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Further Considerations on the use of Parallel Matrix Compression and Calderon Preconditioning in the Method of Moments code EIGER

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Overview

- EIGER description
- Compression algorithm description
- Compression implementation
- Calderon preconditoner
- Conclusions/ Future Work

EIGER Description

- **Method of moments code**
  - **Rao-Wilton-Glisson basis functions**
- **Object oriented design**
  - **Written in Fortran 90**
- **Sub-cell slot and wire models**
- **Configured for massively parallel platforms**

Compression Algorithm

- **Adaptive Cross Approximation (ACA)**
  - **Bebendorf (2000), Zhao (2005)**

- **Basic Idea – some matrix blocks are approximated by a lower rank matrix (without forming the full matrix)**

$$Z^{m\times n} \approx \tilde{Z}^{m\times n} = U^{m\times r} V^{r\times n} = \sum_{j=1}^{r} u_j^{m\times 1} v_j^{1\times n}$$

Compression Algorithm
Block Decomposition

- **Meshed model is enclosed in an oct-tree, equivalent to 1 level Fast Multipole Method**

- **The matrix is described as:**

$$Z = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} Z_i^{com}$$

MOM_Blocks – Moment method matrix blocks

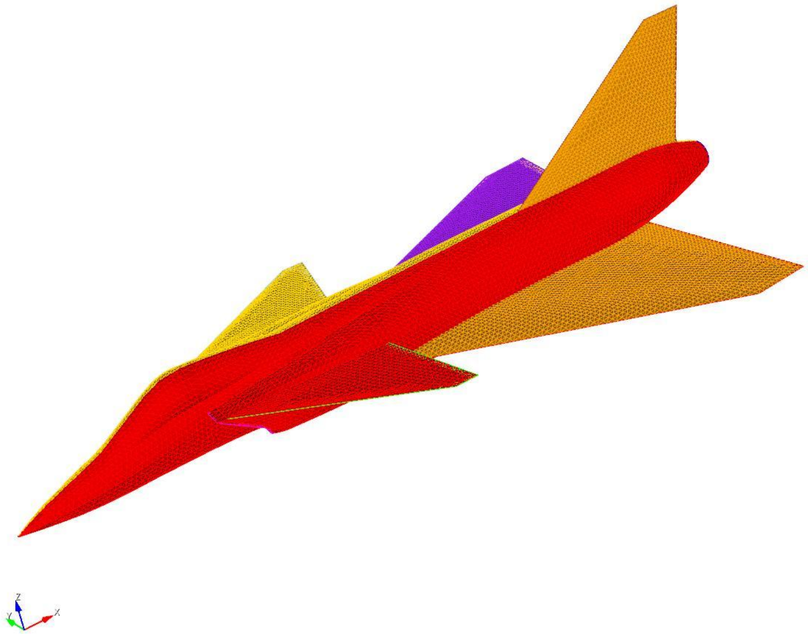
COM_Blocks – Compressed matrix blocks

Matrix Compression in EIGER

Block Decomposition

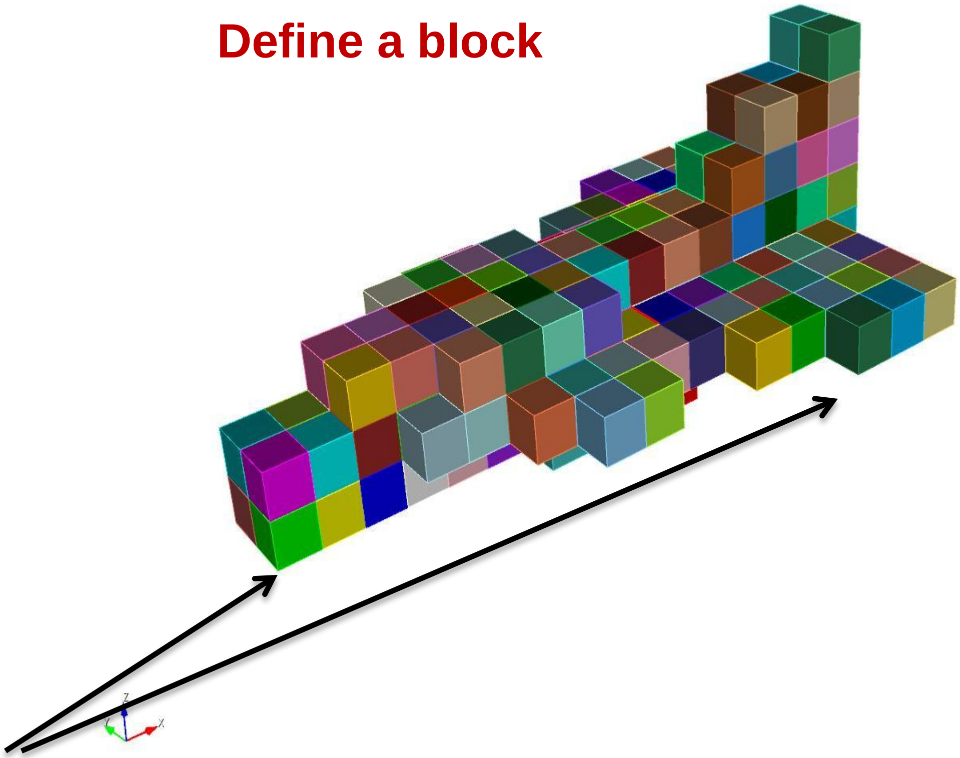
Meshed Object

VFY 218



Interaction Boxes

Define a block



Unknowns that are in these interaction boxes can be compressed.

Compression Algorithm

Block Computation

- **Once first row is obtained, maximum of row is found, that determines the next column, process continues until:**

$$\|u_p\|\|v_p\| \le \varepsilon \|\tilde{Z}^{(p)}\|$$

- **Error control**
  - **Choice of blocks for compression**
    - **proximity**
  - **Error parameter**

Solution Methods

- Direct – demonstrated by John Schaeffer(2007)
  - Modified LU solver
    - Lower Upper Triangular

- Iterative (matrix vector product)
  - GMRES
    - Generalized Minimum Residual
  - TFQMR
    - Transpose Free Quasi-Minimal Residual
  - Preconditioning

Parallelization

- **Blocks are split among the processors.**

- **For either MOM or COM matrix blocks:**

$$Number\_of\_blocks_{processor(i)} = \frac{Total\_blocks}{Number\_of\_processors}$$

- **Note actual algorithm enforces:**
  - **No processor can have more or less than one block than any other processor.**
  - **Processors have both MOM and COM blocks.**

Example Problem - 1

- **VFY 218**
  - **58383 unknowns**
- **Combined Field Integral formulation**
  - **No preconditioner**
- **Compression tolerance 1.e-03**
- **Solution method**
  - **TFQMR Solver tolerance 1.e-04**

