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Modeling ice-sheets dynamics: forward and inverse problems

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joint work with M. Gunzburger[2], M. Hoffman[3], S. Price[3], A. Salinger[1], G. Stadler[4]

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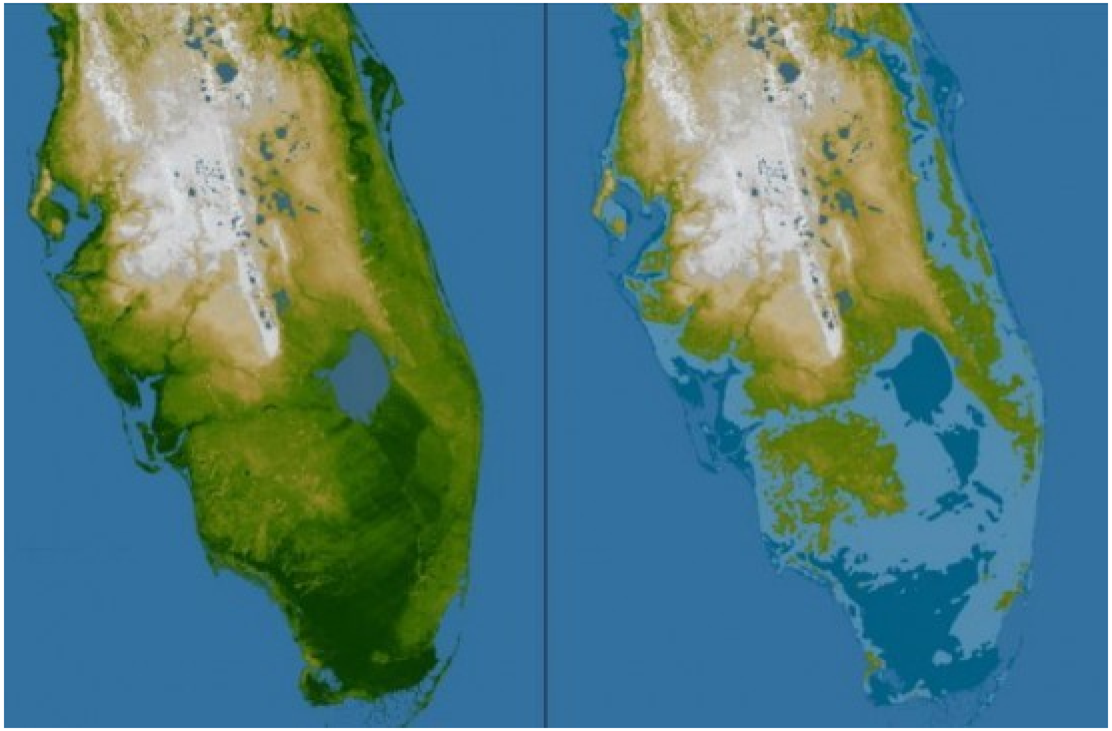
Motivations

- Glaciers and ice sheets influence the global climate, and vice-versa
- Melting of land ice determines the **sea level rise**
  melting of the Greenland ice sheet: 7 m
  melting of the Antarctic ice sheet: 61 m



South Florida projection for a sea levels rise
of 5m (dark blue) and 10m (light blue)

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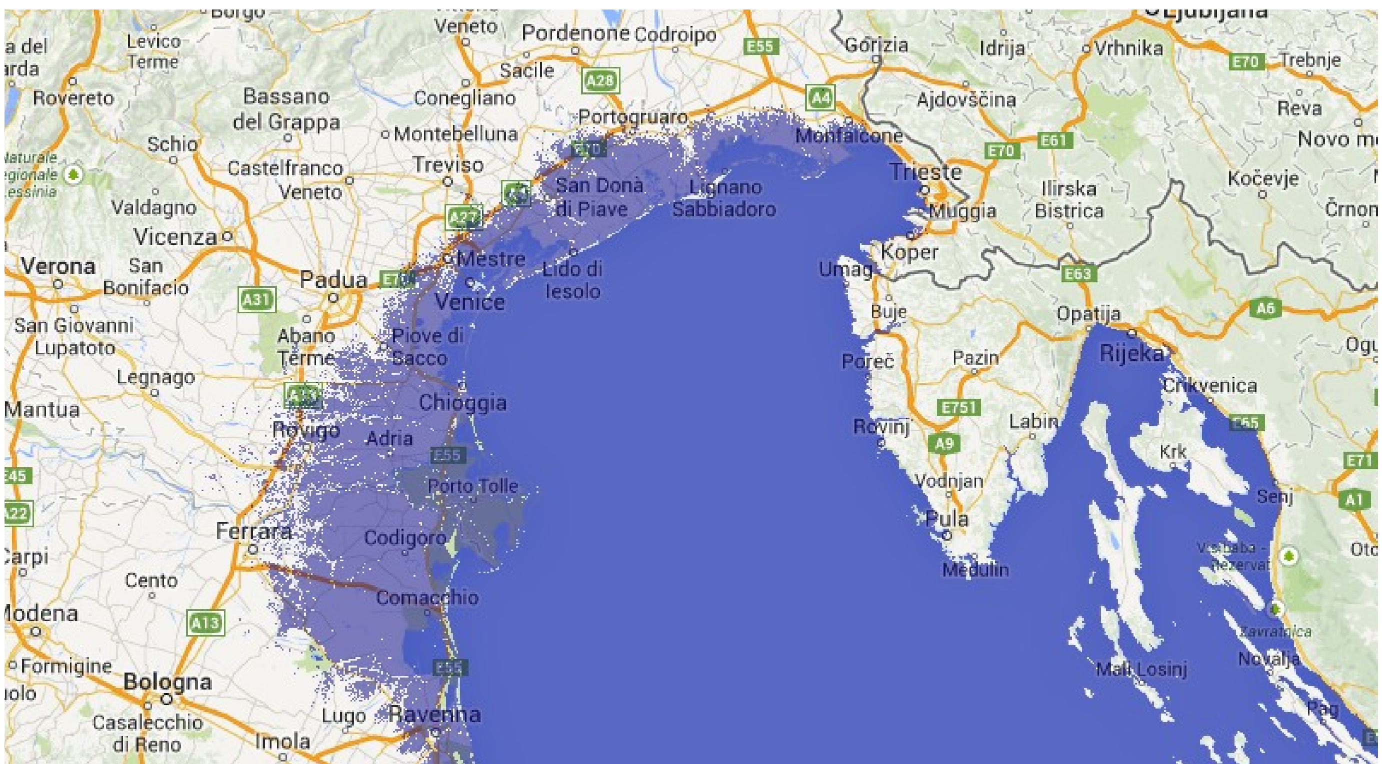

Italy, projection for a sea levels rise of **1 m** (A.Tinlge, Google Maps/NASA)

Motivations

- Glaciers and ice sheets influence the global climate, and vice-versa
- Melting of land ice determines the **sea level rise**
  melting of the Greenland ice sheet: 7 m
  melting of the Antarctic ice sheet: 61 m
- The Fourth Report of the Intergovernmental Panel on Climate Change (**IPCC 2007**) declared that the current models and programs for ice sheets did not provide credible predictions

Ice Sheet Modeling

Main components of an ice model:

- Ice flow equations (momentum and mass balance)

$$\begin{cases} \rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) - \nabla\cdot\sigma = \rho\mathbf{g} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

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$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

with:

$$\sigma = 2\mu - \Phi I, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Non linear viscosity:

$$\mu = \frac{1}{2}\alpha(T)\,|\mathbf{D}(\mathbf{u})|^{(p-2)}, \quad p \in (1,2] \quad (\text{tipically } p \simeq \frac{4}{3})$$

Viscosity is singular when ice is not deforming

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Ice Sheet Modeling

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- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

- **Model for the evolution of the boundaries** (thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot \int_z \mathbf{u} \, dz$$

- **Temperature equation**

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) - \rho c \mathbf{u} \cdot \nabla T + 2 \dot{\varepsilon} \sigma$$

- **Coupling with other climate components (e.g. ocean, atmosphere)**

Stokes Approximations

“Reference” model: **STOKES**[1]

$O(\delta^2)$ ***FO,*** Blatter-Pattyn first order model[2] (3D PDE, in horizontal velocities)

$O(\delta)$ Zeroth order, depth integrated models:
SIA, Shallow Ice Approximation (slow sliding regimes) ,
SSA Shallow Shelf Approximation (2D PDE) (fast sliding regimes)

$\simeq O(\delta^2)$ Higher order, depth integrated (2D) models: ***L1L2***[3], (L1L1)...

[1] *Gagliardini and Zwinger, 2008. The Cryosphere.*

[2] *Dukowicz, Price and Lipscomb, 2010. J. Glaciol.*

[3] *Schoof and Hindmarsh, 2010. Q. J. Mech. Appl. Math.*

First order equation.

