Final Technical Report

Institution: Princeton University
Grant: *Ultra-high Intensity Magnetic Field Generation in Dense Plasma*
US DOE/OFES #DE-FG02-05ER54838
Time Period: August 2005 – December 2008
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Date: January 8, 2014

I. Grant Objective

The main objective of this grant proposal was to explore the efficient generation of intense currents. Whereas the efficient generation of electric current in low-energy-density plasma has occupied the attention of the magnetic fusion community for several decades, scant attention has been paid to carrying over to high-energy-density plasma the ideas for steady-state current drive developed for low-energy-density plasma, or, for that matter, to inventing new methodologies for generating electric current in high-energy-density plasma. What we proposed to do was to identify new mechanisms to accomplish current generation, and to assess the operation, physics, and engineering basis of new forms of current drive in regimes appropriate for new fusion concepts.

II. Accomplishments

To explore the possibility of new methodologies, we derived important constraints in the regimes of high energy density plasma, including new ways of describing wave particle interactions. Under this grant we also explored a number of mechanisms whereby waves push particles in plasma in unusual ways, and we drew helpful analogies to quantum systems. The papers cited below acknowledge support from this grant.

In particular, we showed how localized regions of intense large-scale radiofrequency fields, acting like effective ponderomotive potential barriers, exhibit quantum-like behavior in transmitting particles [1,3]. This effect can also be used for beam slicing and a number of other unusual phenomena with quantum-like properties [4]. Not directly connected to the ponderomotive effects, but important in establishing limits for what currents could in fact be generated, we derived a correction to the Alfvén-Lawson criterion for relativistic electron beams [2].

We also formulated Manley–Rowe relations for a discrete Hamiltonian system with an arbitrary number of resonances [6]. We derived reduced nonlinear equations from the oscillation amplitude and the energy of a charged particle accelerated in a plasma channel. This allowed us to calculate the maximum energy gain, as limited by dissipation [7].

Finally, we derived the generalized effective potential for a nonrelativistic classical particle undergoing arbitrary oscillations in external fields [8]. This calculation has now been the basis for a number of succeeding efforts, both by us and by other groups, in the period following the grant funding period, in elucidating the waves in plasma with trapped particles and negative mass type instabilities.

III. Work Supported under the Grant

[1] I. Y. Dodin and N. J. Fisch,
Nonadiabatic Ponderomotive Potentials,
Physics Letters A **49**, 356-369 (January, 2006).

[2] I. Y. Dodin and N. J. Fisch,
Correction to the Alfven-Lawson criterion for the stability of relativistic electron beams,
Physics of Plasmas **13**, 103104 (October, 2006).

[3] I. Y. Dodin and N. J. Fisch,
Particle manipulation with nonadiabatic ponderomotive forces,
Physics of Plasmas **14**, 055901 (May, 2007).

[4] I. Y. Dodin and N. J. Fisch,
Stochastic extraction of periodic attosecond bunches from relativistic electron beams,
Physical Review Letters **98**, 234801 (June, 2007).

[5] I. Y. Dodin and N. J. Fisch,
Positive and negative effective mass of classical particles in oscillatory and static fields,
Physical Review E **77**, 036402 (March, 2008).

[6] I. Y. Dodin, A. I. Zhmoginov, and N. J. Fisch,
Manley-Rowe relations for an arbitrary discrete system,
Physics Letters A **372,** 6094-6096 (August, 2008).

[7] I. Y. Dodin and N. J. Fisch,
Charged particle acceleration in dense plasma channels,
Physics of Plasma **15**, 103105 (October, 2008).

[8] I. Y. Dodin and N. J. Fisch,
Dressed-particle approach in the nonrelativistic classical limit,
Physical Review E **79**, 026407 (February, 2009).

[9] I. Y. Dodin and N. J. Fisch,
Non-Newtonian mechanics of oscillation centers,
in *Frontiers in Modern Plasma Physics* (AIP, New York, 2008),
AIP Proc. 1061, **263** (2008), (October, 2008).

IV. Further Description of the Major Results

Limitations on existing methods of manipulating charges with electromagnetic fields stem not only from natural technological limitations specific to particular techniques but also from fundamental properties of field-particle interactions. A classic example here is the well-known Earnshaw theorem that prohibits confinement of charged particles by means of solely electrostatic fields. A typical solution to this is to rely on different, *effective* forces that are not subject to such restrictions. Particularly flexible in this regard are oscillating fields, which can produce so-called ponderomotive potentials seen by particles on times scales large compared to the oscillation period.

Conceptual breakthroughs in methodologies of particle manipulation can flow from advancing the existing understanding of such effective forces. For instance, previous studies identified that ponderomotive potentials are not always single-valued or conservative, unlike true potentials; on the other hand, they are also bound to *heat* particles, so the energy cost of the particle average acceleration (or current) is a nontrivial function that is not necessarily optimized in existing applications. This warrants a fundamental study of ponderomotive effects that is not limited to specific settings found in typical experiments (since new configurations may turn out more promising) but rather focus on what effective forces can and cannot do *in principle*.

To explore these issues, we performed multidisciplinary theoretical research that: (i) established connections between ponderomotive effects in areas that are commonly considered unrelated and range from atomic physics to relativistic-beam physics; (ii) identified important theorems that constrain rigorously ponderomotive effects; and (iii) proposed novel methods of particle manipulation by means of electromagnetic waves.

Our research can be divided into three main parts that we describe below in logical (rather than chronological) order.

1. In the first part of our research, we expanded the existing understanding of so-called adiabatic (phase-independent) ponderomotive forces. In particular, we showed how they could make the particle time-averaged dynamics quite different from the relatively simple characteristic examples that were addressed in literature previously.

