

Warm Dense Matter, Saint Malo, France  
June 25, 2013

# Phase Transitions of MgO Along the Hugoniot

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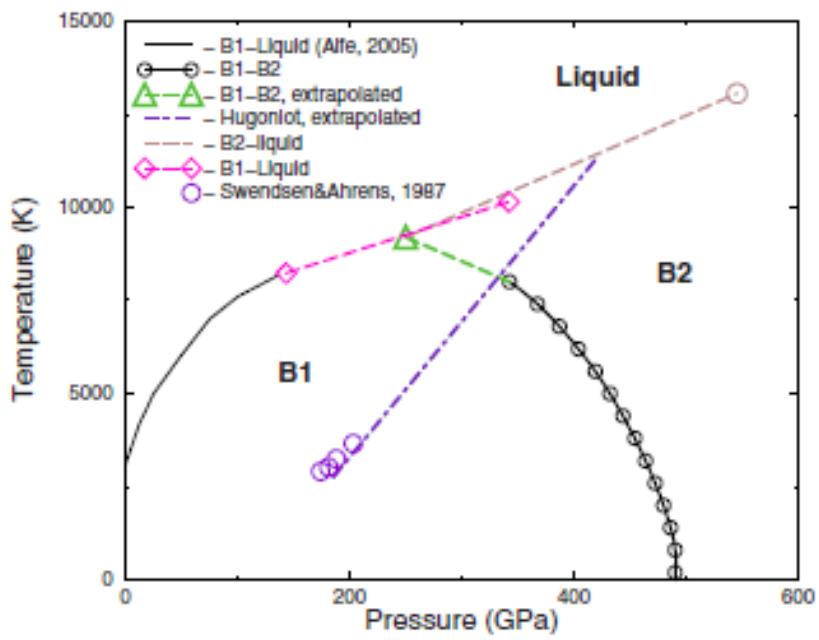
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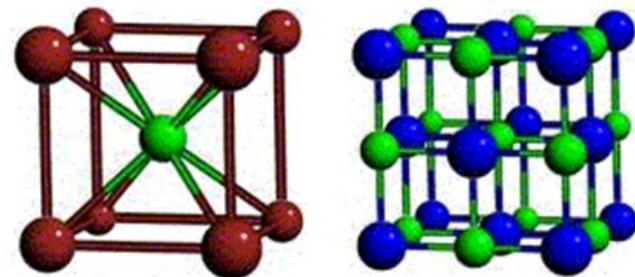
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# MgO Background

- MgO is abundantly found the Earth's mantle and likely other terrestrial exo-planets
- Understanding the high P-T behavior of MgO is important for modeling Earth's interior and planetary formation
- Static pressure data show no phase transition up to 227 Gpa at ambient temperature
- Hugoniot data (starting at ambient temperature) to  $\sim 200$  GPa – no phase transition
- Belonoshko et al predict a B1-B2 phase transition near 350 GPa and melt near 5 Mbar and 12000 K.
- McWilliams et al predict B1-B2 transition at 440 GPa and melt at 650 GPa



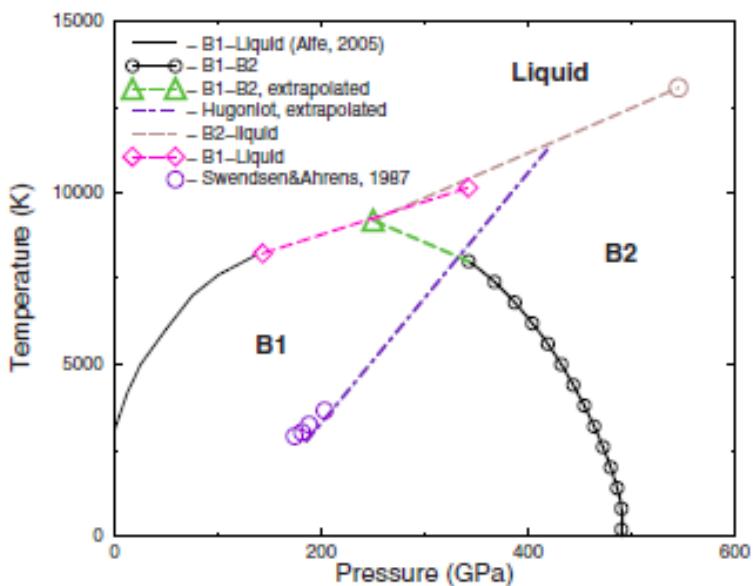
Belonoshko et al., Phys. Rev. B 81, 054110 (2010)



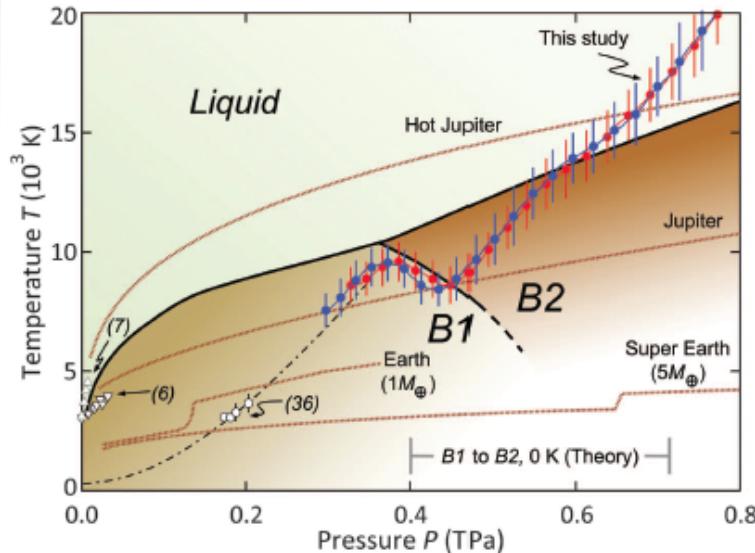
CsCl

NaCl

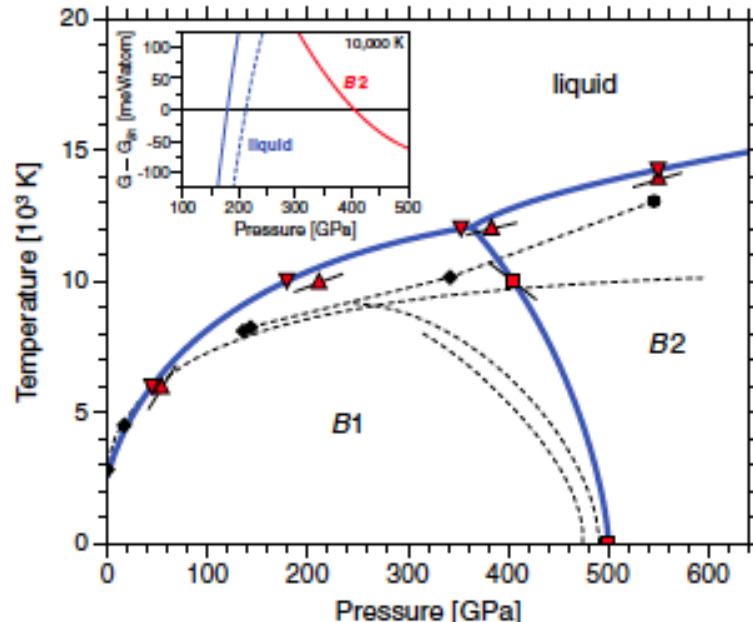
# MgO Phase Diagrams



Belonoshko *et al.*, PRB **81**, 054110 (2010)



