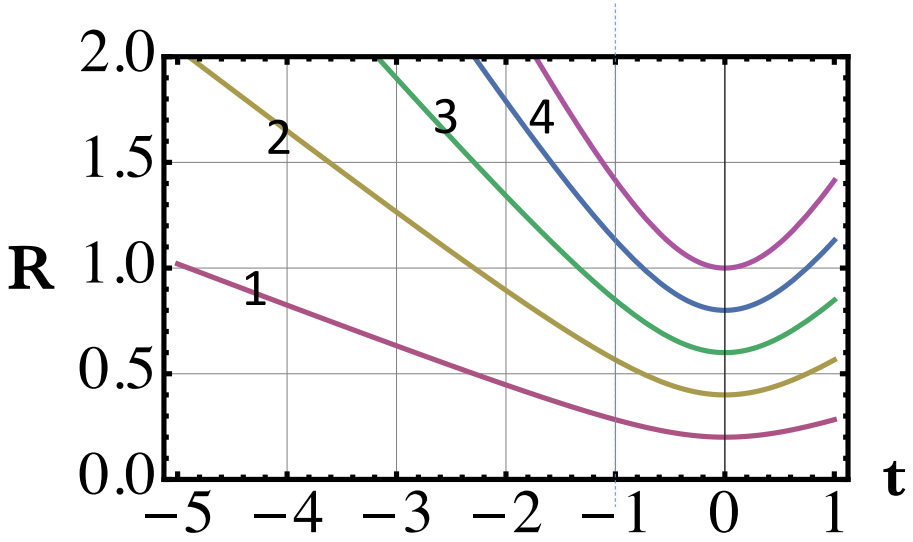
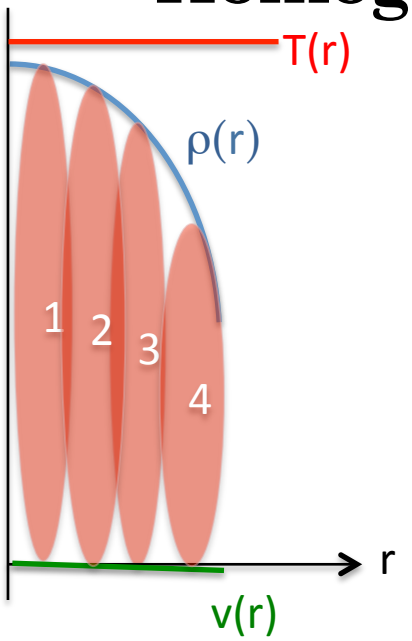
Homogeneous (shockless) stagnation

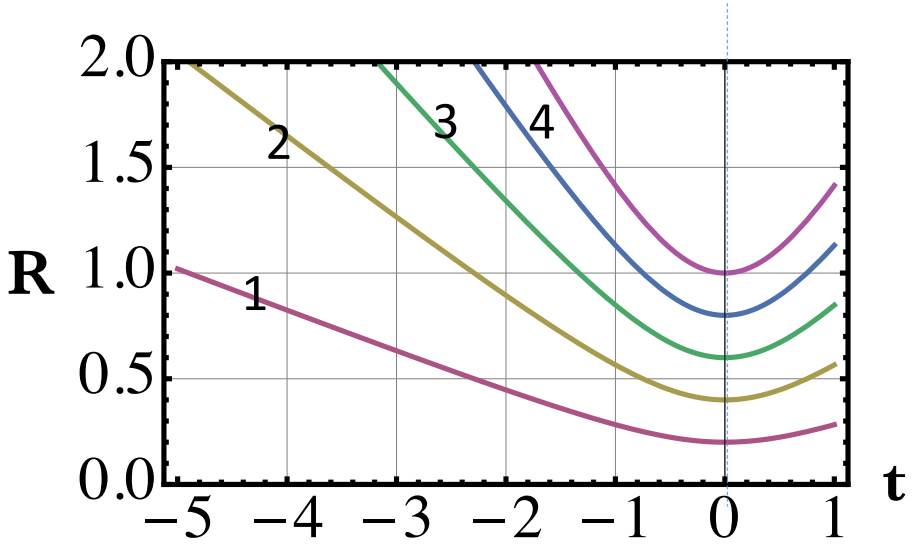




While the particles compress, they gradually decelerate due to the pressure gradient. This pdV work converts kinetic energy to internal energy.