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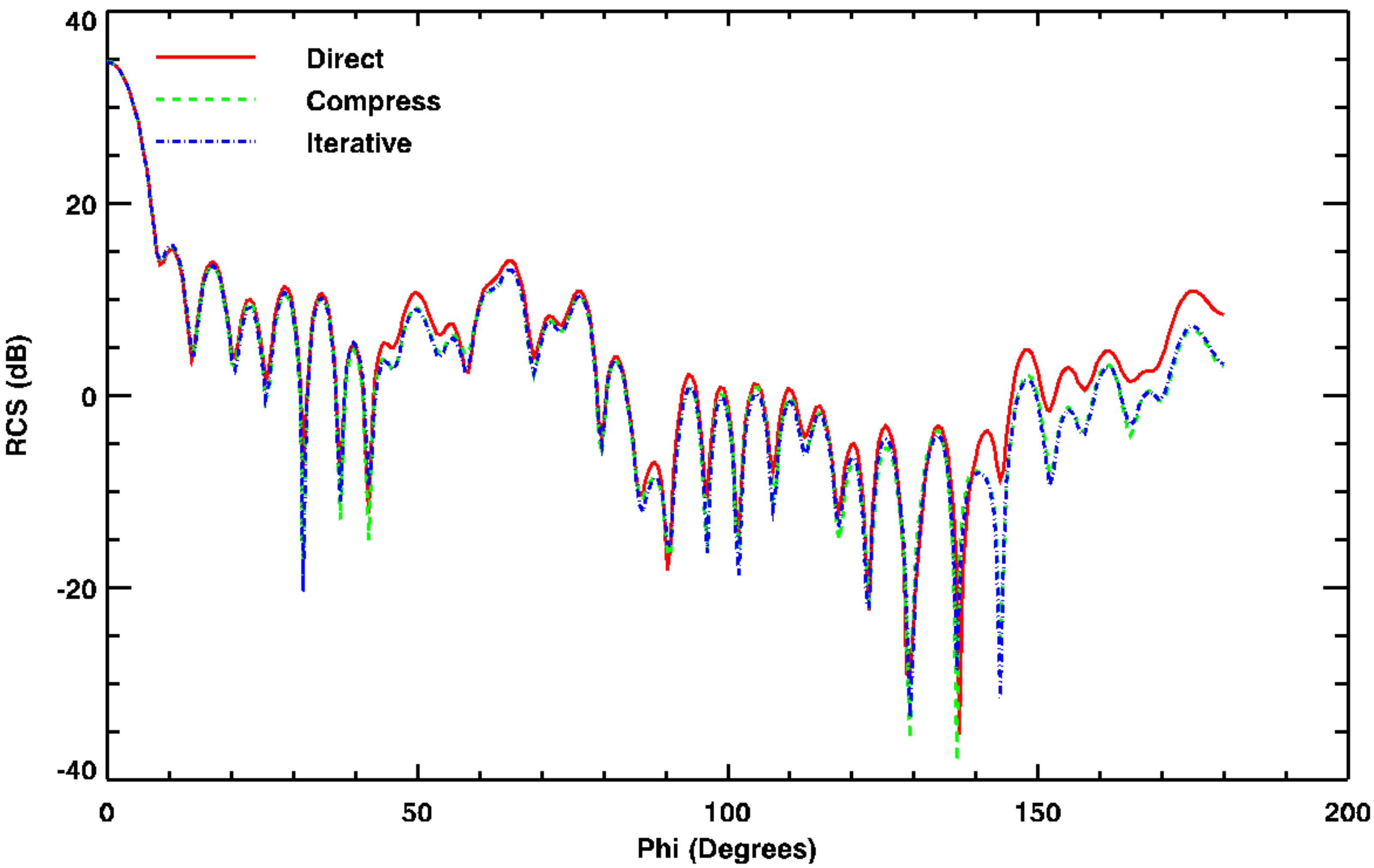
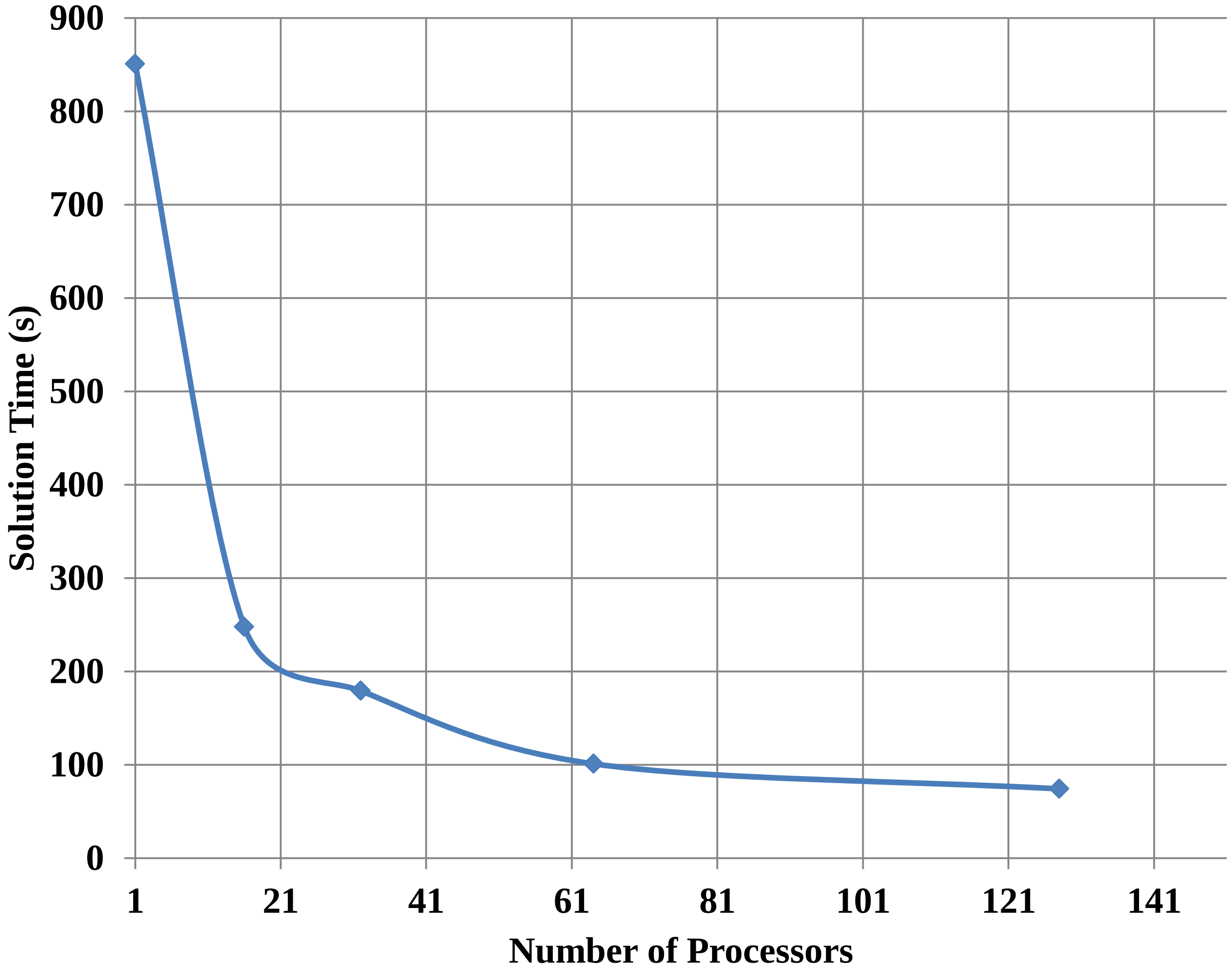
Results

