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Operational Availability: Mission, Instantaneous, and Steady State

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Overview

- Operational Availability (A_O) is conceptually simple.
  - It is the portion of time that a system is either operating or capable of operating.
- Estimating A_O, however, is not so simple
- Several equations can be developed or found in literature that can be used to estimate A_O
  - Not all of the equations are mathematically equivalent
  - Some equations apply to specific usage scenarios
  - Equations are often incorrectly used interchangeably

Ao Defined

- Operational Availability indicates the percentage of time that a system or group of systems within a unit are operationally capable of performing an assigned mission. A_O can be expressed as (uptime/(uptime + downtime)). (DoD RAM-C para 1.2)

$$A_O = \frac{uptime}{uptime + downtime}$$

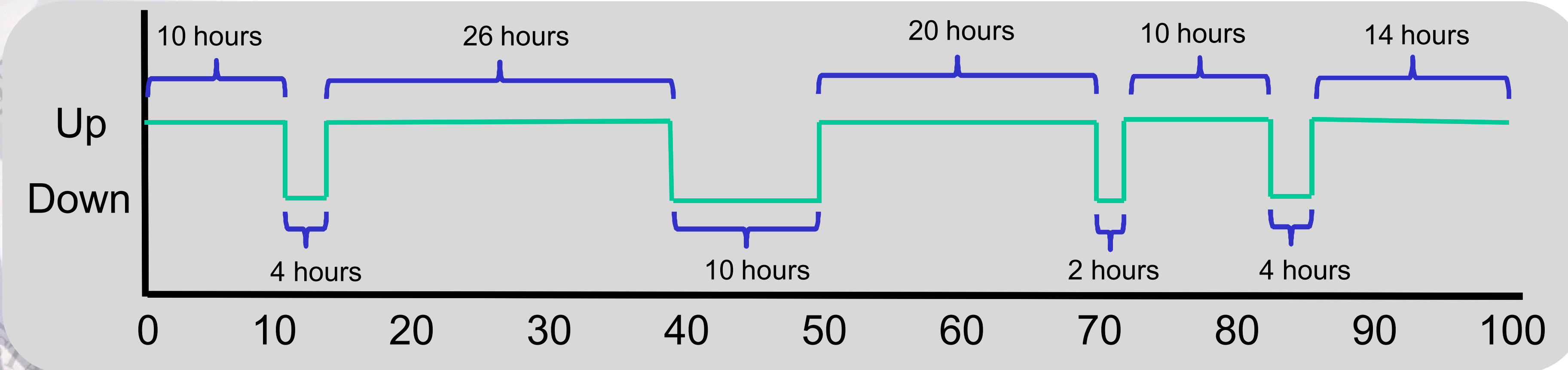
- An alternative form of the A_O equation is often used

$$A_O = \frac{MTBDE}{MTBDE + MDT}$$

MTBDE = Mean Time Between Downing Events
MDT = Mean Down Time

- The specific acronyms used in the alternate equation vary based on chosen terminology (MTBF, MTBCF, MTBOMF, MRT, MTTR) but the equation is valid as long as the first term is the average time between events causing the system to go down, and the second term is the average down time. MTBDE is the most general term since it can include any downing event including required scheduled maintenance downtime.

Ao Example



- A system operates for 100 hours as shown in the above timeline
  - Uptime = 10+26+20+10+14=80
  - Downtime = 4+10+2+4=20 hours
  - Ao = uptime/(uptime+downtime)=80/(80+20)=0.80
- Using the alternate method
  - MTBDE = 80/4=20
  - MDT=20/4=5
  - Ao = 20/(20+5)=0.80

Equation Comparison

$$A_O = \frac{MTBDE}{MTBDE + MDT} = \frac{\frac{Uptime}{Number\ of\ Downing\ Events}}{\frac{Uptime}{Number\ of\ Downing\ Events} + \frac{Downtime}{Number\ of\ Downing\ Events}} = \frac{Uptime}{Uptime + Downtime}$$

- Over a specified period of time, when MTBDE and MDT are calculated based on observed data, the alternative equation is exactly equivalent to the general equation

Steady State Availability

- **Often, various mean time between failure and mean repair time measures are calculated for a system through analysis**
  - If the analysis focuses specifically on failures that cause system downtime, then the terms Mean Time Between Operational Mission Failures (MTBOMF) or Mean Time Between Critical Failures (MTBCF) are often used
  - The corresponding average downtime for these failures is the MDT
- **These calculated values are long term, steady state averages**
  - Using them in the alternate Ao equation results in the steady state average for availability

$$\frac{MTBOMF}{MTBOMF + MDT} = Steady\ State\ Ao$$

  - This is the availability that would be observed if the systems were to run continuously for a long period of time

