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Full-scale 2D and 3D simulations of electron beam acceleration for the LANL dielectric wakefield accelerator experiment

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Motivation

- The pre-conceptual design for Matter-Radiation Interactions in Extreme (MaRIE) future signature facility is underway at LANL, with the design of the 12 GeV electron linear accelerator being one of the main research goals.

- The primary limitation of the conventional accelerator technology is the accelerating gradient that is unable to exceed a few tens of MV/m. Technologies that have orders of magnitude increases in accelerating gradient (e.g., plasma wakefield acceleration) tend to degrade the accelerated beam quality.

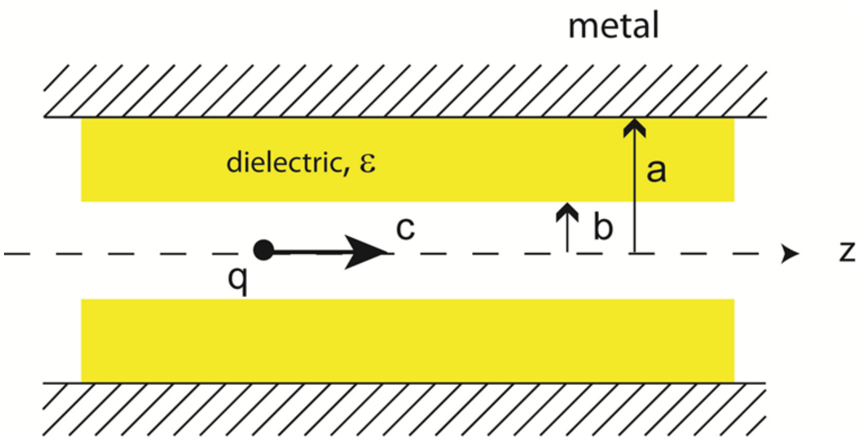
- The electron beam accelerator for MaRIE has to deliver the high quality electron beam with the electron bunch charges of 0.1 to 1 nC, normalized rms emittances of 0.1 to 1 mm, and rms energy spreads of less than 0.1%. Reaching energy spreads of less than 0.1% in conventional linacs is problematic due to the time variation of the accelerating electric field and wakefields.

- The wakefields which cause the dominant energy spread in beams in conventional linacs can be used in carefully designed structures (e.g., dielectric loaded waveguides) to generate extraordinary gradients and potentially small energy spreads in electron beam acceleration.

- This simulation study is to assess the characteristics of DWA and its potential applications in advance of experimental validation later on in our project.

Design principles and goals of the dielectric wakefield accelerator

- We attempt to combine the two proven concepts of the Dielectric Wakefield Accelerator (DWA) and the Emittance Exchanger (EEX) in a unique way to experimentally demonstrate a high-brightness DWA with an acceleration gradient exceeding 100 MV/m and with less than 0.1% induced energy spread in the accelerated beam.
- A transverse mask preceeds EEX to simultaneously produce a drive bunch/main beam pair. We additionally customize the main beam shape to minimize the wakefield-induced energy spread as it is accelerated.
- We conducted the preliminary design of possible beam configurations. We computed wakefields excited by a given charge distribution moving on the axis of the dielectric-loaded (silica, ε=3.75) waveguide.
- It seems that the accelerating gradient close to 200 MV/m with reasonable energy spread is possible with ideal timing and shaping of the bunches.



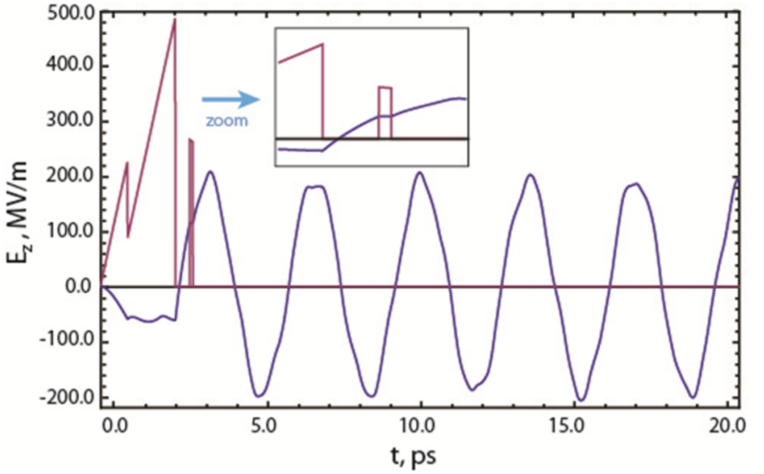
A schematic of the dielectric wakefield accelerator