Homogeneous (shockless) stagnation





Stagnation is complete. All kinetic energy has converted to internal energy. The key parameter:

$$\epsilon = \frac{\int p dV}{\int \frac{1}{2}\rho v^2 dV}$$

← Initial thermal pressure

← Initial kinetic energy

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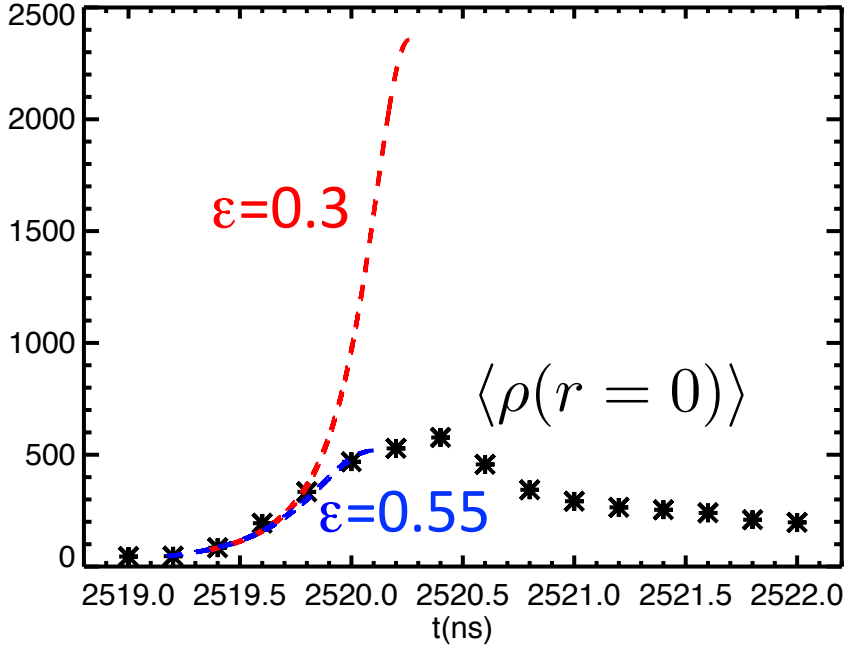
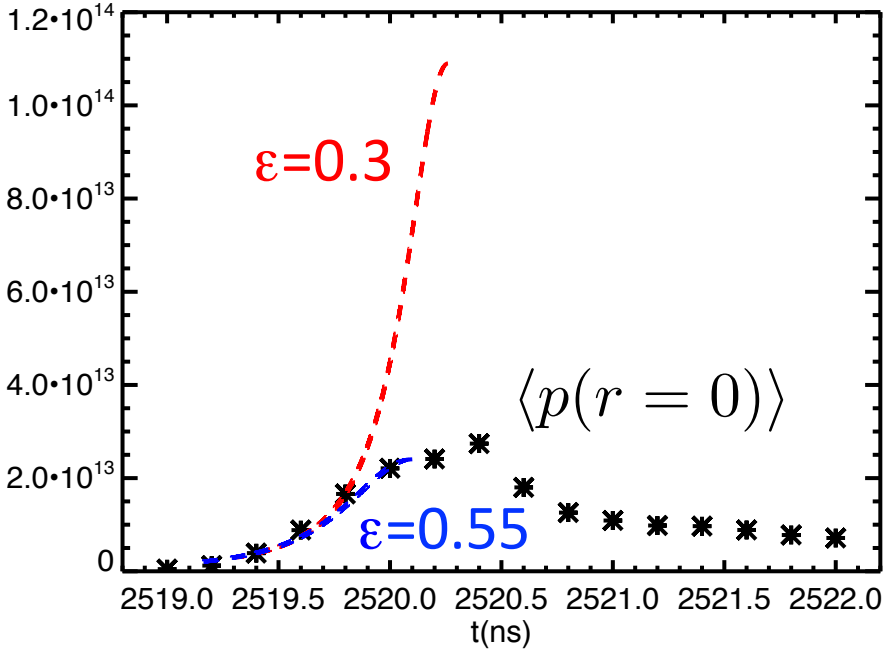
Comparison of homogeneous stagnation with 3D simulation





$$\epsilon = \frac{\int p dV}{\int \frac{1}{2}\rho v^2 dV}$$

Computing ε from simulation, theory predicts too high a compression.