Instantaneous Availability

- The instantaneous availability of a system is described as the probability that the system is "up" at a specific time (t)

- If the system starts in an "up" condition, its instantaneous availability begins at 1.0 and approaches the steady state availability after a few failure/repair cycles

- For systems that operate continuously, once the system passes an initial start-up period, the instantaneous availability equals its steady state availability

- In many analysis cases, the period of interest is such that the startup transient is negligible and is ignored

Availability During Mission Time

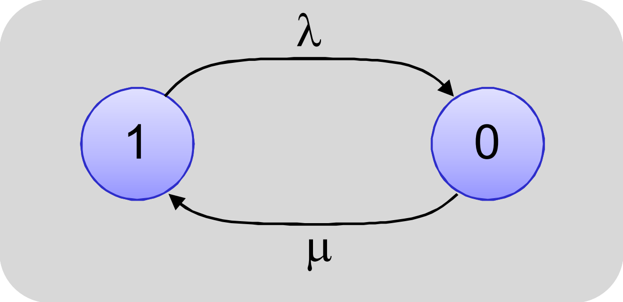
- Often in military applications, the availability of a system over a limited deployed period (specified mission) is desired.

- If the system starts in an "up" state, and the mission length is small relative to the MTBDE, then transients are important

- As a general rule, the availability will start approaching the steady state availability after a time period of approximately four times the average time to failure (Reliability Hotwire, Issue 79, September 2007)

Two State Markov Analysis

- Consider a two state model in which a system is either "up" represented by a 1, or "down" represented by 0



- The system moves from state 1 to state 0 at a rate λ and from state 0 to 1 with rate μ
- λ = 1/MTBDE and μ = 1/MDT
- The instantaneous availability can be calculated as a function of time. It is the probability of being in state 1 at time t.

$$P_1(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)t}$$

Two State Markov (continued)

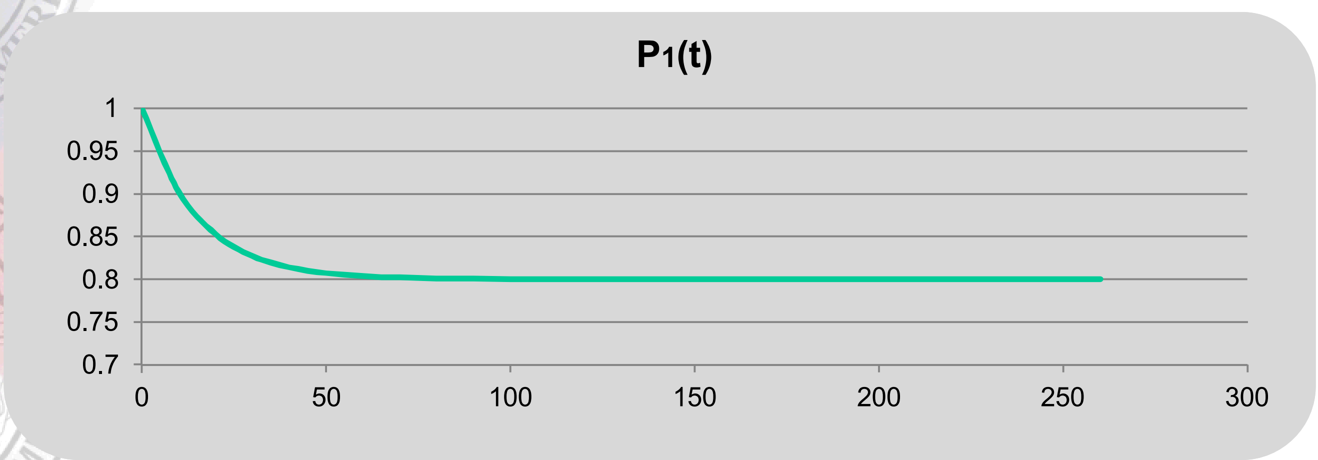
- **The average availability, or uptime availability, is the uptime percentage through time t and can be calculated as**

$$Ao(t) = \frac{1}{t}\int_0^t P_1(u)du$$
$$= \frac{1}{t}\int_0^t \left[\left(\frac{\mu}{\mu+\lambda}\right) + \left(\frac{\lambda}{\mu+\lambda}\right)e^{-(\lambda+\mu)u}\right]du$$
$$= \frac{\mu}{\mu+\lambda} + \frac{-\lambda}{t(\mu+\lambda)^2}\left(e^{-(\lambda+\mu)t} - 1\right)$$