McWilliams *et al.*, Science **338**, 1330 (2012)



B. Boates and S. Bonev, PRL **110**, 135504 (2013).

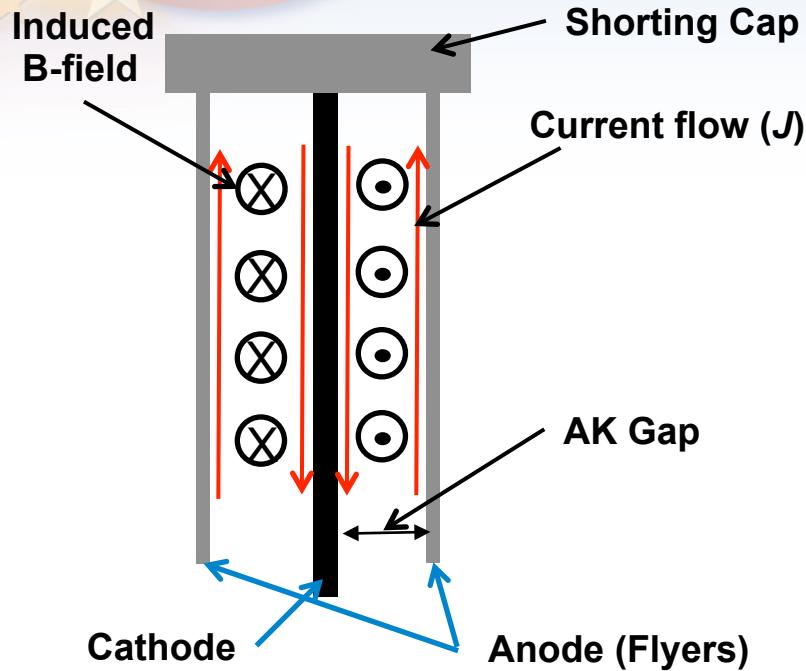
- Multiple phase diagrams published
- B1-B2 phase boundary in disagreement in all phase diagrams
- Melt boundary in disagreement
- No consensus on phase boundaries



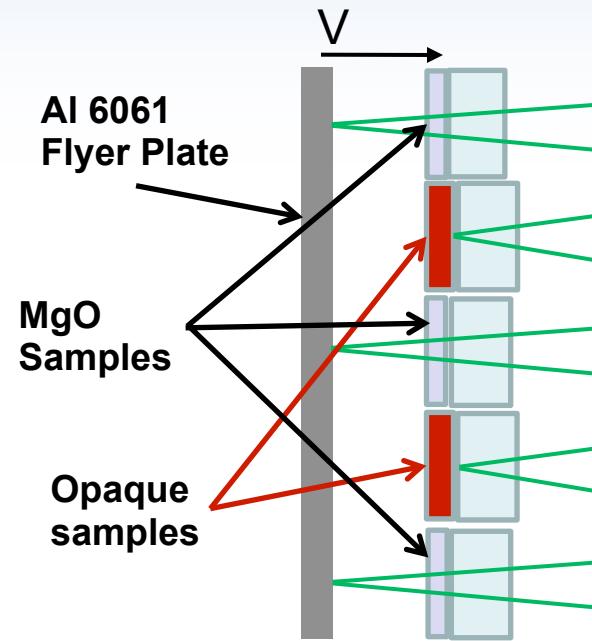
# Objectives

- Measure the MgO Hugoniot to 12 Mbar using the Sandia Z - Machine
- Experimental determine the proposed solid-solid phase transition
- Determine melt on the Hugoniot
- Apply Density Functional Theory methods to corroborate experimental findings
- Update the MgO phase diagram

# Experimental Approach



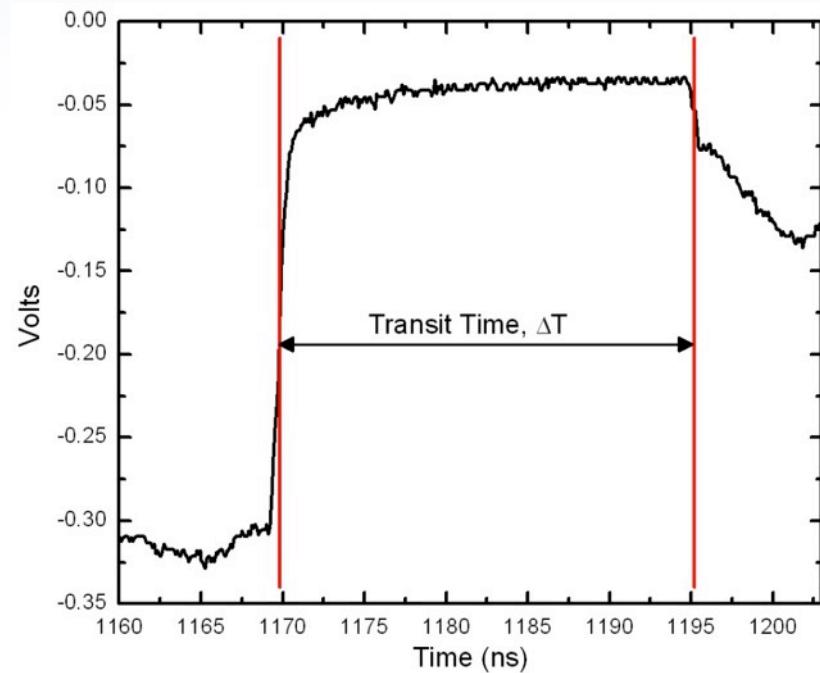
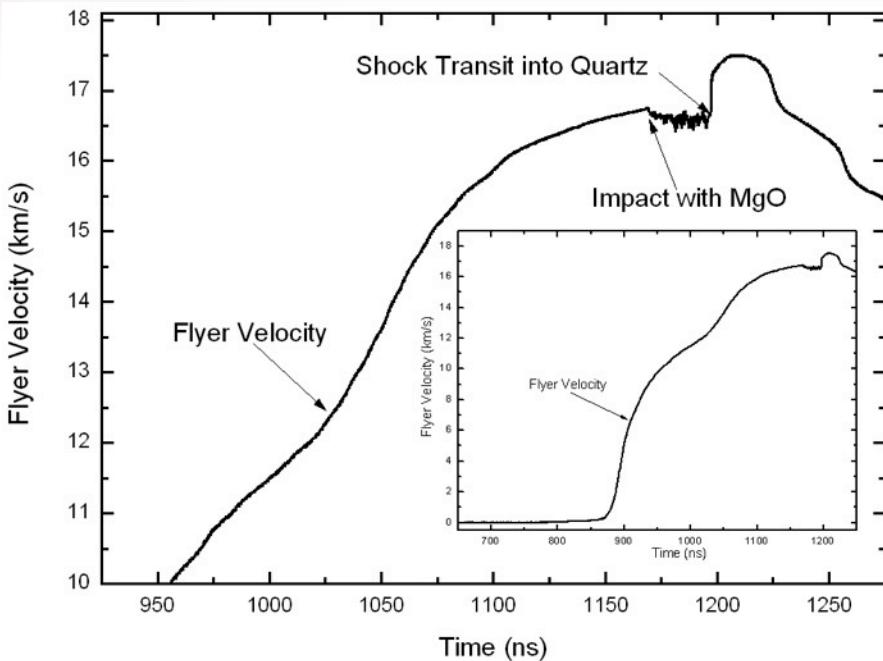
- Current pulse loops through shorting cap inducing a  $B$  – field.
- Resulting  $J \times B$  force accelerates anodes (flyers) outward up to 40 km/s
- Asymmetric AK Gaps result in two different flyer velocities (two Hugoniot points per experiment)



