SAND2012-6678C

Joint Modeling and the Extraction of Joint Parameters

W2: Nonlinear System Identification with Applications to Structures with Mechanical Joints

IDETC, Chicago, Il
August 12, 2012

Dan Segalman and Michael Starr
Sandia National Laboratories†

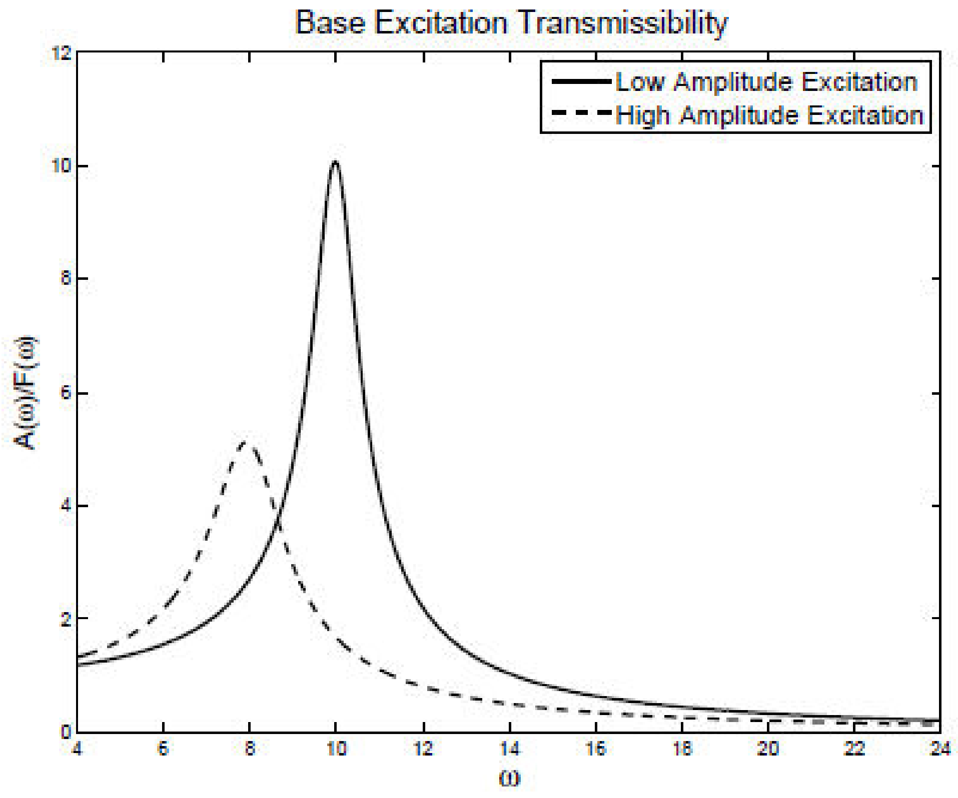
† Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Common Features Observed in Tests of Joints and Jointed Structures

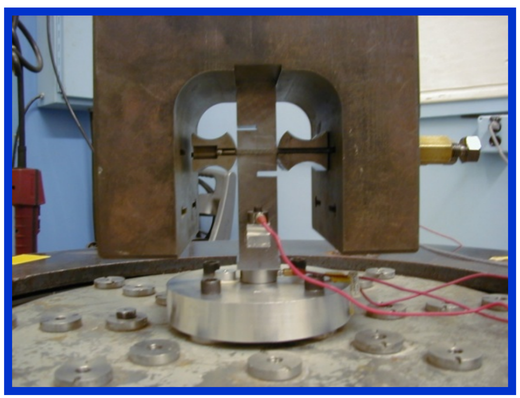
1. Under harmonic loading, joint dissipation per cycle increases approximately as a power of load amplitude and that power generally lies between 2.2 and 2.8.

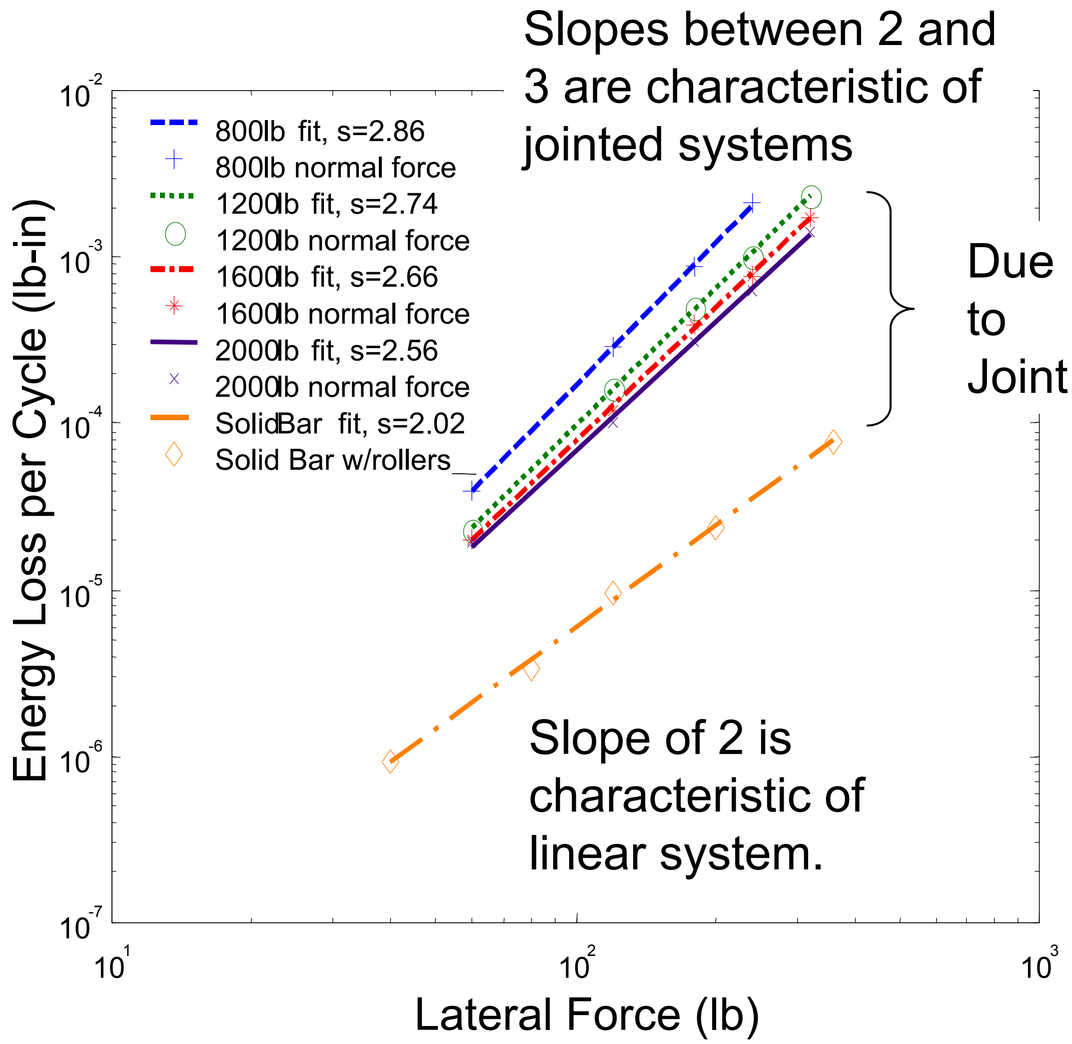
2. Also under harmonic loading and modest load amplitudes, the effective stiffness decreases slightly with amplitude of excitation.

3. Though dissipation per cycle shows a strong dependence on load amplitude, it has little rate dependence.



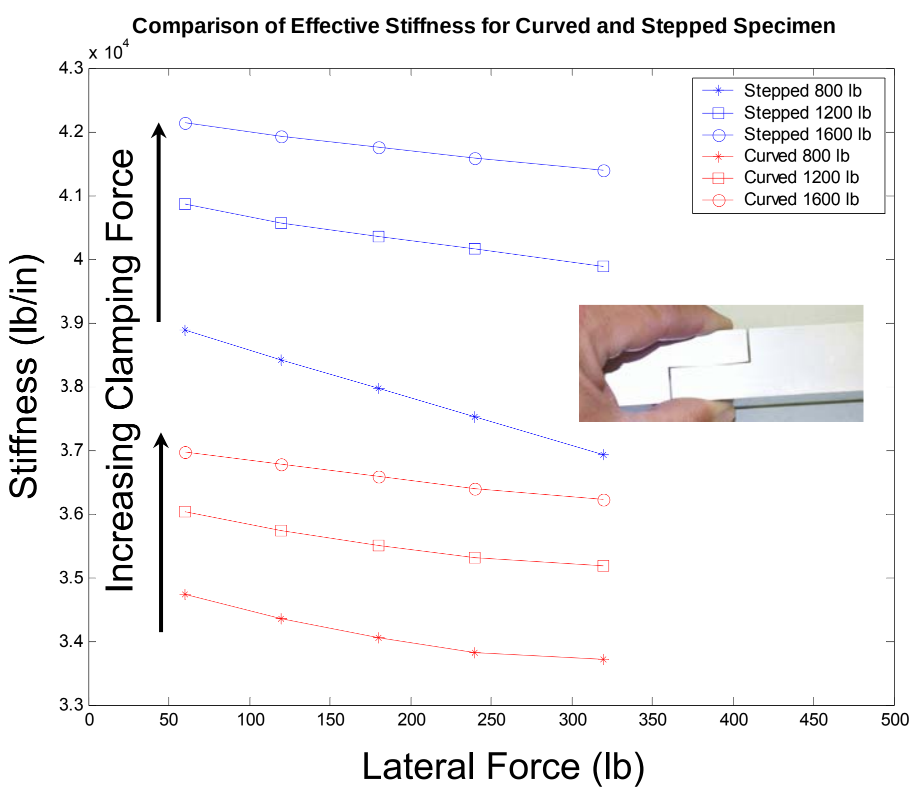
Energy Loss per Cycle for Simple Shear Loading





