

# A Phase-field Model of Substrate Anisotropy Effects in BiFeO<sub>3</sub> Thin Films

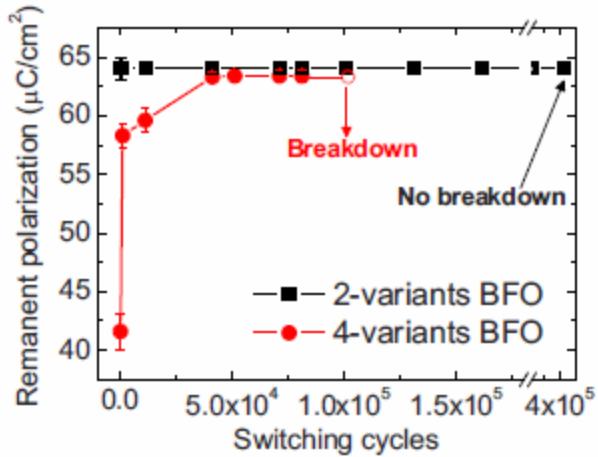
Ben Winchester

# Outline

- Introduction: the importance of ferroelectric domain structures and one way we might control them
- The Approach: a Phase Field model
- Results
- Conclusions & future work

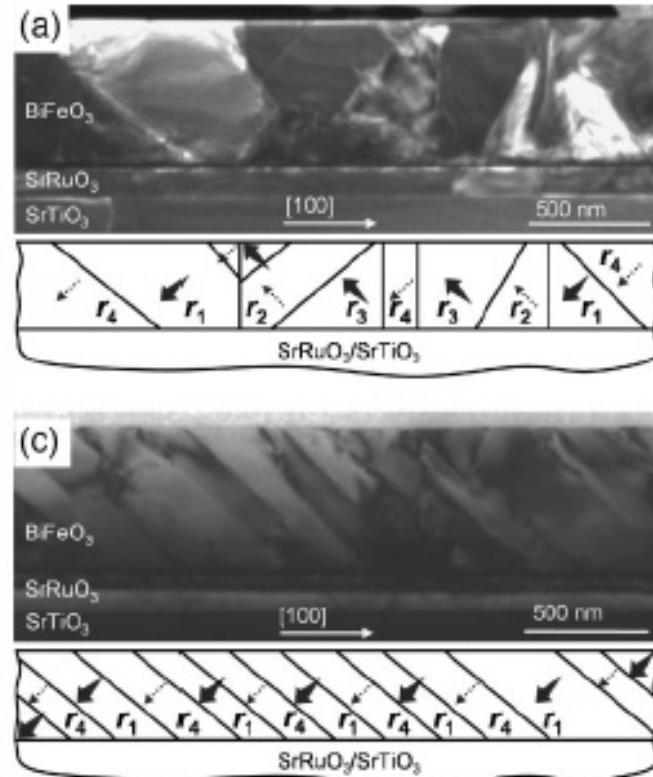
# The importance of domain structure

Between bulk and interfacial properties, that's just about everything.



Breakdown behaviors of 2-variant and 4-variant BFO films.

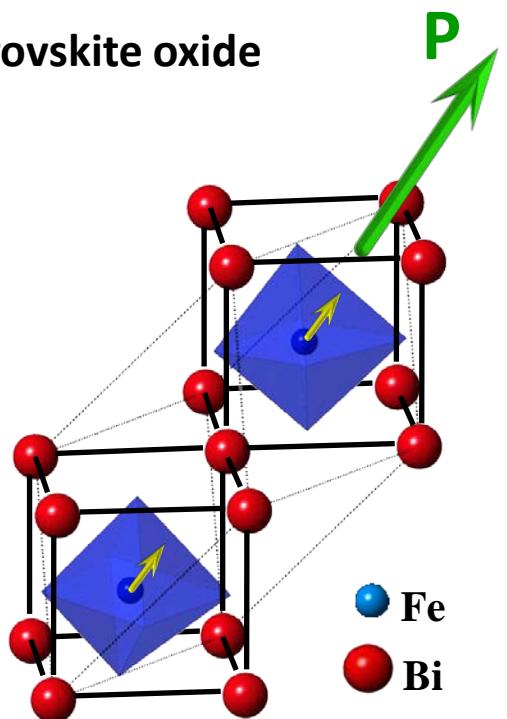
- Two versus four variants in  $\text{BiFeO}_3$
- Four-variant film has decreased  $P_r$ , decreased switching lifetime.



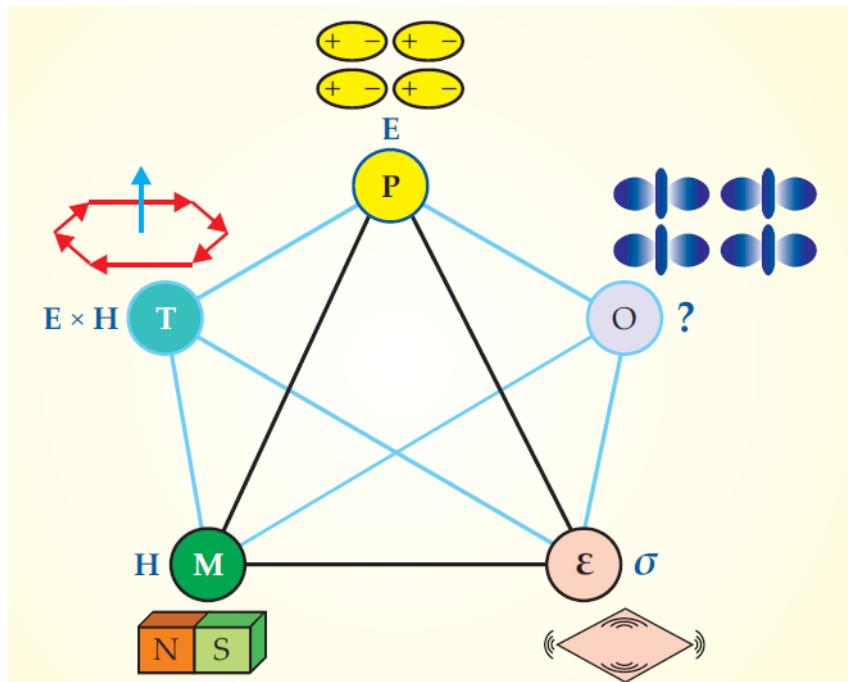
Cross-sectional TEM images of  $\text{BiFeO}_3$  films 600nm thick on a), exact, and c), miscut  $\text{SrTiO}_3$ .