- Multiple samples per experiment
- MgO windows are initially transparent and are backed by quartz, TPX plastic, or aerogel windows
- VISAR used to measure flyer velocity
- Multiple VPFs per sample – reduces uncertainty

Method produces steady shock state in the MgO

# Experimental Measurements

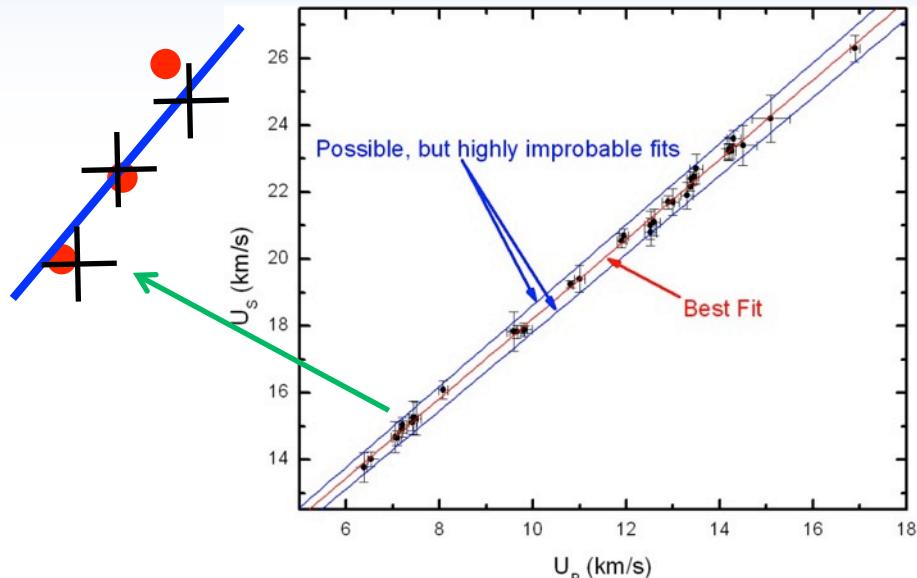


- VISAR tracks flyer plate velocity up to impact
- Shock front in MgO not reflective – Loss of contrast in VISAR signal
- Clear impact and shock transit fiducials – transit time analysis
- At higher pressures, MgO shock front reflective and shock velocity measured directly
- Shock front in back window is reflective – release state for MgO is determined (not discussed here)

# Monte Carlo (MC) Impedance Matching

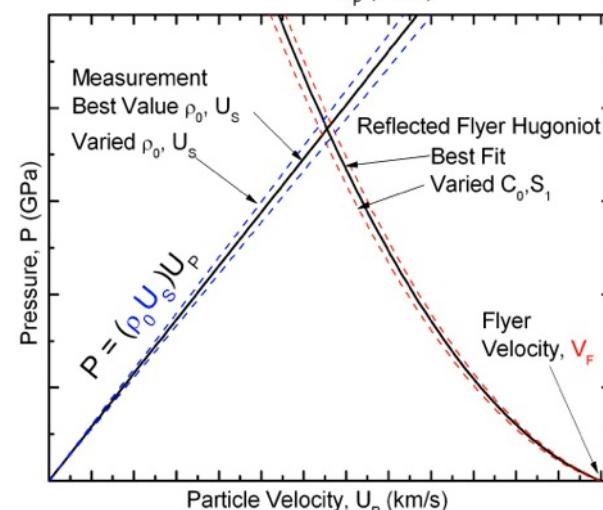
## Aluminum

- Uncertainty in experimental data (Knudson et al., JAP 2003)
- Vary each  $U_s$ - $U_p$  point by an uncorrelated random number with  $\sigma = \text{expt. Uncertainty}$
- Solve for linear fit parameters
- Determine mean,  $\sigma$ , and correlation of fit parameters



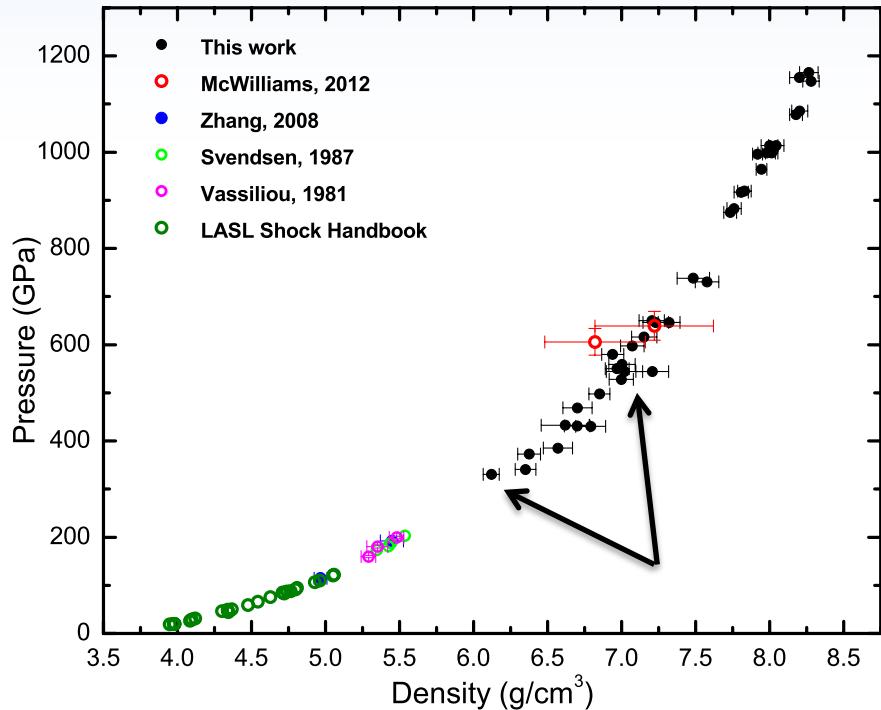
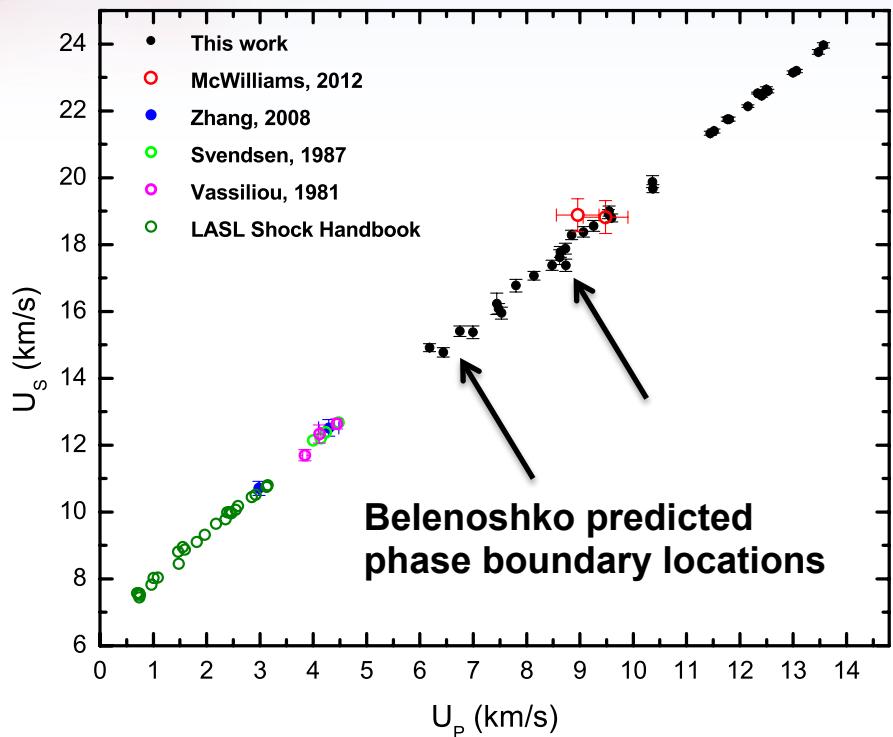
## MgO