FO is a nonlinear system of elliptic equations in the horizontal velocities:

$$\begin{cases} -\nabla \cdot (2\mu\, \dot{\boldsymbol{\varepsilon}}_1) = -\rho g \dfrac{\partial s}{\partial x} & \mu = \dfrac{1}{2} A^{-\frac{1}{n}} \dot{\varepsilon}_e^{\left(\frac{1}{n}-1\right)} \\ -\nabla \cdot (2\mu\, \dot{\boldsymbol{\varepsilon}}_2) = -\rho g \dfrac{\partial s}{\partial y}, & \dot{\varepsilon}_e = \sqrt{\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{xx}\dot{\varepsilon}_{yy} + \dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{xz}^2 + \dot{\varepsilon}_{yz}^2} \end{cases}$$

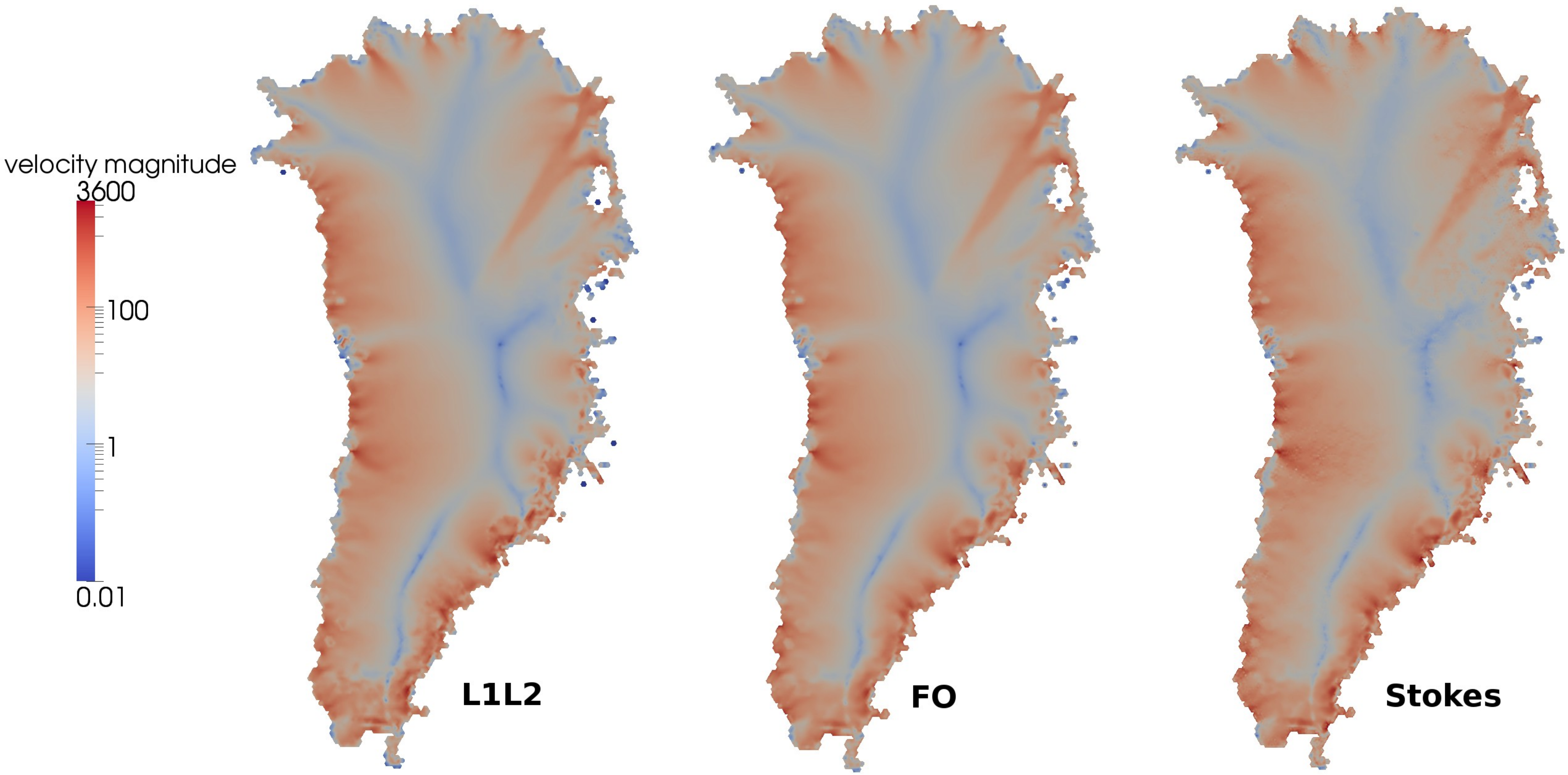
where s is the ice surface and,

$$\dot{\varepsilon}_{i,j} = \frac{1}{2}\left(\partial_j u_i + \partial_i u_j\right), \quad i, j \in \{x, y, z\}, \quad \dot{\boldsymbol{\varepsilon}}_1 = \begin{bmatrix} 2\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{xz} \end{bmatrix}, \; \dot{\boldsymbol{\varepsilon}}_2 = \begin{bmatrix} \dot{\varepsilon}_{yx} \\ \dot{\varepsilon}_{xx} + 2\dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{yz} \end{bmatrix}$$

Remark The nonlinear viscosity μ is singular when $\dot{\varepsilon}_e = 0$, however, $\mu\, \dot{\boldsymbol{\varepsilon}}_1$ is not singular and the PDE is well defined.

Viscosity regularization: $\quad \dot{\varepsilon}_e^{-\left(1-\frac{1}{n}\right)} \approx \left(\sqrt{\dot{\varepsilon}_e^2 + \delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$

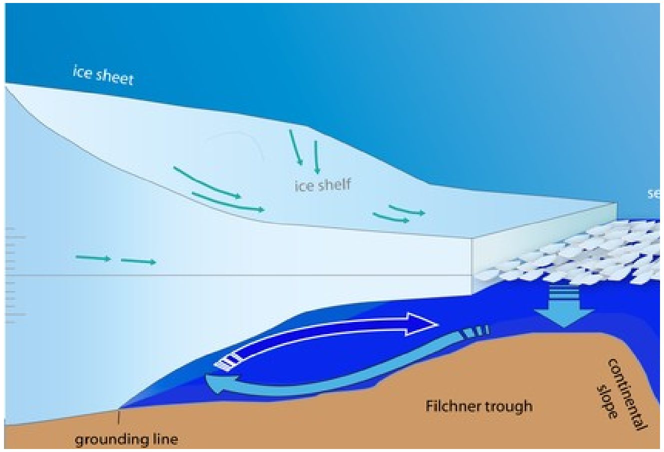
Greenland, steady state, model comparison.



(W. Leng, L. Ju)

(Numerical) Modeling Issues

- Computationally challenging, due to complexity of models, of geometries and large domains
  - design of linear/nonlinear solvers, preconditioners, etc.
  - mesh adaptivity especially close to the grounding line.



- Boundary conditions / coupling (e.g. with ocean)
  - Friction at the bedrock,
  - Subglacial hydrology,
  - Heat exchange / phase change.

- Initialization / parameter estimation.

- Uncertainty quantification.

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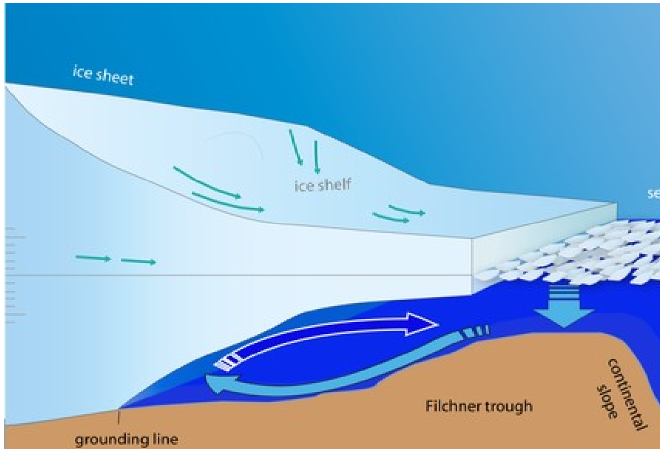

- Boundary conditions / coupling (e.g. with ocean)
  - Floating/calving
  - Basal friction at the bedrock,
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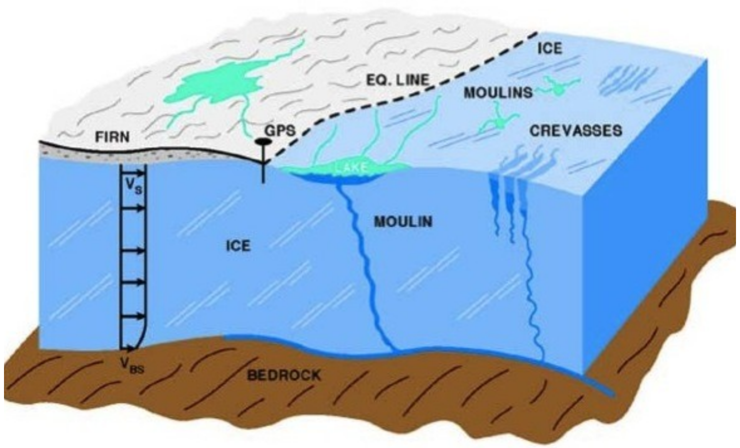

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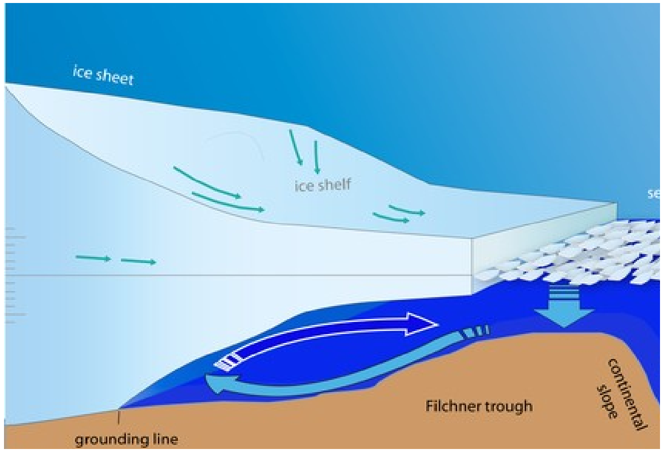

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Implementation Overview (Felix)

- Felix (Finite Element Land Ice eXperiments) is a C/C++ finite element implementation of land ice models. It relies on Trilinos for data structure, for the solution of linear/nonlinear solvers and for adjoint/UQ capabilities.

- Models currently implemented are SIA, SSA, L1L2 and FO, which have been tested against Ismip-Hom experiments and CISM simulations.

