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AQUARIUS™

Adiabatic quantum architectures in ultracold systems

Architecture team overview

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U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

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Architecture team goals

- Strong experimental support
  - Noise modeling
  - Experiment design
  - Simulation
- Working towards robust AQC
  - Identify critical noise processes
  - Extend error suppression techniques
  - Estimate scale at which error correction/
- Hardware realizable AQC
  - Interactions beyond QUBO
  - QUBO

AQC 2013 Results

- Simulations from ETH using GPU
    - Specialized NVIDIA hardware
    - 512 qubits
    - Quantum speedup and slowdown bimodal distribution
    - Where are hard instances?
    - Exp(-alpha sqrt(N))
- Sergio not doing simulated annealing, indistinguishable from PIMC
    - Hastings paper on differentiating from quantum path integral monte carlo
- Federico entanglement signatures
- Fault tolerance still not well defined

Milestones

Year 1

- Identify two-qubit adiabatic quantum optimization (AQO) problem to implement in hardware
- Create noise models for neutral-atom, silicon hardware
- Create adiabatic quantum computing (AQC) simulation software
- Explore universal AQC Hamiltonians for neutral-atom, silicon hardware
- Identify dynamical decoupling (DD), quantum error suppression (QES) schemes to integrate with AQC architectures
- Identify electronics and hardware constraints for AQC in neutral-atom, silicon hardware
- Simulate DD impact on AQO algorithm in neutral-atom, silicon hardware
- Simulate QES impact on AQO algorithm in neutral-atom, silicon hardware

Milestones

Year 2 - previous

- Hardware realizable QUBO AQC
  - Semiconductors
  - Neutrals
- Hardware realizable UAQC Hamiltonians
  - Semiconductors
    - Ground State Quantum Computing
    - (T)erhal-(O)liveira & (B)iamonte-(L)ove
      - Layout, Interactions, Graph Degree
  - Neutrals
    - TOBL
      - Layout, Interactions, Graph Degree
    - Holonomic quantum computing
      - Interactions, Robustness
- Fault tolerance of N-Variable QUBO AQC
  - Hardware realizable QES
  - Hardware realizable robustness against measurement errors
  - Initialization of AQC with QES
  - Design guidelines for QEC for AQC

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Year 2 - current

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Year 3 - previous

- Optimize UAQC Hamiltonian for neutrals, semiconductors
  - Reduce Interaction Types, Degree of the TOBL graph for Neutrals and Semiconductors
- Extend fault tolerance techniques to UAQC Hamiltonian
  - Hardware realizable QES
  - Power of QES via SGP
  - Does GSQC (Teleportation 2) provide robustness against noise (what kind?)
  - Can AQC be made robust against leakage?
  - Feasibility of QEC implemented as algorithmic cooling
- Estimate the benefit of fault tolerant UAQC
  - Given hardware noise model, parameters, estimate the “maximum instance” of an adiabatic algorithm that can be realized.
    - Dominant Noise: Leakage in Neutrals
    - Phonon bath in Semiconductors. Ising problem size with and without Boson bath
  - Estimate the increase in the benefit of QES via DD
- Optimization of holonomic gates for Neutrals
  - Improved holonomic paths and interpolation schemes

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Publications in pipeline

- Collaborations with Silicon experimental effort
  - Energy-dependent relaxation protocols
  - Adiabaticity vs. relaxation for charge qubits
- Robust adiabatic quantum computation
  - Towards fault tolerance in adiabatic quantum computation
  - Practical quantum error suppression with the [[6k,2k,2]] family of codes
  - Decoherence in the transvers-field Ising chain
  - Adiabatic topological quantum computation
  - Optimal control for adiabatic quantum computation
- Algorithms
  - Scaling and randomization
  - Tighter bounds on the complexity of adiabatic quantum computation based on the gap
  - Partial differential equation constrained combinatorial optimization
  - Gap amplification for circuit-analog adiabatic quantum computation

Technical overview

- Experimental support
  - Semiconductor specific
    - Interactions beyond QUBO
    - Back action of measurement with pulse measurements
  - Neutrals specific
    - Bounds on largest QUBO instances
    - Off-resonant Rydberg gates
- Handling errors
  - Landau-Zener transitions
    - Gap amplification
  - Control errors
  - Environmental noise
- Algorithms
  - *Rolando's talk*
- Quantum isothermal algorithm
  - *Vadim's talk*

Semiconductor support

- Charge DQD interactions beyond ZZ
  - See **Poster** by Toby Jacobson
- Relaxation tests
  - Measurements of phonon-mediated relaxation in charge DQDs
    - Good agreement between microscopic and phenomenological models of relaxation
  - See **Talk** by Malcolm Carroll
- Guidance for future experiments
  - Charge DQD characterization
  - Adiabaticity hallmark tests
  - Tests of excited state adiabaticity
  - See **Talk** by Malcolm Carroll

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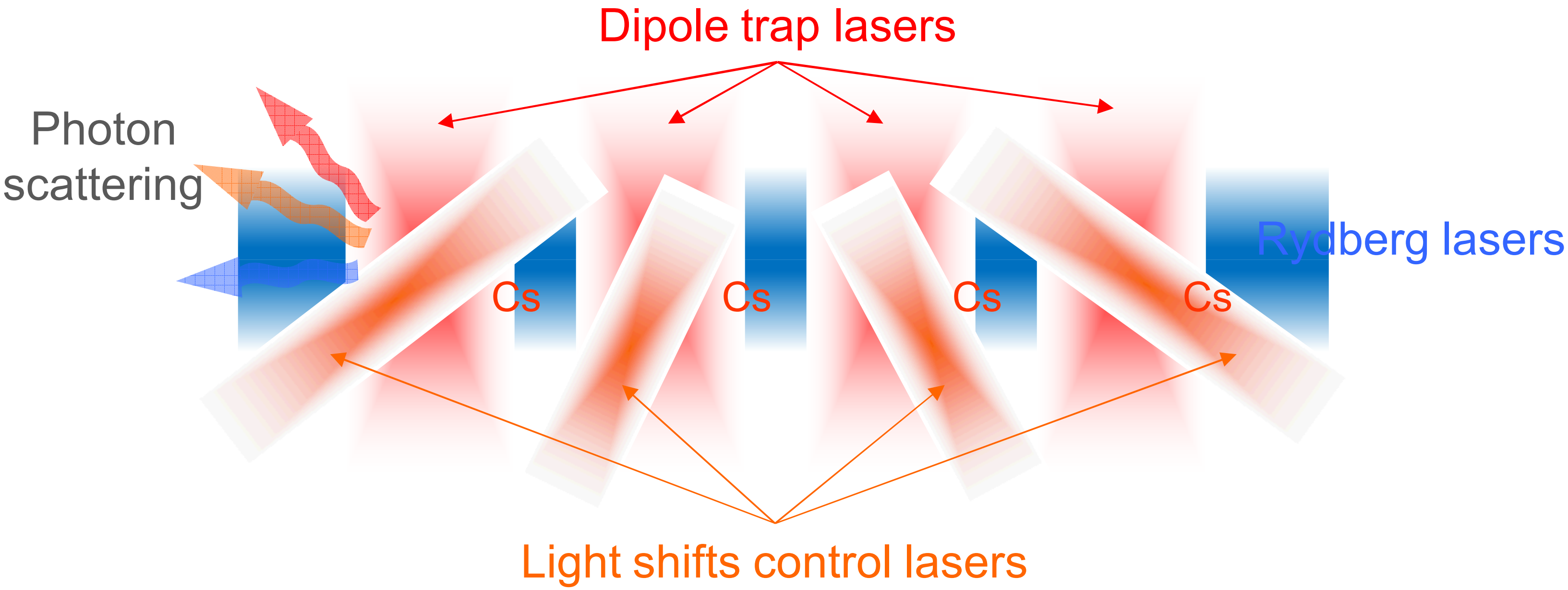
Neutrals support