- **The limit of Ao(t) as t approaches infinity is** $\frac{\mu}{\mu+\lambda}$

Markov Example

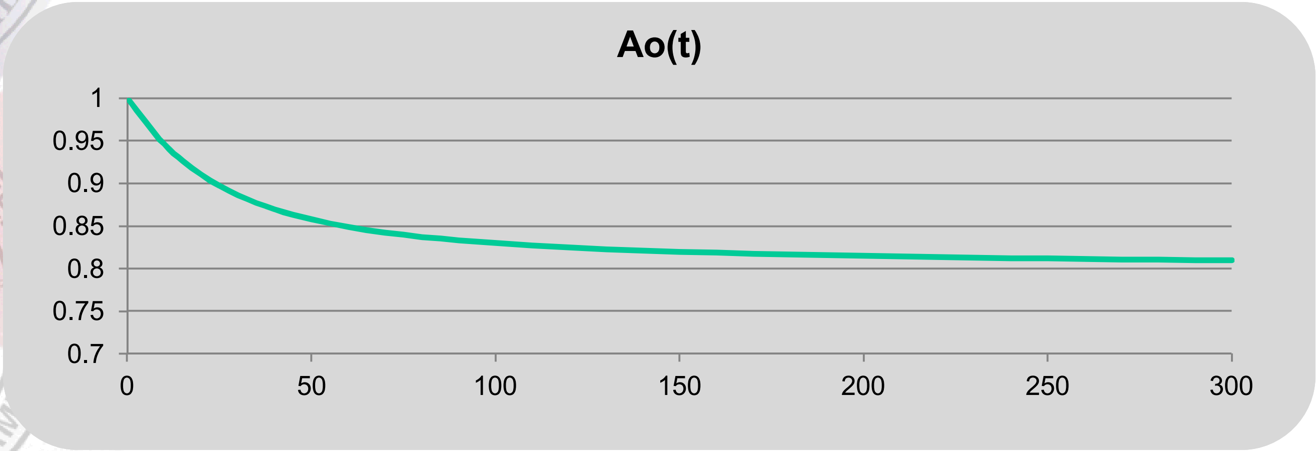
- **Consider a system with MTBDE = 75 and MDT =18.75**
- **The system will be deployed for 280 hours**
  - Its instantaneous availability is plotted below
  - This system has a steady state availability of 75/(75+18.75)=0.8



- **The system *instantaneous* availability settles very quickly to 0.8**
  - The instantaneous availability at t=50 is 0.81

Markov Example

- **The cumulative availability includes affects up to that time and approaches steady state slower**
- **If this system were to be deployed for 50 hours, the expected availability over the period from 0-50 would be about 0.86**
  - The steady state approximation result (0.80) is significantly off



- **Using the developed Ao equation, Ao(280)=0.8107**
  - At this mission length 280, Ao is approaching the steady state value but is still off by 0.01