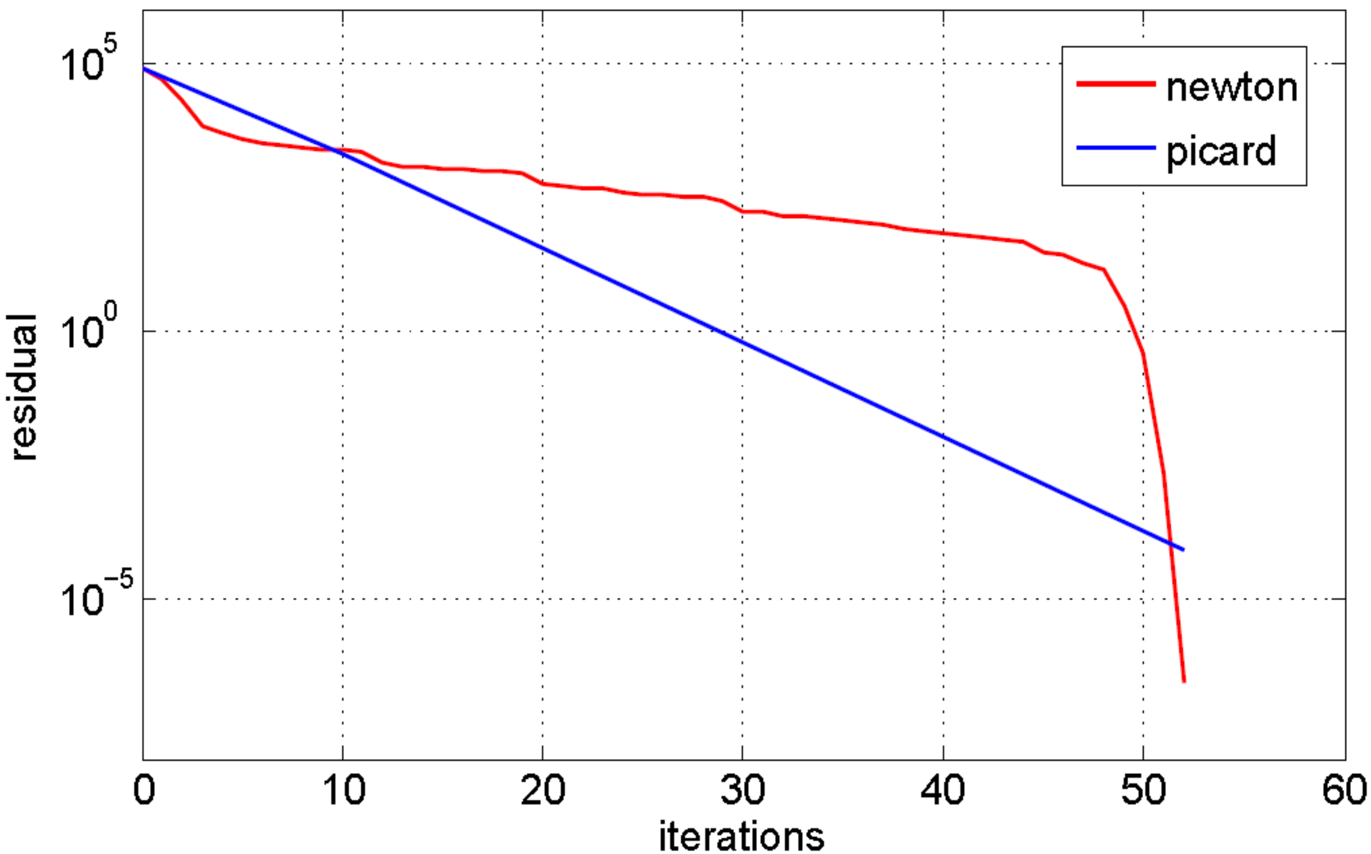
- The nonlinear systems are solved using Newton method with exact Jacobian + continuation of regularization parameters to increase robustness.

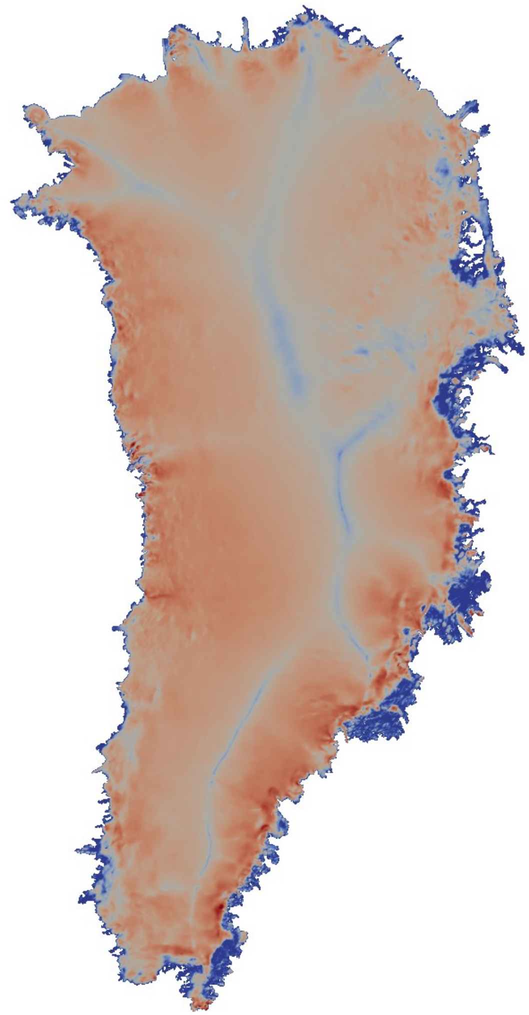
- It is interfaced with the land ice modulus of MPAS (climate library, implements ocean and atmosphere models). Realistic simulation done for ice2sea projects.

- Even if adjoint and UQ capabilities are in early development, Felix can leverage on several trilinos packages which introduce great flexibility. Among these we have:
  - Dakota, MOOCHO (Optimization / UQ)
  - Sacado (Automatic Differentiation)

[1]*Software currently developed under the DOE project PISCEES*
[2]*www.trilinos.sandia.org (albany), www.lifev.org*
[3]*Perego, Gunzburger, Burkardt,* Journal of Glaciology, 2012

Nonlinear solvers
Newton/Picard on Greenland





* In most of the cases Newton works much better than this, but still we want to improve it.

Greenland, FO:
convergence of nonlinear method

Why Newton is nor working well?
Simplified Problem (same kind of nonlinearity):

$$x\ |x|^{-\left(1-\frac{1}{n}\right)} = C. \qquad \text{solution: } \alpha = C|C|^{n-1}$$

Picard method: $x^{k+1} = C|x^k|^{\left(1-\frac{1}{n}\right)}$.
Newton method: $x^{k+1} = (1-n)x^k + nC|x^k|^{\left(1-\frac{1}{n}\right)}$.

Convergence results:

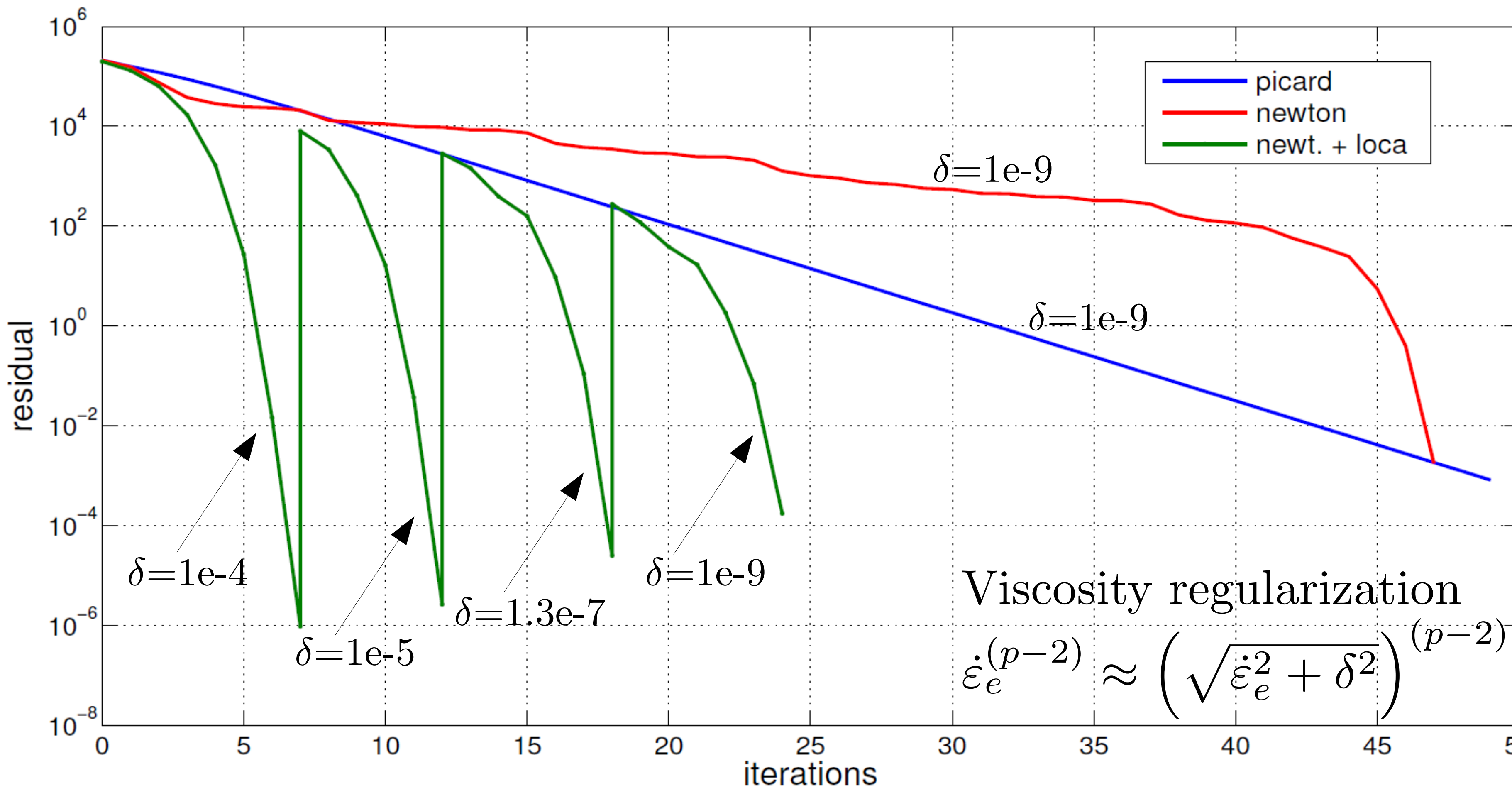
Picard convergence: $\displaystyle\lim_{k\to\infty} \frac{x^{k+1}-\alpha}{x^k-\alpha} = 1 - \frac{1}{n} \quad \left(= \frac{2}{3} \text{ when } n = 3\right)$

