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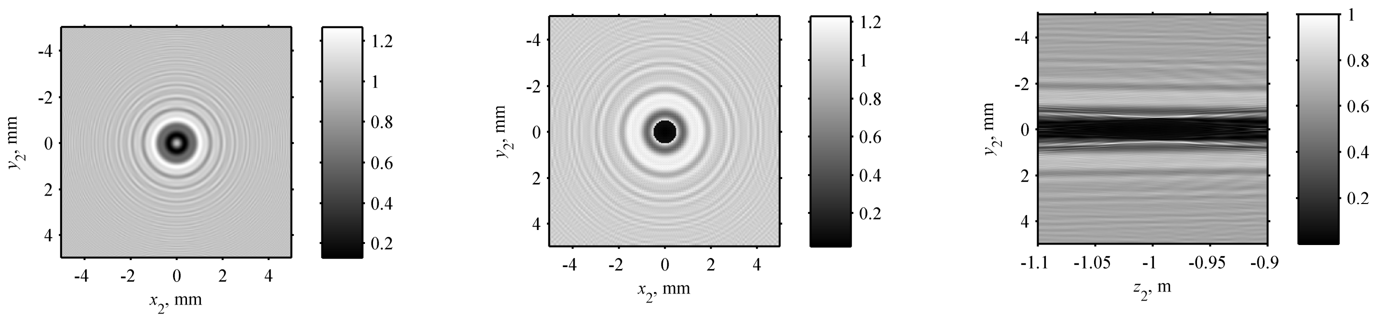

Digital holography reconstruction algorithms to estimate the morphology and depth of non-spherical, absorbing particles

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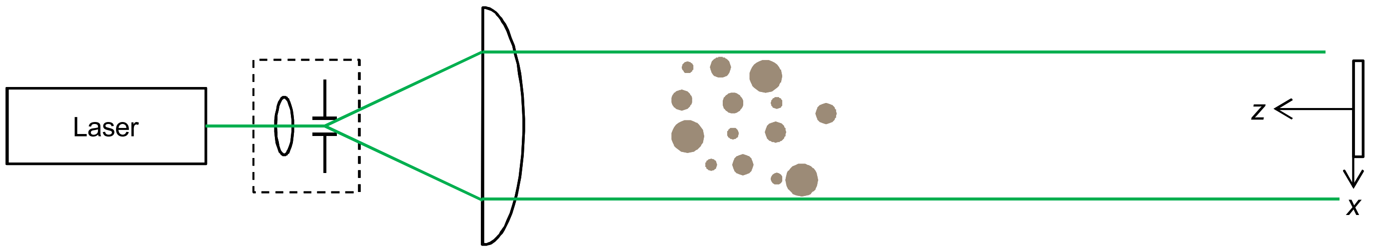
Digital holography of particle fields



Goal: Determine particle location, size and shape in a 3D field

- Record object wave + reference wave: $|E_o + E_r|^2$
- Multiply intensity by complex conjugate of the reference wave: $|E_o + E_r|^2 \cdot E_r^* = \underbrace{(E_o^2 + E_r^2)E_r^*}_{\text{DC term}} + \underbrace{E_r^{*2}E_o}_{\text{virtual image}} + \underbrace{E_r^2 E_o^*}_{\text{real image}}$
- Numerically propagate to the original particle locations
- Measure particle properties from the "in-focus" images

Digital holography of particle fields

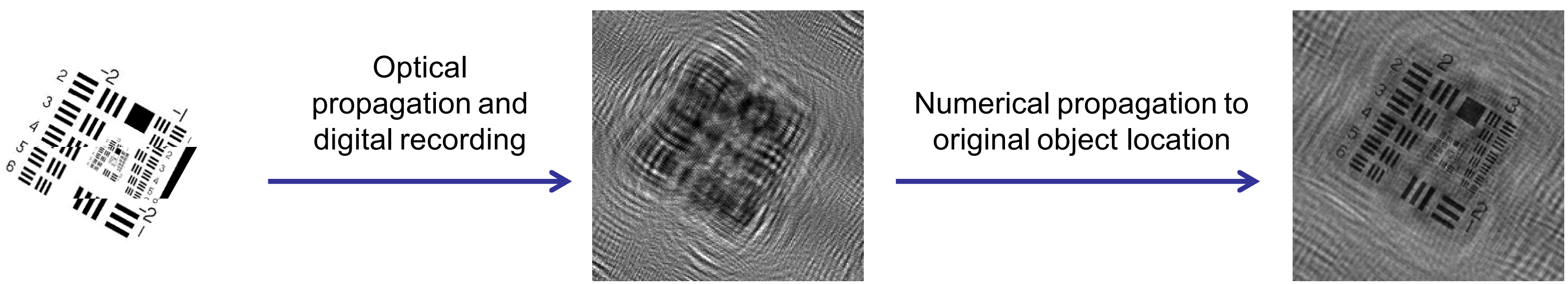
Advantages:

- 3D information from a single image/view
- Simple optical configuration
- Capture non-spherical particles