Energy Dissipation and Interface Stiffness Depend on Many Factors

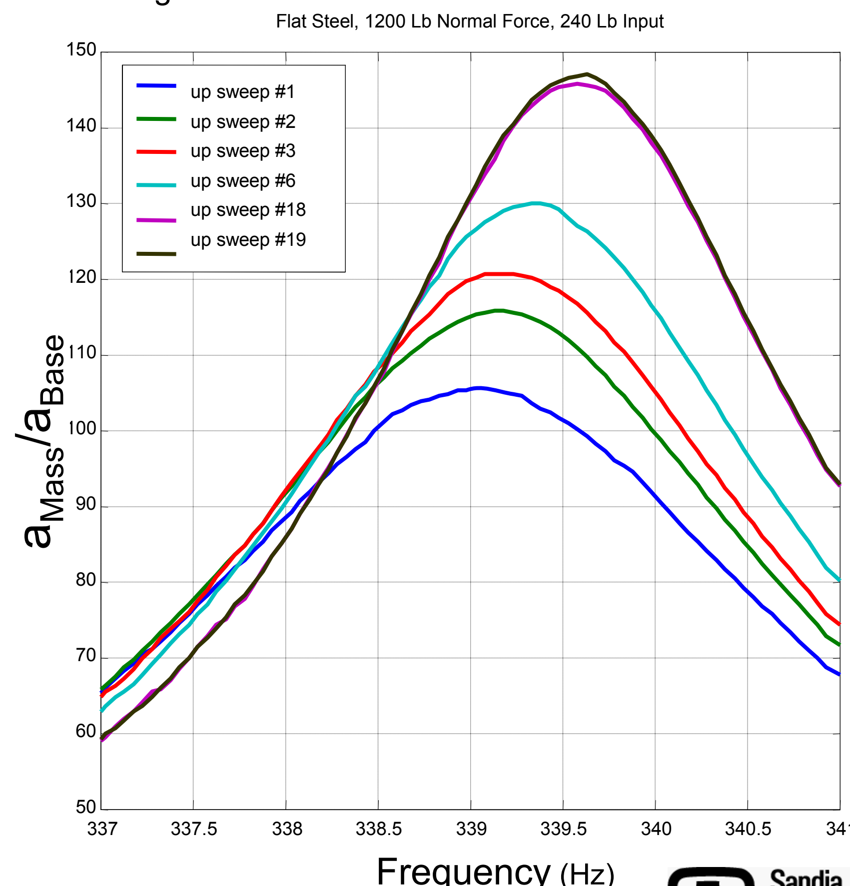
- Normal Load
- Contact Geometry
- Surface Roughness



- Load History

  Dissipation decreases, stiffness increases over first several thousand cycles.

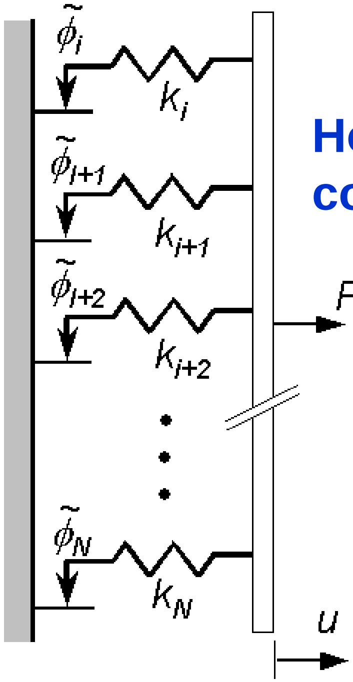
  Disassembly/reassembly returns structure to original state.



Sandia National Laboratories

Mathematical Foundations of the Joint Constitutive Model

The model is defined by a population density $\rho(\phi)$ of Jenkins elements of strength ϕ.



Homogenize into a continuous model

Monotonic loading (backbone) force

$$F = \int_0^u \phi\rho(\phi)\mathrm{d}\phi + u\int_u^\infty \rho(\phi)d\phi$$

Dissipation per cycle during oscillation

$$D = \int_0^{u_0} 4[u_0 - \phi]\phi\rho(\phi)\mathrm{d}\phi$$

Invert and solve for ρ

The selection/derivation of the distribution function, $\rho(\phi)$, allows full characterization of the system.

$$\Rightarrow \quad \rho(\phi) = -\frac{\mathrm{d}^2 F}{\mathrm{d}\phi^2} = \frac{1}{4\phi}\frac{\mathrm{d}^2 D}{\mathrm{d}\phi^2}$$

Segalman, D. J. and Starr, M.J., 2008. Inversion of Masing Models via Continuous Iwan Systems, *International Journal of Nonlinear Mechanics*, **43**, 74-80.

Modeling of Individual Joints in a Finite Element Analysis

Look for the simplest:

- rate independent constitutive model,
- which is capable of manifesting softening and dissipation behavior,
- and that lends itself to mathematical analysis.

Bauschinger, Prandtl, Ishlinskii, Iwan Model (BPII) form:

$$f(t) = \int_0^\infty \rho(\phi)[u(t) - x(t,\phi)]\, d\phi \qquad \dot{x}(t,\phi) = \begin{cases} \dot{u} & \text{if } |u - x(t,\phi)| = \phi \text{ and } \dot{u}\,(u - x(t,\phi)) > 0 \\ 0 & \text{otherwise} \end{cases}$$

The joint properties are characterized by $\rho(\phi)$, which has the following properties:

- Nearly linear at low amplitude.
- Physically reasonable model capable of representing any Masing plasticity model.
- Manifests micro- and macro-slip and power law energy dissipation.
- Creation of a 4-parameter expression for $\rho(\phi)$ fits experimental data.

Sandia National Laboratories

Deducing Model Parameters from Experiments

From laboratory testing:

$$D(F_0) = \upsilon F_0^{\alpha} \qquad \text{where} \qquad 2 < \alpha < 3$$

The manifestation of power-law dissipation behavior requires:

$$\rho(\phi) \approx R\phi^{\chi} \qquad \text{where} \qquad \chi = \alpha - 3$$

An assumption of small deformations provides an expression for tangent stiffness at low loads:

$$F(t) = u(t)\int_0^{\infty} \rho(\phi)d\phi \qquad \longrightarrow \qquad K_T = \int_0^{\infty} \rho(\phi)d\phi$$

The Sandia 4-Parameter Model is populated with data as described above and from the force required to initiate macroslip

$$\rho(\phi) = R\phi^{\chi}\left(H(\phi) - H(\phi - \phi_{\max})\right) + S\delta(\phi - \phi_{\max})$$

This model simplifies to Palmov's model, a three parameter BPII model:

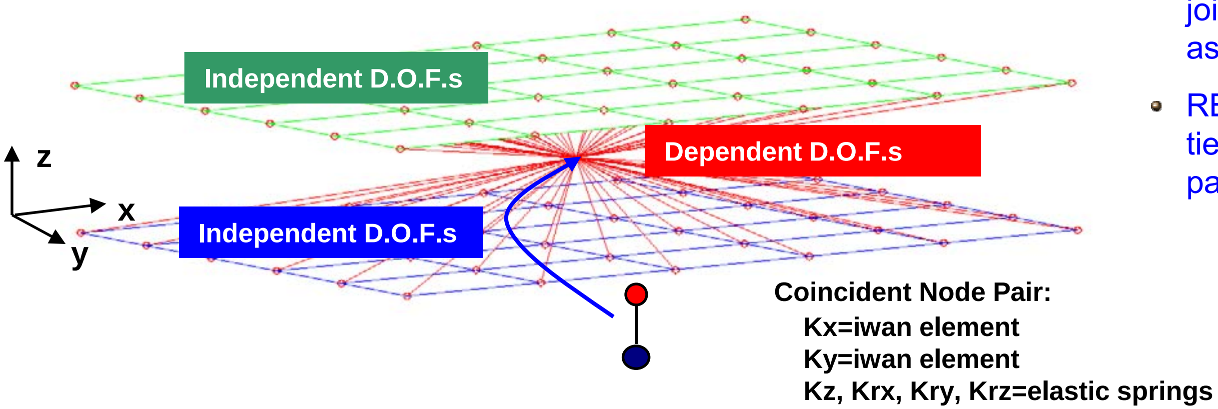
$$F(t) = K_T u(t) + \int_0^{\infty} R\phi^{\chi} x(t,\phi)d\phi$$

Sandia National Laboratories

Whole-Joint Constitutive Model is a Significant Departure From the Standard Damping Approach

- Standard approach for modeling energy dissipation in structural dynamics models is through proportional/modal damping parameters. Main short-coming - models are only useful for a limited operating range (loads/boundary conditions) over which the calibration was performed.
- Relaxed whole-joint model uses RBE3 multi-point constraint equations to define motion of joint2g nodes as an interpolated average of all the nodes specified in the contact patches, (overlapping contact patches do not result in model over constraint).
- Weighting functions are used to tailor the lateral interface stiffness and resulting shear stress profiles.