In Ref. [5], we showed that general ponderomotive interactions do not just create a time-average *force* on a particle; rather they effectively modify the particle *mass*. The effective mass shaped this way is not necessarily positive, even for nonrelativistic interactions. For example, "anti-gravity" can be realized in the sense that a gravitational force on a particle interacting with a homogeneous stationary "pump" wave can, in fact, accelerate the particle upward rather than downward. This phenomenon was explicitly demonstrated, both analytically and numerically, for weakly relativistic electrons in the vicinity of their cyclotron resonance. The pump wave in this case modifies the particle dispersion (i.e., the canonical energy-momentum relation) such that it becomes N-shaped instead of U-shaped; that renders certain initial conditions unstable. Importantly, this nonlinear instability does not rely on dissipation and represents a purely adiabatic single-particle effect.

In Ref. [9], these ideas were explored further. It was shown, for instance, that, when the slow motion of pump-driven plasma particles with N-shaped dispersion is modulated by a low-frequency signal, intriguing effects are produced such as adiabatic, 100%-efficient generation of a second harmonic of the low-frequency signal.

In Ref. [8], we studied related properties of compound classical particles, i.e., classical oscillators, both linear and nonlinear. We showed how the dressed-particle approach, commonly adopted in atomic physics, could be applied to describe the dynamics of classical particles too. The ponderomotive force on a nonlinear oscillator with a single frequency was then demonstrated to exhibit hysteresis. Also it was shown that the ponderomotive force on a particle with more than one internal frequency could be manipulated (e.g., can in fact be made to change sign) simply by reallocating energy within the particle. For example, one can create one-way walls this way. Several groups researching atomic physics have now demonstrated experimentally one-way traps in atoms, an effect whose prediction was in fact stimulated by our work in plasmas.

2. In the second part of our research, we focused on nonadiabatic ponderomotive barriers.

A general theory of such barriers for linear oscillators (such as particles in dc magnetic field, atomic clusters, and individual atoms described classically) was developed in Ref. [1]. An approximate integral of the Manley-Rowe type was found there for a particle moving in a high-frequency field that is allowed to be resonant with the particle natural oscillations. An effective ponderomotive potential was introduced accordingly and was shown capable of capturing nonadiabatic particle dynamics. In particular, we demonstrated that such barriers could confine particles indefinitely even when their interaction with the field is strongly nonadiabatic, i.e., when significant heating occurs. Nonadiabatic ponderomotive barriers were also shown capable of temporarily trapping classical particles and scattering them in a quantumlike manner.

These findings were later summarized in an invited review paper, Ref. [3].

An even more general treatment of resonant ponderomotive interactions, formulated as an abstract mathematical theorem, was later reported in Ref. [6]. To our knowledge, it represents the most general and concise form of the Manley-Rowe conservation theorem for an arbitrary discrete Hamiltonian system with any number of resonances. Specifically, it is formulated as follows. Assuming that the resonances are defined as $R\cdot\omega = 0$, where R is a $n \times n$ integer matrix of rank $r < n$, and $\omega \equiv (\omega_1 \ldots \omega_n)$ is the frequency vector (n being the number of degrees of freedom), the projection of the corresponding action vector $J \equiv (J_1 \ldots J_n)$ on the kernel of R is an adiabatic invariant. Hence $n - r$ independent integrals exist, from where the conventional Manley-Rowe relations (i.e., those for a three-wave resonance) follow as a particular case. In later papers, we explicitly showed how this formulation is applicable to various physical systems, including nonadiabatic ponderomotive barriers for magnetized plasma particles and even the alpha channeling effect in tokamaks.

3. The third part of our research was devoted to studying manipulations of relativistic electron beams.

In Ref. [4], we showed how the ideas that we previously contemplated for nonadiabatic ponderomotive barriers could be extrapolated to ways of manipulating ultrarelativistic electron beams. We showed, in particular, that ultraintense standing laser waves can *slice* such beams, thereby producing attosecond bunches. We also discussed related ideas for lower-energy beams in Ref. [3].

In addition, in Refs. [2, 7], we explored important limitations on electron-beam propagation that come from time-averaged oscillatory dynamics of particles too, albeit in static rather than high-frequency fields.

In Ref. [2], the so-called Alfven-Lawson criterion for relativistic electron beams was challenged and corrected. Our findings correct what is known as Lawson's amendment to the well-known Alfven limit on the total current that can be carried by a self-magnetized relativistic electron beam without filamentation. Specifically, we found that a parameter range exists in which a stationary beam can carry arbitrarily large current, regardless of its transverse structure. This result is derived from first principles using single-particle Hamiltonian mechanics in the beam magnetic and electric fields. As a spin-off, the zoology of possible oscillation orbits of beam particles was also revised.

In Ref. [7], we studied dilute relativistic beams propagating in prescribed plasma channels. Examples of such channels are those formed by crystal lattices in solid media. Reduced equations of electron motion, averaged over the bounce period, were derived for the oscillation amplitude and the particle energy that include the effect of a longitudinal accelerating force, Langevin collisions, and the radiation reaction force. Using those equations, the maximum energy gain, as limited by dissipation, was described by three different scalings depending on the channel parameters.

In summary, our grant-sponsored research as summarized above has now established a solid foundation for a general, first-principles theory of particle manipulations by time-averaged forces. Many of the findings in the course of this research are now serving as a basis for succeeding efforts. For example, the general Lagrangian methods that came out of this grant were then extended and applied to describing the dynamics of linear and nonlinear wave-particle interactions in a variety of contexts in already over 20 papers published by us subsequent to the grant expiration in 2008.