Newton convergence: $\displaystyle\lim_{k\to\infty} \frac{x^{k+1}-\alpha}{(x^k-\alpha)^2} = \frac{1}{2\alpha}\left(\frac{1}{n} - 1\right)$

Regularization: $\displaystyle x\ |x|^{-\left(1-\frac{1}{n}\right)} \approx x\ \left(\sqrt{x^2+\delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$

Nonlinear solvers
Increased Robustness with LOCA continuation method

M. P., A. Salinger

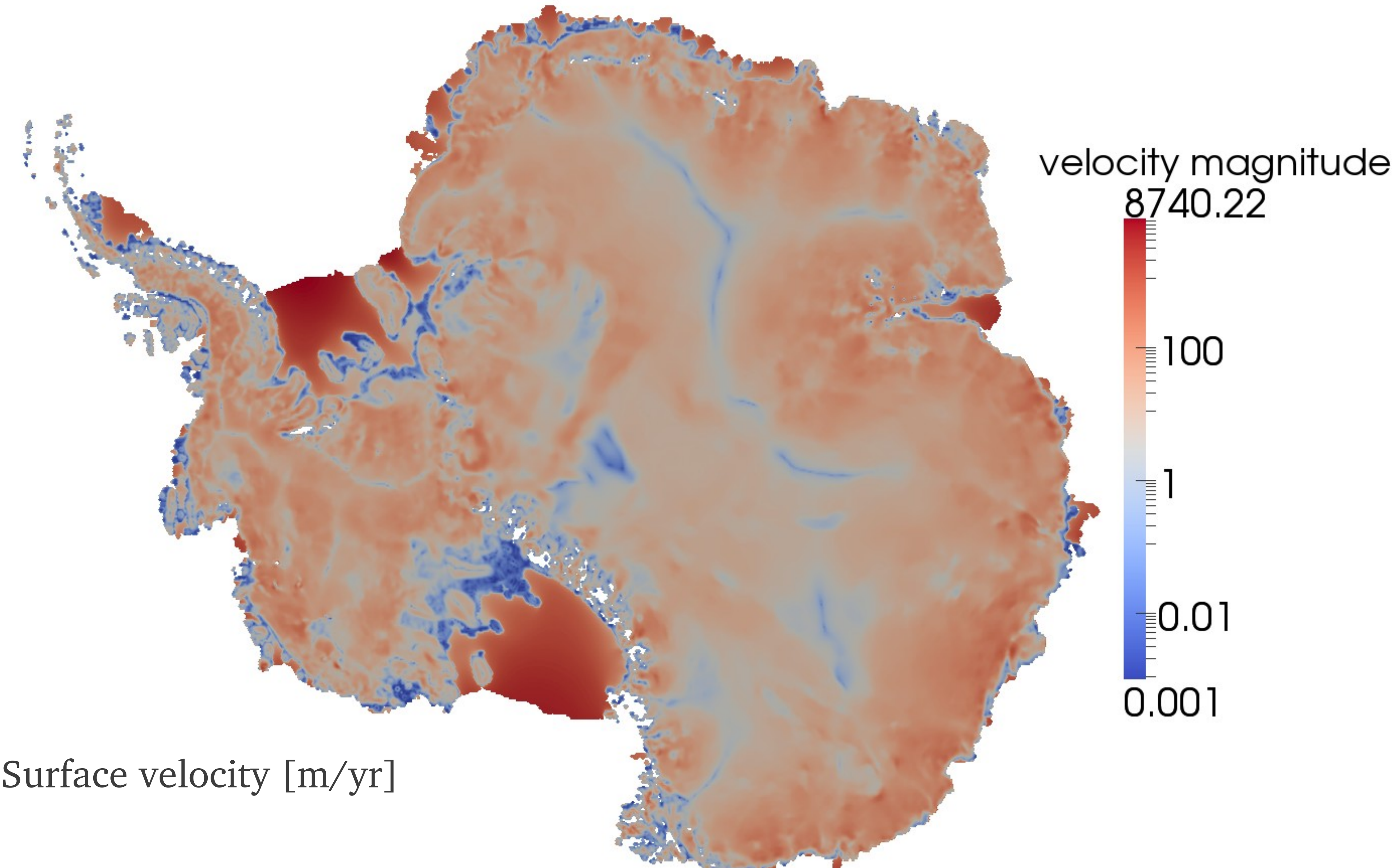


The parameter δ is decreased by LOCA from 1e-4 to 1e-9

I'm feeling lucky approach: for subsequent time steps, try solving Newton first, with a limited maximum number of iterations (say 10), if Newton does not converge, then use LOCA.

Nonlinear solvers
Increased Robustness with LOCA continuation method

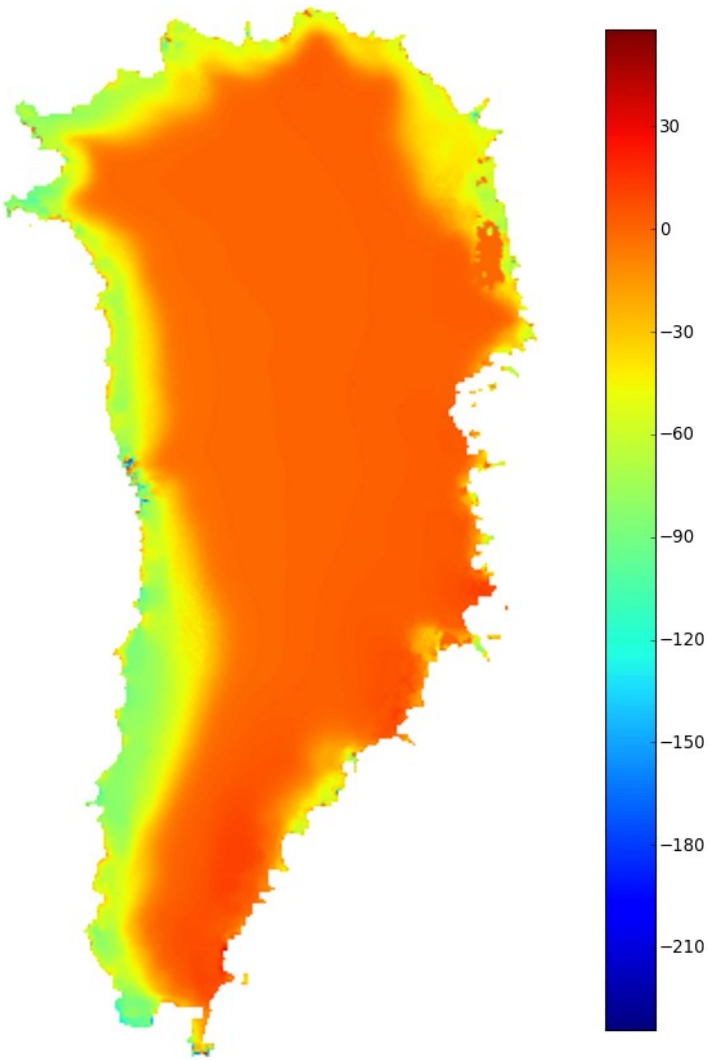
Antarctica continuation on sliding coefficient



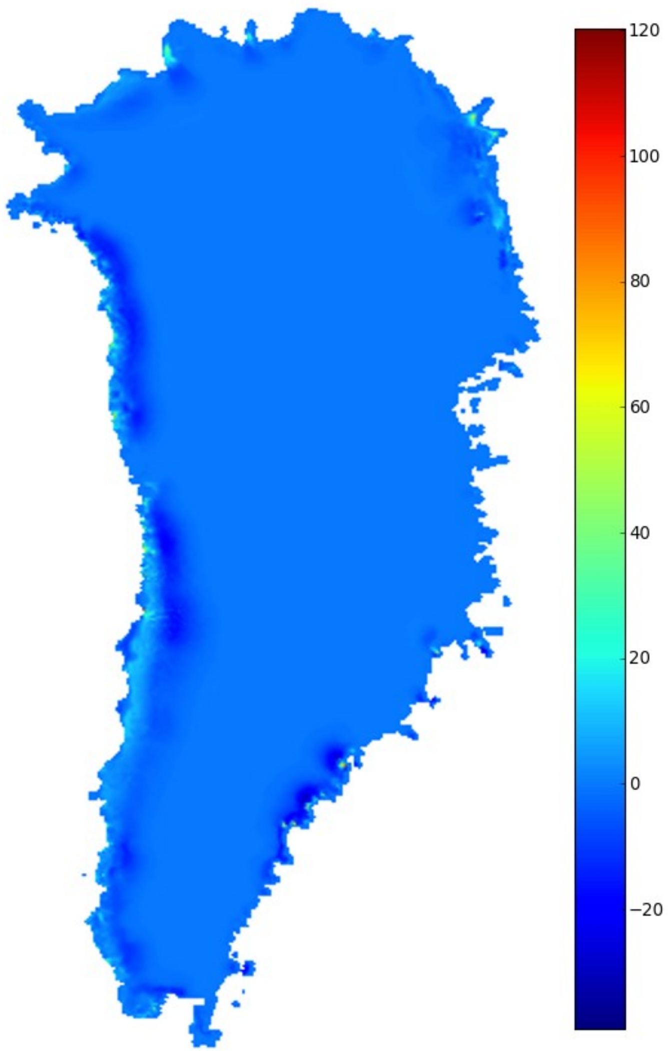
Surface velocity [m/yr]