# Multiferroic $\text{BiFeO}_3$

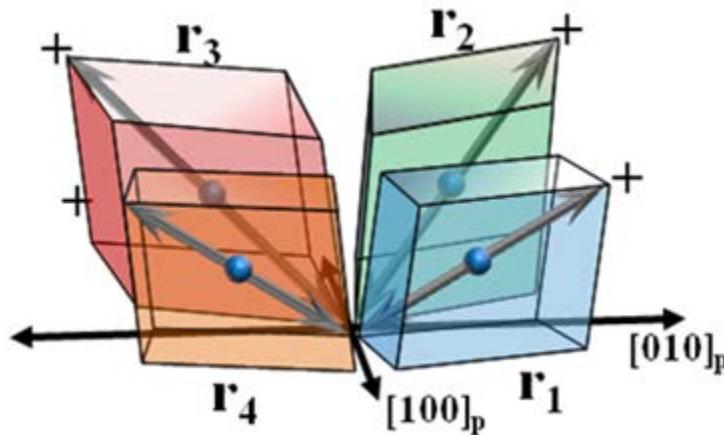
Perovskite oxide



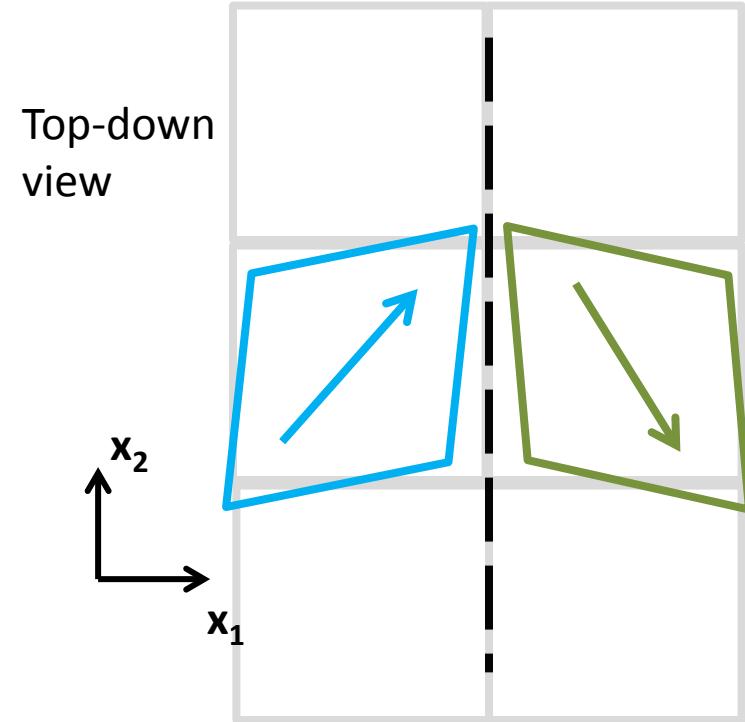
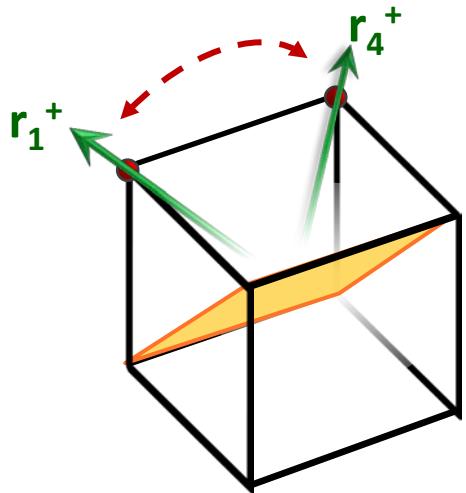
Multiferroicity:



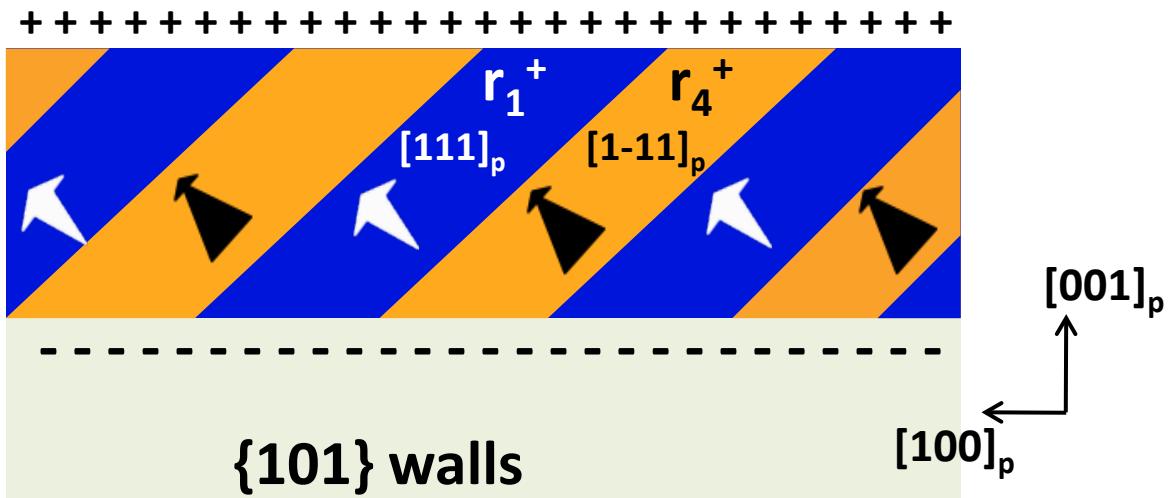
Four ferroelastic variants



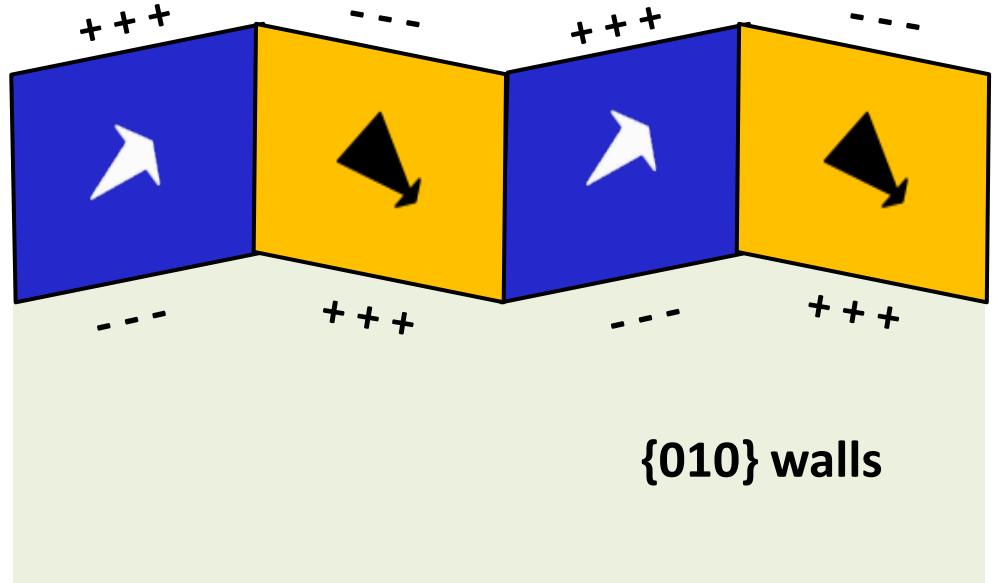
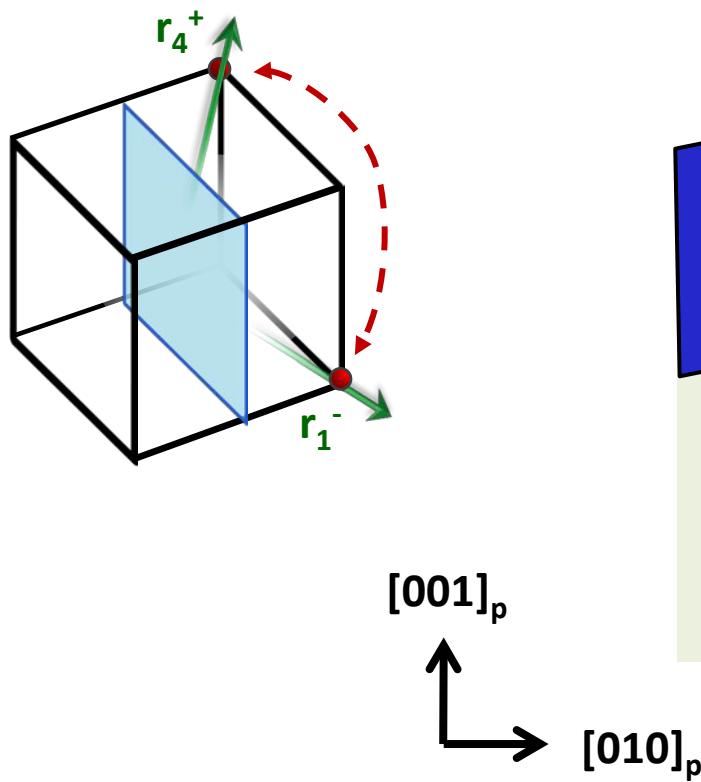
# 71° Domain Wall



Compensates in-plane stresses, but leaves surfaces charged.



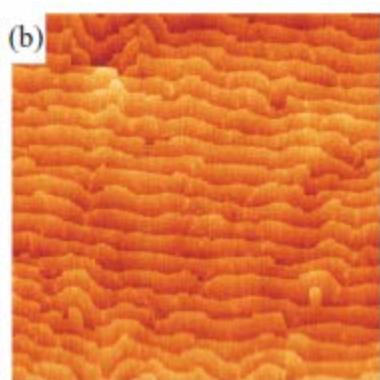
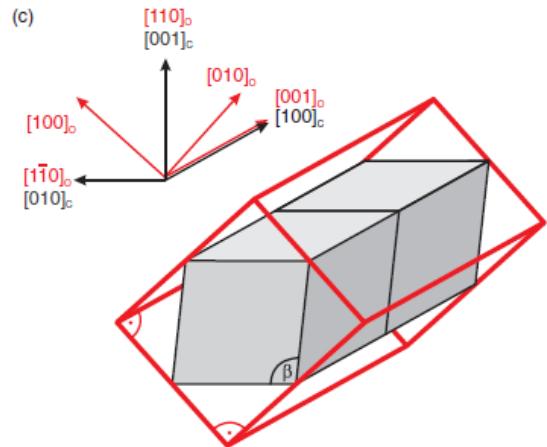
# 109° Domain Wall



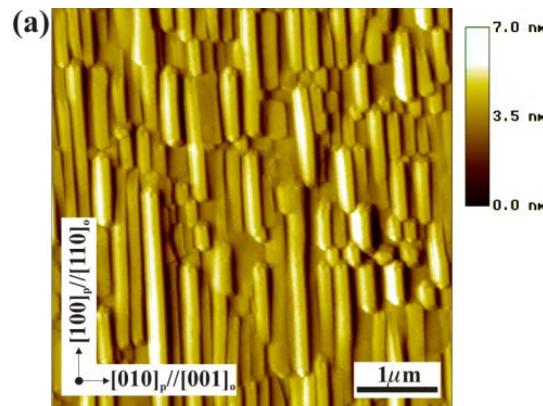
Compensates electrical charge, but creates out-of-plane stresses.

# Why do some substrates tend to produce certain variants?

Orthorhombic rare-earth scandates ( $\text{TbScO}_3$ ,  $\text{DyScO}_3$ ,  $\text{GdScO}_3$ , etc.)