Design parameters of the dielectric wakefield accelerator

Dimensions of the 300 GHz dielectric loaded waveguide, timing between the two bunches and transformer ratios for a possible experimental configuration

Beam pipe ID, *2b*	1.14 mm
Dielectric tube ID, *2a*	1.24 mm
Waveguide cutoff	298 GHz
Charge of the drive bunch	5 nC
Length of the drive bunch	2.350 ps
Charge of the witness bunch	250 pC
Length of the witness bunch	100 fs
Time between the bunches	2.8017 ps
Transformer ratio	3.38
ΔG/G	$9*10^{-6}$

High transformer ratio wake excited by a double triangular drive beam and a trapezoidal witness bunch exposed to flat accelerating gradient.



- Tolerances. For ΔG/G < 10-4 we must have

4.998 nC < drive charge < 5.002 nC
249.9 pC < witness charge < 250.1 pC
2.3494 ps < length of the drive bunch < 2.3507 ps
2.8014 ps < time between the bunches < 2.8020 ps
99.95 fs < length of the witness bunch < 100.1 fs

Full dispersion relation and eigen modes in dielectric wall waveguide

- **In general dielectric wall waveguide supports infinite number of discrete Hybrid EM modes**
  - *TE, TM modes decoupled in axial symmetric excitation (m=0).*
  - *Resonant TM_{01} mode most useful for co-linear particle acceleration*

Flesher & Cohn 1951
Chang & Dawson 1970

$$C(s) = \frac{m^2 s k^2}{a^2}\beta^2\left(\frac{\varepsilon - 1}{1-\beta^2}\right)^2 - s^3\left(\frac{kS'_m(sa)}{S_m(sa)} + \frac{sI'_m(ka)}{I_m(ka)}\right)\left(\frac{sI'_m(ka)}{I_m(ka)} + \frac{\varepsilon k R'_m(sa)}{R_m(sa)}\right) = 0$$

$$R_m(sr) = N_m(sb)J_m(sr) - J_m(sb)N_m(sr), \quad R'_m(sr) = N_m(sb)J'_m(sr) - J_m(sb)N'_m(sr),$$

$$S_m(sr) = N'_m(sb)J_m(sr) - J'_m(sb)N_m(sr), \quad S'_m(sr) = N'_m(sb)J'_m(sr) - J'_m(sb)N'_m(sr).$$

$$k^2 = \frac{\omega^2}{v^2}(1-\beta^2)$$

$$s^2 = \frac{\omega^2}{v^2}(\varepsilon\beta^2 - 1)$$

$$k_0 = \frac{\omega}{c}$$

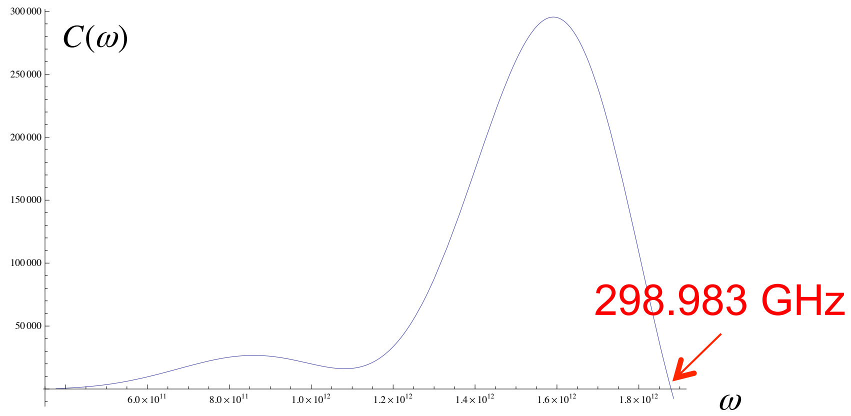
- **Eigen modes can be solved from Bessel's equation with boundary matching conditions at the dielectric-vacuum interface. These modes are orthonormal basis functions for the solution of the inhomogeneous equation with a moving source (i.e., symmetric about z-vt).**