Potential diagnostic for high-speed shrapnel fields

Challenges:

- Out of focus virtual image adds noise to reconstructed signal
- Large depth of focus due to relatively large pixel sizes
- Particle detection algorithms are not yet mature and uncertainties are often unknown



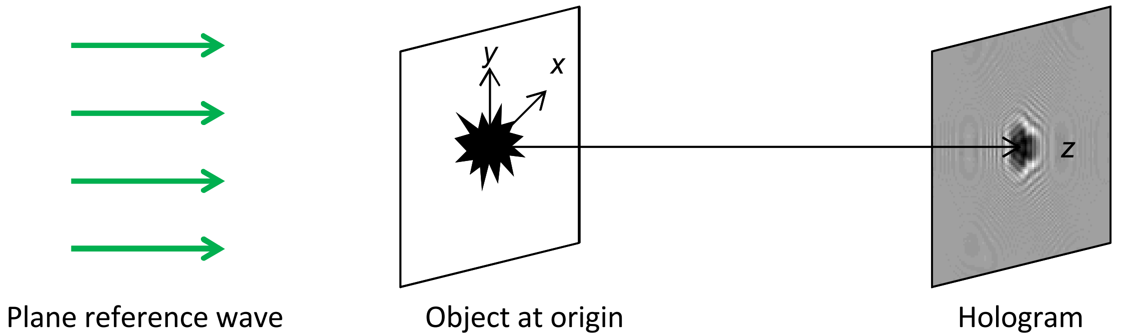
Object | Recorded hologram | Reconstructed, in-focus image

Project goals

1. Simulate holograms with known particle size and position
2. Evaluate accuracy of particle detection techniques
   - Focus on non-spherical, opaque particles
3. Propose improved techniques
4. Experimentally validate results

Hologram simulation

Goal: Simulate holograms of individual particles with known size and position



Diffraction equation: $E(x,y;z) = E(x,y;0) \otimes g(x,y,z)$

- $g(x,y,z)$ is the diffraction kernel (Rayleigh-Sommerfeld, Fresnel, etc.)

1. Discrete solution: $E(x,y;z) = \mathfrak{I}^{-1}\left\{\mathfrak{I}\left\{E(x,y;0)\right\}\mathfrak{I}\left\{g(x,y,z)\right\}\right\}$
   - Errors arise from discretization, signal windowing, periodicity of FFT
   - Governing equation for coherent light propagation in digital holography

Hologram simulation

2. Exact solution to (somewhat simpler) Fresnel equation

- Calculated from known solutions for an aperture combined with Babinet's principle
  - E.g. disk, radius a: $I_0(\rho;Z_a)/I_r=|1-j\pi\exp\{-j\pi\rho^2/Z_a\}[L(u,v)-M(u,v)]/Z_a|^2$
    - I_0 is the intensity at the hologram plane
    - I_r is the intensity of the reference wave
    - $Z_a = \lambda z/a^2$ is non-dimensional distance
    - $\rho = (x^2+y^2)^{1/2}/a$; $u = 2/Z_a$; $v = 2\rho/Z_a$
    - $L(u,v)$ and $M(u,v)$ are solutions to Lommel's integral
- Note, the equation is fully non-dimensional
- Paper contains similar equations for the hologram of an opaque rectangle

Hologram simulation

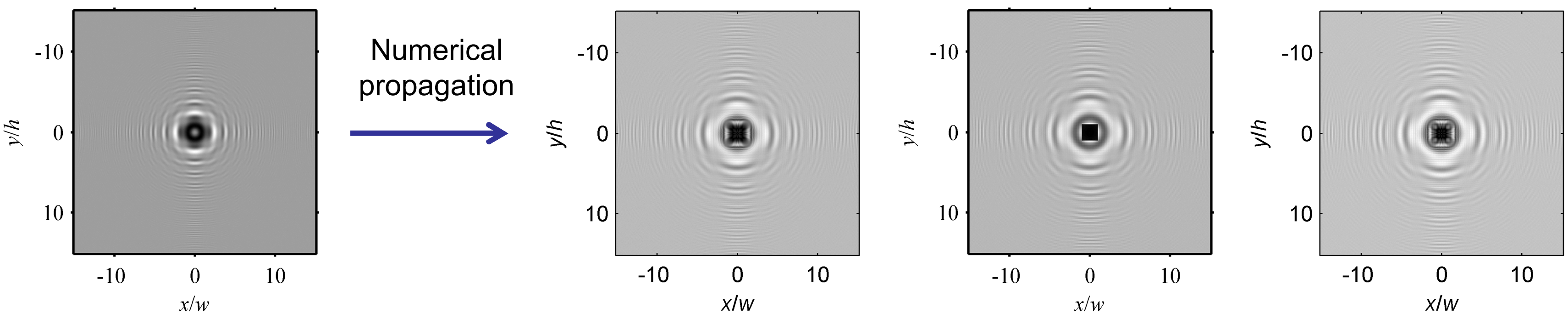
- Common noise sources added to simulated signal:
  - Discretization into $N \times M$ pixels of size $\Delta x \times \Delta y$
  - Analog to digital conversion with n bits of accuracy
  - Additional random noise modeled with a Gaussian distribution