Photon scattering



- Photon scattering-induced leakage from the computational basis states limits scaling
  - We have not identified coding strategy capable of handling leakage
  - Leakage rate is only weakly temperature-dependent:

$T\,[\mathrm{K}]$	300	77	0
$\Gamma\,[\mathrm{Hz}]$	81	35	17

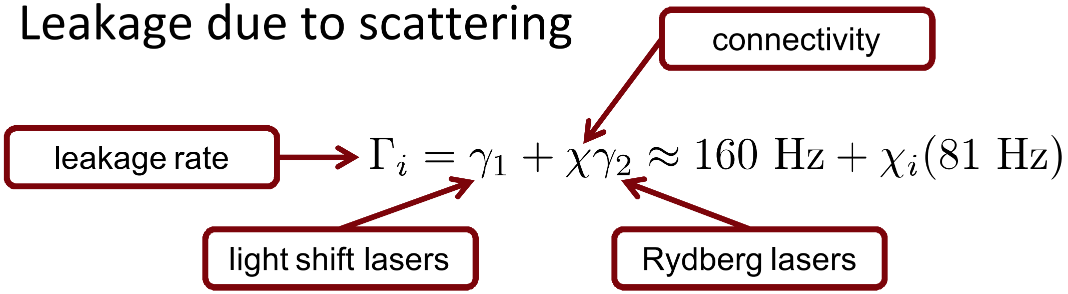
Neutrals support

Leakage impact on QUBO scaling

- Photon scattering/leakage is a Poisson process

$$p_{\text{success}}(T) = \prod_{i=1}^{n} e^{-\Gamma_i T} = e^{-\sum_i \Gamma_i T}$$

- Leakage due to scattering



$$\Gamma_i = \gamma_1 + \chi\gamma_2 \approx 160\ \text{Hz} + \chi_i(81\ \text{Hz})$$

- For 50% probability of success for *n*-qubit QUBO: $nT(n) \approx \dfrac{\ln(2)}{160 + 81\chi}$
  - Assuming all qubits have same connectivity

- Assume running time scales as: $T(n) = \alpha n^q$

- Criterion to run *n*-qubit QUBO: $\alpha \approx \dfrac{\ln(2)}{n^{q+1}(160 + 81\chi)}$

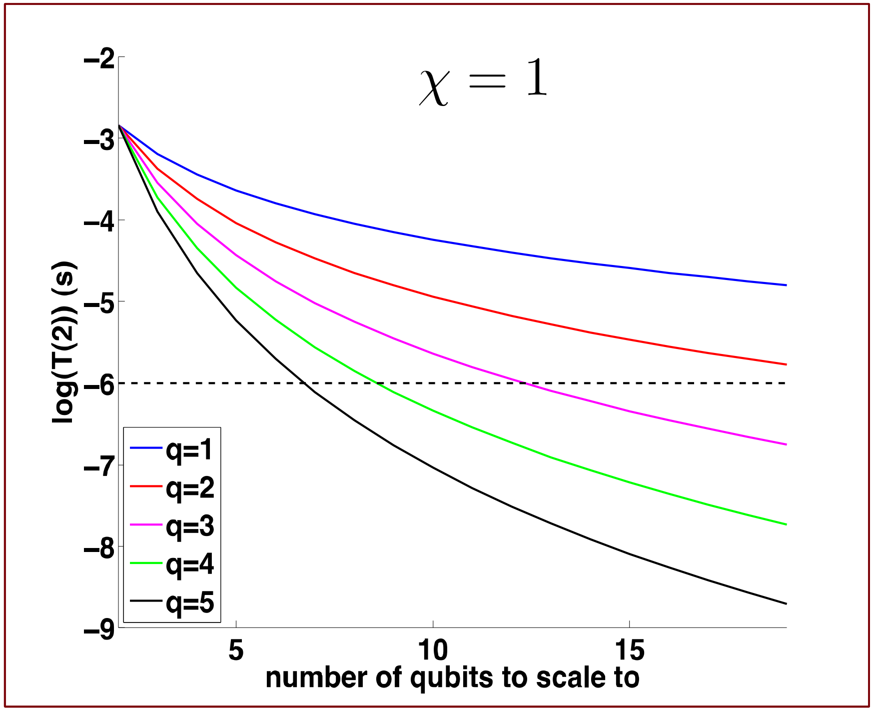
Neutrals support

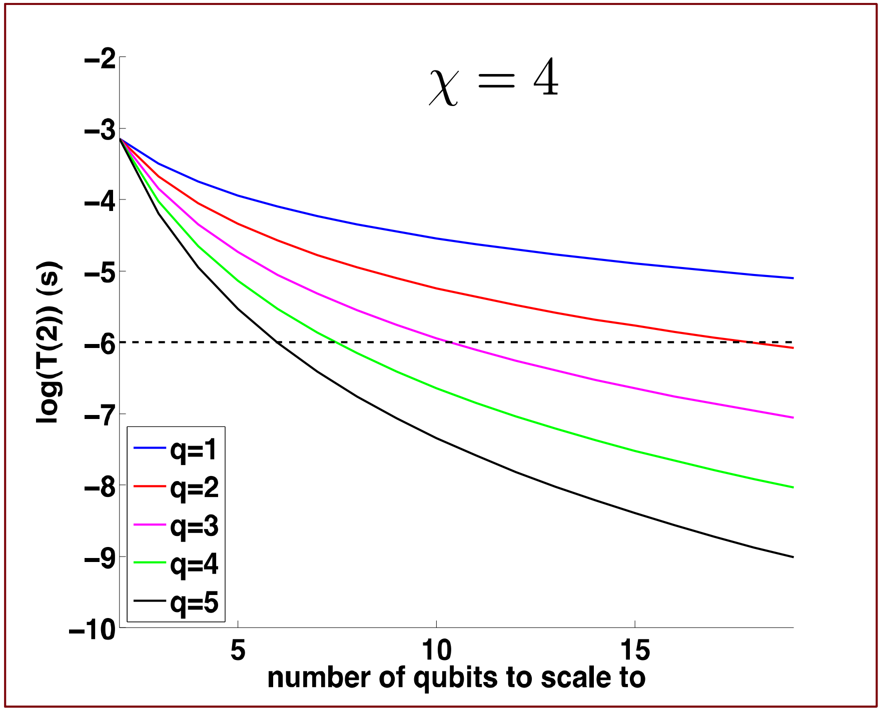
Leakage impact on QUBO scaling

- Estimate alpha by considering two-qubit example
- Dashed line indicates fastest possible run time for two qubit problem (limited by slew rates, electronics, etc.