- Vary measured parameters ( $V_F$ ,  $U_s$ ,  $\rho_0$ ) with uncorrelated random numbers,  $\sigma = \text{experimental uncertainty}$
- Vary AI fit parameters using correlated random numbers
- Calculate  $U_p$ ,  $P$ , and  $\rho$
- Determine mean and  $\sigma$



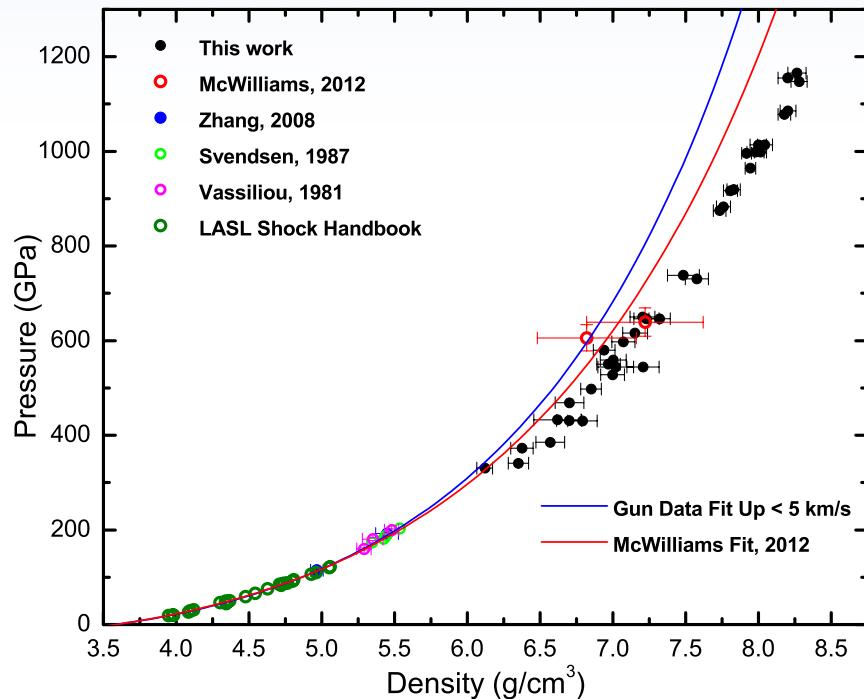
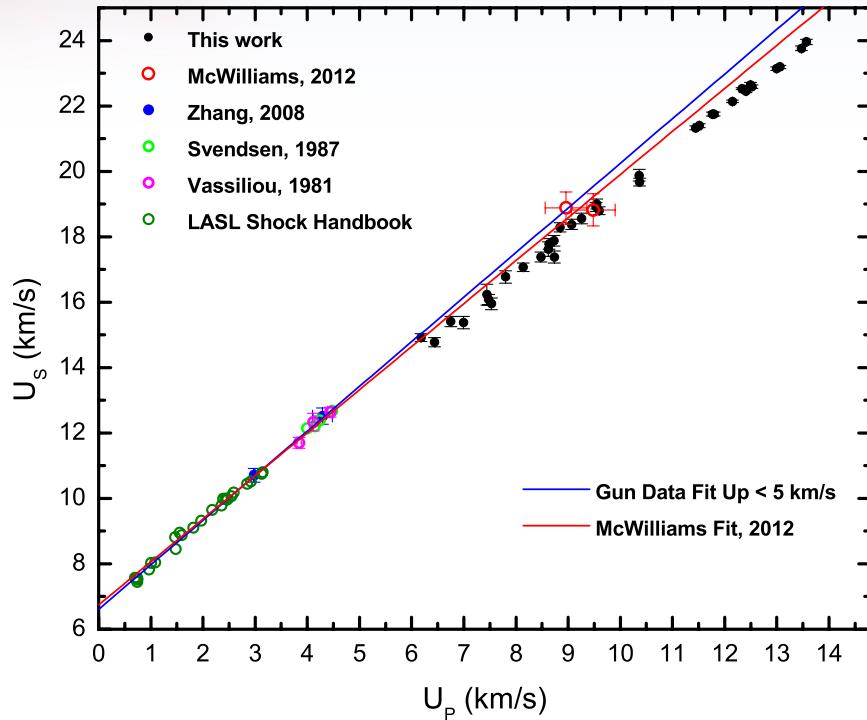
**Monte Carlo technique accounts for all experimental uncertainty and propagates the AI standard error into the MgO data.**

# Experimental Results



- Total of 37 Hugoniot measurements
- Highest pressure Hugoniot measurement - 11.6 Mbar
- Lowest pressure Hugoniot measurement – 330 Gpa
- No reflective shock front below 7 Mbar
- Data spans the B1-B2 transition and the B2 - melt transition

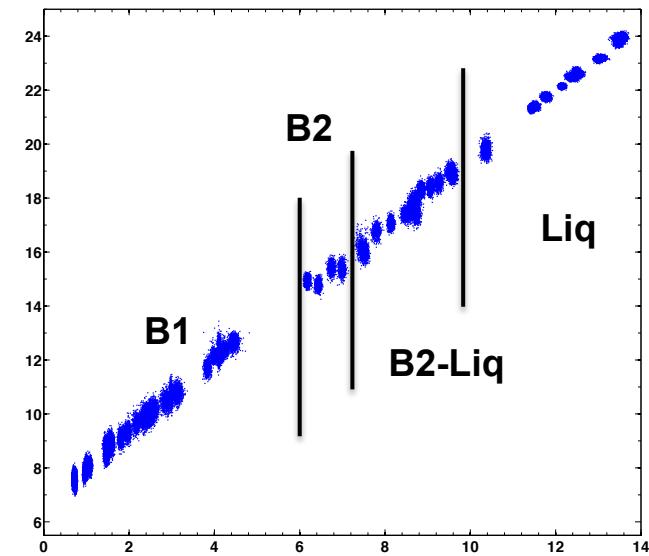
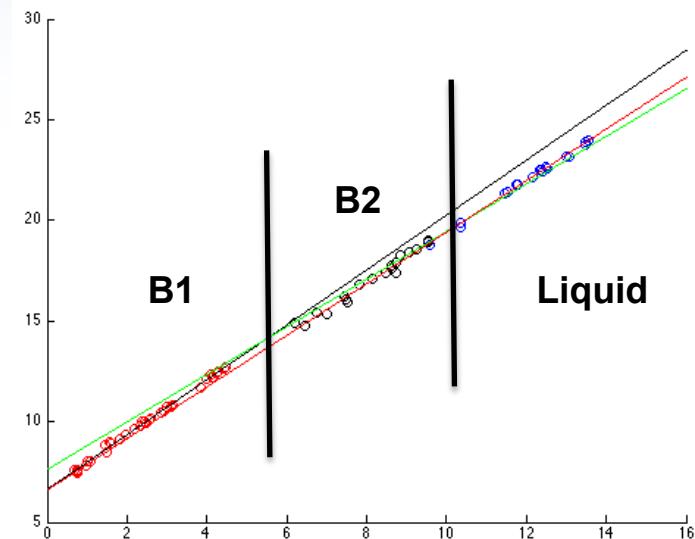
# Comparison To B1 Solid Fit