But suppose we artificially enhance ε, to account for the centrifugal pressure from the vortices.

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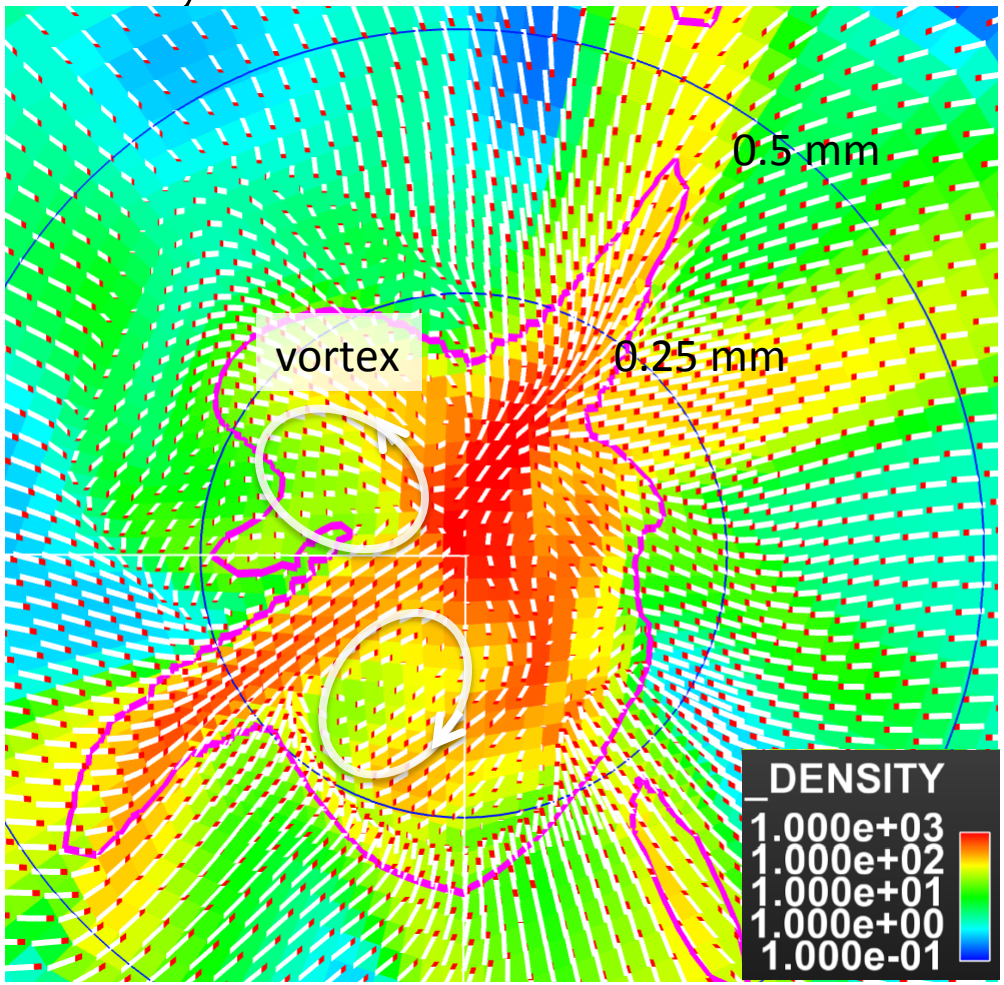
Summary

-We have examined two purely hydrodynamic 1D stagnation solutions and how they apply to a 3D simulation (no radiation).

-Fluid motion is complicated. Lack of symmetry generates vortices that result in centrifugal pressure and enhanced thermal conduction.