Ice2Sea* experiment: thickness change with basal friction

**S. Shannon et al, PNAS, 2013, to be considered for the IPCC 2014 report .*



Thickness [m] change after 100 yr



Thickness difference [m], when doubling the basal friction coefficient

Inverse Problem

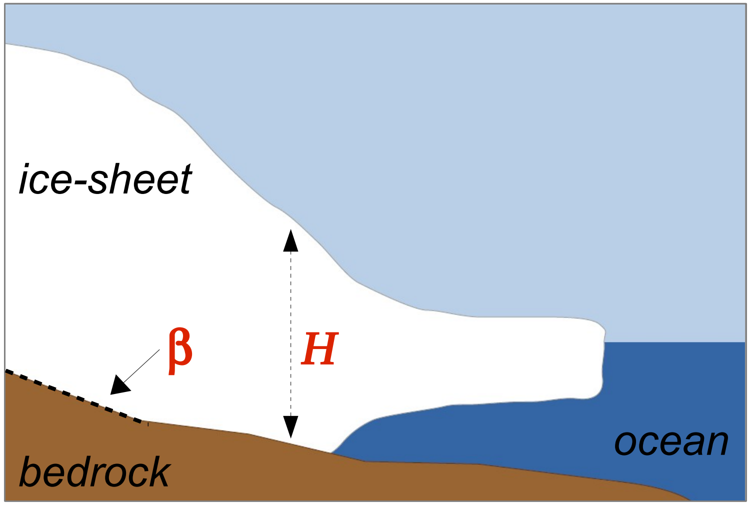
Estimation of ice-sheet initial state

(w/ G. Stadler, UT, and S. Price, LANL)

Problem: what is the initial thermo-mechanical state of the ice sheet?

Data we have:

- *ice extension and surface topography*
- ***surface velocity***
- ***Surface Mass Balance*** *(SMB: accumulation/melt rate)*
- ***ice thickness H (very noisy)***



Data we do NOT have:

- ***basal friction β***

Additional information:

- *ice fulfills* ***nonlinear Stokes equation***
- *ice is almost* ***at thermo-mechanical equilibrium***

Assumption (for now):

- *given* ***temperature field***

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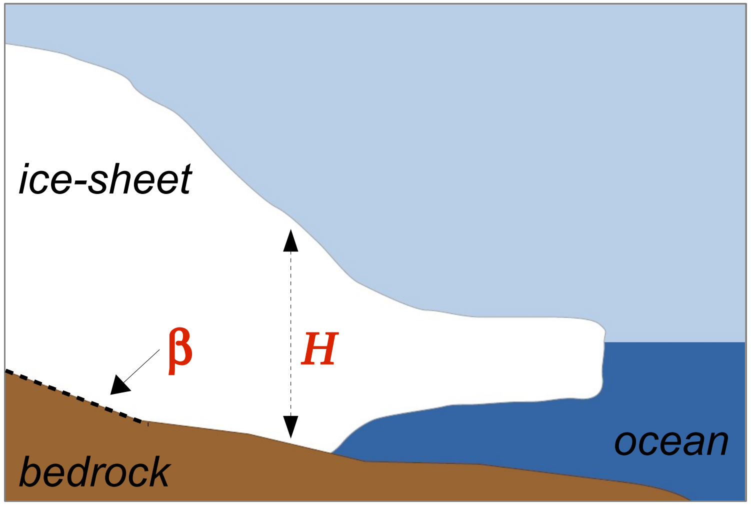

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Inverse Problem
Estimation of ice-sheet initial state

G. Stadler (UT), M. P. and S. Price (LANL)

How to prescribe ice-sheet mechanical equilibrium:

divergence flux

$$\frac{\partial H}{\partial t} = -\mathrm{div}\,(\mathbf{U}H) + \tau_s, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz.$$

Surface Mass Balance

At equilibrium: $\mathrm{div}\,(\mathbf{U}H) = \tau_s$

Boundary condition at ice-bedrock interface:

$$(\sigma\mathbf{n} + \beta\mathbf{u})_{||} = \mathbf{0} \quad on \quad \Gamma_\beta$$

Bibliography*:

Arthern, Gudmundsson, J. Glaciology. 2010

Price, Payne, Howat and Smith, PNAS 2011

Morlighem Thesis 2011

Brinkerhoff, Meierbachtol, Johnson, Harper, Annals of Glaciology, 2011

Morlighem et al. A mass conservation approach for mapping glacier ice thickness, 2013

Pollard DeConto, TCD 2012

Petra, Zhu, Stadler, Hughes, Ghattas, J. Glaciology , 2012.

Inverse Problem
Estimation of ice-sheet initial state

G. Stadler (UT), M. P. and S. Price (LANL)

Problem: find initial conditions such that the ice is almost at thermo-mechanical equilibrium given the geometry and the SMB, and matches available observations.

Optimization Problem:
find β and H that minimizes the functional $\mathcal{J}$

$$\mathcal{J}(\beta, H) = \frac{1}{2}\alpha_d \int_\Gamma |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds + \quad \text{(SMB mismatch)}$$

$$\frac{1}{2}\alpha_v \int_{\Gamma_{top}} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds + \quad \text{(surface velocity mismatch)}$$

$$\frac{1}{2}\alpha_H \int_\Gamma |H - H^{obs}|^2 \, ds + \quad \text{(observed thickness mismatch)}$$

$$\mathcal{R}(\beta) + \mathcal{R}(H) \quad \text{(regularizations).}$$

such that the ice sheet model equations (FO or Stokes) are satisfied

$\boldsymbol{U}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

Inverse Problem
Estimation of ice-sheet initial state

- Settings of the preliminary experiments:

1) Constraint: FO model.

2) No coupling with temperature solver (temperature field is given).

3) Tikhonov regularization both for β and H.   .

- Optimization:

Optimization Package Moocho (Trilinos).

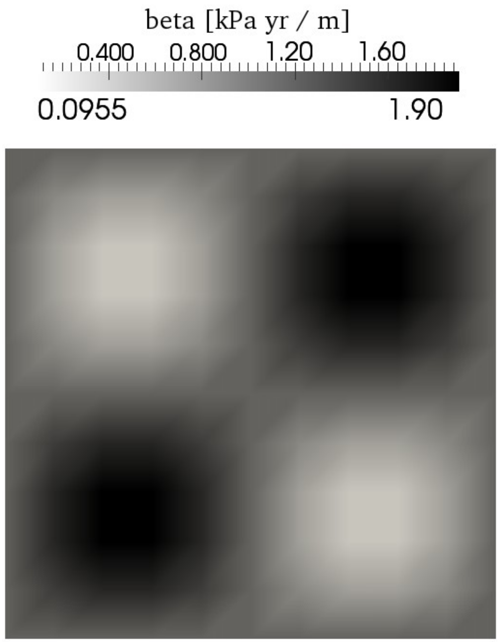
Sequential Quadratic Programming using *LBFGS* for approximating the reduced Hessian.

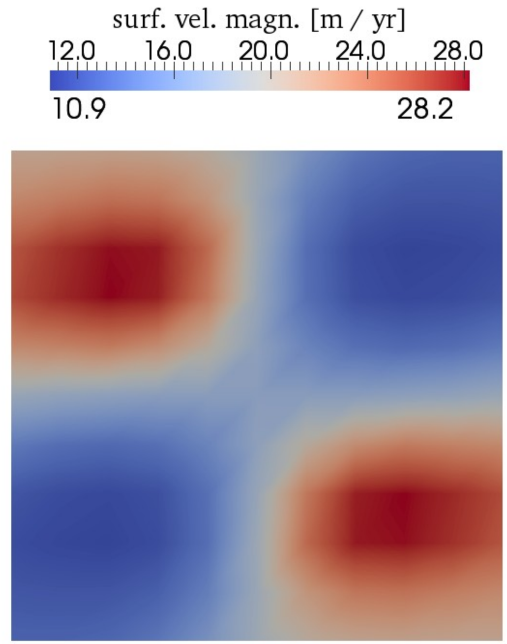
The first derivatives of the constraint and the cost functional are provided by LifeV.

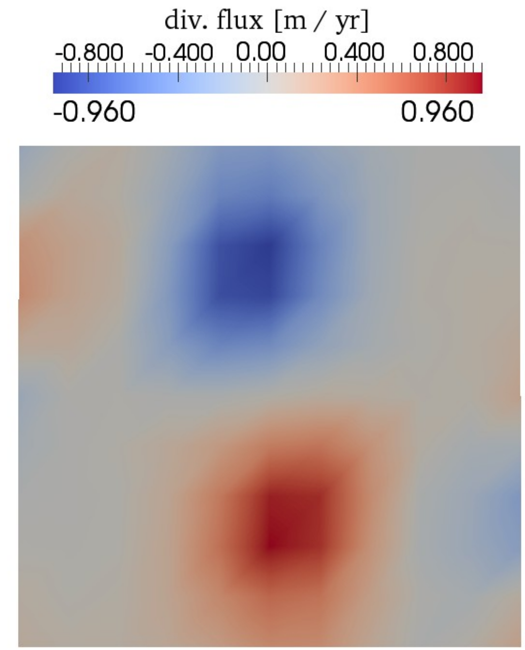
Inverse Problem

Estimation of ice-sheet initial state

Forward problem: ISMIP-HOM test C, with homogeneous Neumann lateral BCs.