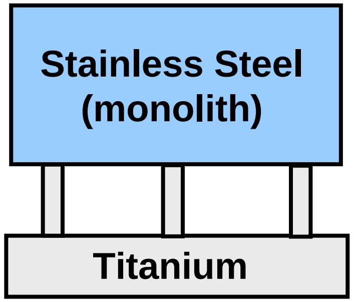
Relaxed Whole-Joint Attachment Method



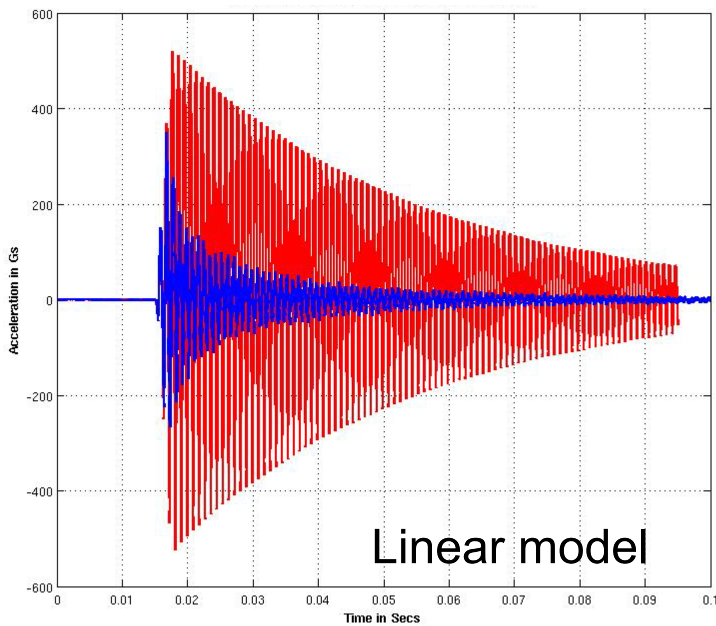
- Coincident node pair referenced via joint2g. Independent D.O.F. associated with these nodes.
- RBARs or RBE3 elements used to tie nodes in pre-defined contact patch zones to coincident node pair.

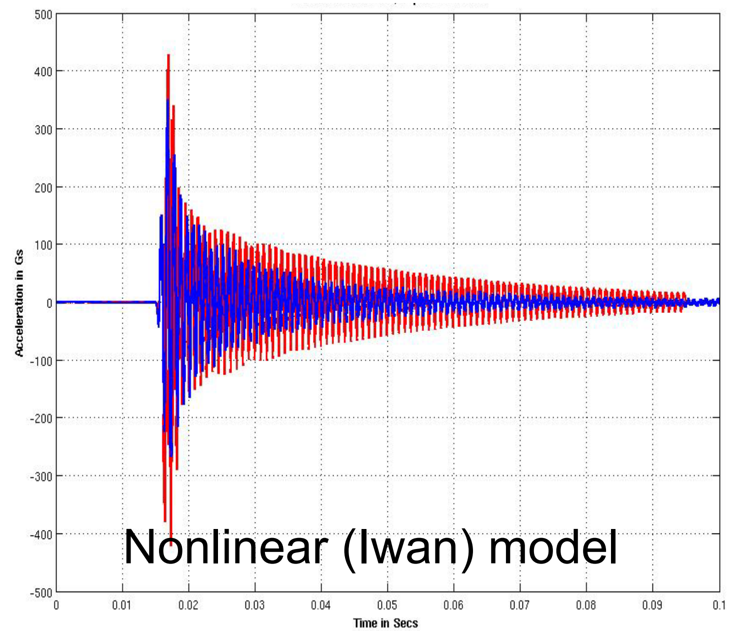
Sandia National Laboratories

Linear and Nonlinear Models Calibrated for Low Load Microslip Predict Much Different Responses at Higher Input Levels



Acceleration predictions at interface joints: Ti-SS 3-leg hardware with shaker dynamics





Experiment

Model

Sandia National Laboratories

Limitations Associated With Modeling Discrete Joints in Built-up Structures

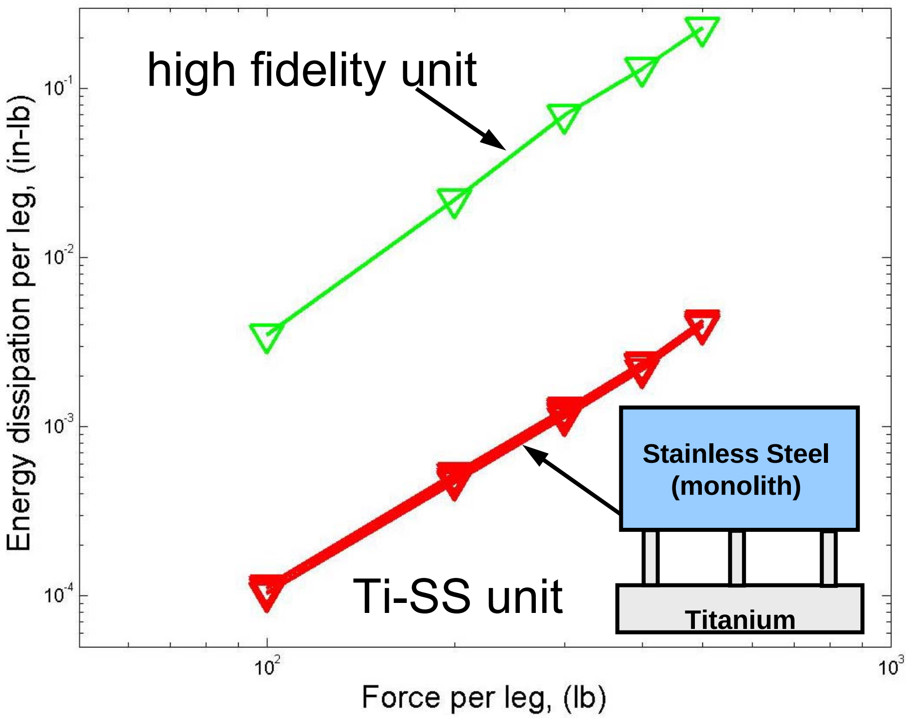
1. Parameterization is required for every joint in the structure.

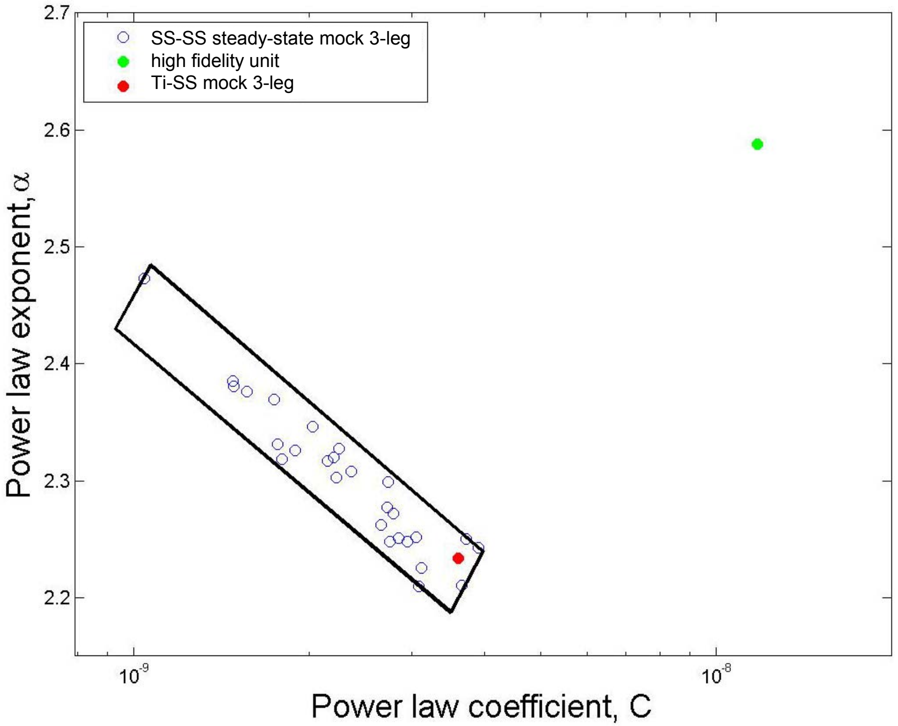
2. These models are still simple and fundamentally one-dimensional.

3. Incorporation into a finite element model quickly turns the problem intractable.

4. High fidelity models and nonlinearity hamper convergence.

Energy Dissipation of Related Systems is Significantly Different





- The dissipation of the high-fidelity unit is very joint-like in nature.
- That dissipation is much more than can be explained by the leg joints alone.

A Proposed Approach to Distributed Nonlinearities

The elementary notion presented here relies on the following observations:

- Even under loads sufficient to cause structures to manifest significant nonlinearity, amplitude-dependent damping and apparent softening, linear eigenmodes generally appear to be preserved.

- Coupling among the modes generally does not appear to become significant until very high loads. (Violation of this is most easily observed when modes appear to be complex.)

- By segregating response modally, we may choose to treat only a subset of those modes (presumably those for which we have some data) in a nonlinear manner and to treat the remainder more conventionally.

Assumptions propagated into a nonlinear modeling approach:

1. Modal forces excite only corresponding modal responses.
2. Modal coordinates evolve according to some simple nonlinear constitutive model.
3. The nonlinear modal constitutive response resolves to linear in the limit of small loads.

Strategy to Model Such Damping and to Incorporate into Structural Dynamics in a Tractable Manner

- Assumption

  The joint cumulative forces project onto only the first H eigen modes.