- Z data deviate from the extrapolated gun data fit at 330 Gpa**
- Z data also deviate from the McWilliams *et al* fit near 330 Gpa**
- Suggests the B1-B2 solid phase transition starts around 330 Gpa**
- No obvious change in Hugoniot as MgO transitions from B2 - Liquid**

# Optimization for Phase Boundaries

- Use Genetic Algorithm (GA) methods to optimize a 3 and 4 piece linear fit.
- 3 – Piece uses B1, B2, Liquid regions
- 4 – Piece uses B1, B2, Coexistence region, and liquid region
- Similar method applied to diamond (Knudson et al, Science 322, 1822 (2008)).
- GA method optimizes ‘break points’ for a multi-piece linear fit using linear regression
- Several optimization runs performed to estimate uncertainty in break points
- 3-piece linear fit suggests a large B2 region inconsistent with previous phase diagrams and has a larger  $\chi^2$  than a 4-piece fit



# Optimization Results

## B1 – B2 Boundary

$U_p = 6.00 \pm 0.20 \text{ km/s}$

$P = 330 - 355 \text{ Gpa}$

## B2 – Liquid Coexistence Region

$U_p = 7.5 \pm 0.3 \text{ km/s}$

$P = 412 - 466 \text{ Gpa}$

## Coexistence Region – Liquid Boundary

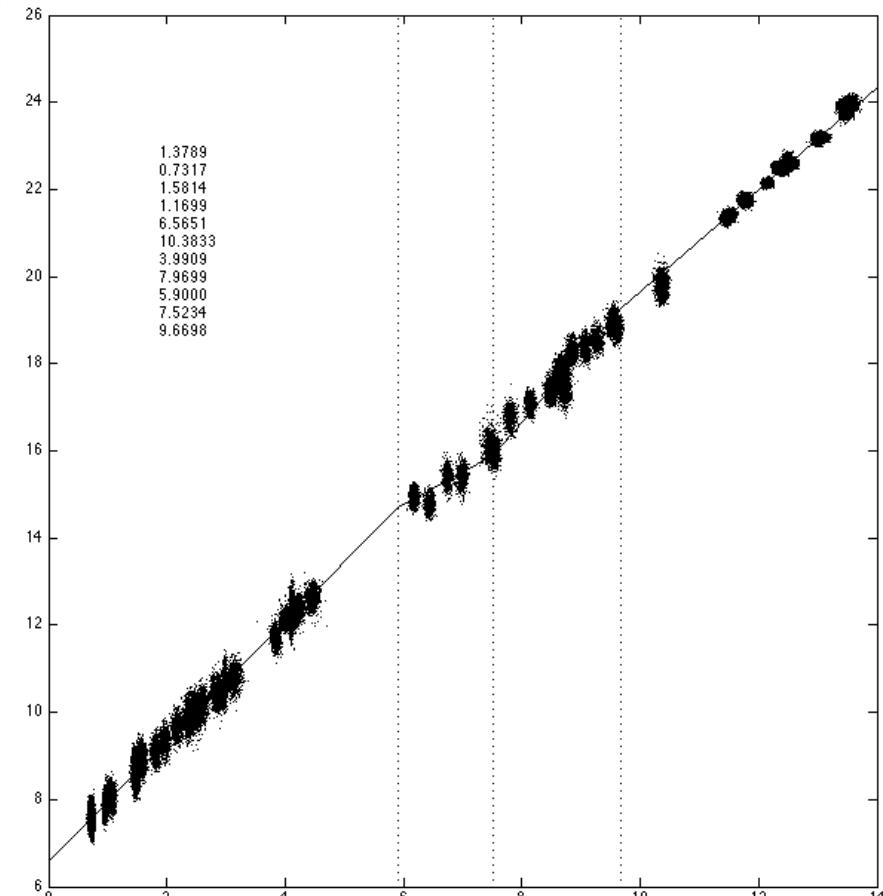
$U_p = 9.7 \pm 0.1 \text{ km/s}$

$P = 660 - 683 \text{ GPa}$

Largest uncertainty in the B2 – coexistence region

More data will improve the result statistics

**Use DFT methods to further elucidate the phase diagram**





# Density Functional Theory

- Use DFT-MD to assess the state of MgO at high pressures
- DFT-MD simulations performed using VASP 5.2.12\*
- Electronic states occupied according to Mermin's finite-temperature formulation
- Calculate energy and pressure for a given density and finite temperature
- Satisfy the Hugoniot Condition:

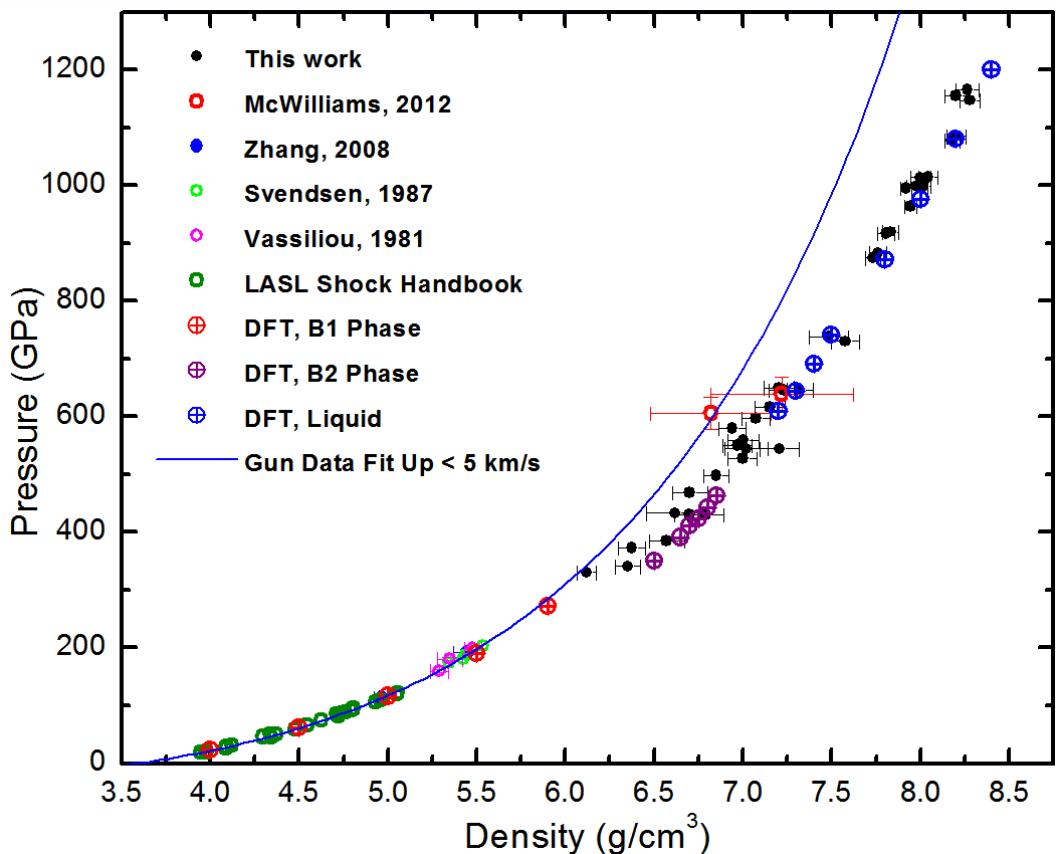
$$2(E - E_{ref}) - (P + P_{ref})(\nu_{ref} - \nu) = 0$$