- 60 different conditions simulated
  - Non-dimensional parameters selected to span expected experimental conditions
  - 20 realizations of random noise simulated at each condition for 1200 total simulations

Particle detection techniques

Goal: Use simulation results to estimate accuracy of particle detection algorithms

- Begin by reconstructing intensity at multiple distances



Hologram

Reconstruction at $Z = 0.9Z_0$

Reconstruction at $Z = Z_0$

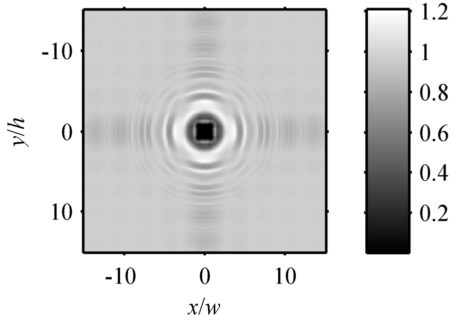
Reconstruction at $Z = 1.1Z_0$

- In-focus objects are characterized by:
  1. Local intensity minima
  2. Local edge sharpness maxima

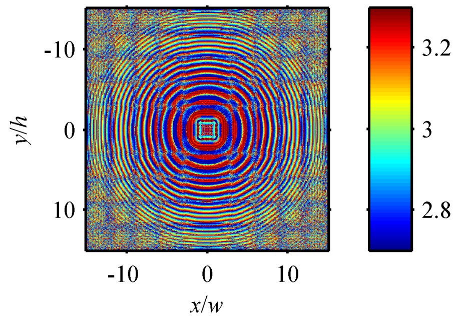
Particle detection techniques

Minimum intensity method:

1. Reconstruct 1000 planes between $Z = 0.9Z_0$ to $1.1\ Z_0$
2. For each pixel, store the minimum intensity and its *Z*-location

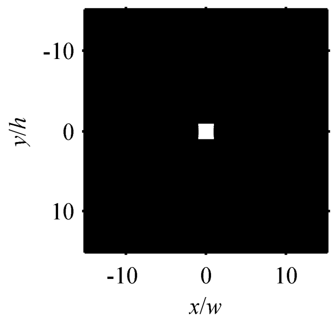


Minimum intensity in *Z*-direction



Z-location of minimum intensity

3. Threshold to find objects

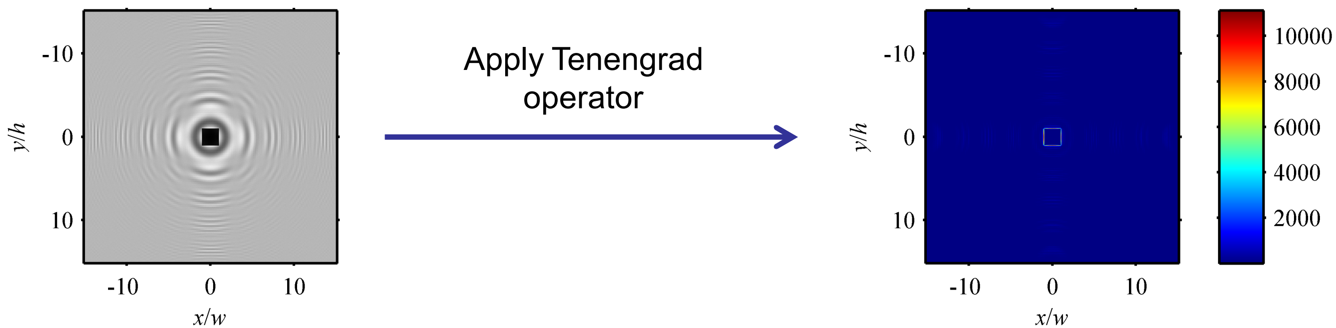


Minimum intensity thresholded at 0.1

Particle detection techniques

Maximum edge sharpness method:

1. Reconstruct 1000 planes between $Z = 0.9Z_0$ to $1.1\ Z_0$
2. Calculate spatial gradients within each reconstructed plane
   - Tenengrad operator: $T(x,y) = \left[I(x,y) \otimes S_x(x,y)\right]^2 + \left[I(x,y) \otimes S_y(x,y)\right]^2$
   - S_x and S_y are horizontal and vertical Sobel kernels