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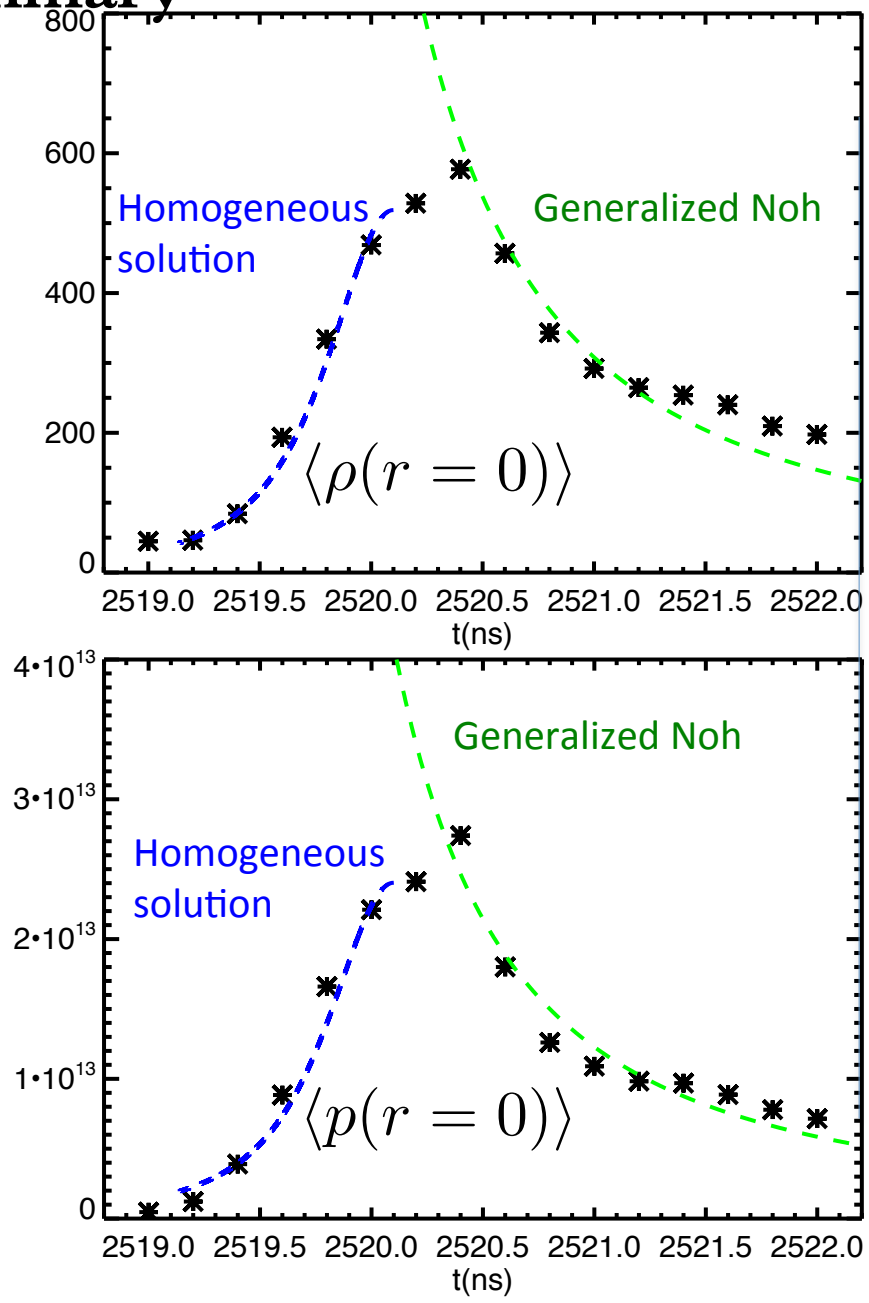
Summary

-We have examined two purely hydrodynamic 1D stagnation solutions and how they apply to a 3D simulation (no radiation).

-Fluid motion is complicated. Lack of symmetry generates vortices that result in centrifugal pressure and enhanced thermal conduction.

-Nonetheless, generalized Noh solution applies to `phase 2' of stagnation, when stagnated core expands into imploding plasma

-Homogeneous stagnation solution with enhanced pressure describes compression phase of stagnation (`phase 1')



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