- Assumption

  When projected onto those eigen-modes, the joint forces have the following diagonal form

$$\Phi^T \left(\Delta K + F^J \right) = \int_0^\infty \mathrm{diag}\left(\left\{ \rho_k \left(\phi \right) \right\} \right) \beta(t,\phi) d\phi$$

where

$$\dot{\beta}_k(t,\phi) = \begin{cases} \dot{\alpha} & \text{where } \dot{\alpha}\left(\alpha_k - \beta_k\right) > 0 \text{ and} \left|\alpha_k - \beta_k\right| = \phi \\ 0 & \text{otherwise} \end{cases}$$

and α_k *is the* k^{th} *modal coordinate*

Implementation in Transient Analysis

$$M\ddot{u} + C\dot{u} + Ku =$$

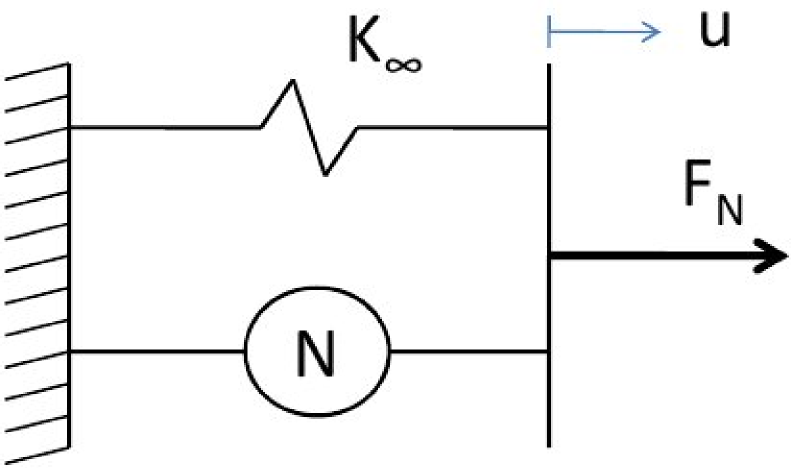
$$F^X + M\Phi \underbrace{\int_0^\infty \text{diag}\left(\left\{\rho_k(\phi)\right\}\right)\beta(t,\phi)\,d\phi}_{\text{Iwan joint force in modal coordinates}}$$

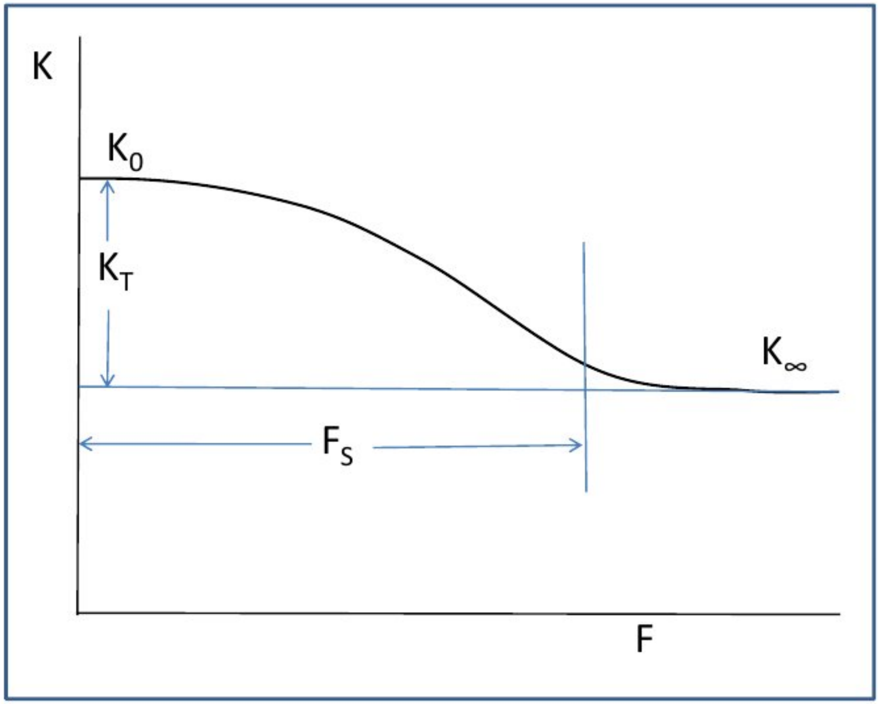
Iwan joint force in modal coordinates

Steps

1. Project physical nodal to get $\dot{\alpha} = \Phi^T M\dot{u}$
2. Evaluate β vector
3. Evaluate modal force vector ϕ
4. Project back to physical space $F^J = M\Phi\phi$

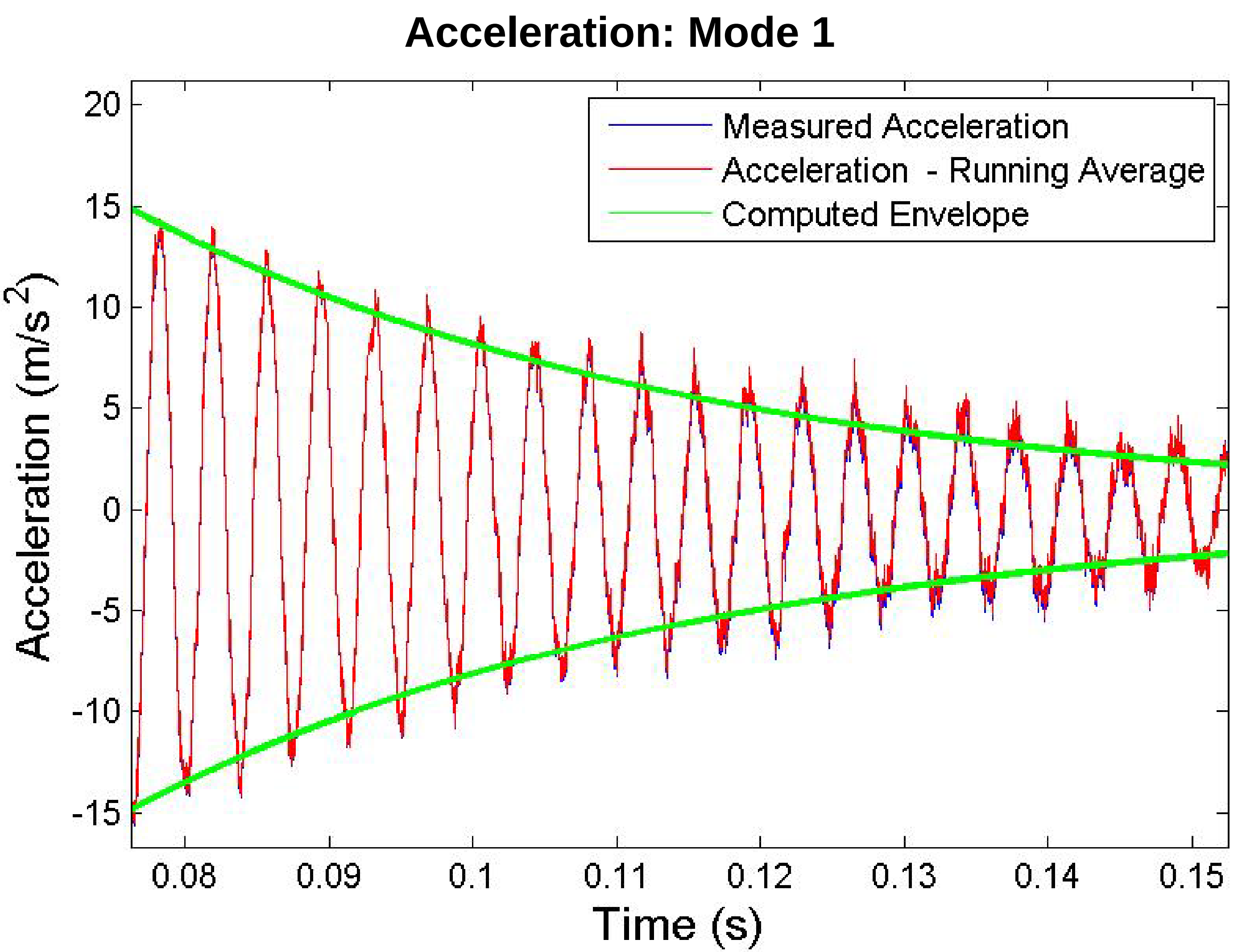
Deducing Modal Parameters for Distributed Damping