$$\alpha \approx \frac{\ln(2)}{n^{q+1}(160 + 81\chi)}$$

$$T(2) = \alpha 2^q$$





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Natural robustness of AQC

Lies we tell ourselves

- Expected to be robust to:
  - **Dephasing errors**
    - Since the state is always an eigenstate, phase coherence between eigenstates is not required
  - Some kinds of **control errors**
    - As long as the path is adiabatic and the final Hamiltonian is close to the target, slight variations in the control path are unlikely to matter
  - **Relaxation**
    - State is already in the ground state, so relaxation (T1 processes) won't happen

Natural robustness of AQC

Lies we tell ourselves

- Expected to be robust to:
  - **Dephasing errors**
    - Since the state is always an eigenstate, phase coherence between eigenstates is not required
    - *Does not imply that local dephasing is unimportant*
  - Some kinds of **control errors**
    - As long as the path is adiabatic and the final Hamiltonian is close to the target, slight variations in the control path are unlikely to matter
    - *Probably not robust to "solving the wrong problem"*
  - **Relaxation**
    - State is already in the ground state, so relaxation (T1 processes) won't happen
    - *Need to consider physical system – qubits are often effective two-level systems embedded in a larger Hilbert space*

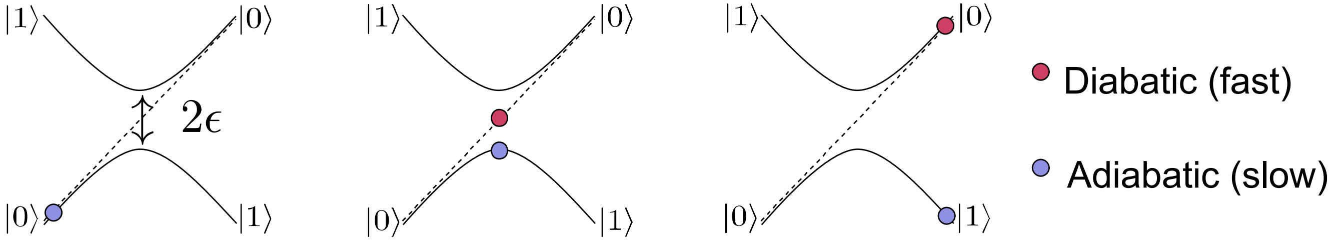
What can go wrong?

- Well, pretty much everything.
  - You implement the wrong Hamiltonian (**Control errors**)

$$H = 0.7532 \sum_i X_i + 0.7011 \sum_i Z_i Z_{i+1}$$

$$H = 0.6923 \sum_i X_i + 0.739 \sum_i Z_i Z_{i+1}$$

  - Blow through an adiabatic transition (**Diabatic errors**)



  - Coupling to environmental degrees of freedom (**noise**)

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k E_k \otimes B_k + H_{\mathrm{Bath}}(t)$$

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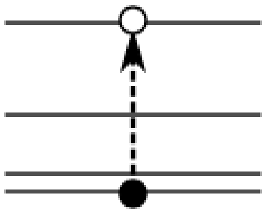

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Experimental & Architectural

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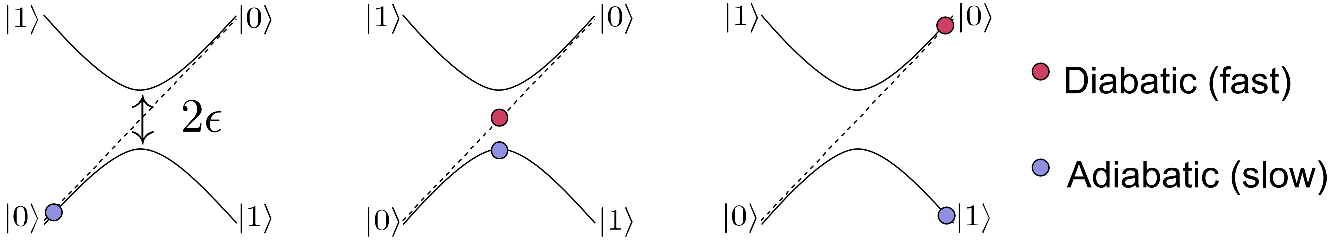

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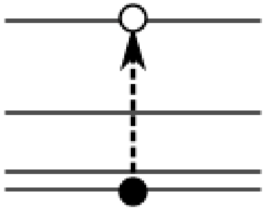


Suppressing control errors

Problem statement

- Control errors modify the final Hamiltonian:

$$H = \sum_{i,j} J_{i,j} z_i z_j + \sum_i h_i z_i$$
$$\to \sum_{i,j} \left(J_{i,j} + \Delta_{i,j}\right) z_i z_j + \sum_i \left(h_i + \delta_i\right) z_i$$

- Ground state of erred Hamiltonian could have high Hamming distance to desired ground state
- If noise is *iid*, then perturbation will be of order

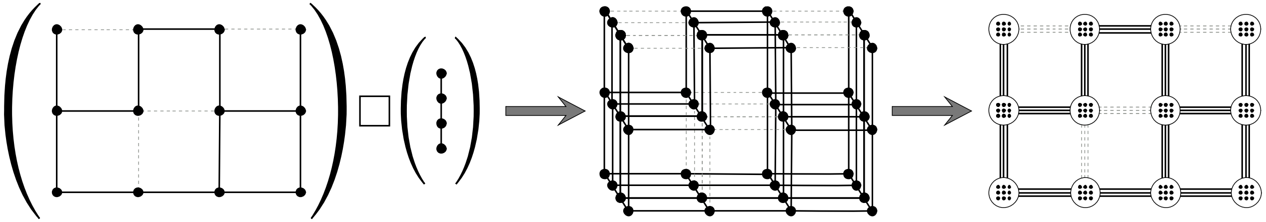
$$\Delta E \simeq \sqrt{||\mathcal{E}||\, \Delta^2_{\mathrm{RMS}} + ||\mathcal{V}||\, \delta^2_{\mathrm{RMS}}} \simeq \sqrt{(d+1)N}\, \epsilon_{\mathrm{RMS}}$$

- If this perturbation is larger than the gap, then there is reasonably large probability that the ground state(s) of the problem Hamiltonian will not be a ground state of the physical Hamiltonian

Suppressing control errors

Encoding

- We can encode the problem Hamiltonian in a K-bit repetition code:



- In the code space, the gap is expanded by a factor of K, while the perturbation increases by sqrt(K), effectively reducing the noise by a factor of sqrt(K)

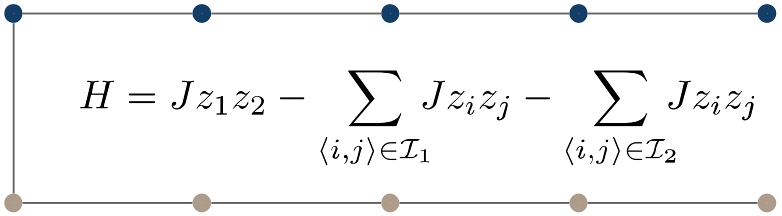
$$H \simeq \sum_{i,j} \left(KJ_{i,j} + \sqrt{K}\Delta_{i,j}\right) z_i z_j + \sum_i \left(Kh_i + \sqrt{K}\delta_i\right) z_i$$