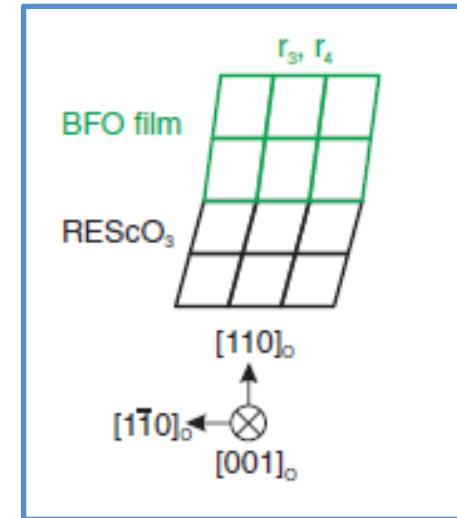


AFM image of  
 $\text{BiFeO}_3$  on  $\text{DyScO}_3$

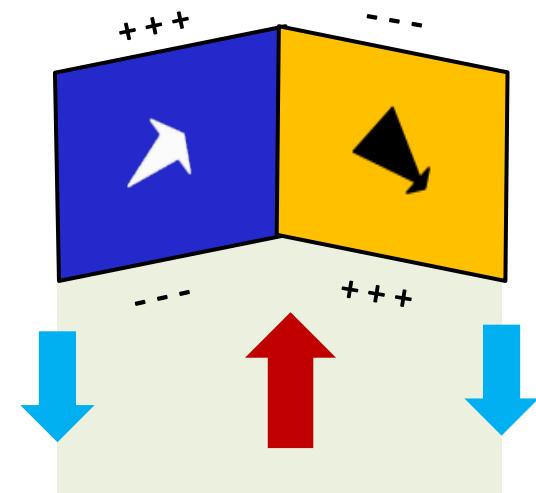


AFM images of  $109^\circ$  walls  
on  $\text{TbScO}_3$

Interfacial effect, or...



Substrate anisotropy?



[1] F. Johann *et al.*, Phys. Rev. B **84**, 094105 (2011)

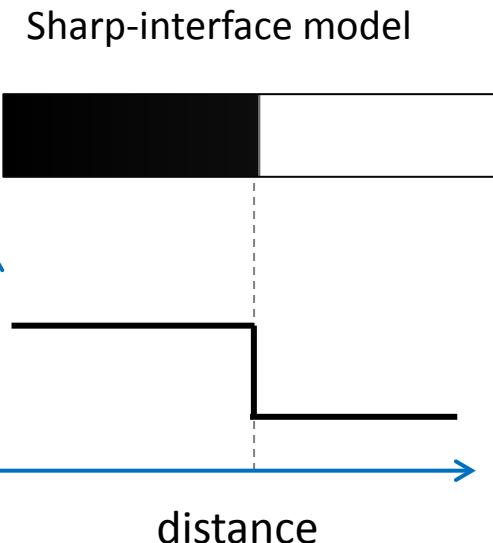
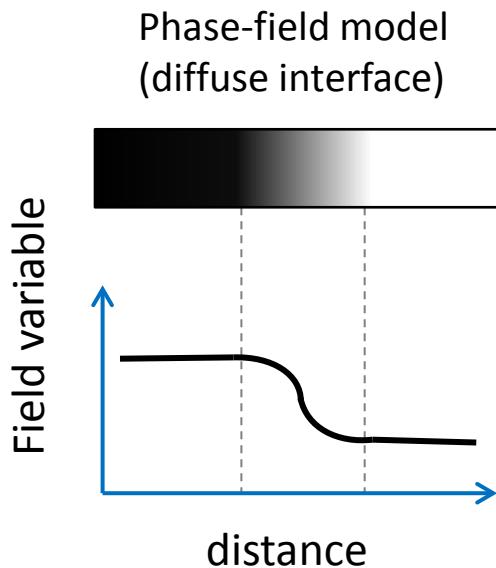
[2] C. M. Folkman *et al.*, Appl. Phys. Lett **94**, 251911 (2009)

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# About the phase-field method

- A meso/continuum-scale method based on thermodynamics
- Uses phenomenological data from experiments and other models.
- Takes some macro “order parameter” that varies in space (superconductivity, magnetism, etc.) and solves its evolution by a variational method.
- We treat the parameter as a continuous variable.



# The Ginzburg-Landau model

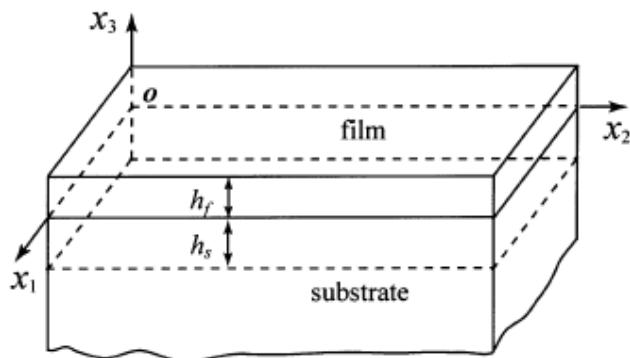
$$F = \int_V (f_{Landau} + f_{elastic} + f_{gradient} + f_{electric}) dV$$

Time-dependent Ginzburg-Landau equation:

$$\frac{\partial P_i(x)}{\partial t} = -L \frac{\delta F}{\delta P_i(x)}$$

## Gradient energy

$$f_{gradient} = \frac{1}{2} g_{ijkl} \frac{\partial P_i}{\partial x_j} \frac{\partial P_k}{\partial x_l}$$



## Landau (bulk) energy

$$\begin{aligned} f_{Landau} = & \alpha_1 (P_1^2 + P_2^2 + P_3^2) \\ & + \alpha_{11} (P_1^4 + P_2^4 + P_3^4) \\ & + \alpha_{12} (P_1^2 P_2^2 + P_2^2 P_3^2 \\ & + P_3^2 P_1^2) + \dots \end{aligned}$$

## Electric energy

$$f_{electric} = -E_i \left( \frac{1}{2} \epsilon_r \epsilon_0 E_i + P_i \right)$$

## Elastic energy

$$\begin{aligned} f_{elastic} = & \frac{1}{2} c_{ijkl} e_{ij} e_{kl} \\ = & \frac{1}{2} c_{ijkl} [\epsilon_{ij} - \epsilon_{ij}^0] [\epsilon_{kl} - \epsilon_{kl}^0] \end{aligned}$$