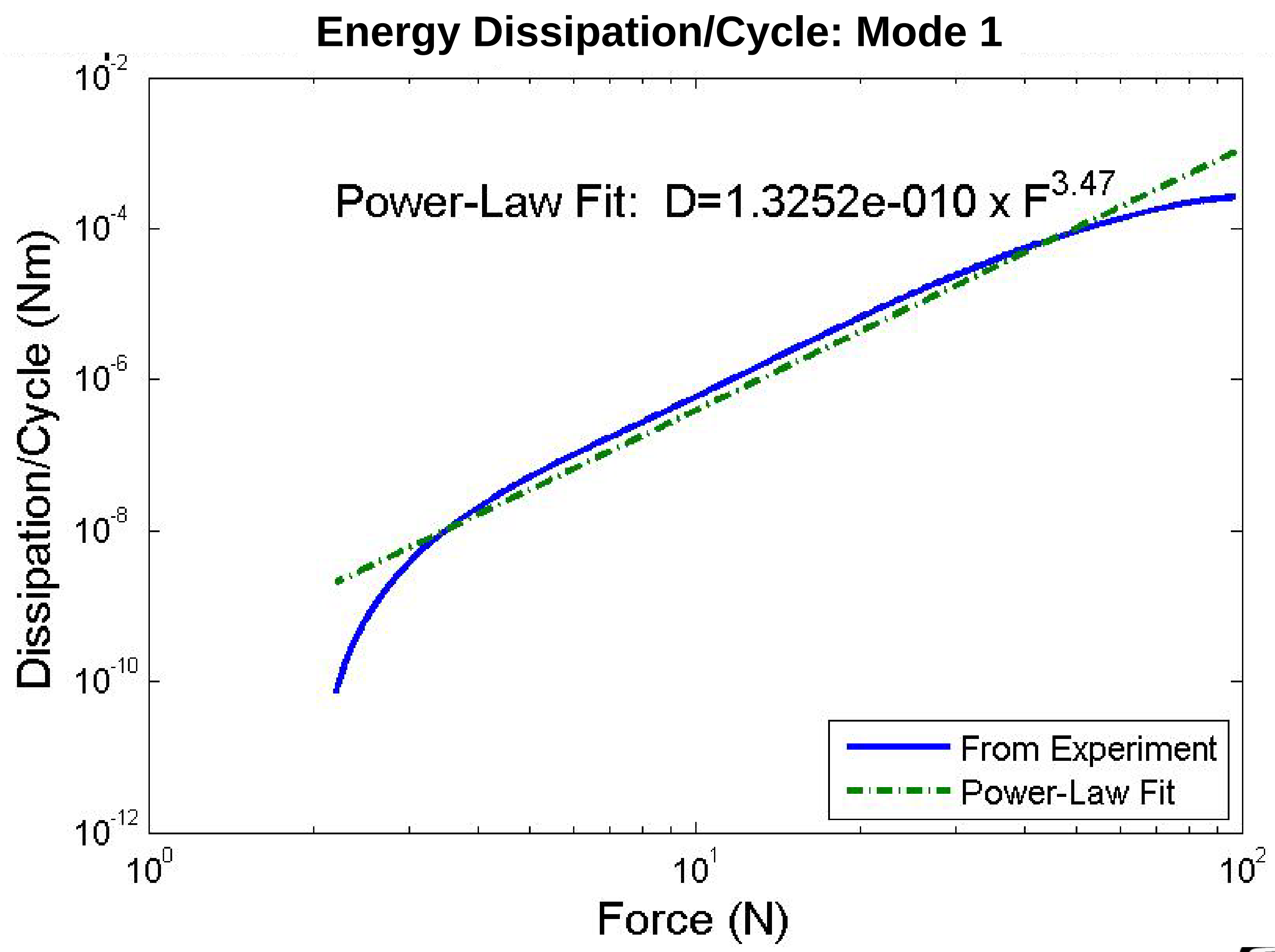
- At each mode, the generalized joint stiffness is assumed small compared to that of the underlying structure