Suppressing control errors

Numerical Example

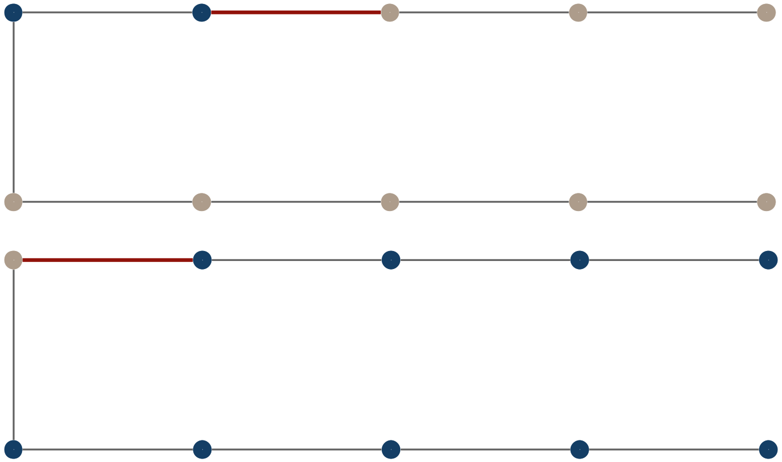
- Example:
  - Horizontal links: Ferromagnetic
  - Vertical link: Antiferromagnetic
  - Uniformly distributed errors on NN's

$$\Delta_{ij} \in \left[-\frac{4J}{5}, \frac{4J}{5}\right]$$



$$H = Jz_1z_2 - \sum_{\langle i,j\rangle \in \mathcal{I}_1} Jz_iz_j - \sum_{\langle i,j\rangle \in \mathcal{I}_2} Jz_iz_j$$

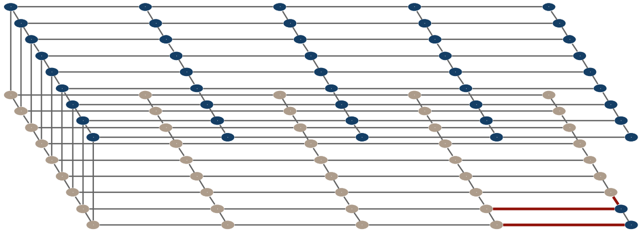
- Likely to cause errors
  - Monte Carlo results:
    - 1000 instances
    - 436 errors
- Solution:
  - Encode with Ising ferromagnet



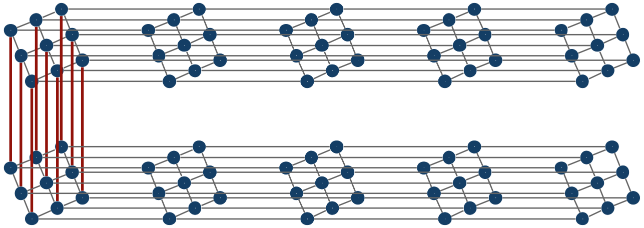
Suppressing control errors

Numerical examples

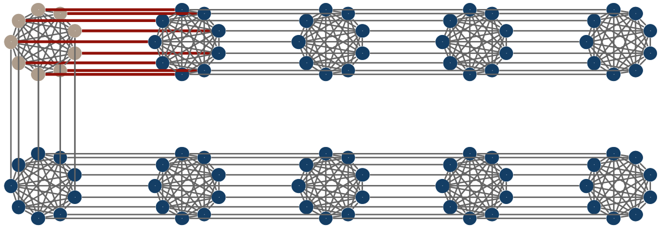
- Encoding examples
  - 1D Ising ferromagnet– 9 spins
    - Errors outside code space
    - 49/1000 instances wrong
    - Easiest to implement



  - 2D Ising ferromagnet – 9 spins
    - Errors in code space
    - 5/1000 instances wrong
    - Goldilocks



  - Complete ferromagnet – 9 spins
    - Errors in code space
    - 4/1000 instances wrong
    - Hardest to implement



Suppressing Control Errors

Lessons learned and open questions

- Lessons
  - Degree of encoding graph should be at least as high as degree of coding graph
    - Helps ensure that blocks stay locked together
  - May supply extra robustness for encoding used by Lidar et al. on D-Wave machine
- Open Questions
  - How does this encoding behave under adiabatic evolution?
    - Logical X operators are many-body and not physically implementable
    - Initial Hamiltonian is therefore not part of the code
    - What happens?!

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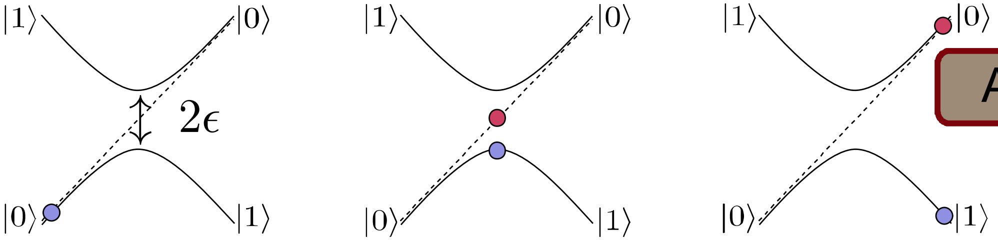
Experimental & Architectural

  - Blow through an adiabatic transition (**Diabatic errors**)



Algorithmic

● Adiabatic (slow)

  - Coupling to environmental degrees of freedom (**noise**)

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_k \gamma_k E_k \otimes B_k + H_{\mathrm{Bath}}(t)$$

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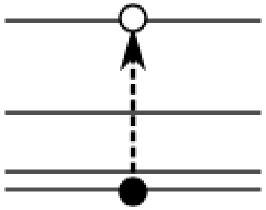

Suppressing diabatic errors

- See **talk** by Rolando Somma
  - Gap amplification
- See **poster** by Constantin Brif
  - Constructing optimal interpolation paths by tracking ground state population in exactly solvable models

$$J_{\mathrm{t}} = \frac{\alpha}{T} \int_0^T |\langle \phi_0(t) | U(t) | \phi_0^{(0)} \rangle|^2 dt$$

  - Hope to use exactly solvable models to
    - Assist in experimental support
    - Assist

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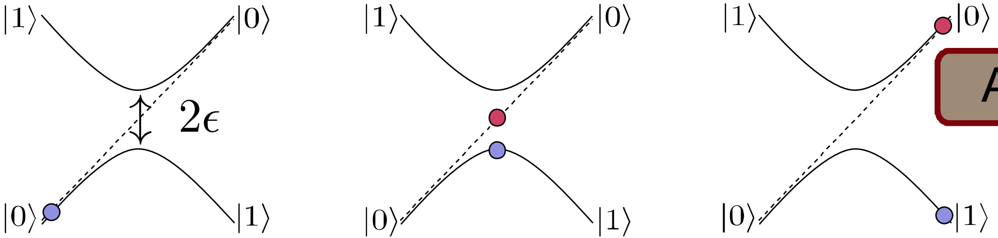

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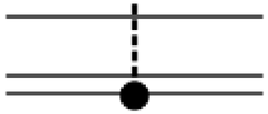


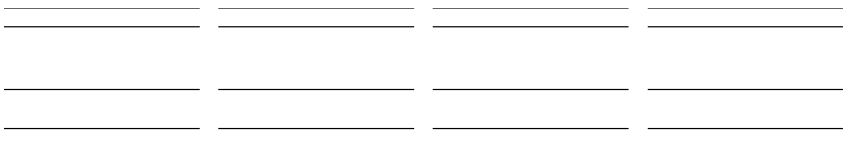
Error correcting stabilizer codes

- Such codes are how we protect information in circuit model QC
- A code comes with:
  - A set of mutually-commuting stabilizer generators
    - An error operator is ***correctable*** if it anti-commutes with a unique set of stabilizer generators. We use the following notation to index detectable errors:

$$E_{\mathbf{k}} S_i = (-1)^{\mathbf{k_i}} S_i E_{\mathbf{k}}$$

  - A set of logical operators
    - Commute with the stabilizer group, so there *exists a basis of simultaneous eigenstates* so the logical degrees of freedom are separated from the degrees of freedom which track errors

$$\bar{L} S_i = S_i \bar{L}$$



  - +1 Eigenspace of the stabilizer generators defines the ***code space***
  - States outside the code space belong to an ***error space***
- *Stabilizer codes permit the detection and correction of an error without destroying the encoded quantum information*