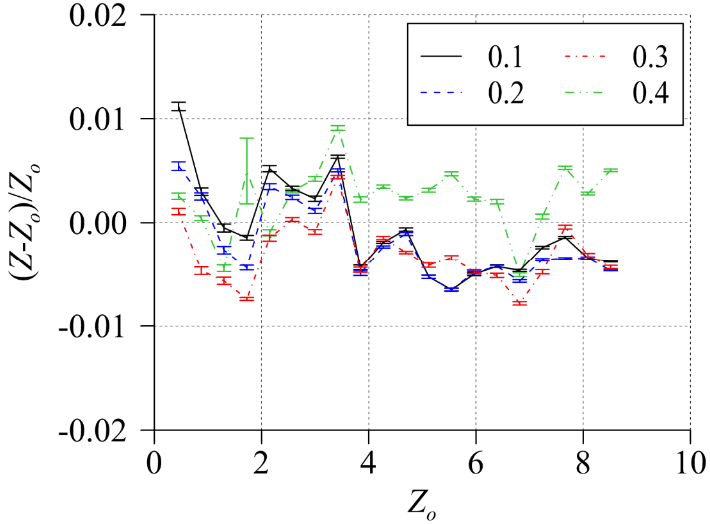


Reconstruction at $Z = Z_0$ — Tenengrad of reconstruction at $Z = Z_0$

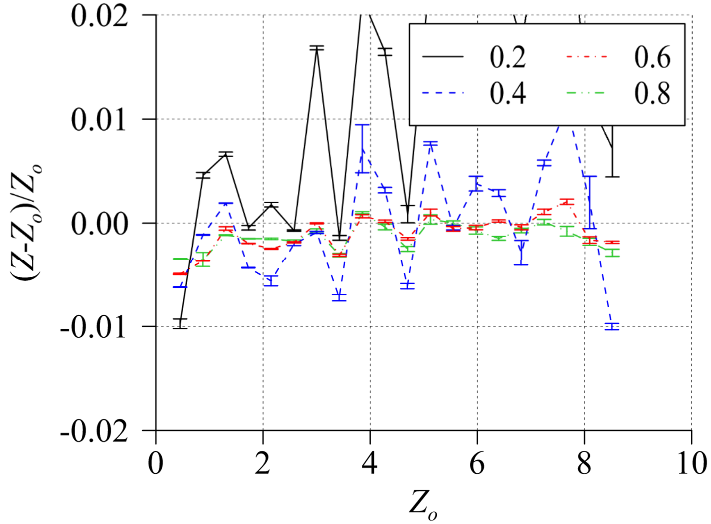
3. For each pixel, store the maximum Tenengrad value and its Z-location
4. Threshold to find objects

Particle detection techniques

Minimum intensity and maximum Tenengrad methods applied to all 1200 simulated holograms



Depth uncertainty for minimum intensity method



Depth uncertainty for maximum Tenengrad method

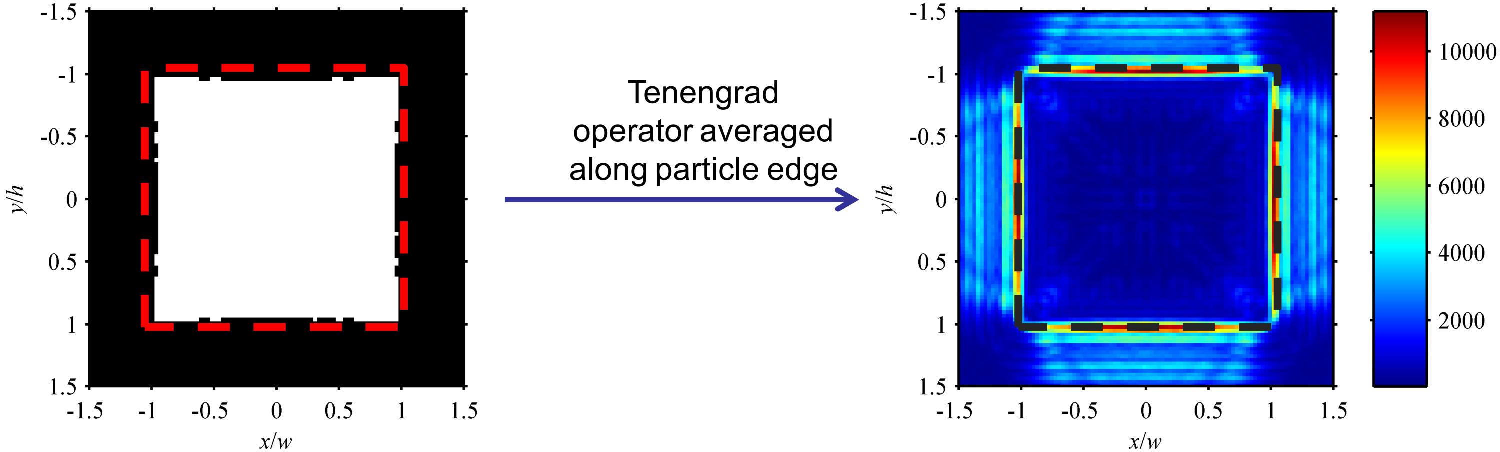
Challenges:

- Optimum threshold is not known *a-priori*
- Results are unstable with respect to *Z*-position

Improved particle detection

Idea: Search for simultaneous intensity minima and edge sharpness maxima

1. Calculate minimum intensity and maximum Tenengrad maps
2. Threshold minimum intensity to find particle shape and quantify edge sharpness using the Tenengrad map