Isolation of Modal Acceleration Signals from Experiment



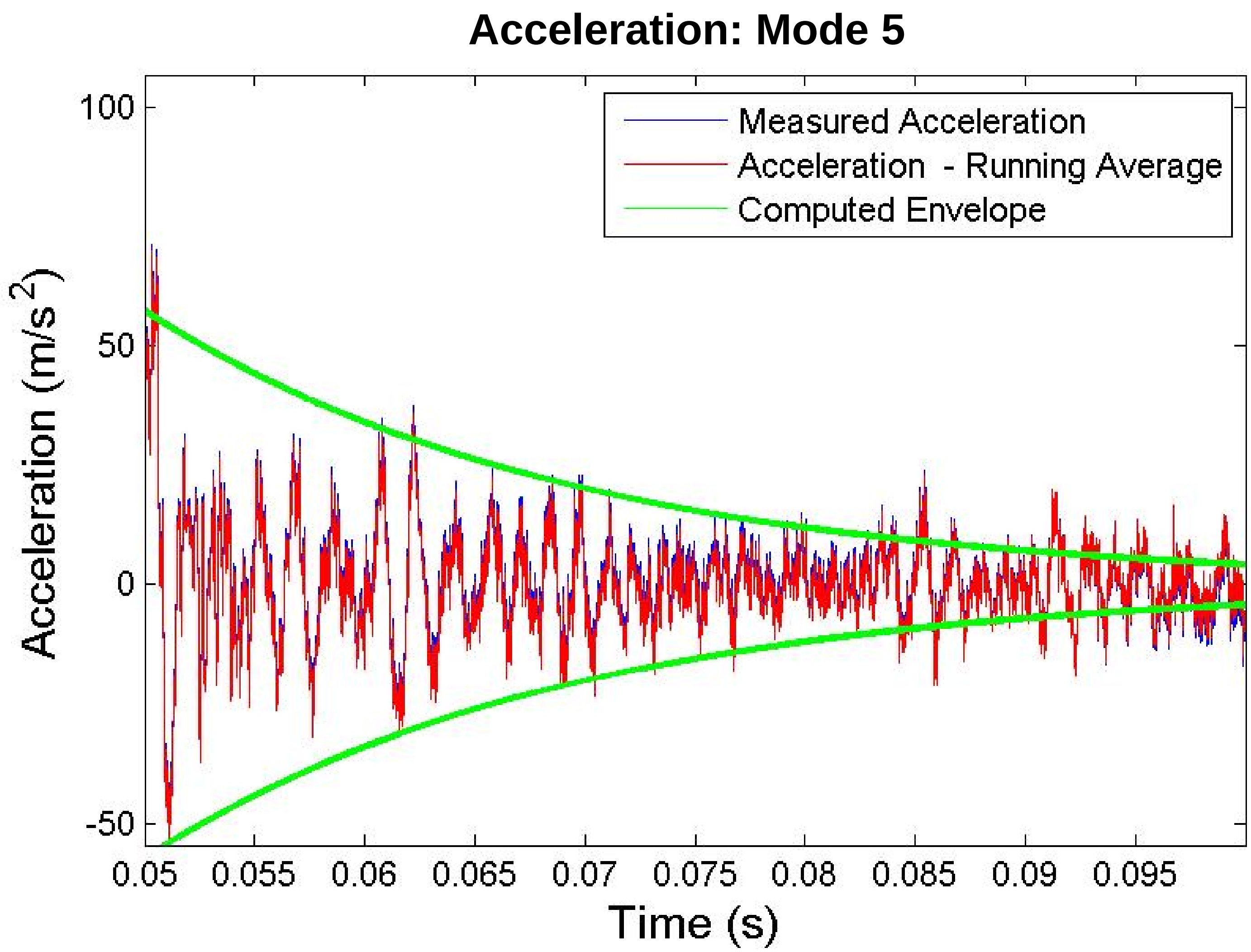
Sandia National Laboratories

Deduce Energy Dissipation as a Function of Net Modal Force

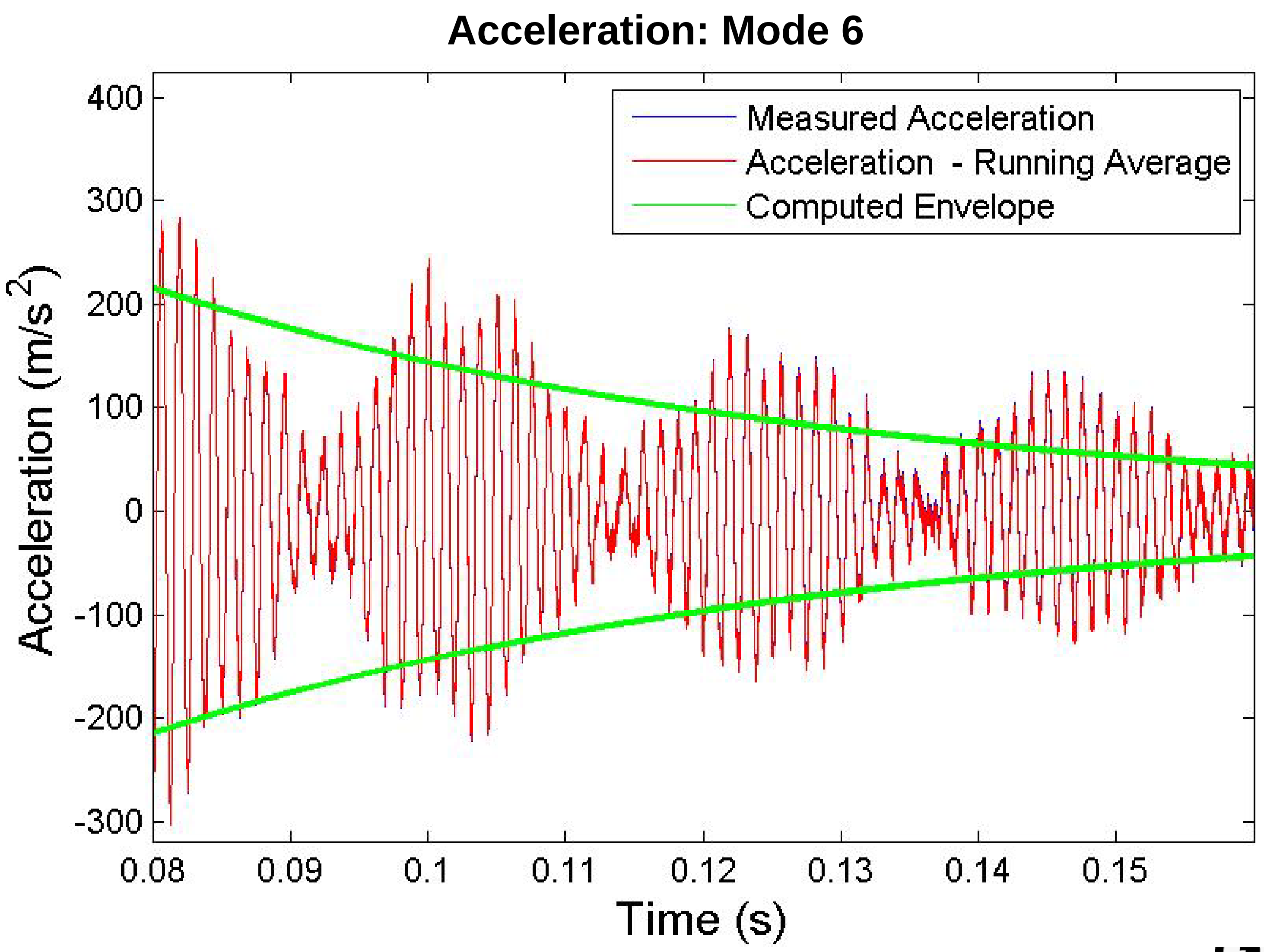


Sandia National Laboratories

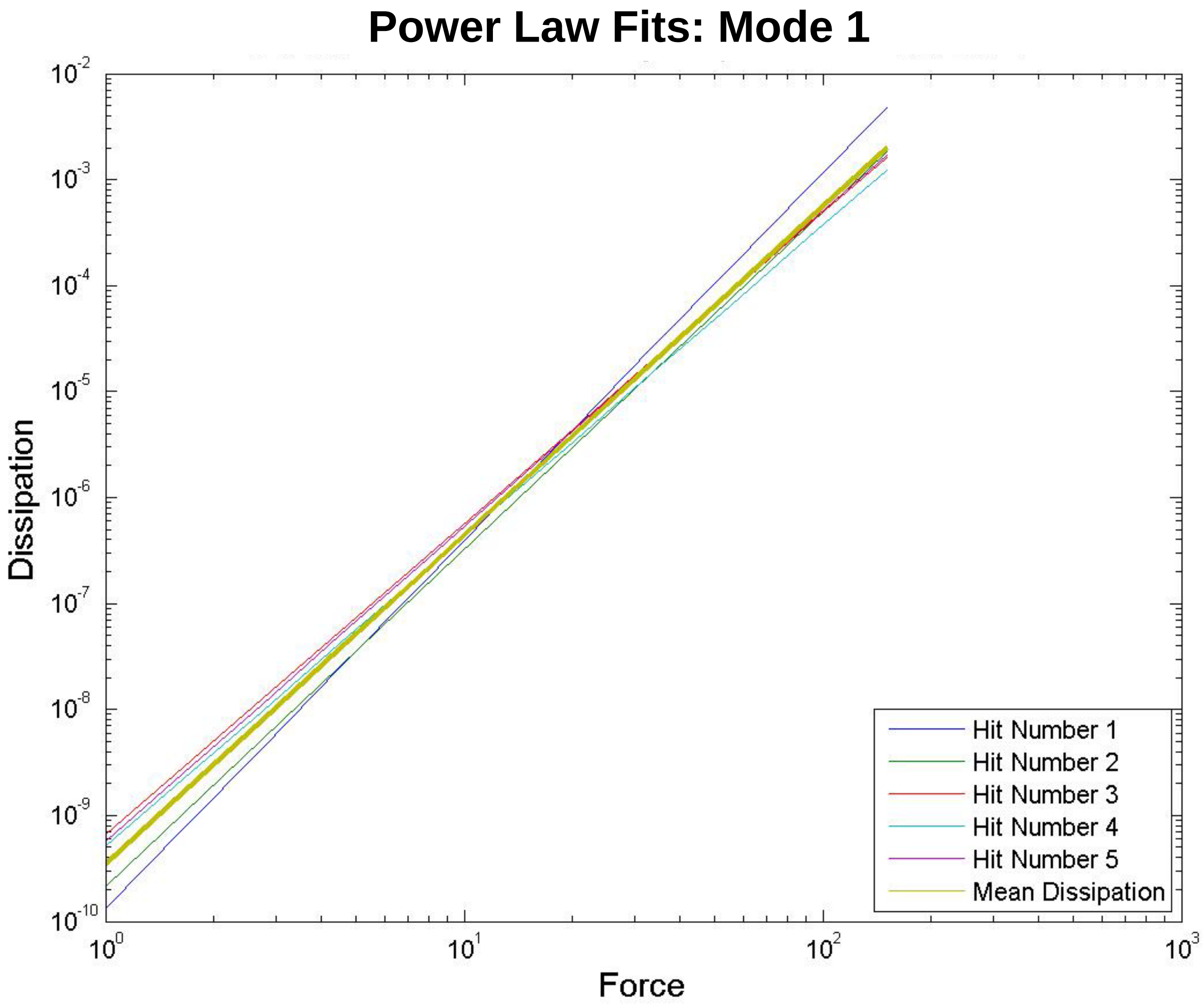
Some Signals are Not As Clean



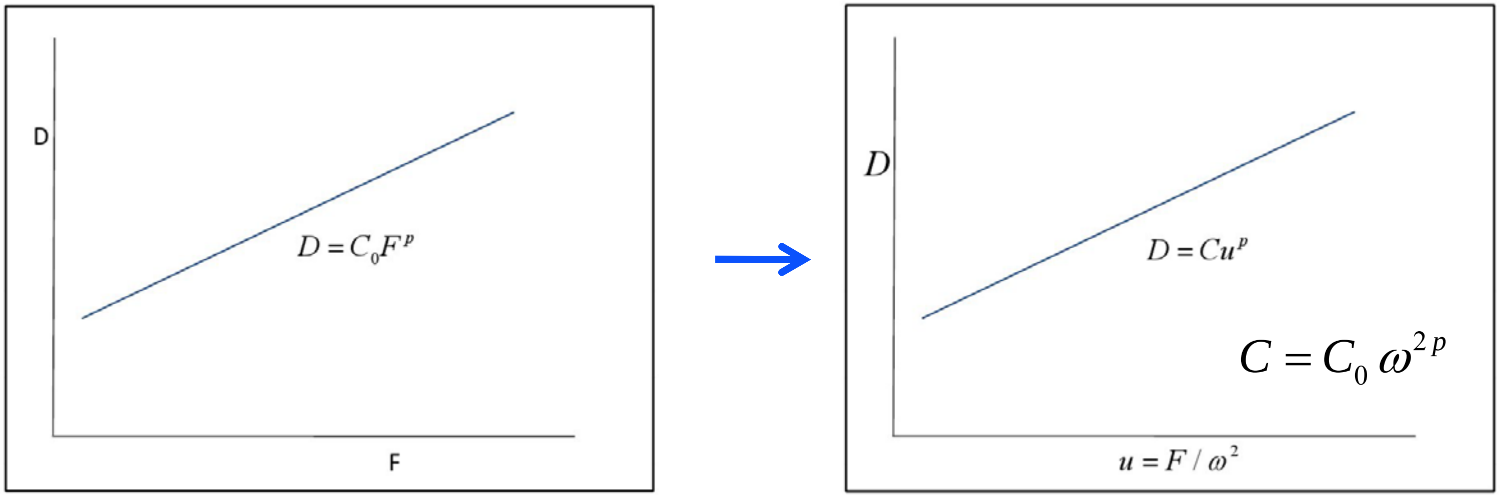
Some Signals are Not As Clean



Variability in Experimental Data is Manifest in Power-Law Dissipation Fits



Dissipation vs Net Force From Experimental Data



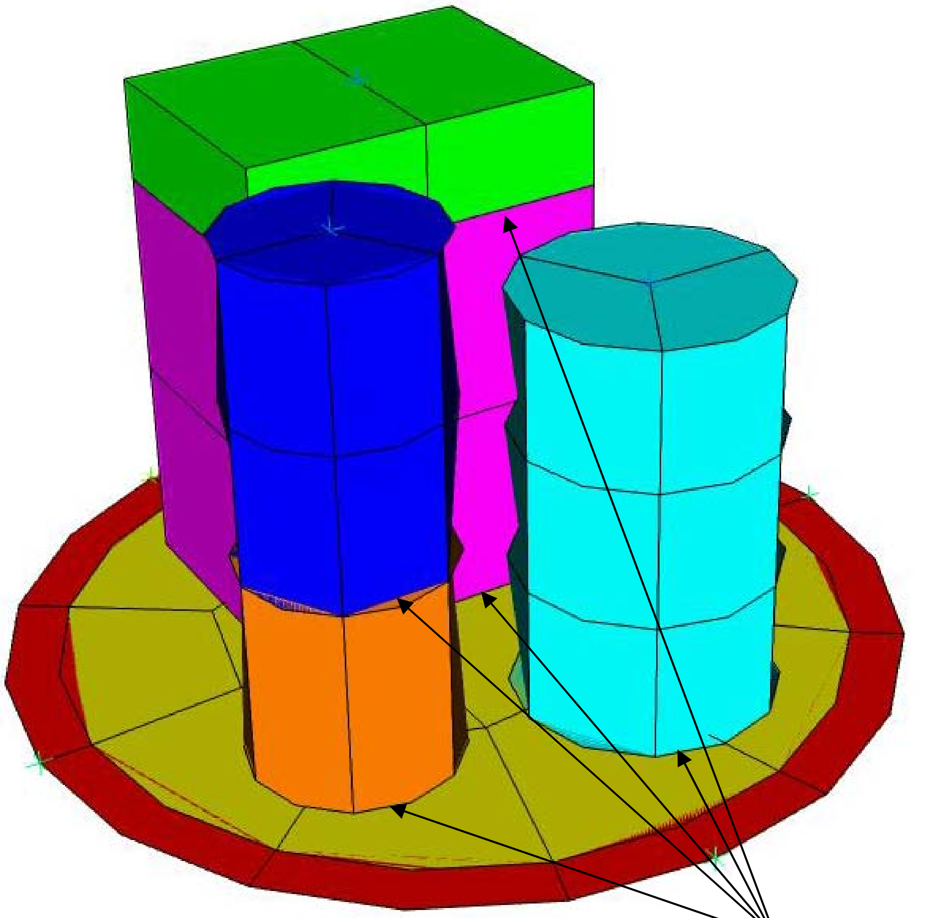
- Experiment yields dissipation vs net force
- Joint displacement is expressed in terms of force
- Dissipation vs joint displacement provides two joint parameters: R, χ
- Set macro-slip above experimental levels of joint displacement

$$\phi_{\max} = 2 \max_t u$$

- Set β so that $K_T \approx K_0$: β =2

Extraction of Palmov Constitutive Parameters from University of Illinois Benchmark Structure