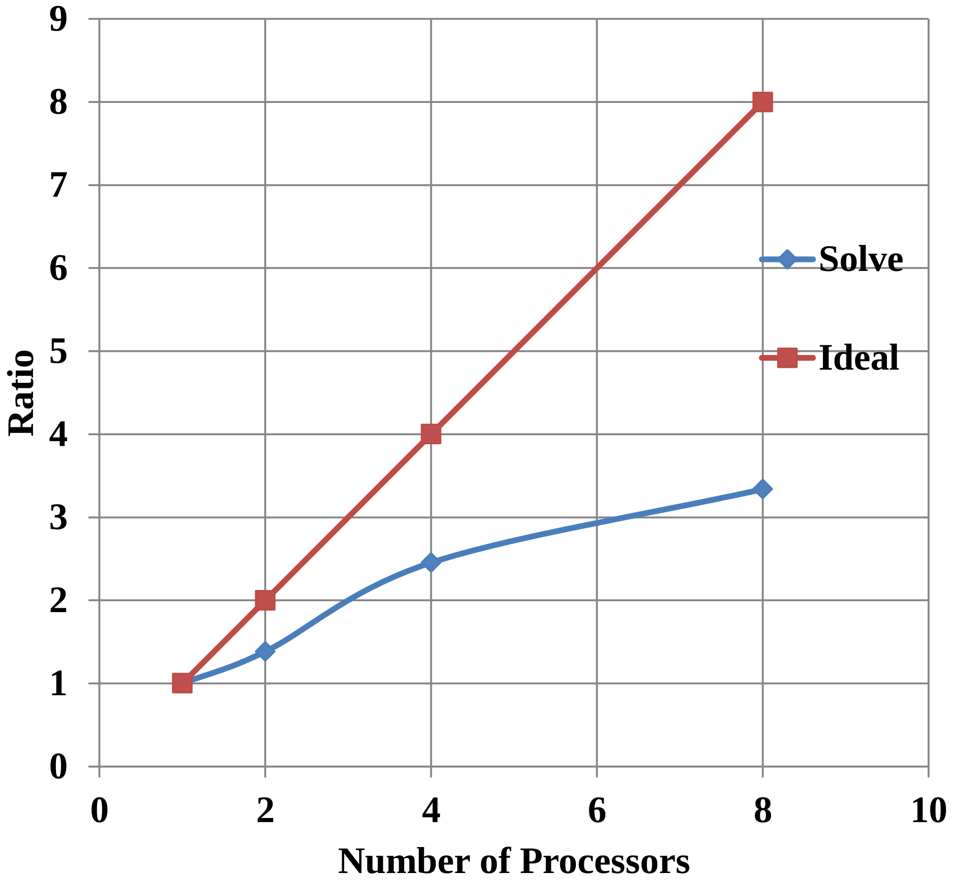


RCS – Radar Cross Section

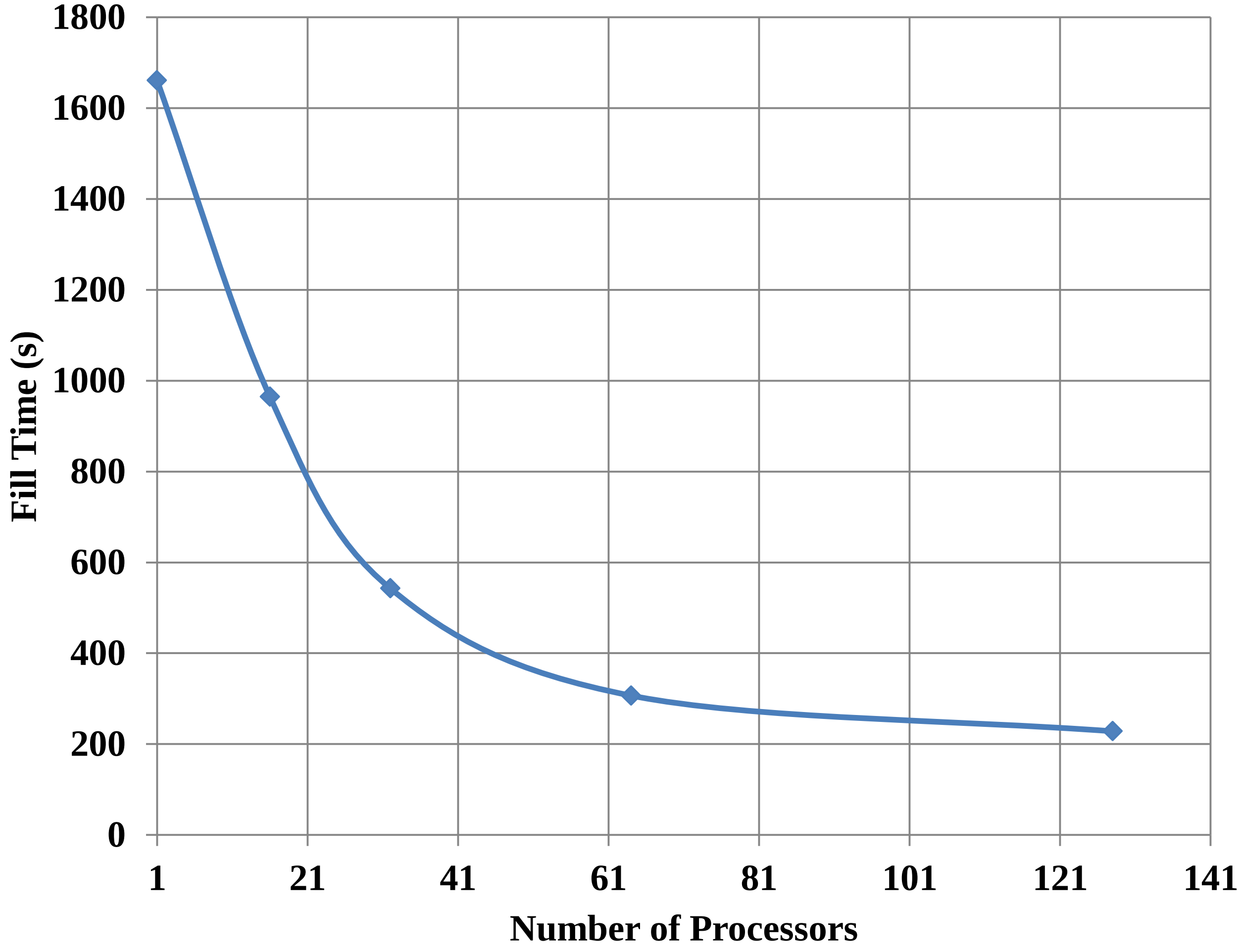
Parallel Timing



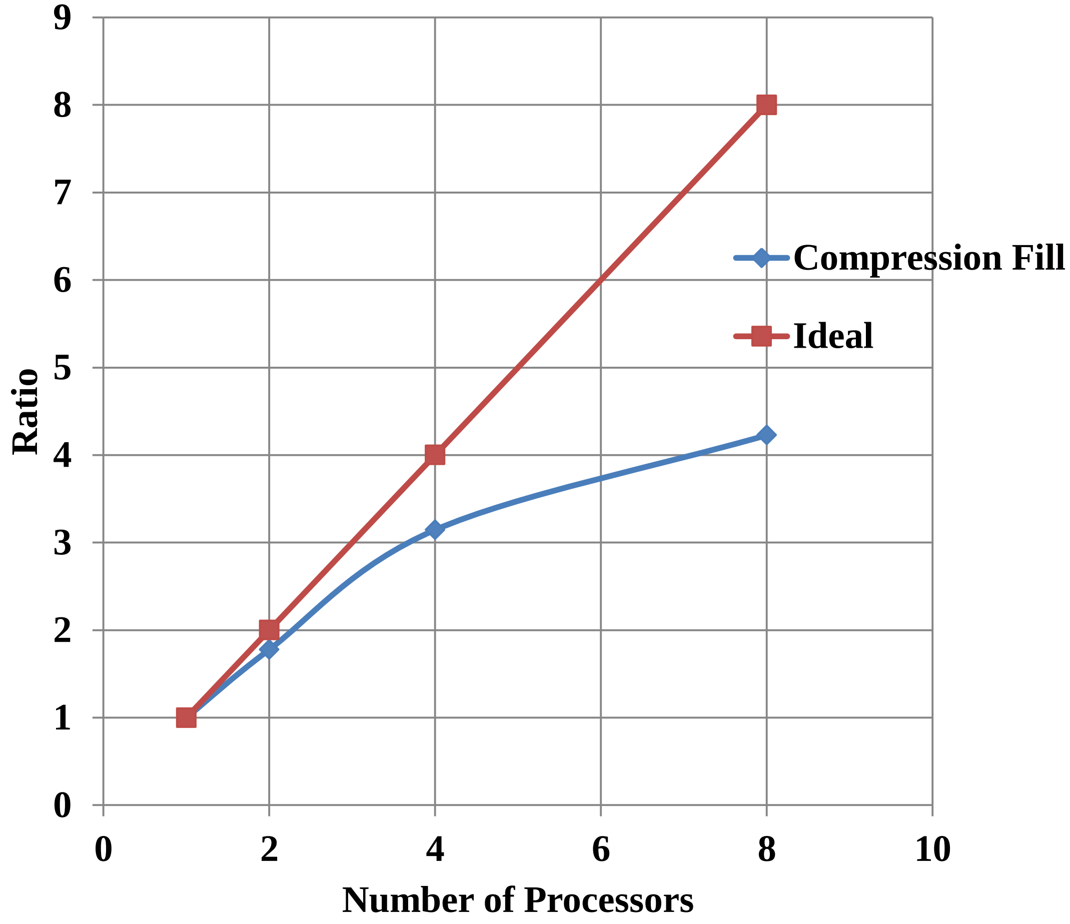
Parallel Timing



Parallel Timing



Parallel Timing



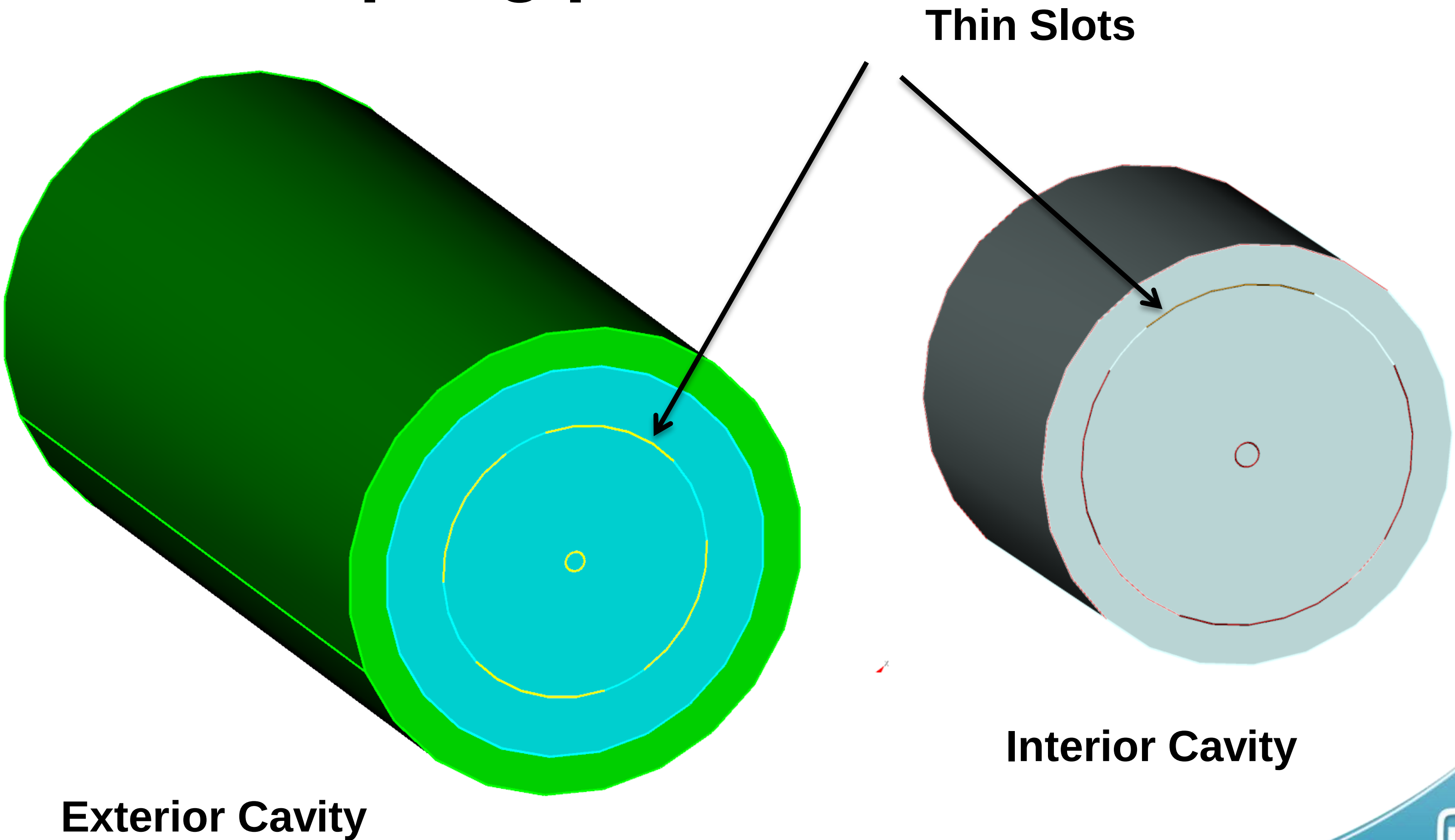