## Coupling term (eigenstrain):

$$\epsilon_{ij}^0 = Q_{ijkl} P_k P_l$$

# Microstructural evolution with adaptive meshing

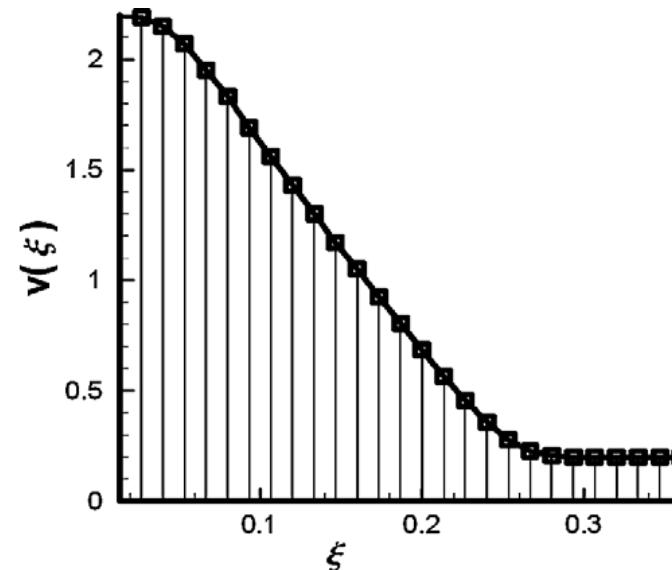
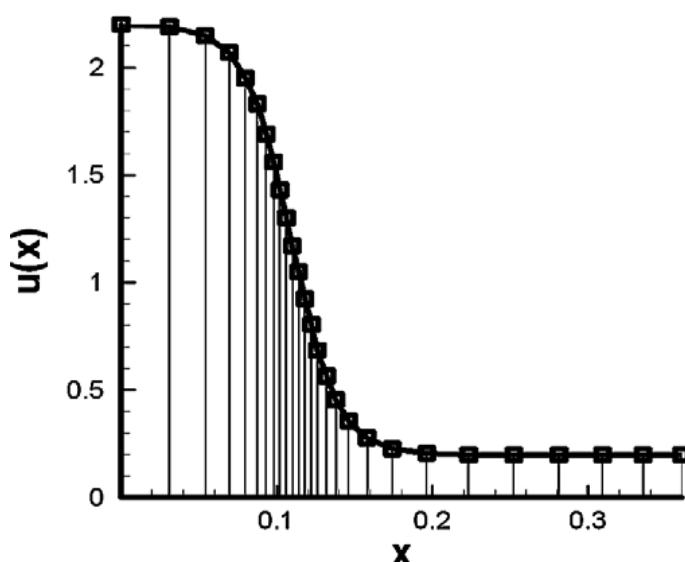
Mesh moves to where the monitor function  $\omega$  is highest.

Variational eq'n

$$I[\xi] = \int \sum_{i=1}^3 \frac{\nabla_x \xi^i \cdot \nabla_x \xi^i}{\omega} dx$$

Monitor function

$$\omega \cong \sqrt{1 + \beta^2 |\nabla_x u|^2}$$

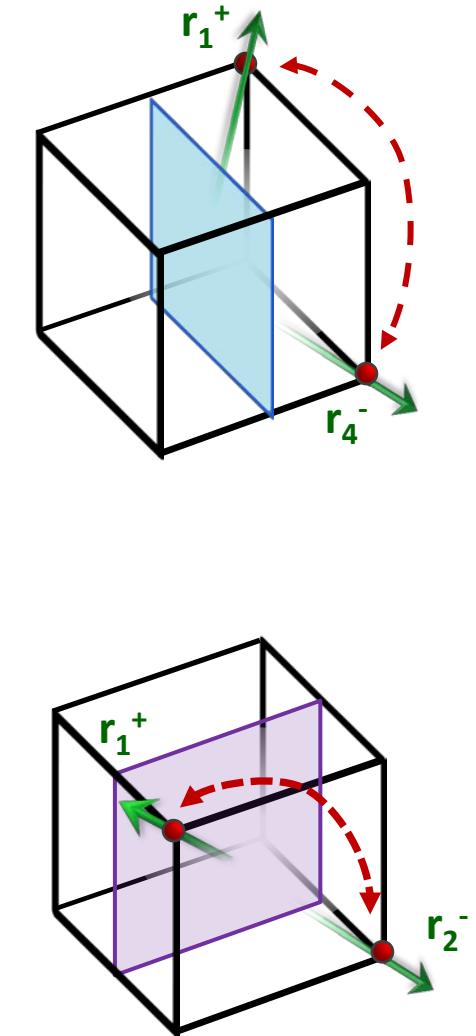
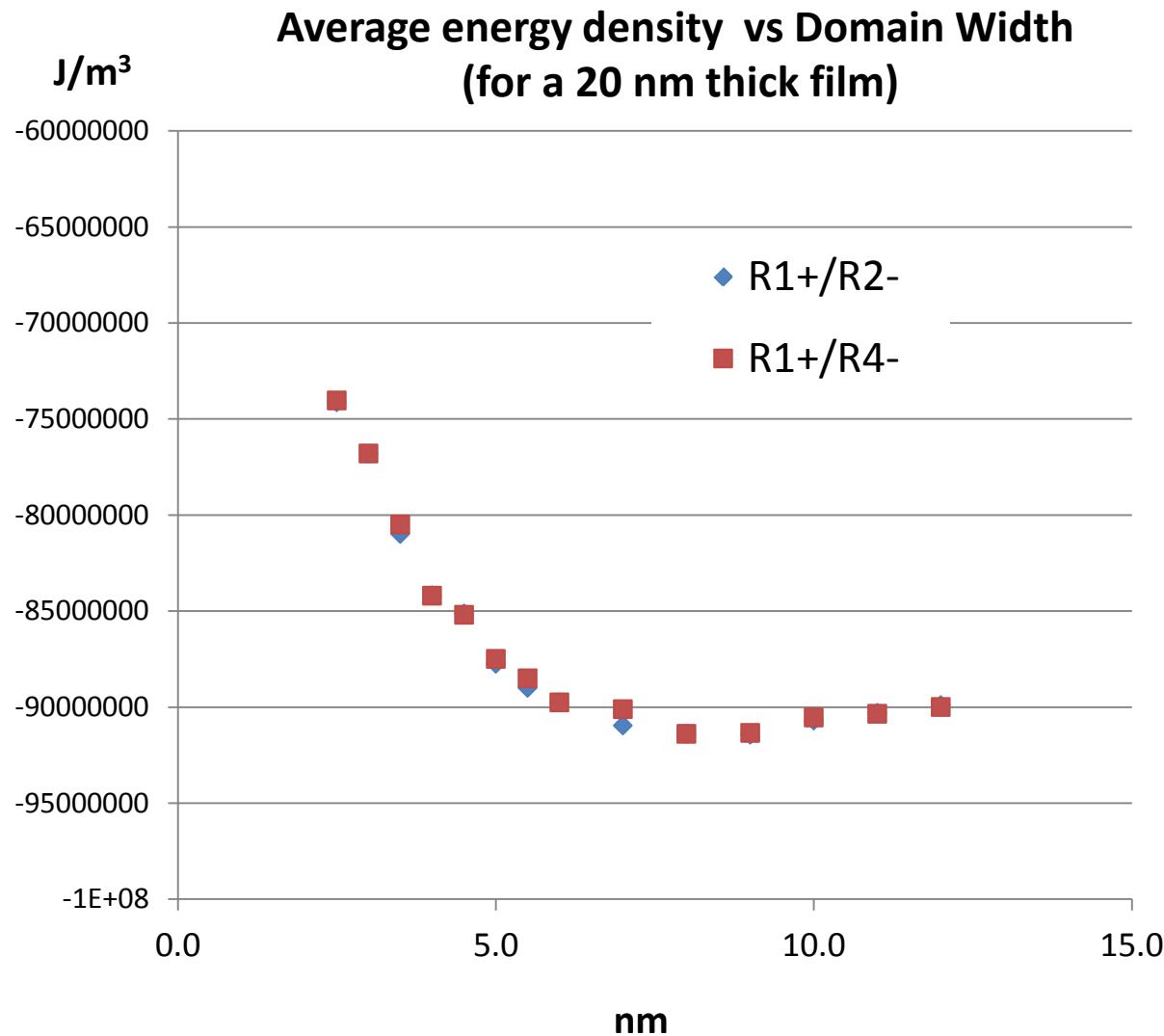