- Simulations start from the B1 and the B2 phase
- 216 atoms per simulation (B1 phase); 250 atoms (B2 phase)
- AM05 (Armiento-Mattsson) exchange correlation functional
- VASP PAW potentials: Mg(3s<sup>2</sup>) and O(2s<sup>2</sup>p<sup>4</sup>) for low temp (< 1eV)  
Mg(2s<sup>2</sup>2p<sup>6</sup>3s<sup>2</sup>) and O(2s<sup>2</sup>p<sup>4</sup>) for (> 1eV)

\* G. Kresse and J. Hafner, Phys. Rev. B **47**, 558 (1993) and Phys. Rev. B **49**, 14251 (1994).

# DFT Hugoniot Results

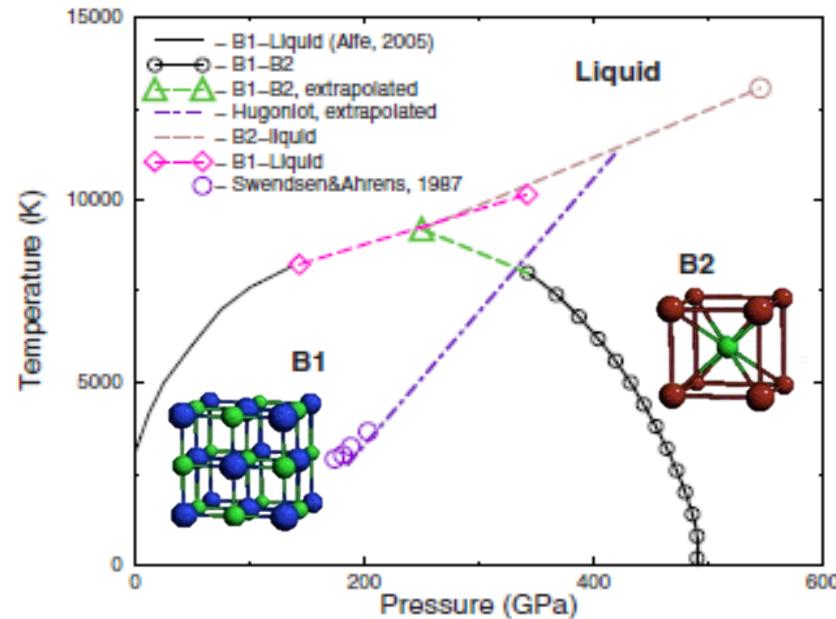
- DFT simulations to 12 Mbar
- Simulations start from B1, B2, or liquid phase
- Excellent agreement with the experimental data over the entire range
- DFT data suggest a large region for B2 – Melt coexistence on the Hugoniot



- Examine the phase boundaries

# Calculation of Solid-Solid Phase Boundary

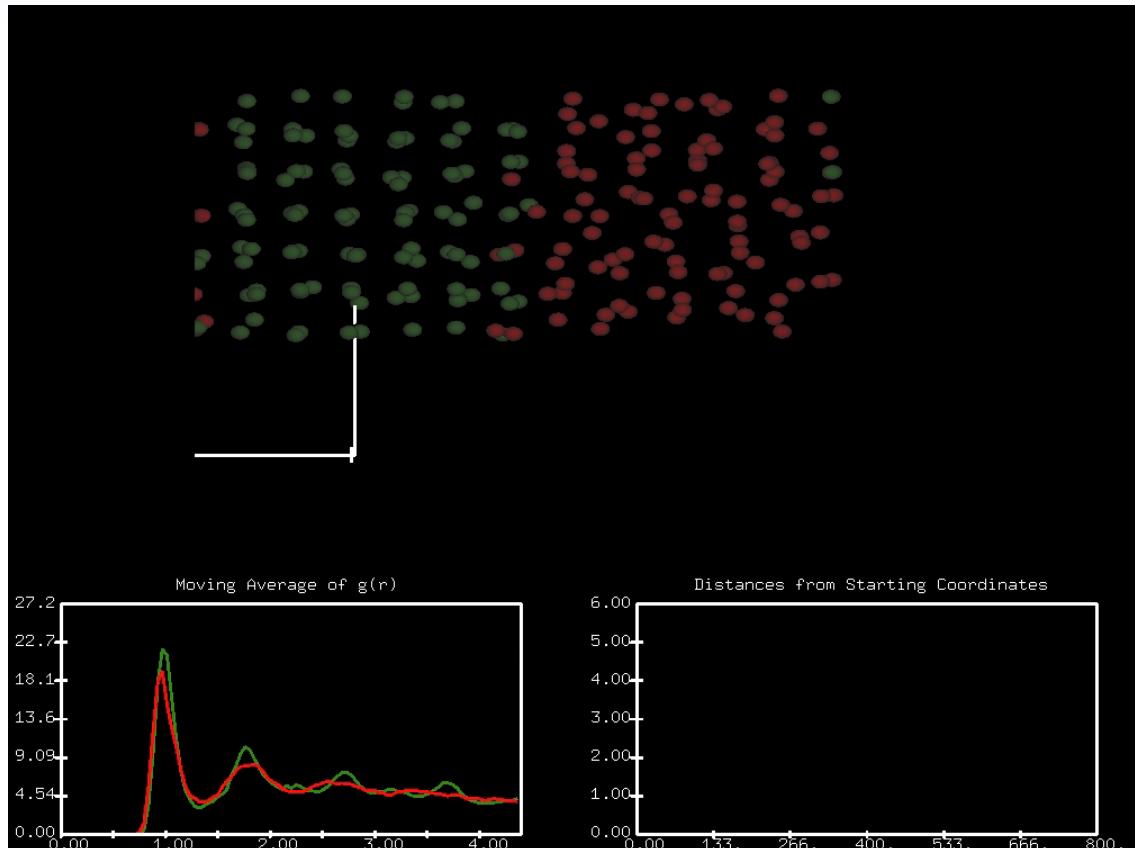
- Pressure and Energy are directly calculated in DFT-MD
- For finite temperature phase transitions entropy is necessary
- At low temperatures, harmonic phonon approximation provides solution
- Use finite displacements in a supercell following the approach of the Phon code  
(Alfe, Computer Physics Communications. 180, 2622 (2009))
- Entropy can be calculated directly using analogy to finite temperature quantum harmonic oscillator
- Approximation breaks down for moderate temperatures