Memory Used

- **Original , full matrix**
  - **3.4 Gbytes**

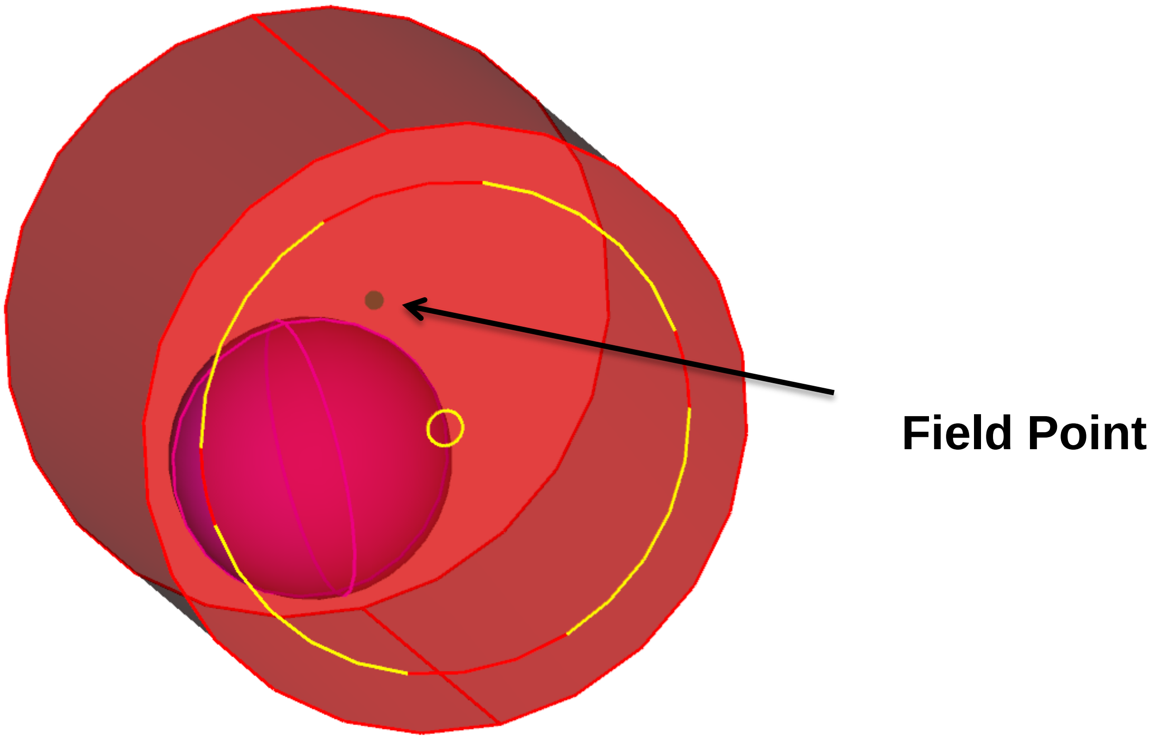
- **Compression**
  - **MOM matrix memory 52.5 Mbytes**
  - **COM matrix memory 22.6 Mbytes**

Example Problem - 2

- **Slot coupling problem**



Example Problem - 2



Interior View

Results

- **Field Results LU**
  - Ex (-79.535, -4.247)
  - Ey (184.40, 20.565)
  - Ez (-18.517, 4262)
- **Field Results TFQMR (Diagonal Preconditioning)**
  - Ex (-79.525, -4.238)
  - Ey (184.38, 20.578)
  - Ez (-18.528, .4305)

Electric Field Components

Calderon Preconditioning

- **Preconditioning with the operator**
  - **(Electric Field Integral Equation)**

$$\mathcal{T}^2(\boldsymbol{J}) = -\frac{\boldsymbol{J}}{4} + \mathcal{K}^2(\boldsymbol{J})$$