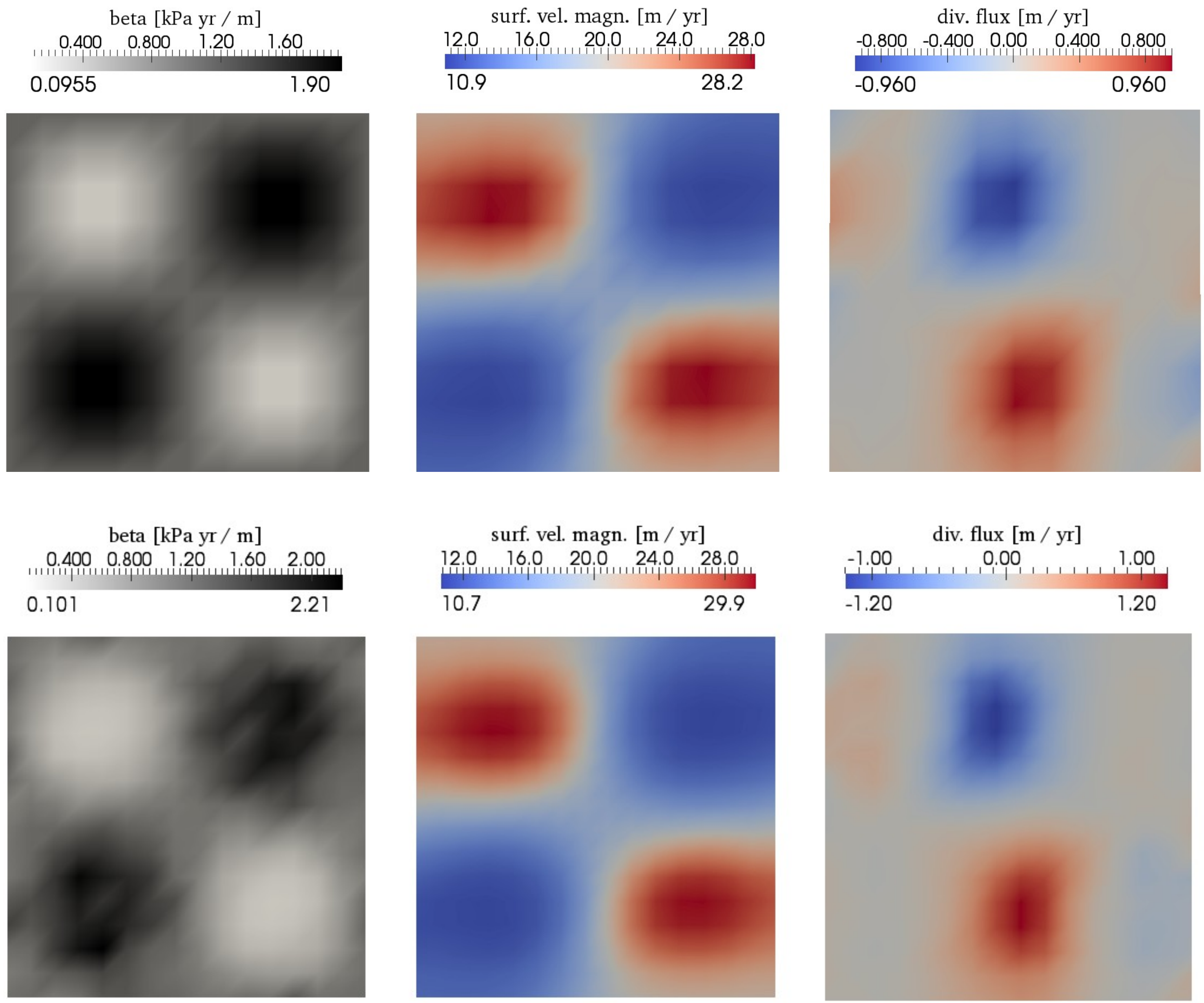
We will add a centered, uniformly distributed noise to the divergence flux and surface velocity obtained with the forward simulation and use them as “measured” SMB and surface velocity.
In particular, the amplitude of the noise added to the divergence flux is *10%* of the divergence flux.
Whereas, the amplitude of the noise added to the surface velocity is 1*%* of the the surface velocity.

<u>Case 1</u>: Minimize only the mismatch between the flux divergence and the noisy SMB

$$\mathcal{J}_1(\beta) = \frac{1}{2}\alpha_d \int_{\Gamma} |\mathrm{div}(\boldsymbol{U}H) - \tau_s|^2 \, ds + \frac{1}{2}\alpha_v \int_{\Gamma_{top}} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds + \mathcal{R}(\beta).$$

Forward Simulation

Estimated beta and reconstructed fields

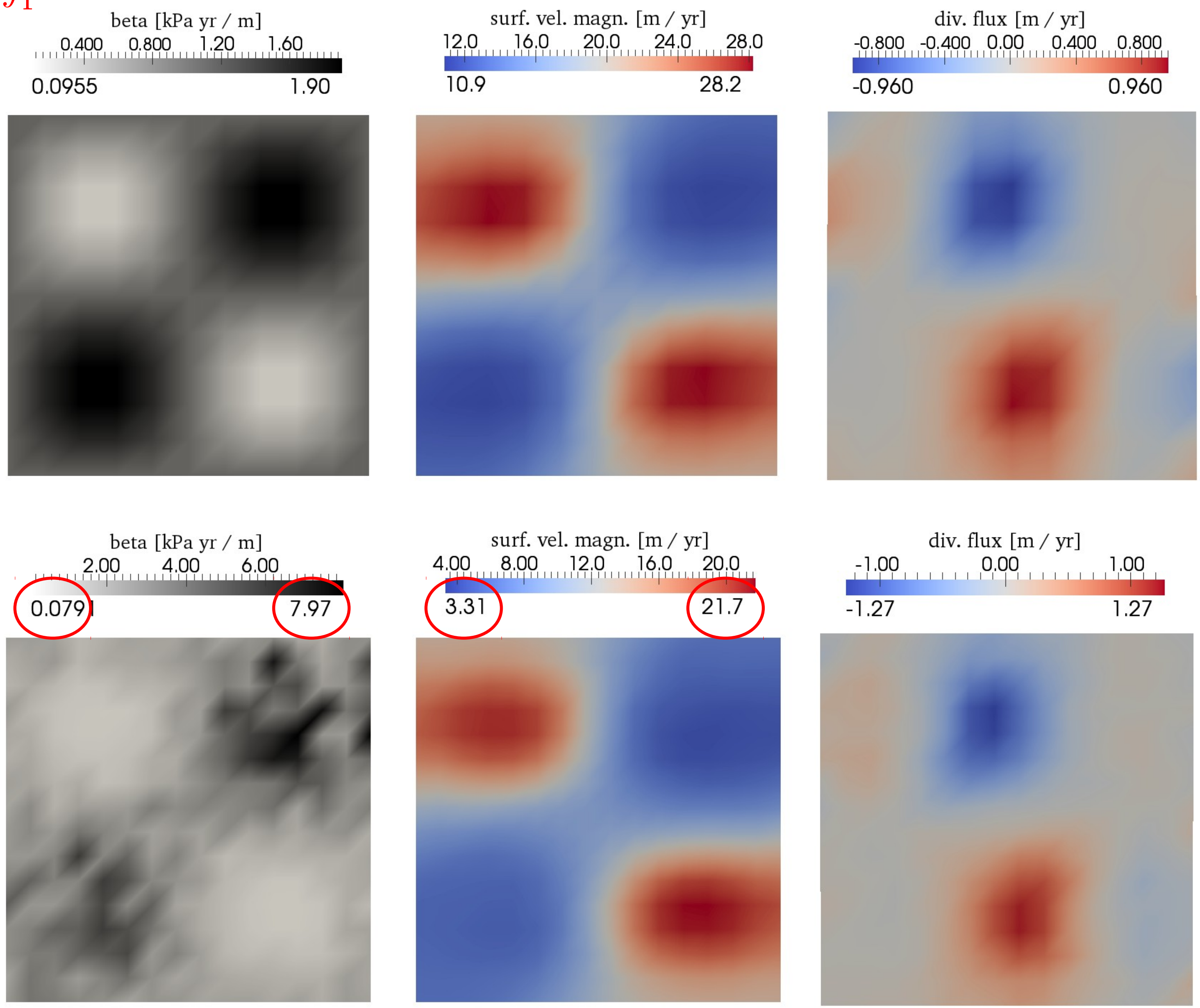


<u>Case 2</u>: Same as case 1, but we add a noise (5%) to the thickness field, to study the sensibility of the estimated beta w.r.t noise in the bedrock topography.

$$\mathcal{J}_2(\beta) = \frac{1}{2}\alpha_d \int_\Gamma |\mathrm{div}(\boldsymbol{U}H) - \tau_s|^2 \, ds + \mathcal{R}(\beta).$$

