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Solution of Stochastic Media Transport Problems by Means of Deterministic Generation of Realizations

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1. Introduction

Exact solution of particle transport problems in stochastic media generally requires the random generation of a large number of material realizations and subsequent transport calculations for each of them. This can be quite expensive in order to obtain good statistics. We describe a deterministic approach to the generation of representative realizations that involves numerical quadrature rules, which for some problems can be much more computationally efficient.

2. Theory

We begin by defining a binary Markovian medium whose material distributions are governed by average chord lengths λ_1 and λ_2 . We can define an effective combined chord length $\lambda_c = \lambda_1\lambda_2/(\lambda_1 + \lambda_2)$; this is the average chord length between “pseudo-interfaces”. There are two very useful properties of pseudo-interfaces:

1. The frequency with which I pseudo-interfaces occur is governed by the Poisson distribution $f(I; \lambda_c) = e^{-\lambda_c} \lambda_c^I / I!$
2. The location of pseudo-interfaces is uniformly distributed, and thus independent of the location of other pseudo-interfaces.

Given the above properties we define a two-level process for the deterministic generation of stochastic realizations. At the first level we subdivide the overall stochastic problem into a set of stratified subproblems defined by the number of pseudo-interfaces the realizations contain. Subsequent solutions of each subproblem (described below) may be combined by multiplying them by the (known) Poisson weight; their weighted sum is the solution to the full stochastic problem.

At the second level we need to solve each subproblem. For a given subproblem with I pseudo-interfaces we need to determine the location of each pseudo-interface for each representative realization. We do this by using I numerical quadratures of order N to produce a product quadrature, where the i th quadrature governs the location of the i th pseudo-interface. This process creates $O(2^{N+1})$ unique transport problems, where N is chosen according to the desired accuracy and computational expense. The solution to the I th subproblem is obtained by the quadrature weighting of the solution to each realization generated above.

3. Results

We have revisited the stochastic media transport benchmarks first reported in [1]. These problems consist of nine different sets of material properties in one-dimensional slab geometry and are driven by an isotropic source at one boundary; the quantities of interest are the average reflected and transmitted fluxes. We generate realizations according to the process described in the previous section; we solve the

transport problem for each realization with the SCEPTRE radiation transport code [2]. Some representative results are shown in Table I for six of the problems and a slab width of unity. For these problems both the accuracy and the computational efficiency of our approach compare favorably with Monte Carlo generation of realizations. In other problems, however, the number of pseudo-interfaces encountered in typical realizations and the quadrature order required for good accuracy make this approach more expensive than simple Monte Carlo generation of realizations.

Table I: Average reflection results (number of realizations required).

problem	method		
	quadrature, order 2	quadrature, order 4	Monte Carlo
4	0.1196 (210)	0.1201 (4578)	0.123 (49700)
5	0.07503 (210)	0.07481 (4578)	0.0744 (1500)
6	0.1424 (210)	0.1425 (4578)	0.145 (10200)
7	0.3221 (90)	0.3219 (818)	0.319 (8200)
8	0.008829 (90)	0.008864 (818)	0.00888 (10700)
9	0.2444 (90)	0.2442 (818)	0.242 (6900)

4. Future Work

As noted above the expense of our method grows rapidly as the complexity of the problem grows. Thus we do not expect that this method will be directly useful for typical stochastic media applications, e.g. modeling a full pebble bed reactor. But it may find utility in performing localized calculations in which a small number of interfaces are expected to occur. In particular, we are interested in creating improvements to the LP closure [1]. The LP closure assumes that the average angular fluxes at a given spatial location are unaffected by the presence or absence of a material interface there. We hope to explicitly model the behavior of the angular fluxes in a small region centered about a potential interface location in order to relax this assumption. Such a model would account for other nearby interfaces, presumably limited in number, which if combined with our method could yield a relatively inexpensive improvement to the LP closure.

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References

- [1] M. Adams, E. Larsen and G. Pomraning, "Benchmark Results for Particle Transport in a Binary Markov Statistical Medium," *J. Quant. Spectrosc. Radiat. Transfer*, **42**, pp. 253-266 (1989).
- [2] S. Pautz, B. Bohnhoff, C. Drumm and W. Fan, "Parallel Discrete Ordinates Methods in the SCEPTRE Project," *Proc. Int. Conf. on Mathematics, Computational Methods and Reactor Physics*, Saratoga Springs, NY, May 3-7, 2009.