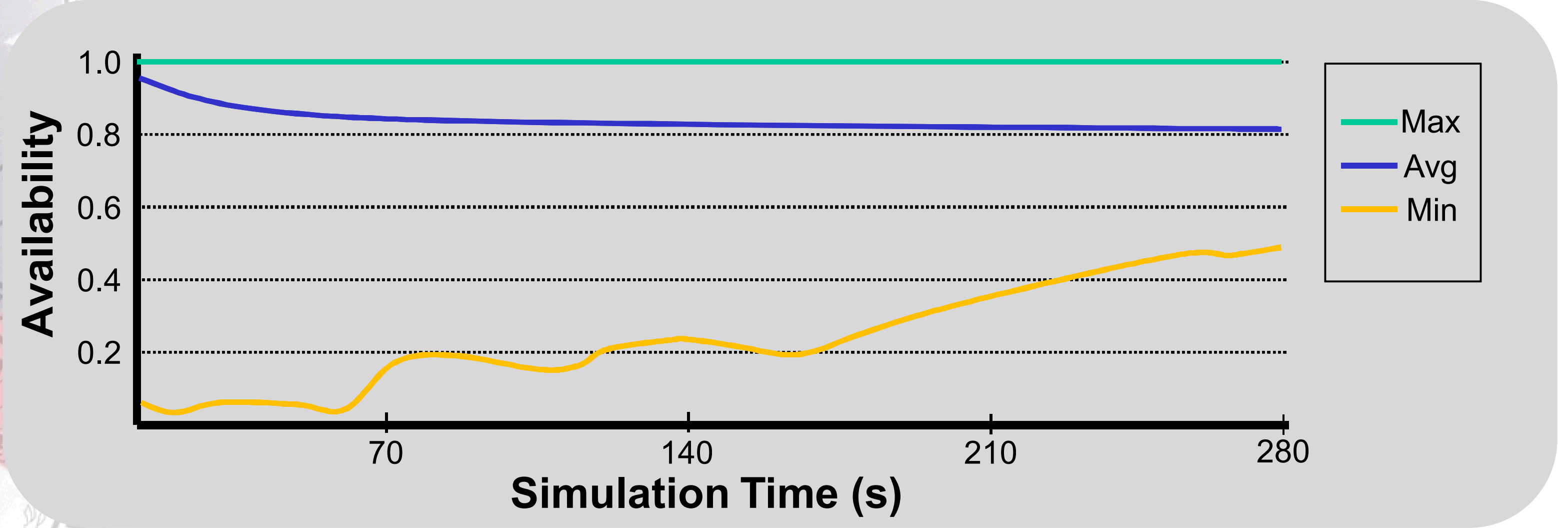
Simulation to Solve Ao

- **The two-state Markov analysis is a close approximation of a deployed system on a short mission**
- **One significant difference is in the repair distribution.**
  - The Markov analysis assumes an exponential distribution for the time to repair, while in practice, a lognormal distribution is typically used
- **Simulation allows for estimating the Ao without the need for the simplifying assumption**

Simulation of Example

- **Replicating Known Solution case**

  Failure Distribution: Exponential(75)
  Repair Distribution: Exponential(18.75)

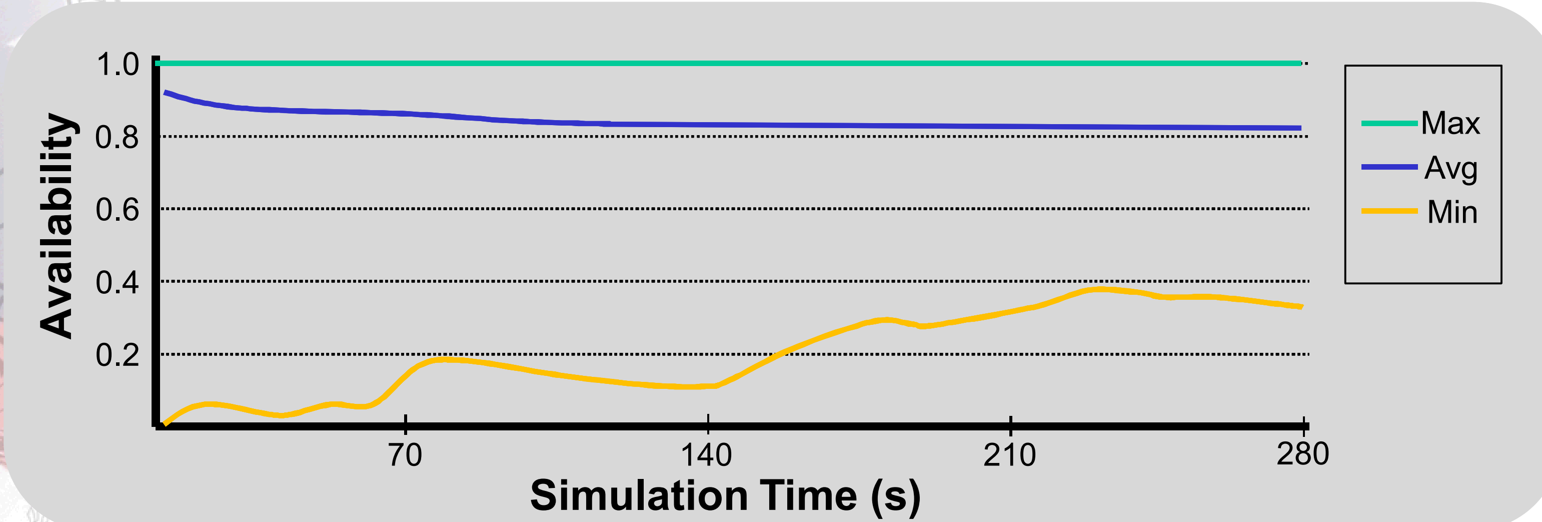


- **Results from 1000 trials**
  - Ao(280) = 0.8097 +/- .0041 (1 SEM)
  - Known solution of 0.8107 is in range of simulation result (.8056 - .8138)
  - Individual trials still reflect wide range of Ao results

Simulation of Example

- **Simulation using lognormal repair (exact solution unknown)**

  Failure Distribution: Exponential(75),
  Repair Distribution: Lognormal(18.75. 18.75)



- **Results from 1000 trials**
  - Ao(280) = 0.8141 +/- .0039 (1 SEM)
  - Result is very close to known solution for exponential repair case of 0.8107
  - Graph – Transient region very similar to exponential case

Conclusions

- The steady-state availability equation yields an Ao that can be significantly different than what is actually observed during a short mission
  - Don’t blindly apply Ao equations!
- As a good guideline, systems are not approaching steady state behavior until they have operated through four failure cycles
  - If the mission length is less than 4*MTBDE, be careful!
- If applied to short missions that start in an operable state, steady-state Ao can be too small
  - You can end up under-selling the performance of the system.