Forward Simulation

Estimated beta and reconstructed fields

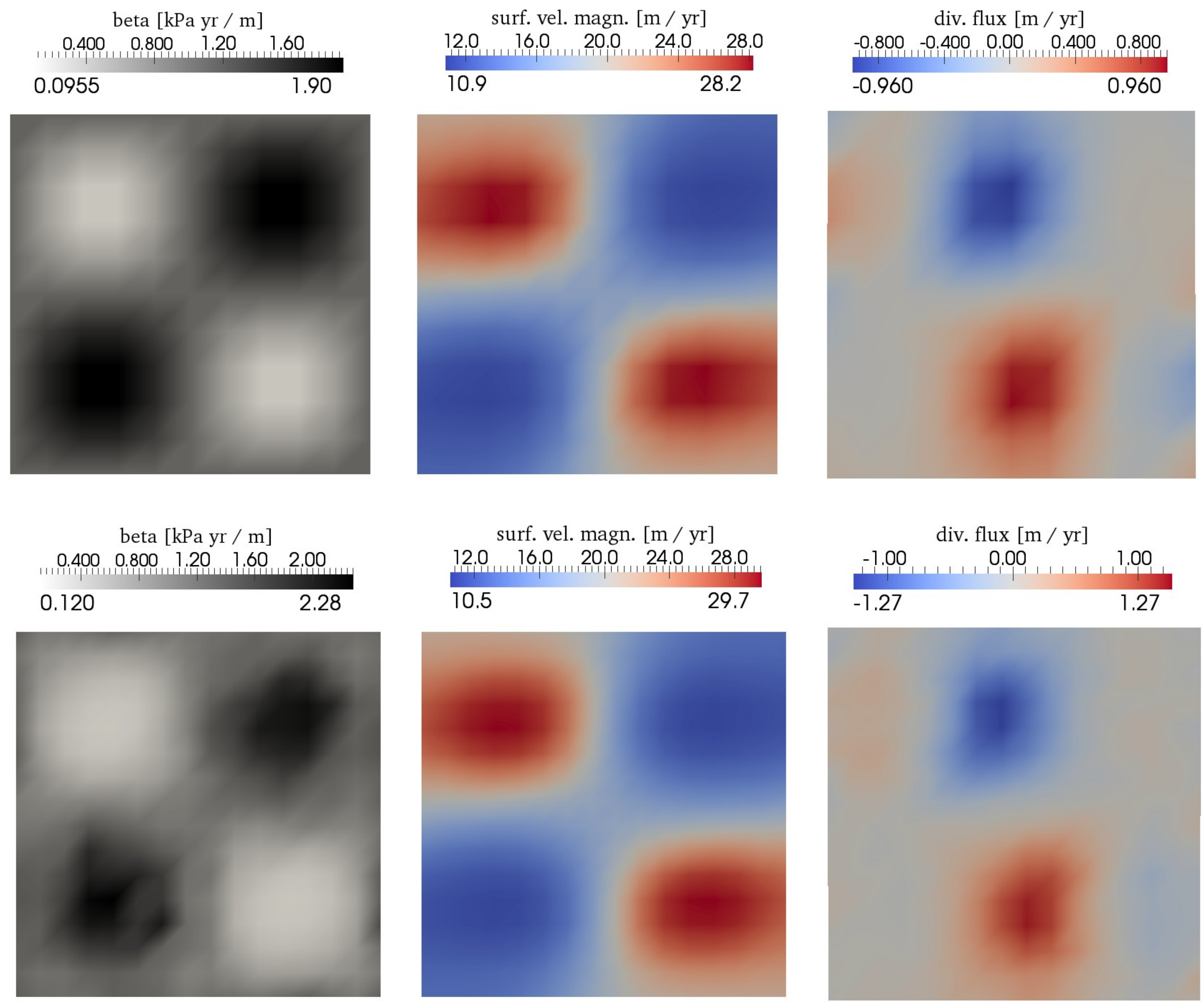


Case 3: **Same as case 2, but tune the thickness.**

$$\mathcal{J}_3(\beta, H) = \mathcal{J}_1(\beta) + \frac{1}{2}\alpha_H \int_\Gamma |H - H^{obs}|^2\, ds + \mathcal{R}(H).$$

Forward Simulation

Estimated beta and reconstructed fields



Inverse Problem
Estimation of ice-sheet initial state of Greenland ice sheet

Target surface mass balance and ice geometry from the data*.

Temperature field and target surface velocity from *ice2sea* forward simulation.

Cases:

1) keep the geometry (the thickness) fixed, control variable: β

2) Allow change in thickness up to 50m, control variables: β, H

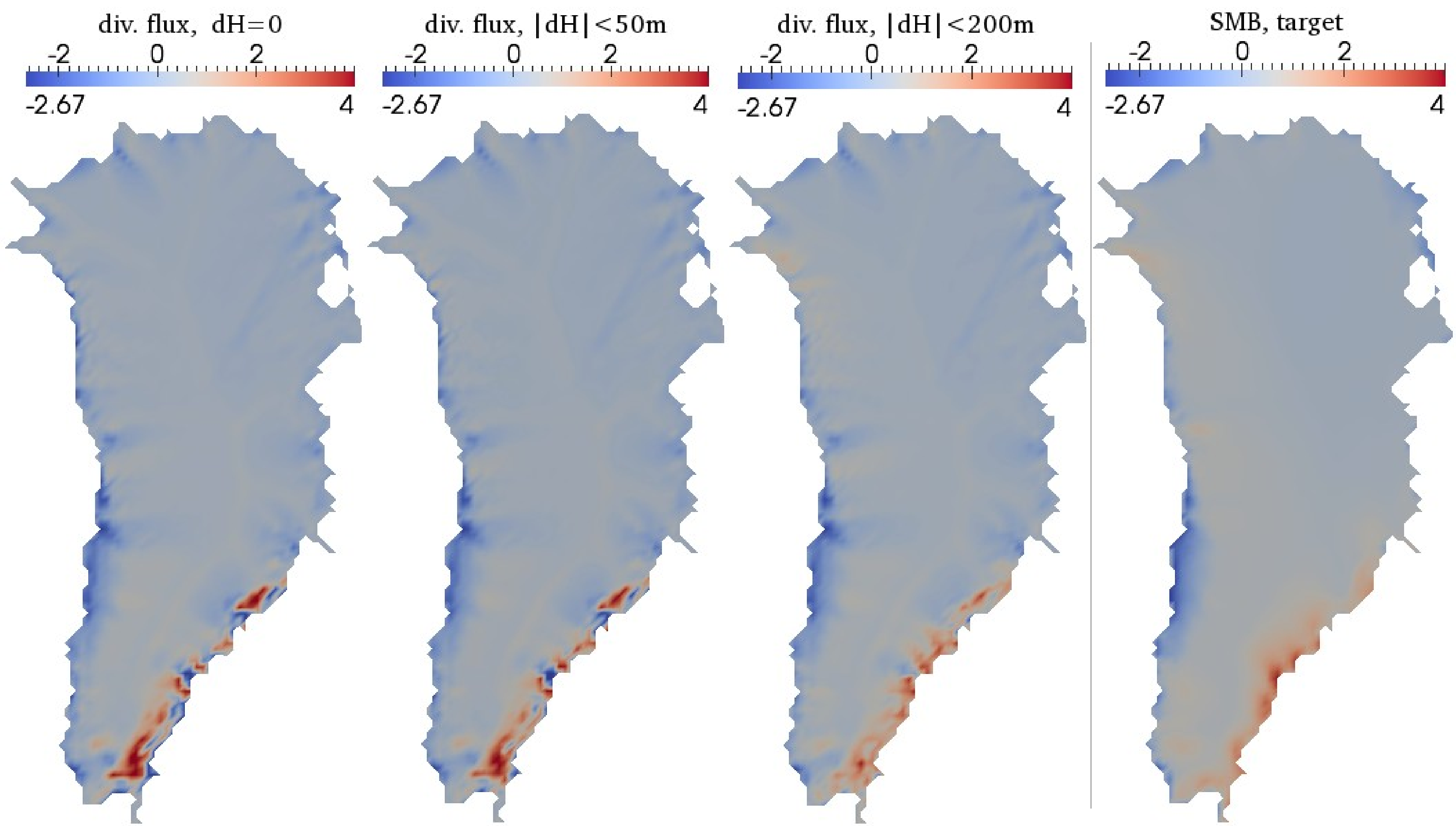
3) Allow change in thickness up to 200m, control variables: β, H

**Price, Payne, Howat and Smith*, PNAS 2011

Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Reconstructed div. flux[m/yr], with max change in thickness forced to be 0, 50 or 200m



var. range (-15.8, 9.6)

var. range (-8.9, 8.1)

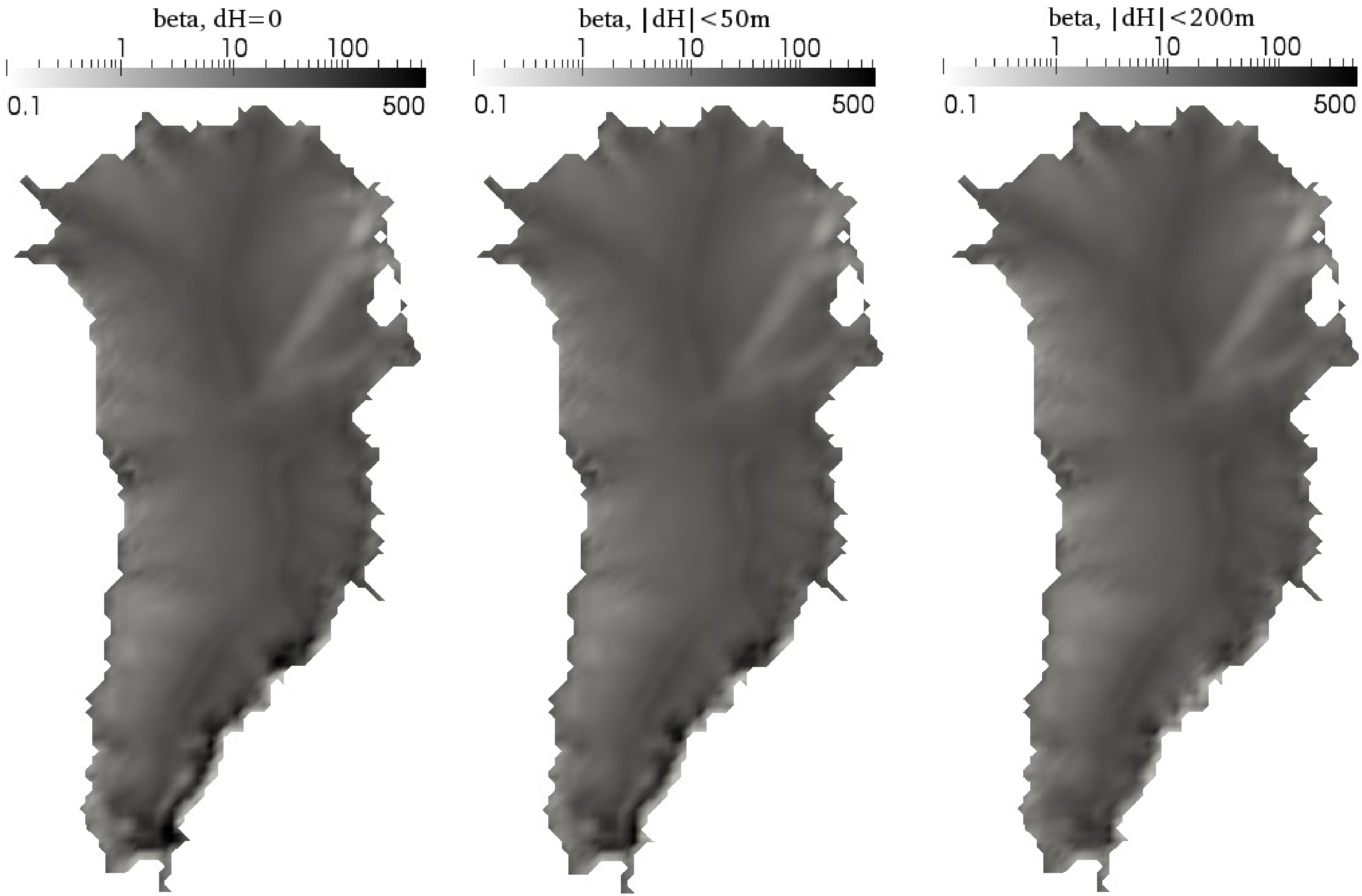
var. range (-3.3, 4.3)