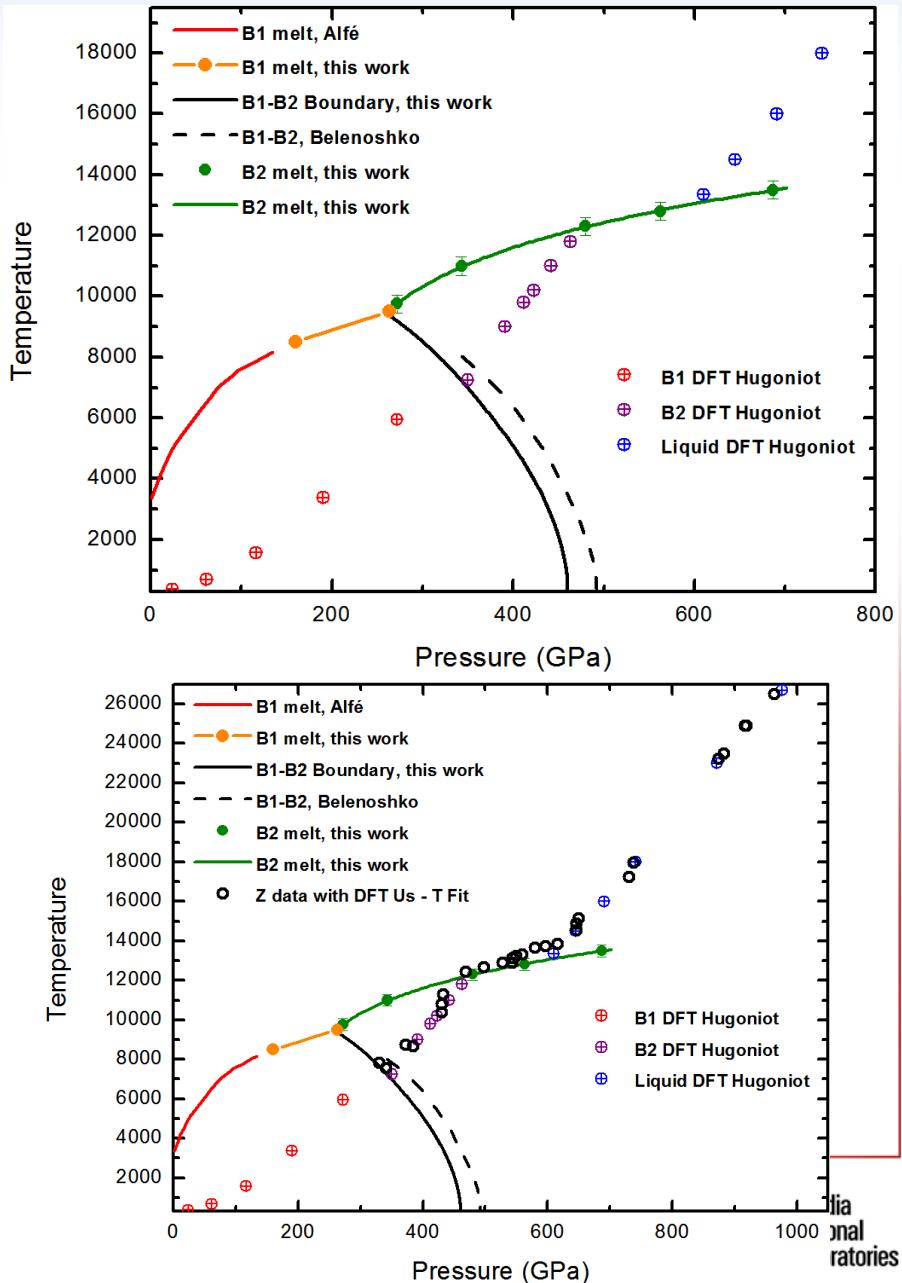
# Solid – Melt Boundaries

- For melting boundary use two phase coexistence simulations
- Place solid and liquid in contact with each other
- Run at different temperatures or starting energies and watch phase boundary
- Relative heat capacities and enthalpy of melting determine range of phase coexistence
- Follow work of Belonoshko, but include quantum calculations of B2 phase melting



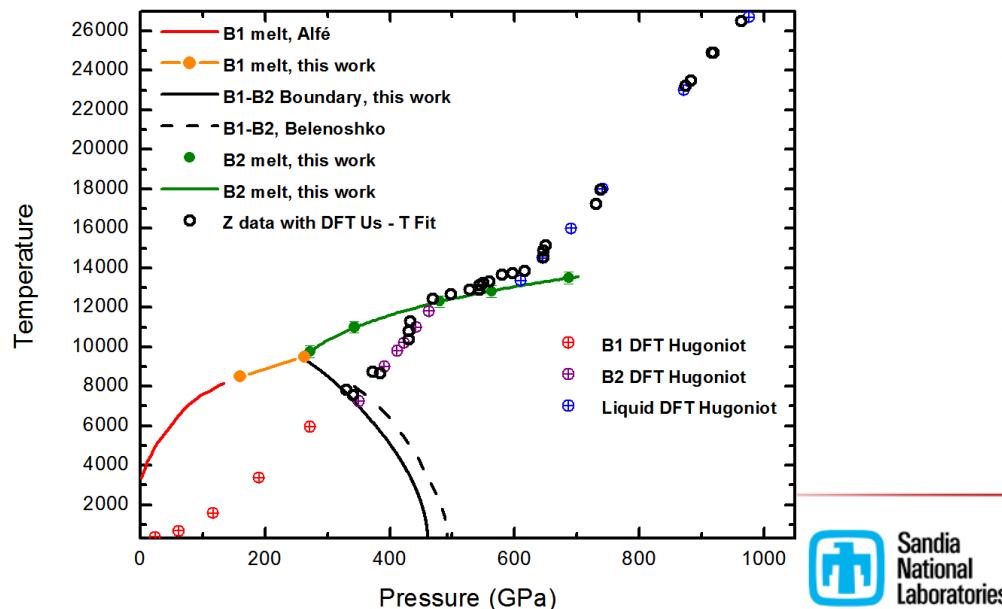
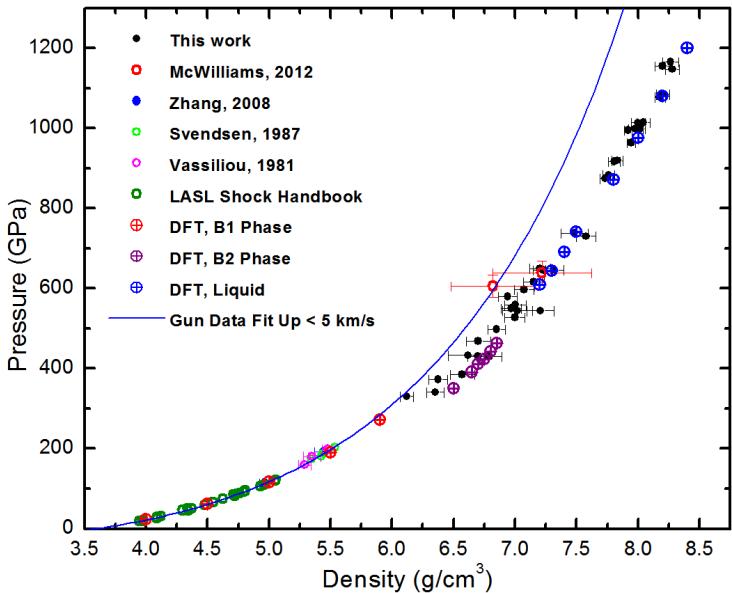
# MgO Phase Diagram

- **B1 – B2 phase boundary occurs lower than predicted by Belenoshko**
- **B1 – melt boundary consistent with Alfe**
- **DFT Hugoniot shows a large region of coexistence on the B2 – melt boundary**
- **US – Temperature Fit applied to experimental data**
- **Region of B1 – B2 coexistence from DFT ranges from 330 – 350 GPa**
- **Region of B2 – melt coexistence ranges from 430 – 660 GPa**