A Simplified Subsystem Test Case

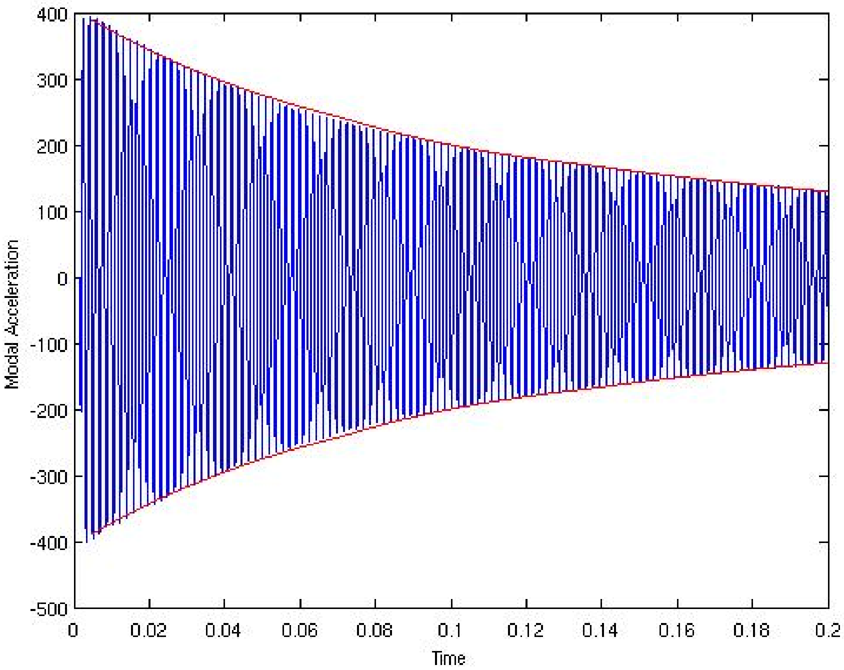


- A very coarse mesh of a generalized multi-component flanged system was created.
- A nonlinear transient analysis was performed. The model was excited using discrete Iwan interfaces at low force input level.
- Distributed Iwan parameters were deduced from these low-level numerical tests.

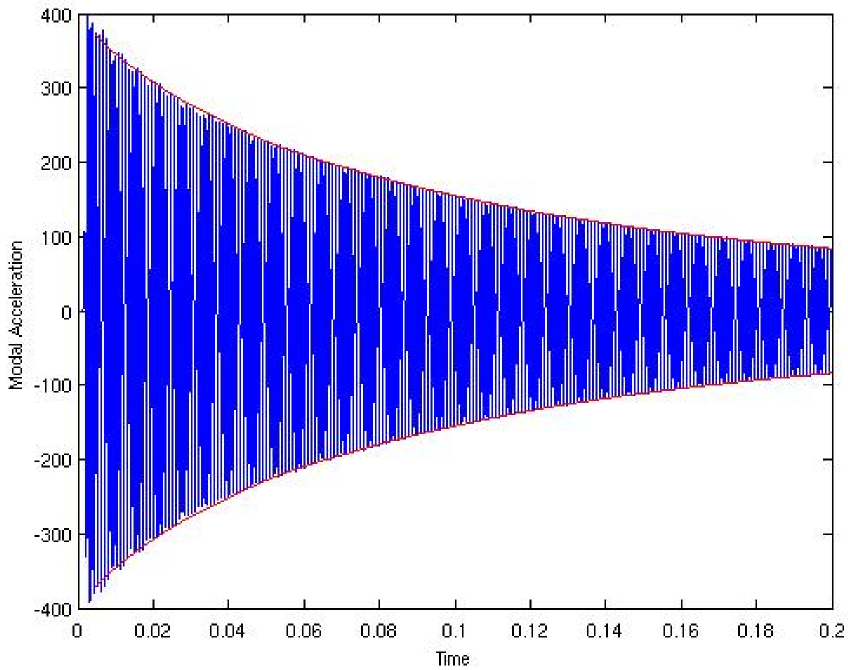
Sandia National Laboratories

Approximate Modal Acceleration Time Histories Were Extracted and Fit With Convex Envelopes

Mode 9:



Mode 11:

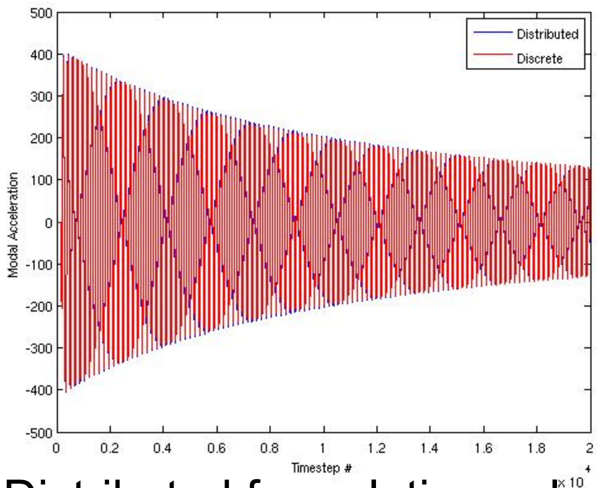


- **Model parameters were determined from these envelopes for the first 20 modes.**
- **This distributed Iwan formulation was used to perform a nonlinear transient simulation of the subsystem in Salinas.**

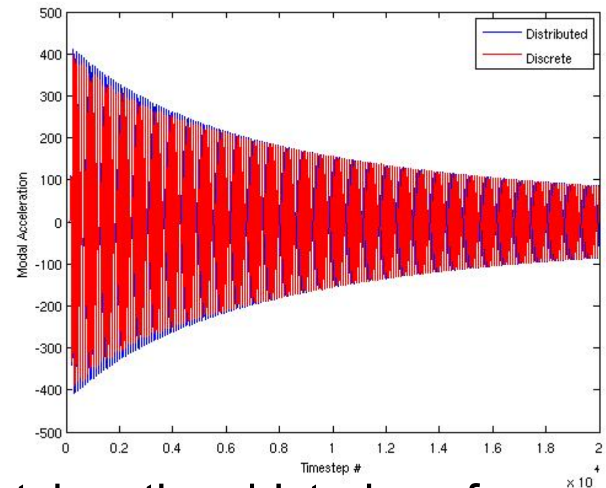
Sandia National Laboratories

Distributed Formulation Matches Time Histories of Important Modes Well

Mode 8

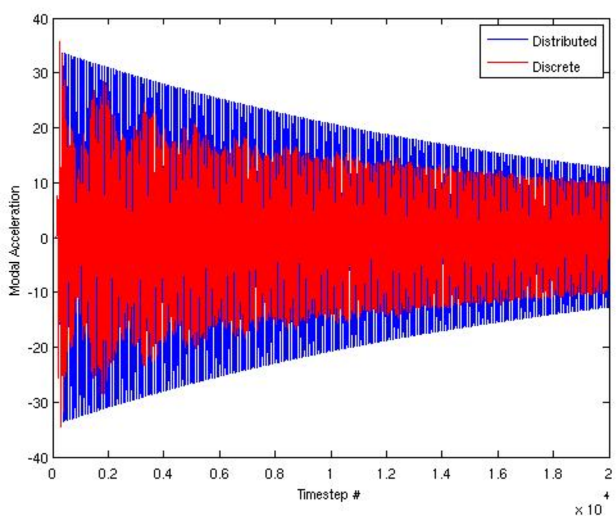


Mode 11

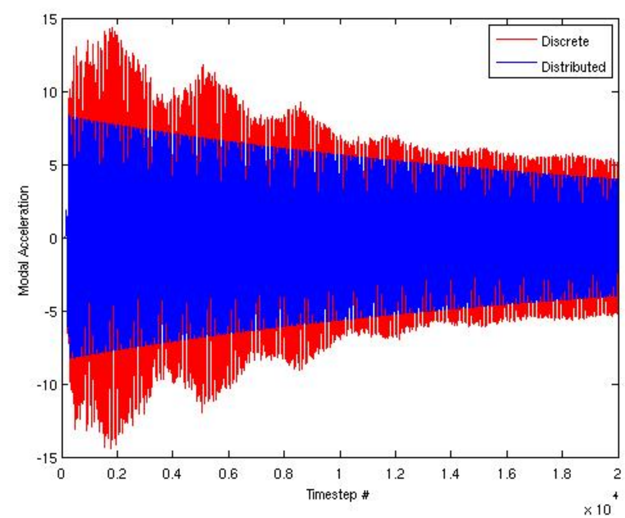


- Distributed formulation only roughly matches time histories of some modes, but these are generally less important modes