Step 1: Suppressing errors

- Several strategies for suppressing errors:
  - **Energy Gap Protection**
    - Add stabilizer generators to Hamiltonian, increasing energy gap to excited states

$$H \rightarrow H - \sum_i \kappa_i S_i$$

S. Jordan, Error-correcting codes for adiabatic quantum computation, PRA 74, 052322 (2006)

    - If noise power spectrum drops exponentially, then so does the transition rate out of the code-space
  - **Dynamical Decoupling**
    - Apply stabilizer generators periodically as unitary operators

$$H \rightarrow S_i H S_i$$

D. Lidar, Towards fault tolerant adiabatic quantum computation, PRL, 100, 160506 (2008)

    - Error terms are suppressed in the average Hamiltonian
  - **Zeno Effect Suppression**
    - Frequently measure stabilizer generators
    - Zeno effect tends to keep the state in the code-space

G. Paz-Silva, Zeno effect for quantum computation and control, PRL, 108, 080501 (2012)

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DD and QES relationship

- The encoded system with control is,

$$H(t) = H_{\mathrm{AQC}}(t) + \sum_{\mathbf{k}} \gamma_{\mathbf{k}} E_{\mathbf{k}} \otimes B_{\mathbf{k}}(t) + \mathrm{control}$$

$$\mathrm{control} = \begin{cases} H_{\mathrm{QES}} = \frac{\Omega}{2} \sum_i S_i & \\ U_{\mathrm{DD}} \in \{S_i\} & \text{at frequency } \Omega \end{cases}$$

- Move to **interaction picture** with respect to control

$$\tilde{H}(t) = U_{\mathrm{ctrl}}(t)\ \left(H(t) - H_{\mathrm{ctrl}}(t)\right)\ U^{\dagger}_{\mathrm{ctrl}}(t)$$

$$\tilde{H}_{\mathrm{int}}(t) = \begin{cases} \sum_{\mathbf{k}} \gamma_{\mathbf{k}} (-1)^{g_{\mathbf{k}}(t)} E_{\mathbf{k}} \otimes B(t) & \mathrm{DD} \\ \sum_{\mathbf{k}} \gamma_{\mathbf{k}} \exp\left(2i\Omega t\, \mathbf{k} \cdot \vec{S}\right) E_{\mathbf{k}} \otimes B(t) & \mathrm{QES} \end{cases}$$

- In rotating frame, both QES and DD result in periodic modulation of noise.
- Effective noise rate is similar in both cases.
    - Fermi Golden Rule yields roughly equal leakage from the ground state.

Step 1: Suppressing errors

Master equation

- Master equation formalism gives insight into dynamics and highlights differences between suppression mechanisms
  - We can derive a **master equation** describing the population remaining in the code space,

$$\frac{d\left\langle \mathbf{P}(t)\right\rangle}{dt} = 2\sum_{j=1}^{N_e}\int_0^{\tau} ds\,\mathrm{Re}\left\{C_j(s)m_j^*(\tau,s)\mathrm{Tr}\left[E_j\mathbf{P}E_j(t,s)\mathbf{Q}\rho(s)\mathbf{Q}\right]\right\} - \mathrm{Re}\left\{C_j(s)m_j(\tau,s)\mathrm{Tr}\left[E_j\mathbf{Q}E_j(t,s)\mathbf{P}\rho(s)\mathbf{P}\right]\right\}$$

  - Population left in code space (derived assuming *no logical Hamiltonian*)

$$\left.\frac{\mathrm{d}\left\langle \mathbf{P}(t)\right\rangle}{\mathrm{d}t}\right|_{H_{\mathrm{AQC}}=0} = -\sum_k r_k^+(t)\left\langle \mathbf{P}(t)\right\rangle + \sum_k r_k^-(t)\left\langle \mathbf{Q}_1(t)\right\rangle$$

  - Rates are **modified by error suppression** (DD ignores detailed balance)

$$r_j^+(t) = 2\mathrm{Re}\left\{\int_0^t ds\,C_j(s)m_j^*(t,s)\right\} \qquad m_j^{\mathrm{DD}}(t,s) = (-1)^{p(t)-p(t-s)}$$

$$r_j^-(t) = 2\mathrm{Re}\left\{\int_0^t ds\,C_j(s)m_j(t,s)\right\} \qquad m_j^{\mathrm{EGP}}(t,s) = e^{2i\alpha s\omega_j}$$

Step 1: Suppressing errors

DD and QES major difference

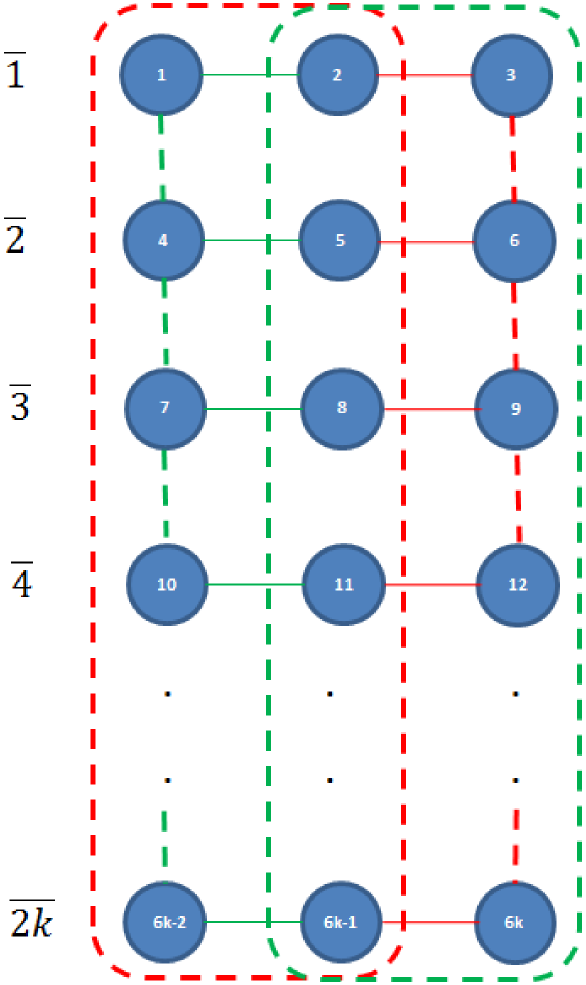
- Dynamical Decoupling $$E_{\mathbf{k}} e^{-i\Omega t} \otimes \sum_{\omega} B_{\omega} e^{i\omega t} \overset{\text{RWA}}{\simeq} E_{\mathbf{k}} \otimes B_{\Omega}$$
- Energy Gap Protection $$E_{\mathbf{k}} \operatorname{sgn}(\cos(\Omega t)) \otimes \sum_{\omega} B_{\omega} e^{i\omega t} \overset{\text{RWA}}{\simeq} E_{\mathbf{k}} \otimes [B_{\Omega} + B_{-\Omega}]$$