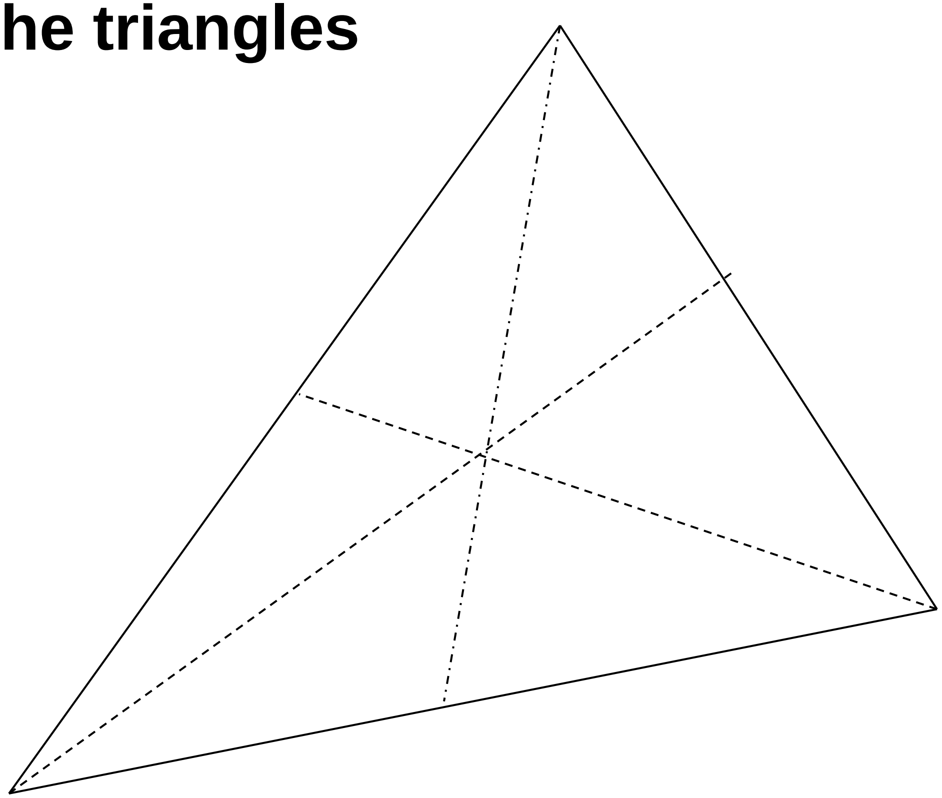
- **Spectrum is bounded**

Multiplicative Calderon Preconditioning

- Based on the Buffa-Christiansen(BC) basis functions.
- Implemented by Francesco P. Andriulli, Kristof Cools, Femke Olyslager, Eric Michielssen
- The BC basis functions are div-conforming, quasi-curl-conforming basis functions
  - RWG basis functions defined on the barycentric mesh

Barycentric Mesh

6 times the triangles



Matrix Equation

$$\left(\bar{\bar{\mathbf{P}}}^T\ \bar{\bar{\mathbf{Z}}}^B\ \bar{\bar{\mathbf{Q}}}\ \bar{\bar{\mathbf{Z}}}^B\ \bar{\bar{\mathbf{R}}}\right)\bar{\mathbf{I}} = \left(\bar{\bar{\mathbf{P}}}^T\ \bar{\bar{\mathbf{Z}}}^B\ \bar{\bar{\mathbf{Q}}}\right)\bar{\mathbf{V}}^B$$

$\bar{\bar{Z}}^B$ **Matrix on barycentric mesh**

$\bar{V}^B$ **RHS on barycentric mesh**

$\bar{I}$ **Unknowns on original mesh**

Matrix Equation

$$\left(\bar{\bar{\mathbf{P}}}^T\ \bar{\bar{\mathbf{Z}}}^B\ \bar{\bar{\mathbf{Q}}}\ \bar{\bar{\mathbf{Z}}}^B\ \bar{\bar{\mathbf{R}}}\right)\bar{\mathbf{I}} = \left(\bar{\bar{\mathbf{P}}}^T\ \bar{\bar{\mathbf{Z}}}^B\ \bar{\bar{\mathbf{Q}}}\right)\bar{\mathbf{V}}^B$$

$\overline{\overline{\mathrm{R}}}$ **Maps RWG space to the barycentric RWG**

$\overline{\overline{\mathrm{Q}}}$ **Maps curl-conforming RWG space to the div-conforming barycentric RWG**

$\overline{\overline{\mathrm{P}}}^{\mathrm{T}}$ **Maps div-conforming RWG barycentric and div and quasi –curl-conforming BC functions**

Calderon Preconditioner Implementation

- Construct barycentric mesh
- Construct matrices needed for the solution
  - Compression for matrix Z^B
- Iterative solution

Calderon Preconditioner Implementation - Status

- **Barycentric mesh generator in place**
  - **Basis function information**
- **Other projectors integrated into the code.**
- **Testing continues**

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Conclusions/ Future Work

- **Finish integration of the two techniques described.**
- **Continue testing with focus on appropriate parameters for compression.**
- **Consider multilevel compression algorithms.**
- **Implement an enhanced parallelization strategy.**
  - **Reordering of unknowns**
  - **Block algorithm**
    - **multilevel**