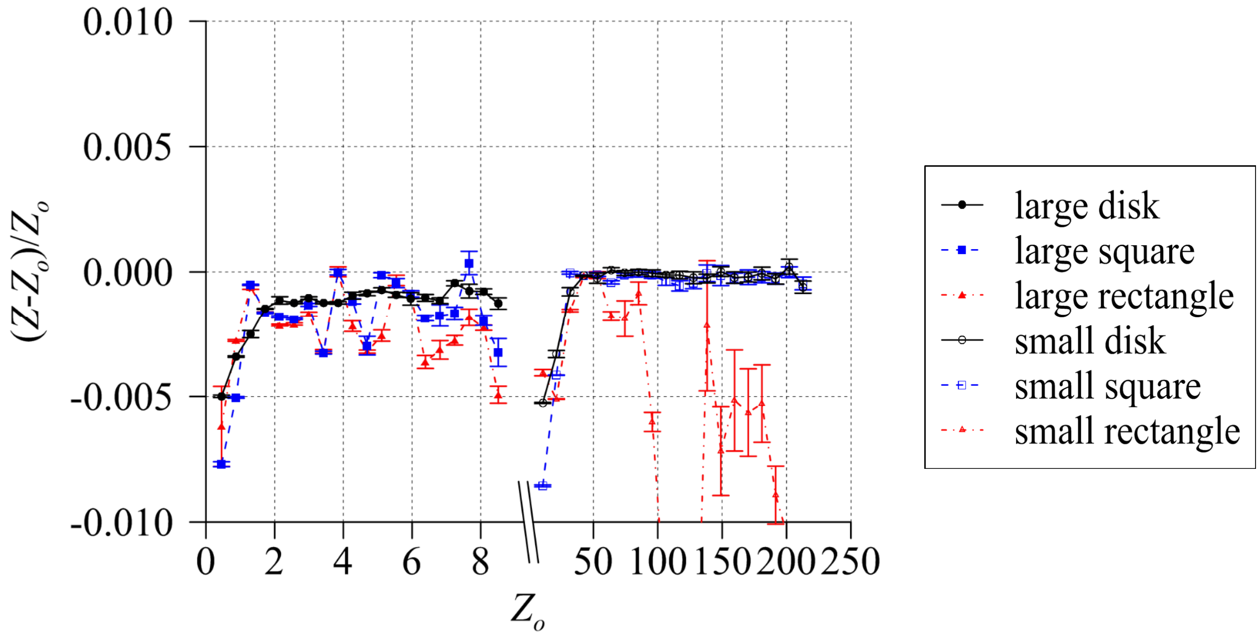


Minimum intensity thresholded at 0.1

Maximum Tenengrad in the *Z*-direction

3. Repeat step 2 until a particle with a maximum edge sharpness is located

Improved particle detection

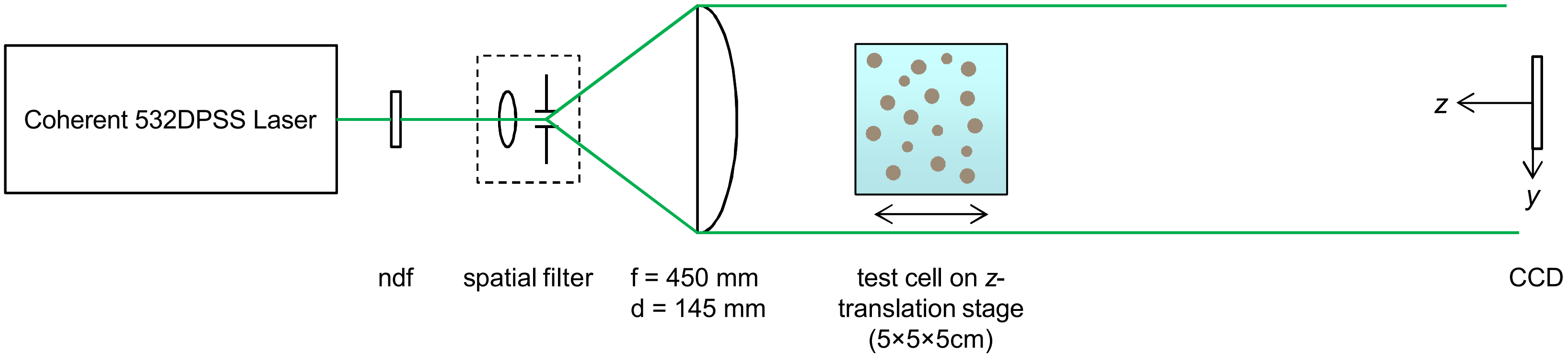


Depth uncertainty for hybrid method

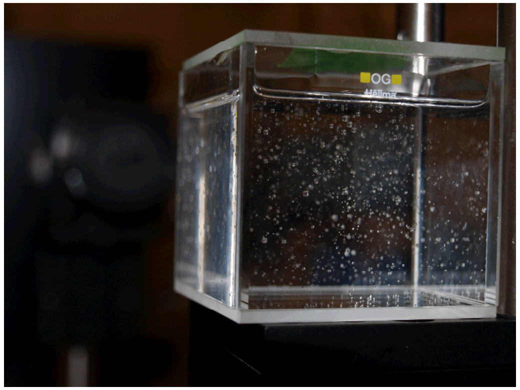
Advantages:

- Improved *Z* accuracy
- No user defined thresholds
- Does not require spherical particles

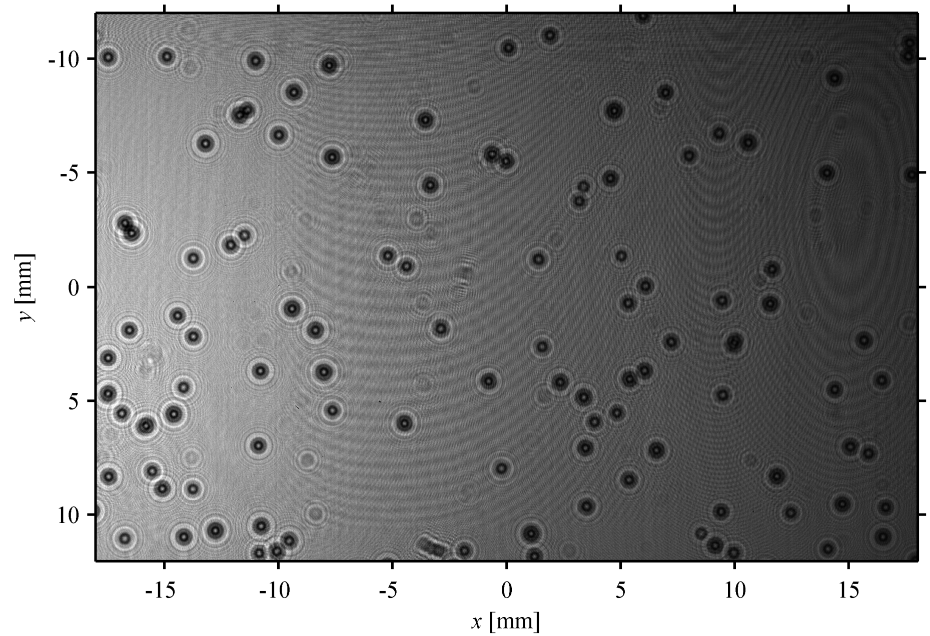
Experimental validation



- Quasi-stationary particle field
  - Polystyrene beads ($\bar{d} \approx 465\,\mu\text{m}$) in 10000 cSt silicone oil
  - Settling velocity ≈ 0.9 μm/s
- ProSilica GE4900 monochrome camera
  - 4872 ×3248 pixels
  - Each pixel 7.4 μm square



Experimental validation

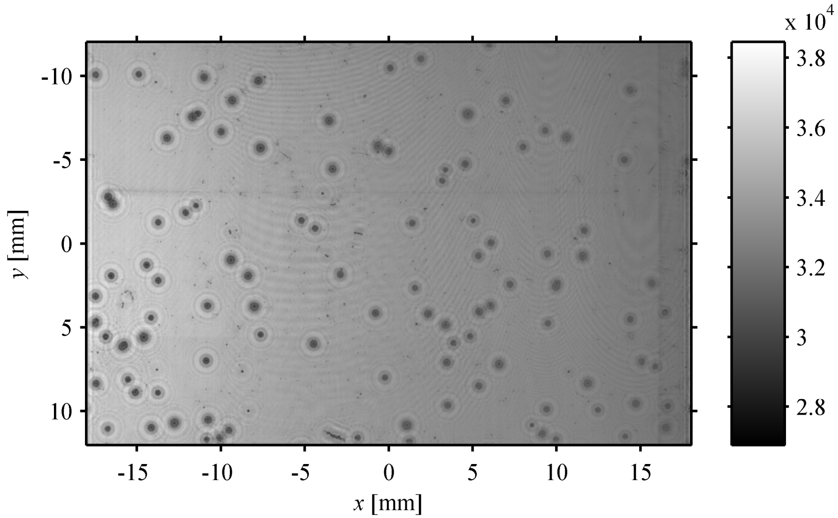


Typical experimental hologram

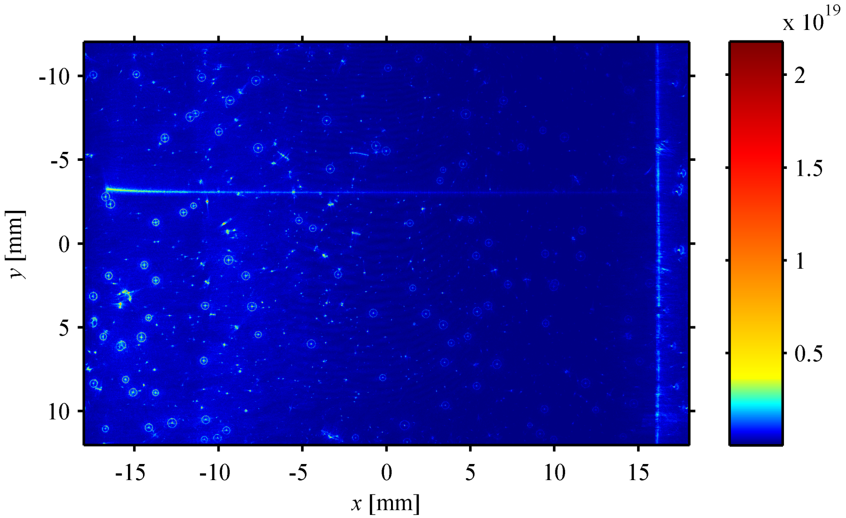
- Diffraction pattern of individual particles clearly visible
- Background variations due to imperfections in test cell and optical configuration