Physical domain (left) and computational domain (right)<sup>[1]</sup>.

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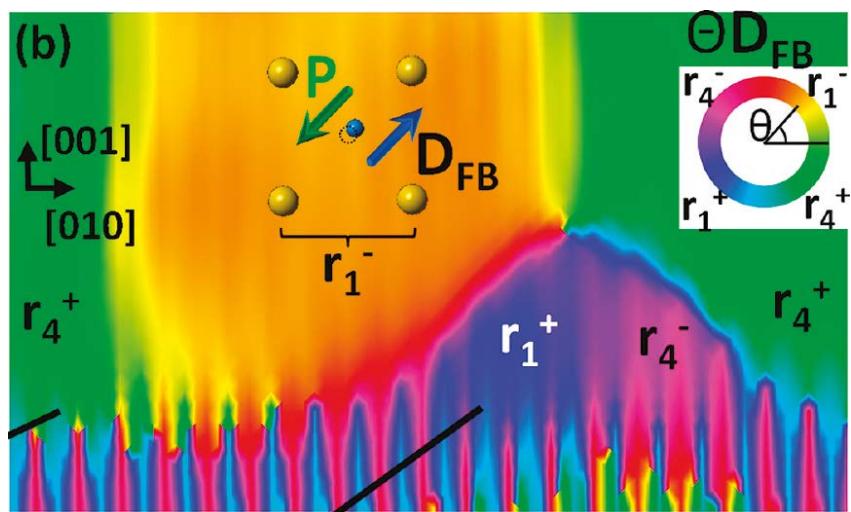
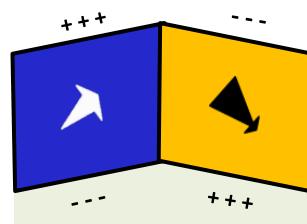
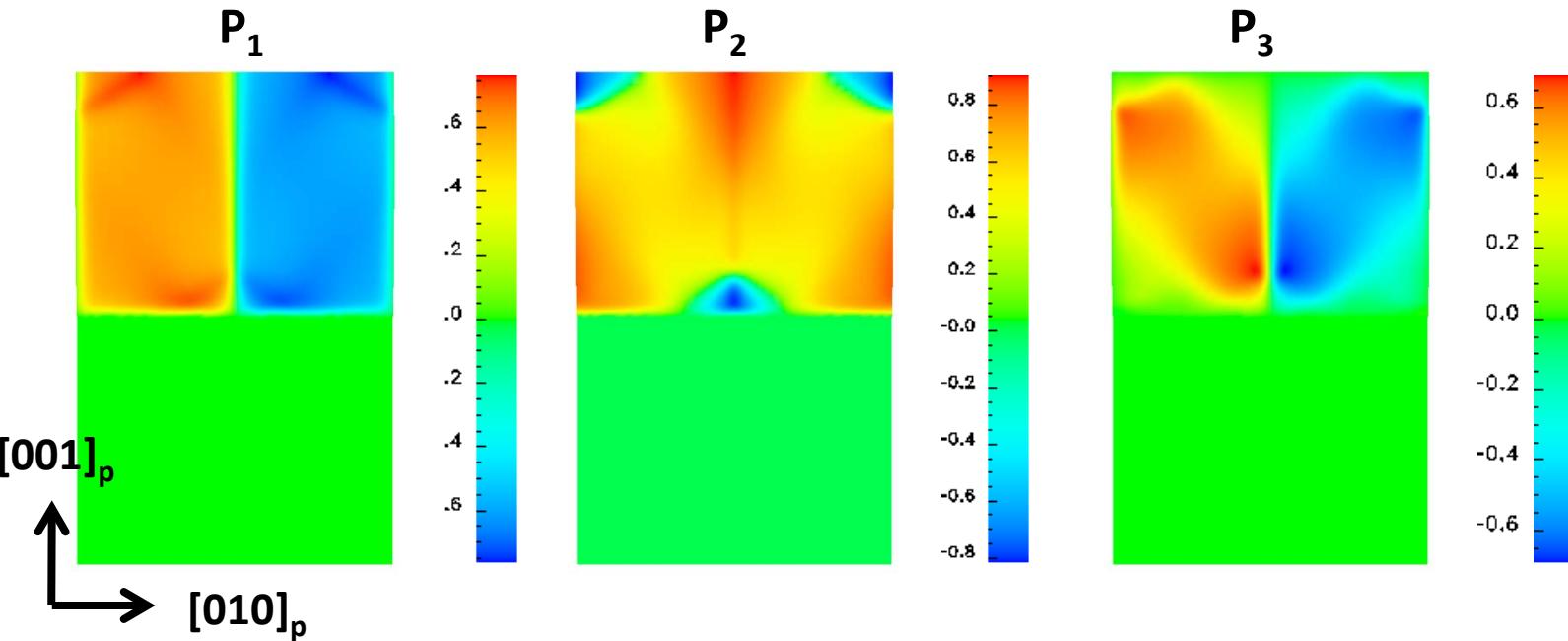
# Examining domain orientation's affect on energy:



**Conclusion:** very little energy difference between the two orientations. Why is that?

# What's going on? A look at some plots.

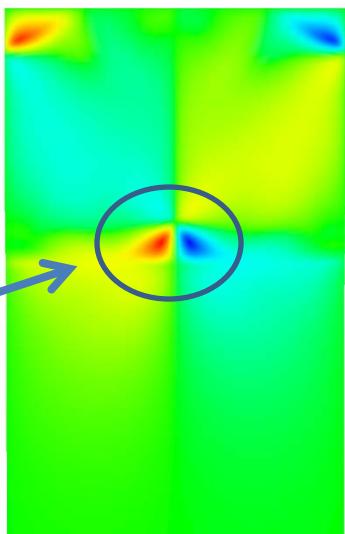
Polarization components ( $\mu C/m^2$ ):



Z-contrast STEM image of two 109° domain walls.

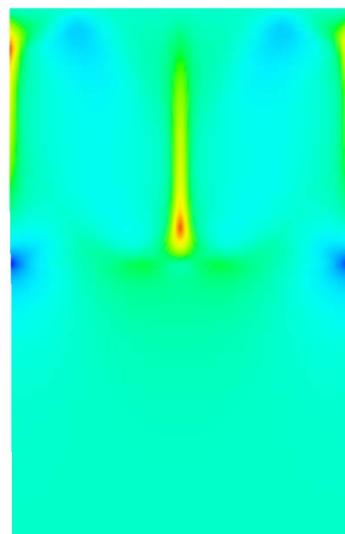
# What's going on? A look at more plots.

stress<sub>4</sub> (10<sup>-9</sup> J/m<sup>2</sup>)

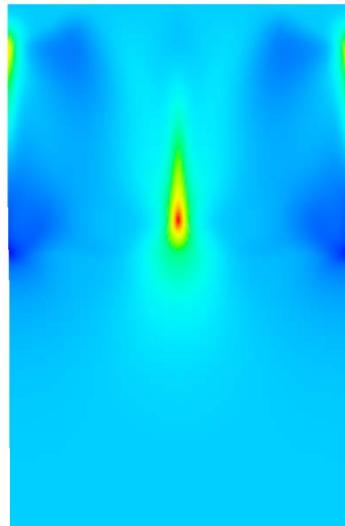


Wrong!

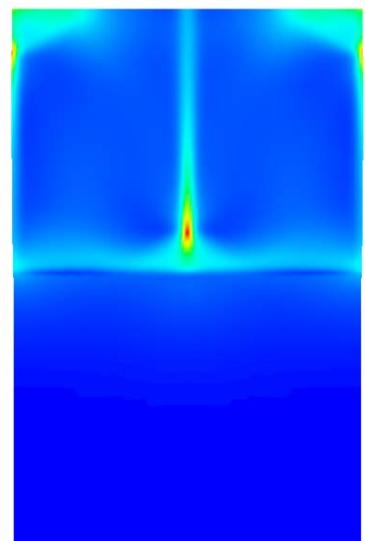
stress<sub>5</sub> (10<sup>-9</sup> J/m<sup>2</sup>)



stress<sub>3</sub> (10<sup>-9</sup> J/m<sup>2</sup>)