DD and EGP modulation functions

- Dynamical Decoupling
  - Sensitive to positive and negative bath frequencies
  - At equilibrium, population spread equally across code and syndrome spaces
    - Coupling to a cold bath doesn't help
- Energy Gap Protection
  - Sensitive only to a single bath frequency
  - At equilibrium, population accumulates in code space
    - Coupling to a cold bath *might* help

Step 1: Suppressing errors

The [[6k,2k,2]] code



- Low-weight logical operators
- Bounded qubit interactions
- Planar
  - Logical operators

$$\overline{Z_i} = Z_{i,1}Z_{i,2}$$

$$\overline{X_i} = X_{i,2}X_{i,3}$$

$$\overline{Z_iZ_j} = Z_{i,2}Z_{j,2}$$

$$\overline{X_iX_j} = X_{i,2}X_{j,2}$$

  - Stabilizers

$$Z^{\otimes 6k}, X^{\otimes 6k} \in \mathcal{S}$$

Step 2: Correcting errors

Adding error correction

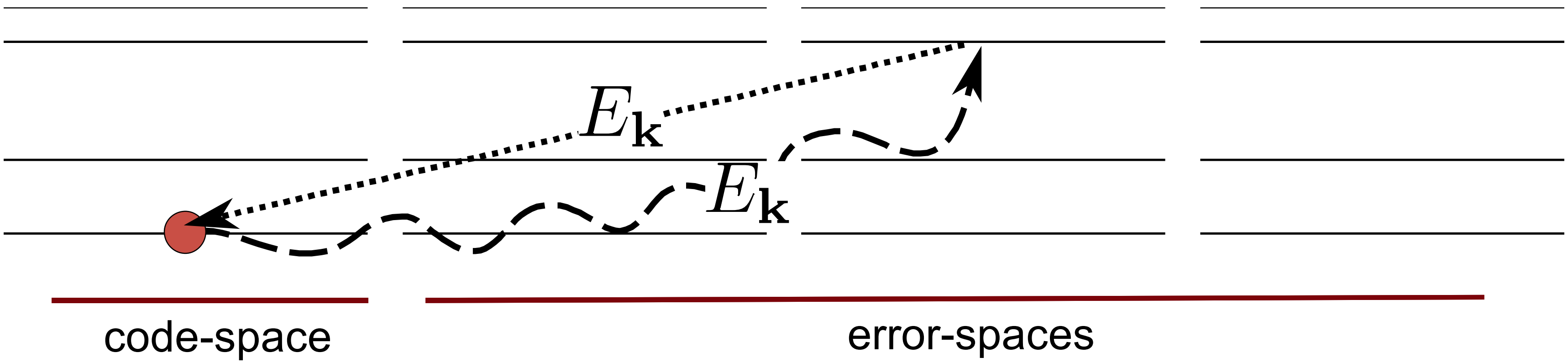
- What resources are we allowing?
  - Measurements and circuits?
    - Measurements of stabilizers could be difficult
    - Requiring gates is tricky
  - Continuous error correction?
    - Appealing for adiabatic

- **Stabilizer Hamiltonians**, Hamiltonians composed entirely of stabilizer elements, naturally fit into a continuous error correction framework because:
  - Local error operators promote the ground state to excited energy eigenstates
  - These errors can be reversed (corrected)
  - We can *cool* the system by local coupling to cold bath

Step 2: Correcting errors

Good intentions

- If an error happens, measuring stabilizers will detect *which* error happened
  - Reach into system and apply correction operator
- But stabilizer generators may be **high weight**
  - Gadgets?
    - Introduces unprotected interactions
  - Gates?
    - A little bit out of paradigm
  - Decode at the end?
    - Limits number of correctable errors.



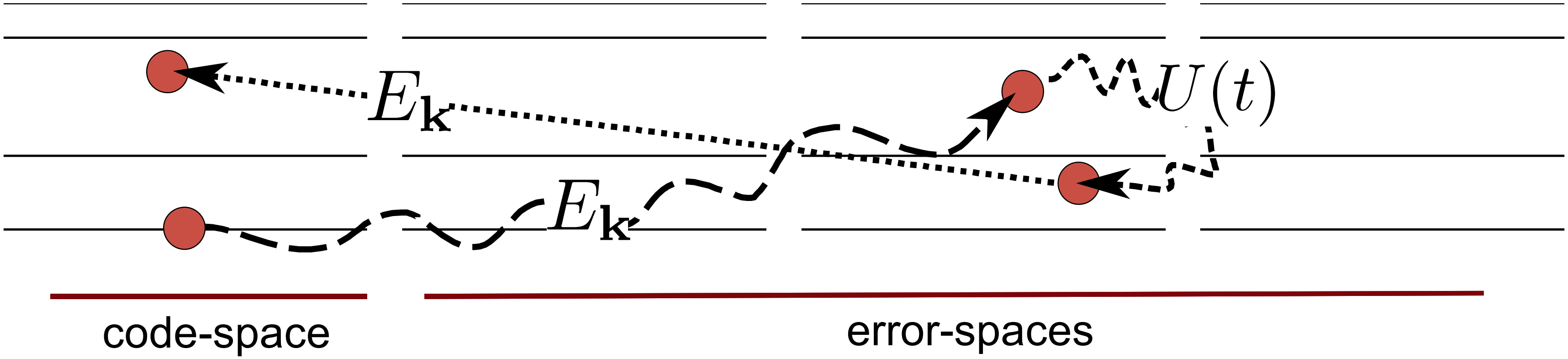
Step 2: Correcting errors

Unintended consequences

- Coherent evolution (by logical Hamiltonian) leads to mixing of states.
  - Hamiltonian and error operator don't exactly commute or anti-commute, so $E_{\mathbf{k}} \left|\psi\right\rangle$ is not necessarily an eigenstate. This state then evolves under the action of the logical Hamiltonian before it is corrected.

$$\boxed{\begin{array}{c} E_{\mathbf{k}} e^{-i(H_{\mathbf{k}}^{+}+H_{\mathbf{k}}^{-})t} E_{\mathbf{k}} \left|\psi\right\rangle = e^{-i(H_{\mathbf{k}}^{+}-H_{\mathbf{k}}^{-})t} \left|\psi\right\rangle \\ H = H_{\mathbf{k}}^{+} + H_{\mathbf{k}}^{-} \qquad [E_{\mathbf{k}}, H_{\mathbf{k}}^{+}] = \{E_{\mathbf{k}}, H_{\mathbf{k}}^{-}\} = 0 \end{array}}$$

  - A low weight physical error gets "dressed" by the (always on) logical Hamiltonian and gets converted into a high-weight uncorrectable error
  - Diffusion rate obeys a Lieb-Robinson bound, but connectivity of encoded system may be high



Step 2: Correcting errors

Limitations of error suppression in AQC

- "Decode at the end" is unlikely to be useful
  - State might be in correctable subspace, but ***logical errors*** have happened
  - Error suppression must be so strong that state is in code-space with high probability.
    - For large computations, this is very unlikely
    - Circuit model results imply the state will almost certainly not be in the code-space
- For EGP implementation of error suppression, require high-weight (many-body) terms to enforce energy penalties.
  - DD does not have this problem:
    - For example:

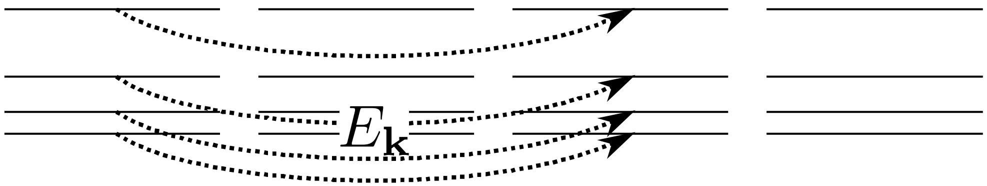
$$XXXXX = e^{-i\pi\sigma_x^{(1)}/2}e^{-i\pi\sigma_x^{(2)}/2}e^{-i\pi\sigma_x^{(3)}/2}e^{-i\pi\sigma_x^{(4)}/2}e^{-i\pi\sigma_x^{(5)}/2}$$

- Our options:
  - Fixing the logical operators
  - Active error correction
  - ***BOTH***?