Experimental validation

- Reconstruction performed at 1000 planes between 150 to 220 mm



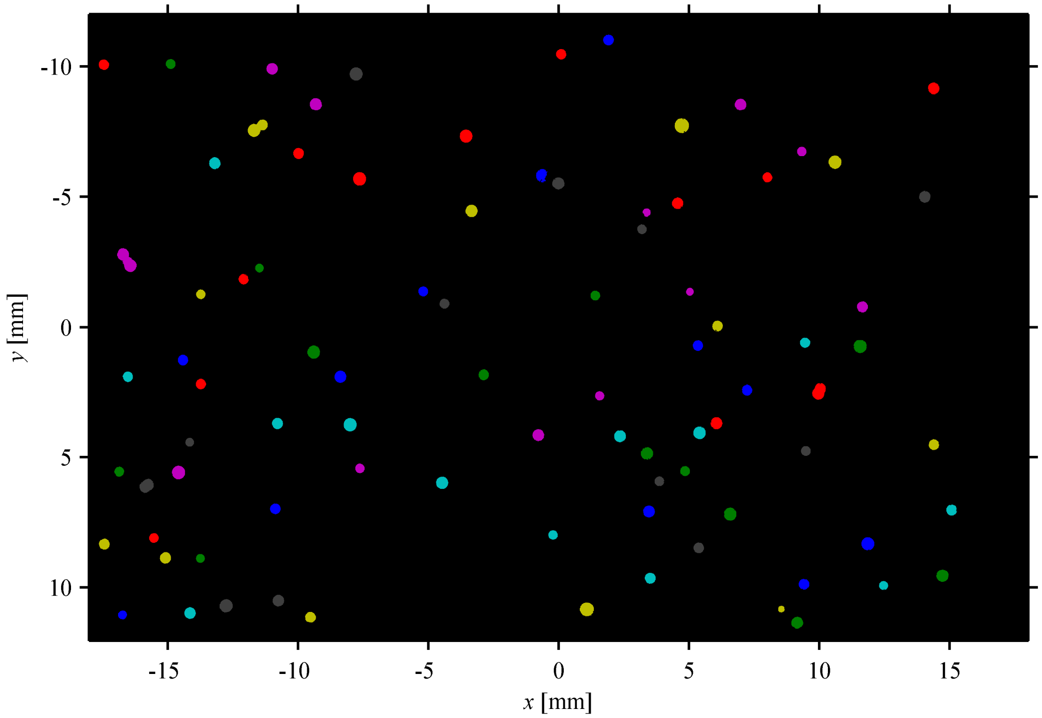
Minimum intensity in *z*-direction



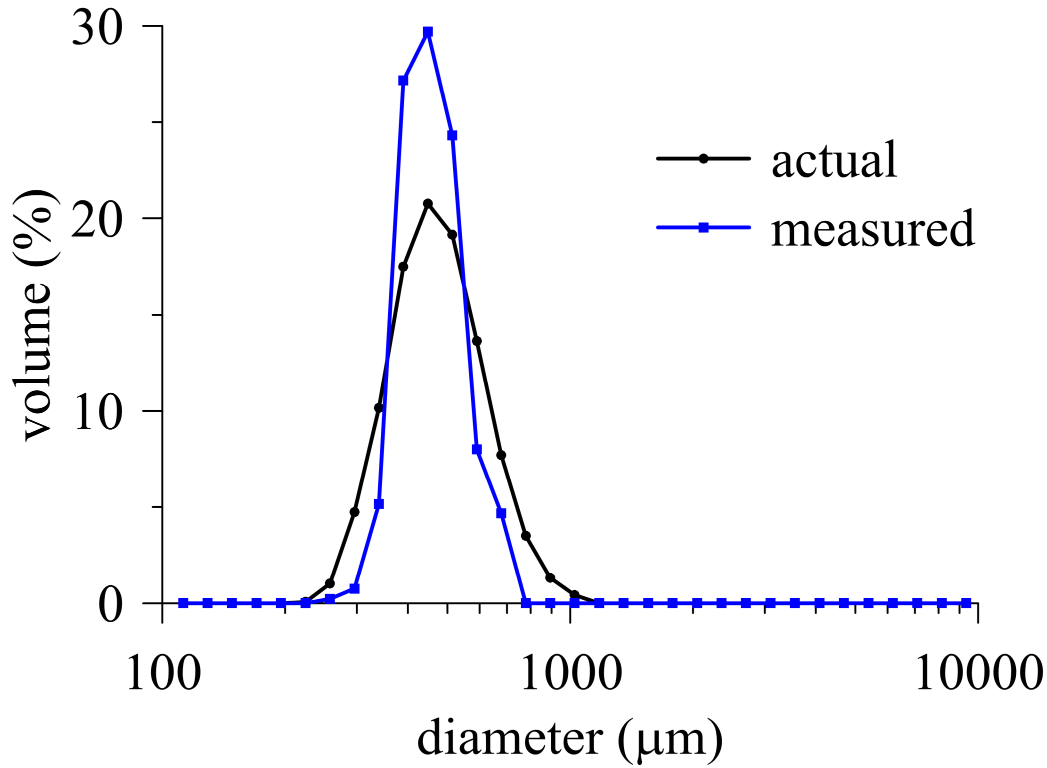
Maximum Tenengrad in *z*-direction

Experimental validation

- 100 different thresholds between 30500 and 32500 used to automatically select the optimum value for each particle
  - Detected particles with d < 200 μm are rejected



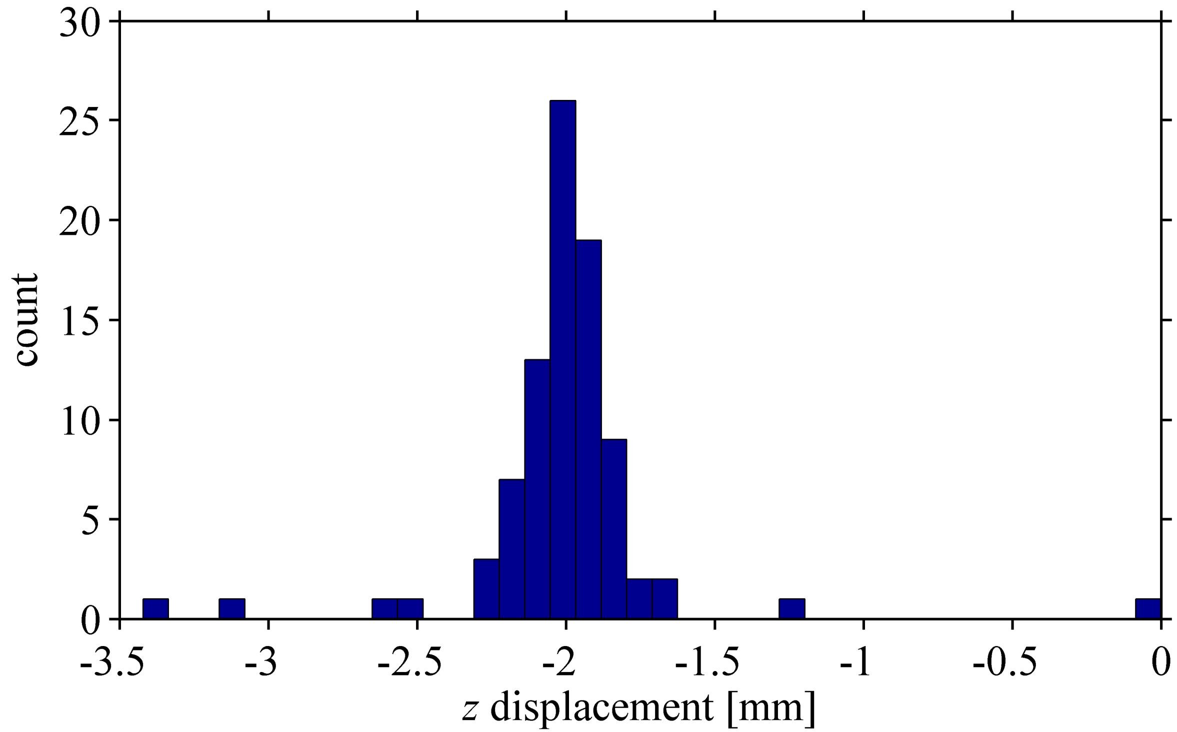
Experimental validation



- 87 particles detected in a volume which is (33.5×23.2×50mm) → 0.0022 particles/mm^3
- Actual particle density is 0.0029 particles/mm^3

Experimental validation

- Experiments repeated with the particle field displaced -2 mm in the *z*-direction
  - Hungarian particle matching algorithm minimizes sum of paired distances (MATLAB® implementation by Jean-Yves Tinevez)



Summary

- Digital holography of particle fields has many advantages:
  - Simple optical configuration
  - 3D particle position from a single view
  - Capture non-spherical particle morphologies

- Significant advancements of this work:
  - Non-dimensional simulation methods to estimate accuracy *and* precision
  - Improved particle detection algorithms which requires minimal *a-priori* knowledge of particle shape or optimal threshold
  - Experimental methods involving *quasi-stationary* particles to verify 3D positional accuracy

Questions?