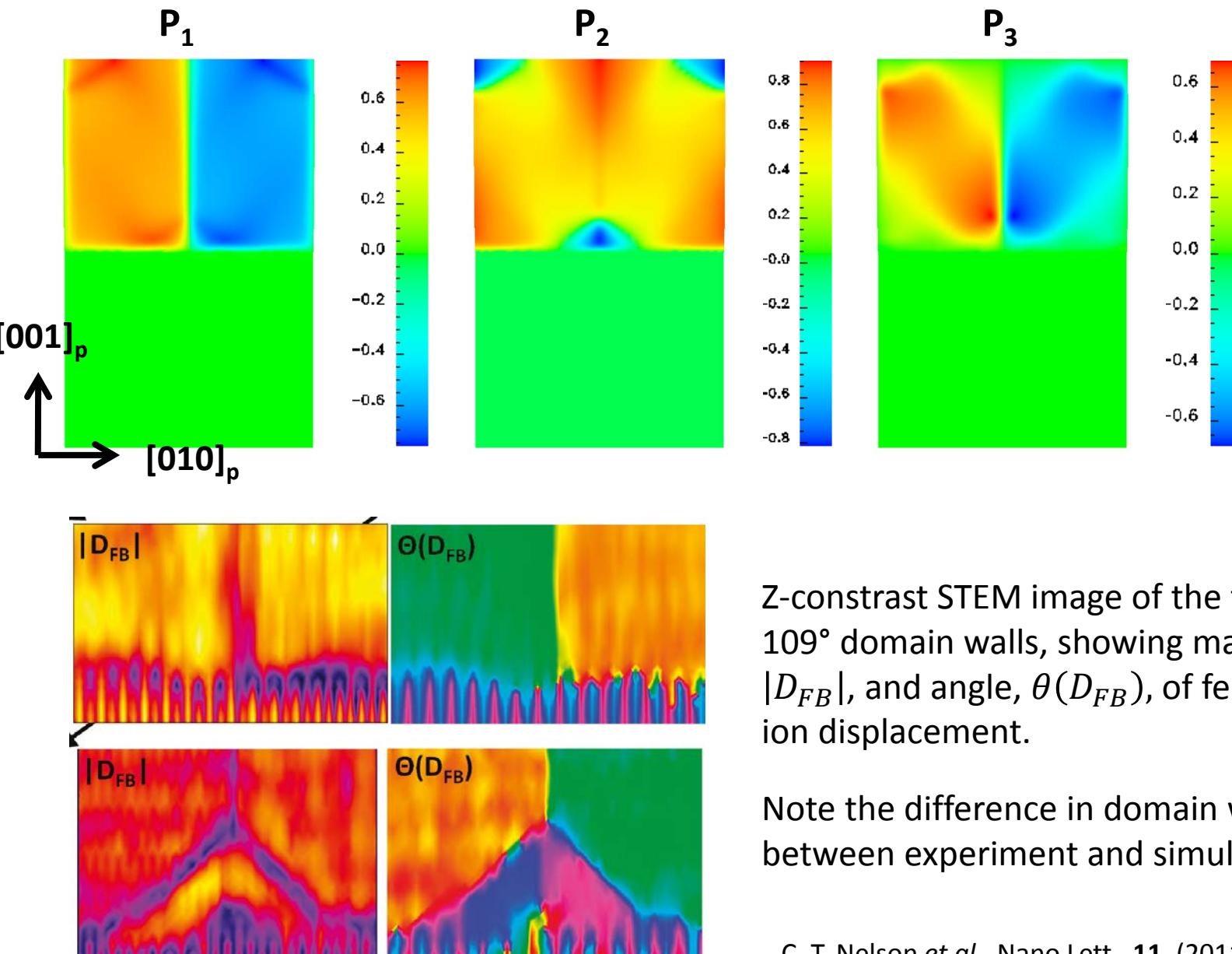


$f_{elastic}$  (10<sup>-9</sup> J/m<sup>3</sup>)



[001]<sub>p</sub>  
↑  
→ [010]<sub>p</sub>

# Comparing experimental and simulation polarizations in more depth.



Z-contrast STEM image of the two different  $109^\circ$  domain walls, showing magnitude,  $|D_{FB}|$ , and angle,  $\theta(D_{FB})$ , of ferroelectric ion displacement.

Note the difference in domain wall sharpness between experiment and simulation.

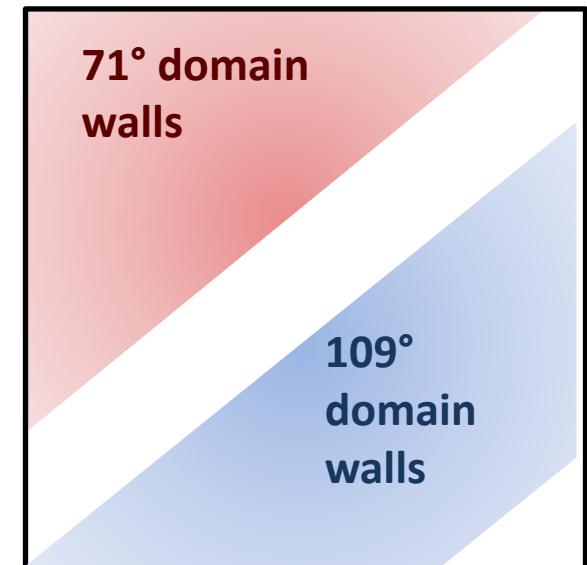
# Conclusions

- We developed an adaptive mesh phase-field model to look at the effects of substrate anisotropy on domain structure.
- The results did *not* indicate that the substrate anisotropy is responsible for the preferential selection of two ferroelastic variants in these films.
- However, the results leave questions as to why.

# Future Work

- Continue to examine potential discrepancies between simulation and experiment.
- Comparison of energetics, for film thickness vs electrical compensation. Where are the different domain structures favored?

(Hypothetical) Electrical Surface Compensation vs. Film Thickness



Film thickness

# Acknowledgements



- We acknowledge generous support from Advanced Simulation and Computing – Physics and Engineering Models (ASC PEM).



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# Elastic anisotropy

Typical elastic stiffness tensor for these rare-earth scandates

(in *Pbnm* notation)

$$\begin{pmatrix} 300 & 130 & 130 & 0 & 0 & 0 \\ 130 & 300 & 130 & 0 & 0 & 0 \\ 130 & 130 & 250 & 0 & 0 & 0 \\ 0 & 0 & 0 & 115 & 0 & 0 \\ 0 & 0 & 0 & 0 & 90 & 0 \\ 0 & 0 & 0 & 0 & 0 & 80 \end{pmatrix}$$

*(convert to pseudo-cubic lattice)*

(with  $[001]_o \parallel [010]_p$ )



(with  $[001]_o \parallel [100]_p$ )

$$\begin{pmatrix} 295. & 130. & 135. & 0. & 0. & 0. \\ 130. & 250. & 130. & 0. & 0. & 0. \\ 135. & 130. & 295. & 0. & 0. & 0. \\ 0. & 0. & 0. & 102.5 & 0. & 12.5 \\ 0. & 0. & 0. & 0. & 85. & 0. \\ 0. & 0. & 0. & 12.5 & 0. & 102.5 \end{pmatrix}$$

$$\begin{pmatrix} 250. & 130. & 130. & 0. & 0. & 0. \\ 130. & 295. & 135. & 0. & 0. & 0. \\ 130. & 135. & 295. & 0. & 0. & 0. \\ 0. & 0. & 0. & 85. & 0. & 0. \\ 0. & 0. & 0. & 0. & 102.5 & 12.5 \\ 0. & 0. & 0. & 0. & 12.5 & 102.5 \end{pmatrix}$$