Step 2: Correcting errors

Protected Hamiltonians

- We want errors to map **eigenstates to eigenstates**



  - Want error to be pseudo-eigenoperator of Hamiltonian

$$[H, E_\mathbf{k}]\Pi_0 = \epsilon_\mathbf{k} E_\mathbf{k} \Pi_0$$

- Exploit freedom in choosing logical operators
  - **Multiplication by stabilizers** does not affect action on code space:

$$S_\mathbf{j} L \simeq L$$

    - Repetition code example:

$$ZZI \cdot XXX = YYX \simeq XXX$$

  - Construct **protected Hamiltonian** by conjugation with correctable errors

$$\bar{H} = \sum_\mathbf{k} E_\mathbf{k} \, H \, \Pi_0 \, E_\mathbf{k} \qquad \Pi_0 = \prod_j \frac{(1+S_j)}{2} = \frac{1}{2^N} \sum_n s_n$$

    - This is ***terribly high weight***

Step 2: Correcting Errors

Protected operators: [[5,1,3]] code

- The quantum [[5,1,3]] code is usually defined by the operators

$$\mathcal{S} = \langle IXZZX, XIXZZ, ZXIXZ, ZZXIX \rangle$$

$$\bar{Z} = ZZZZZ \quad \bar{X} = XXXXX$$

- But standard logical operators *do not act consistently across subspaces*

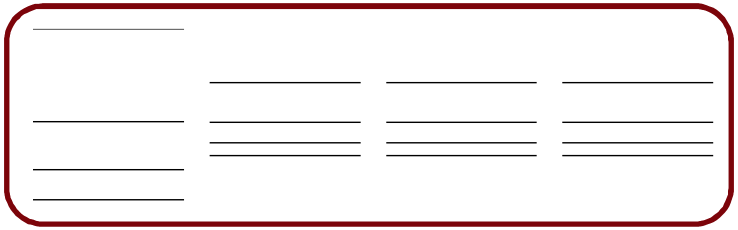
$$\begin{aligned} 4\,U[\bar{X}]U^\dagger = X \otimes (&-IIII + IIIZ + IIZI + IIZZ + IZII - IZIZ + IZZI + IZZZ + \\ &ZIII - ZIIZ - ZIZI - ZIZZ + ZZII - ZZIZ + ZZZI + ZZZZ) \\ 4\,U[\bar{Z}]U^\dagger = Z \otimes (&-IIII + IIIZ + IIZI - IIZZ + IZII + IZIZ - IZZI - IZZZ + \\ &ZIII + ZIIZ + ZIZI + ZIZZ - ZZII + ZZIZ - ZZZI + ZZZZ) \end{aligned}$$

- Improved operators act consistently

$$\begin{aligned} \bar{X}_3 =& IIZXZ + IXIYY + IYYIX + IZXZI + XIYYI + \\ & XZIIZ + YIXIY + YYIXI + ZIIZX + ZXZII \\ \bar{Z}_3 =& IIYZY + IXXIZ + IYZYI + IZIXX + XIZIX + \\ & XXIZI + YIIYZ + YZYII + ZIXXI + ZYIIY \end{aligned}$$

$$U\bar{X}_3U^\dagger = -\frac{1}{2}X \otimes (4\Pi_0 + \mathbb{I})$$

$$U\bar{Z}_3U^\dagger = -\frac{1}{2}Z \otimes (4\Pi_0 + \mathbb{I})$$

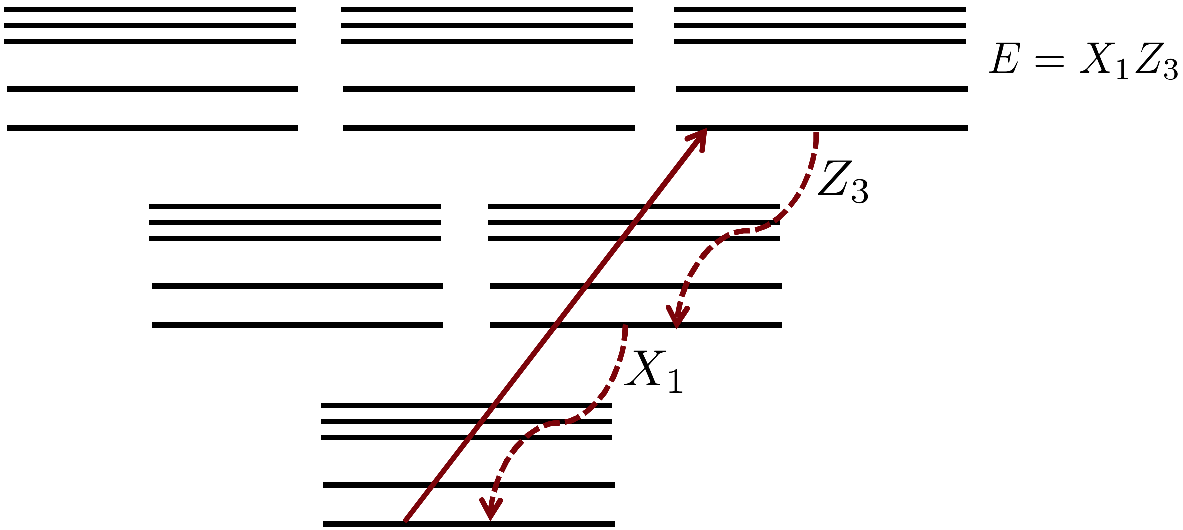


Step 2: Correcting errors

Ideal Error Correction

$$|\Psi\rangle\,|\mathbf{k}\rangle \rightarrow |\Psi\rangle\,|\mathbf{0}\rangle$$