# Summary

- Accurately measured the MgO Hugoniot from 330 GPa to 1160 GPa
- Determine the phase boundary regions for the B1, B2, and Liquid phases of MgO along the Hugoniot
- Show MgO has a large coexistence region along the Hugoniot between B2 and liquid
  - Significant importance to planetary and moon formation
  - Shock pressures of ~7 Mbar or greater needed to completely melt MgO
- Developed an updated phase diagram for MgO



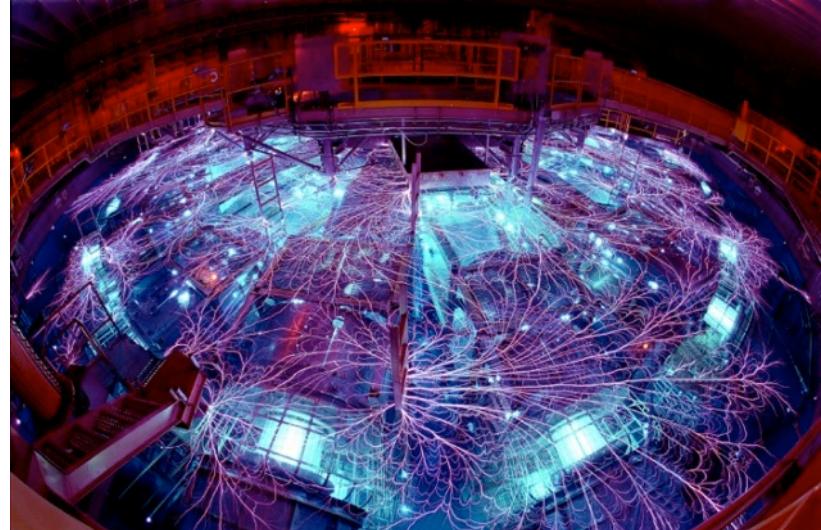
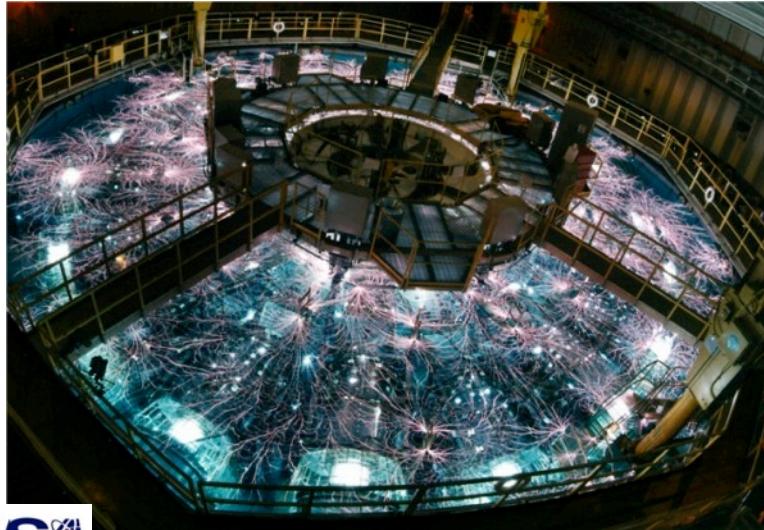


# Acknowledgements

**Dan Dolan for the Optimization routine**

**Mike Desjarlais and Marcus Knudson**

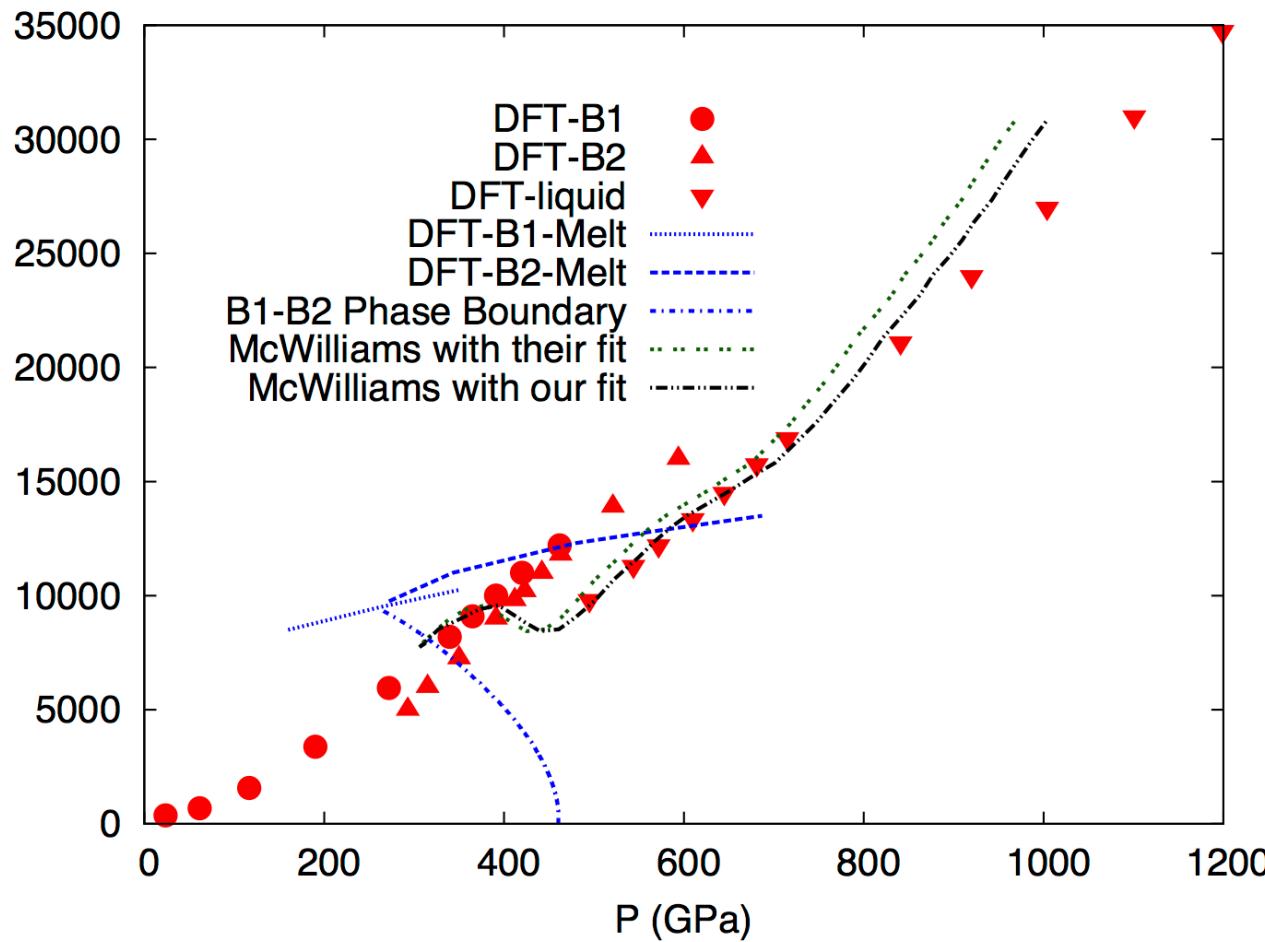
**All members of the Sandia Z-Operations Team  
and Target Fab./Metrology Team**





# Extra Slides

# Comparison to McWilliams



- The B1-B2 transition occurs at lower pressure
- Larger B2 – Liquid coexistence region
- DFT temperatures lower than measured temperatures in the liquid state