Mode 14



Mode 17



Sandia National Laboratories

Fitting a 3-Parameter Palmov Model

Recall the Palmov model, a three parameter BPII model:

$$F(t) = K_T u(t) + \int_0^\infty R\phi^\chi x(t,\phi)d\phi$$

From monochromatic resonance data we can deduce the stiffness of all of the retained modes:

$$K_{T,i} = \omega_i^2$$

Also determined from those resonance experiments is energy dissipation per cycle as a function of force amplitude. In harmonic motion energy dissipation in terms of modal force is

$$D \approx C_i \psi^{\chi_i + 3} \qquad \text{where} \qquad C_i = \frac{4R_i}{K_{T,i}^{3+\chi_i}(2+\chi_i)(3+\chi_i)}$$

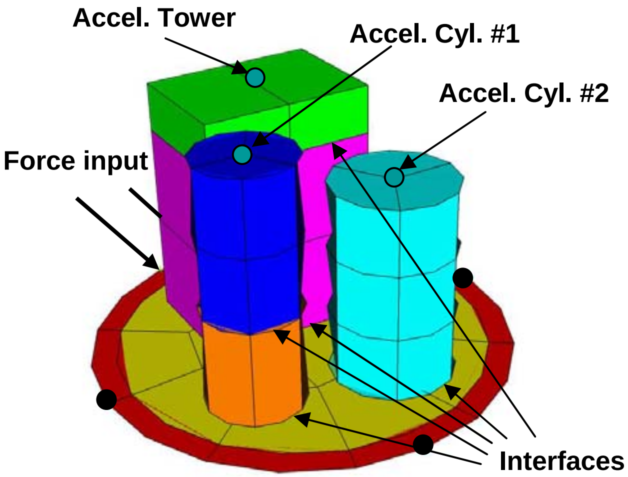
Sandia National Laboratories

Modal Palmov Parameters From Ring-Down Experiments

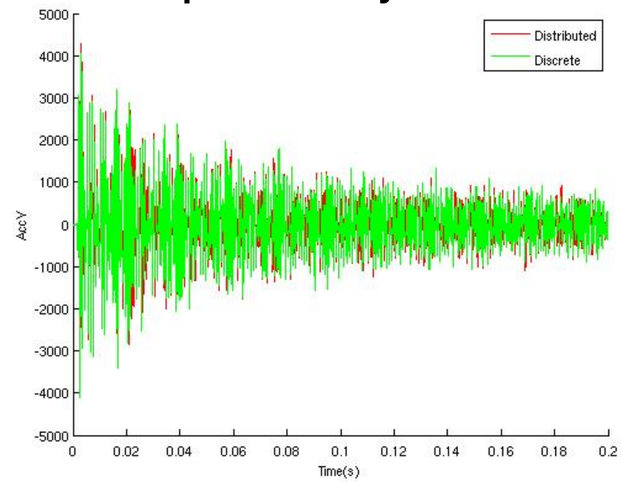
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
$K_{T,i}$	0.18016	0.18017	1.4771	1.4779	3.0388
χ_i	-0.506	-0.5	-0.52957	-0.5	-0.4489
R_i	8.91×10^{-6}	8.75×10^{-6}	9.19×10^{-4}	9.35×10^{-4}	2.88×10^{-3}

Physical Accelerations From the Discrete and Distributed Formulations Agree Well

Comparison at calibration acceleration level



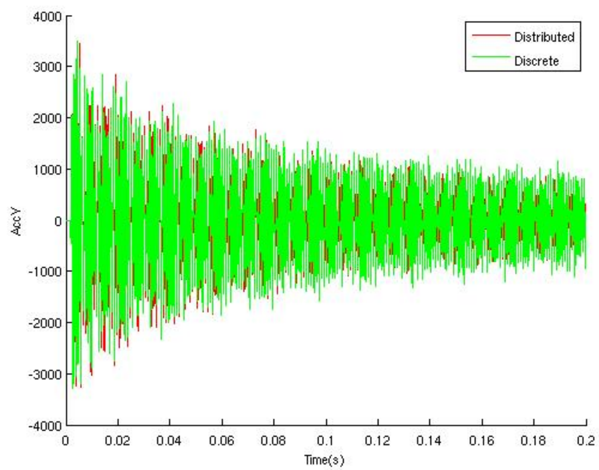
Response on cylinder 1



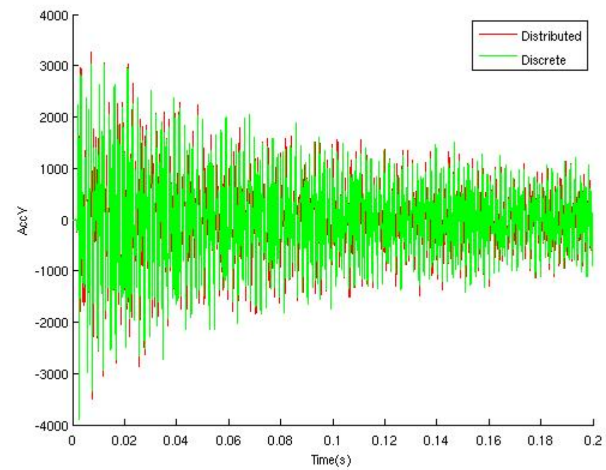
Numerical experiment response

Distributed Iwan prediction

Response on tower

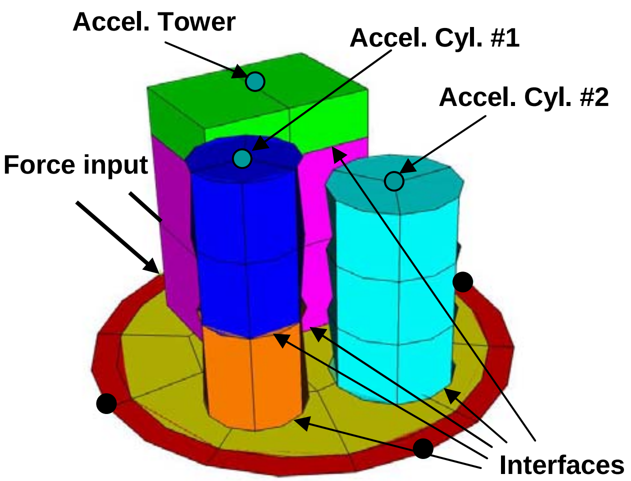


Response on cylinder 2

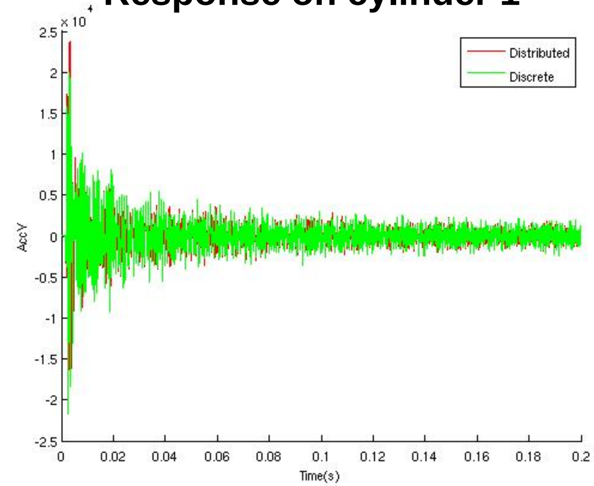


The Distributed Formulation is Accurate Across a Wide Range of Input Levels

Comparison at 5X calibration acceleration level

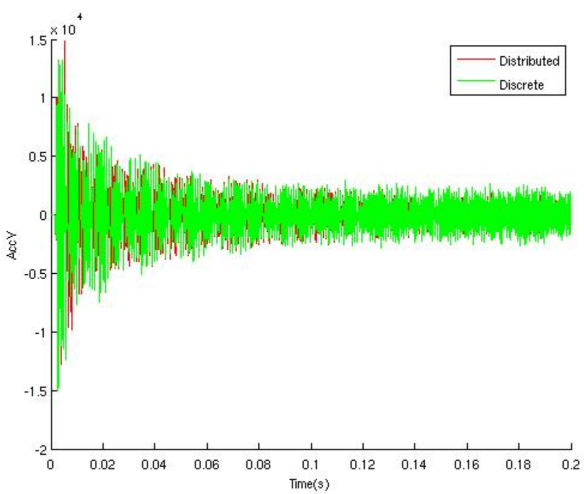


Response on cylinder 1

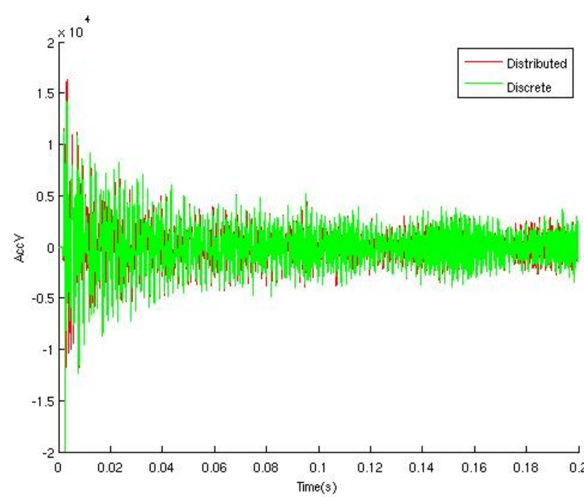


Numerical experiment response (green)

Response on tower



Response on cylinder 2



Distributed Iwan prediction (red)

Sandia National Laboratories