- For local cooling, errors must be penalized according to their **weight**



Weight-2 errors

Weight-1 errors

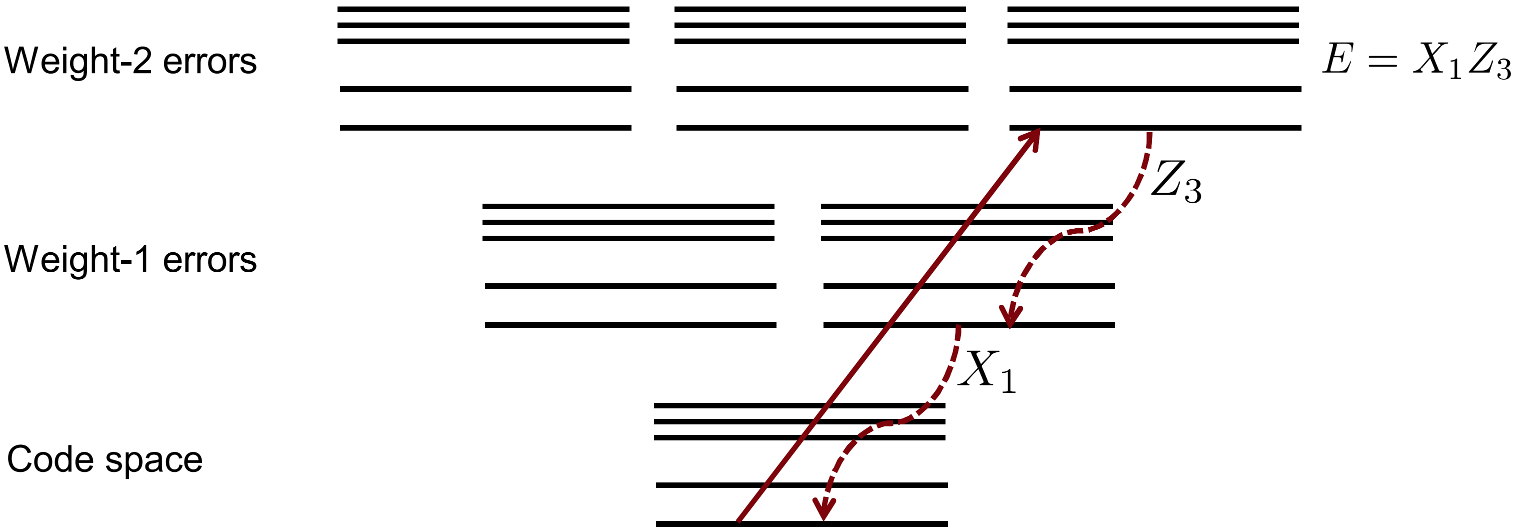
Code space

Step 2: Correcting errors

Ideal Error Correction

$$|\Psi\rangle\,|\mathbf{k}\rangle \rightarrow |\Psi\rangle\,|\mathbf{0}\rangle$$

- For local cooling, errors must be penalized according to their **weight**



- But this is a self **correcting quantum memory**
  - No-go theorem in 2D: Bravyi and Terhal
  - Embed toric code in ferromagnet to get string tension: Loss ?
- This will require nonlocality **somewhere**

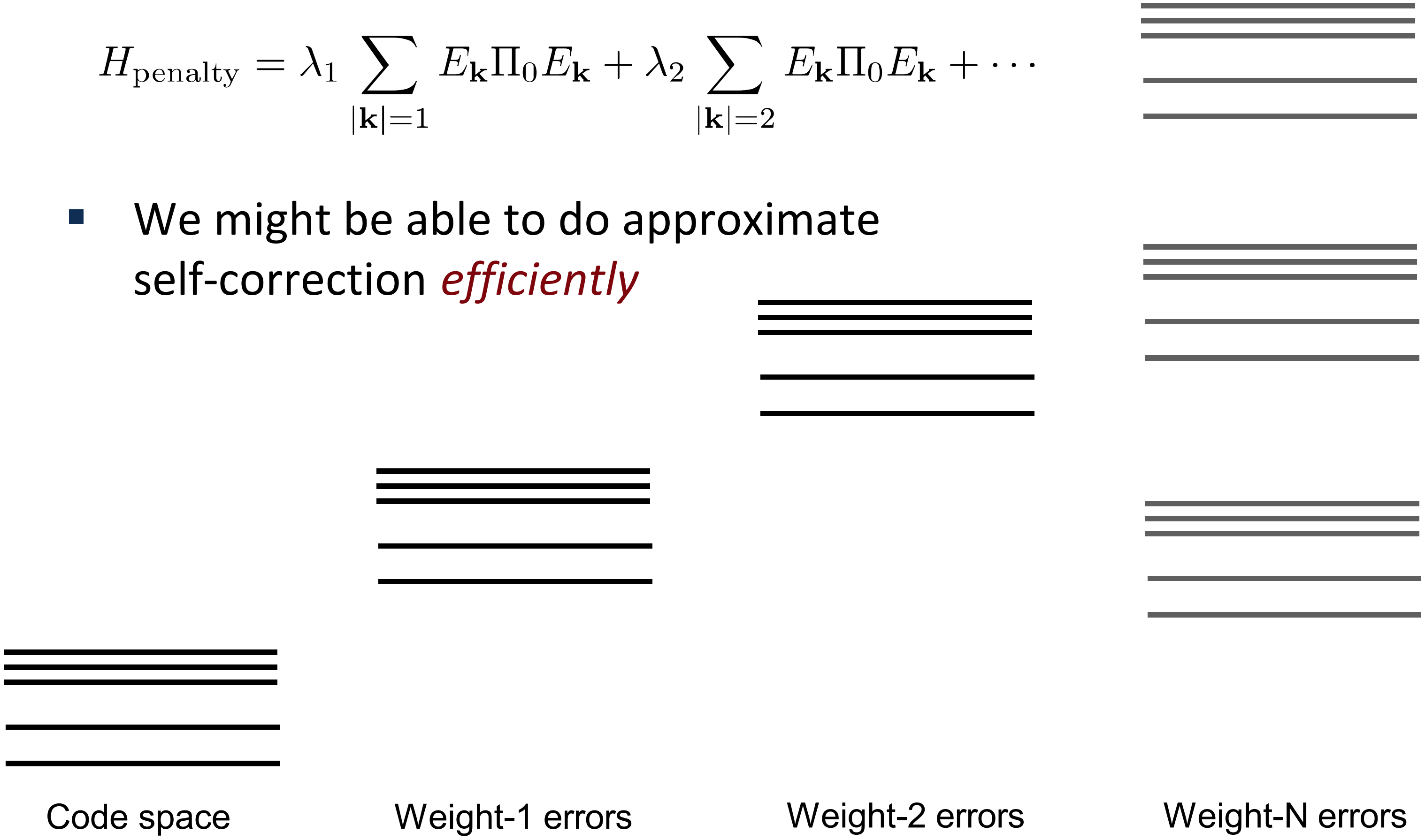
Step 2: Correcting errors

Almost-Ideal Error Correction

- But we can produce **incomplete** self-correcting codes

$$H_{\text{penalty}} = \lambda_1 \sum_{|\mathbf{k}|=1} E_{\mathbf{k}} \Pi_0 E_{\mathbf{k}} + \lambda_2 \sum_{|\mathbf{k}|=2} E_{\mathbf{k}} \Pi_0 E_{\mathbf{k}} + \cdots$$

- We might be able to do approximate self-correction *efficiently*



Step 2: Correcting errors:

Open questions

- Can we make error correction work?
  - Can we do active error correction without gates?
  - Can we locally cool a non-self correcting memory and reduce the active error correction overhead?
  - What is the role of thermalization?

- What new techniques are necessary for AQC fault tolerance?
  - What *is* fault tolerance in this sense?
    - *The results of measurements on the final state of a faulty AQC simulator can be efficiently decodable to the correct answer.*
      - Restrict to computational basis measurement?
  - **Is fault tolerant AQC impossible?**
    - We need high distance codes → high weight logical operators
    - Physics requires locality and low-weight operator

Architecture plans

Remaining six months

- Fault tolerance
- Experimental support