var. range (-2.7, 4)

Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Estimated beta [kPa yr/m], when max change in thickness forced to be 0, 50 or 200m

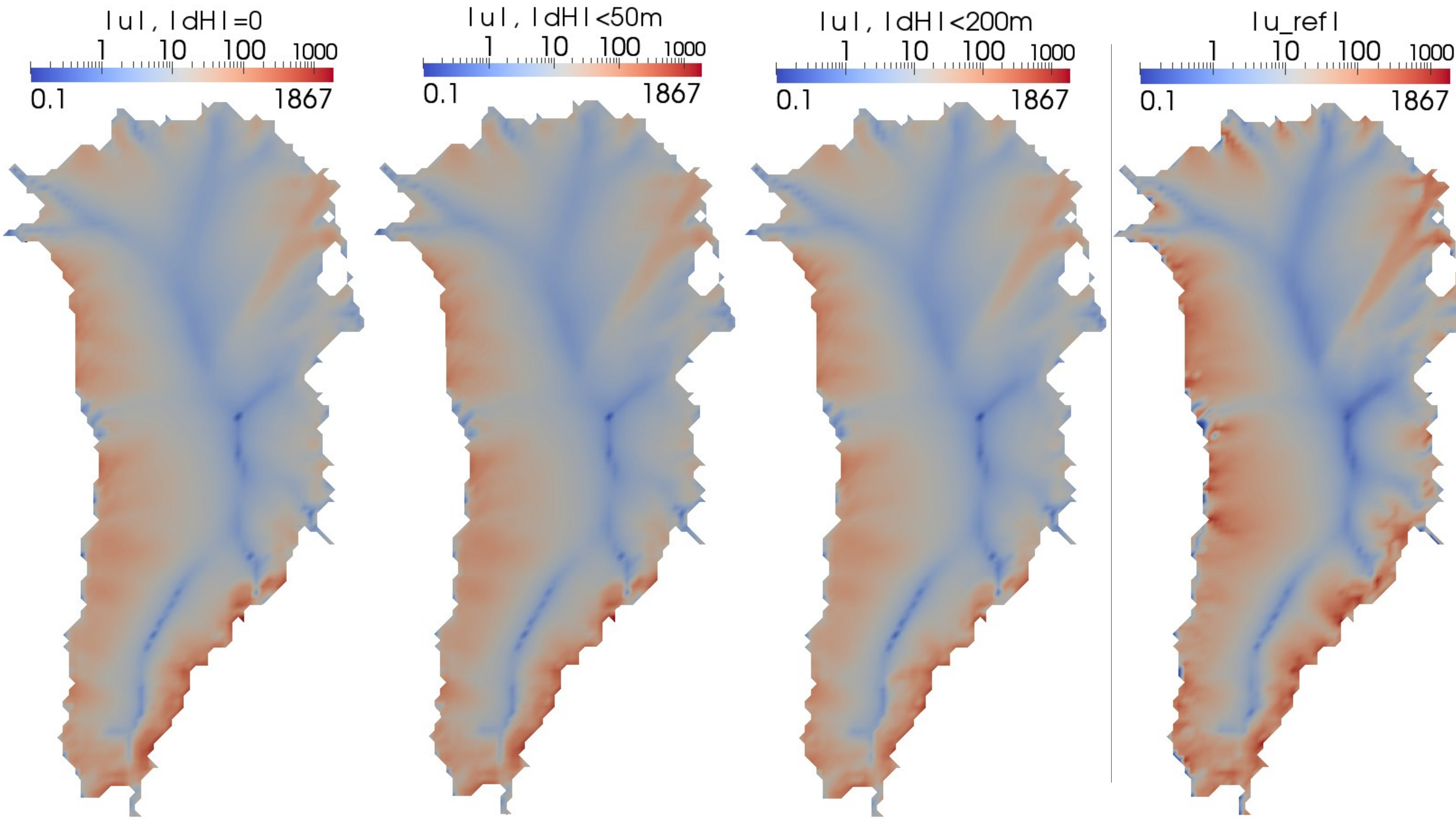


var. range (0.003, 2100) | var. range (0.02, 713) | var. range (0.04, 117)

Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

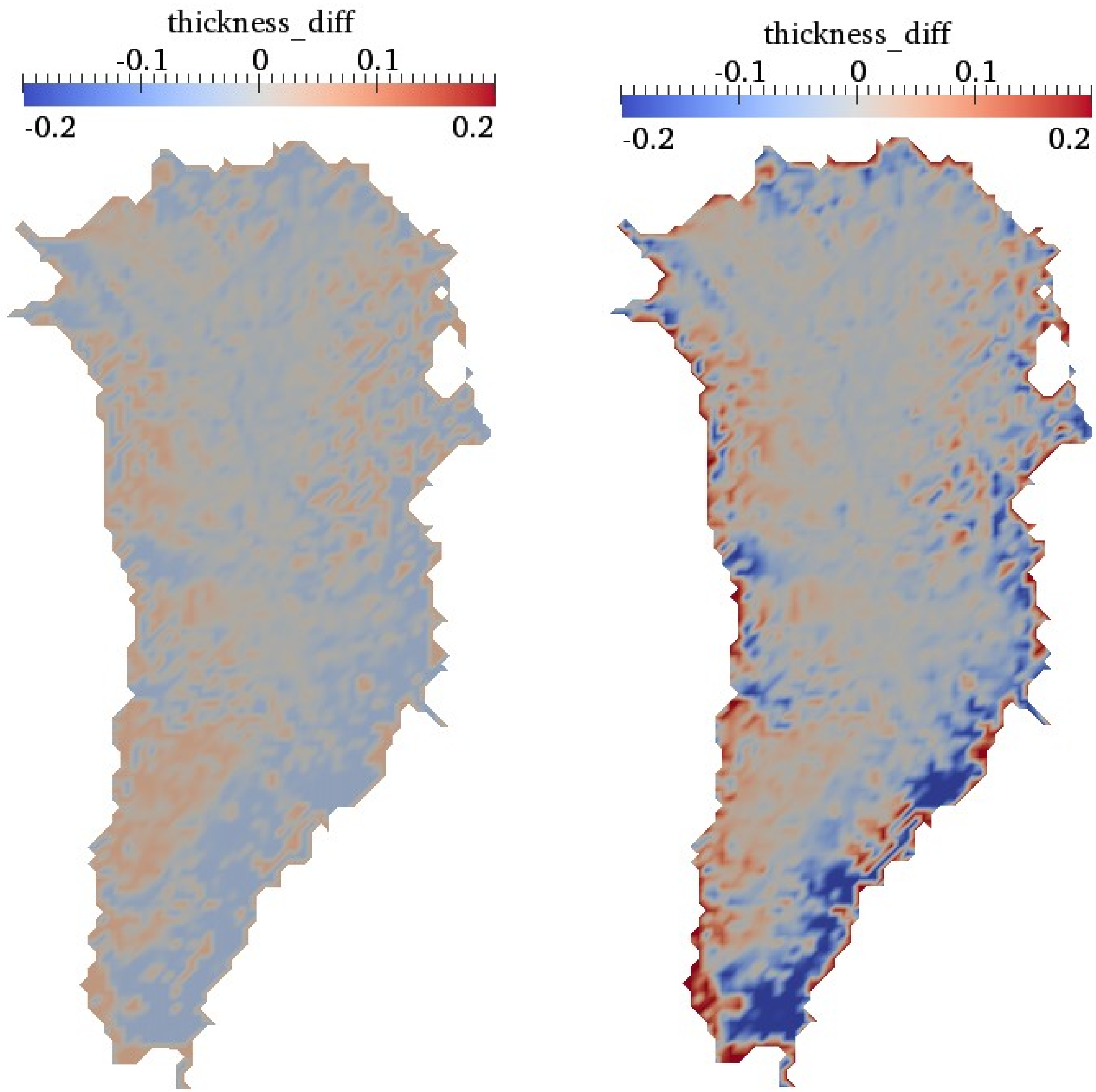
Reconstructed velocity [m/yr], when max change in thickness forced to be 0, 50 or 200m



Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Thickness change [km], when max change in thickness forced to be 50 (left) or 200m(right)



Future development

- Improve robustness and efficiency of Inversion algorithm.
- Port the code into Albany in order to exploit automatic differentiation and built in tools for sensitivity analysis and uncertainty quantification.
- Add coupling with temperature.
- Tackle finer geometries, possibly using different discretizations for the control parameters and the solution.

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Thank you for your attention