$$\left[\frac{d^2}{dr^2} + \frac{d}{rdr} + (s^2 - \frac{m^2}{r^2})\right]\begin{pmatrix} e_z(r) \\ h_z(r) \end{pmatrix} = 0 \qquad \sum_{l=1}^{N}\int_{R_{l-1}}^{R_l} dr \cdot r\left[\varepsilon_l e_{z,i}(r)e_{z,j}(r) + \mu_l h_{z,i}(r)h_{z,j}(r)\right] = \delta_{ij}C_j$$

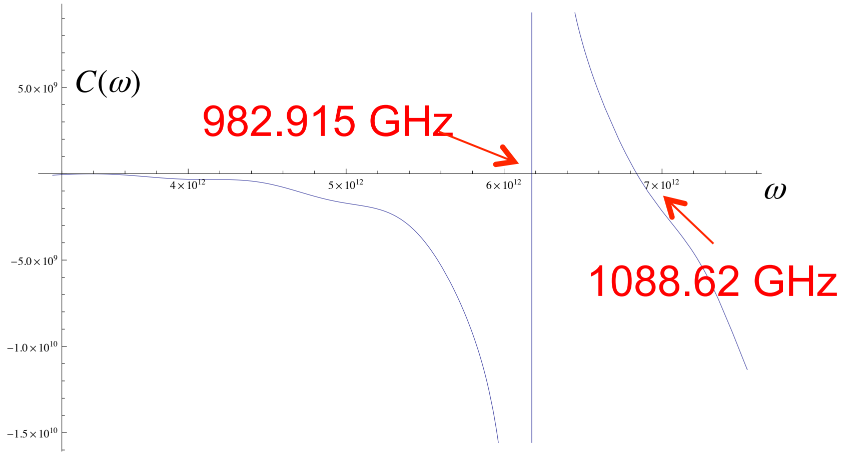
Park & Hirshfield 2000

The dispersion relation of a dielectric wall waveguide can be solved numerically

- **Numerical solution of the TM_{0n} modes in the dielectric waveguide with a=0.057cm, b=0.0662cm and phase velocity $\beta = (1-\gamma^{-2})^{1/2}$ for $\gamma=195.69$ (100MeV)**



298.983 GHz



982.915 GHz

1088.62 GHz

- **Fundamental TM mode is solitary while high order modes appears in pairs. The high order TM mode pairs are degenerate when $\beta=1$.**
- **Frequency of TE_{01} mode is at 51.2691 GHz, high order modes also appears in pairs when $\beta<1$.**

Green’s function for the excitation of wakefield in a dielectric wall waveguide

- **The wakefield of a beam is the convolution of the single particle wake and the beam shape**
  - *Accelerating field in TM$_{mn}$ mode excited by a single particle* $\rho = e\dfrac{\delta(r-r_0)}{r}\delta(\theta)\delta(z-vt),\ j_z = v\rho$

$$E_{z,mn}(r,\theta,z-vt) = -8e\cos(m\theta)\cos\left(\frac{\omega}{v}(z-vt)\right)\times I_m(kr)I_m(kr_0)\times\frac{K_m(ka)}{I_m(ka)}\,\frac{T(s)}{sdC(s)/ds}\bigg|_{s=s_n}$$

Azimuthal Longitudinal Radial Amplitude

$$T(s) = -\frac{m^2k^4\beta^2}{a^2}\left(\frac{\varepsilon-1}{1-\beta^2}\right)^2 - s^2k^2\left(\frac{kS_m'(sa)}{S_m(sa)}+\frac{sI_m'(ka)}{I_m(ka)}\right)\left(\frac{sK_m'(ka)}{K_m(ka)}+\frac{\varepsilon kR_m'(sa)}{R_m(sa)}\right)$$

Rosing & Gai 1990
K.-Y. Ng 1990

  - *Higher order TM mode has larger radial variation (but vanish for β=1)*

- **The Green’s function for a moving source can also be decomposed into eigen modes of the waveguide**
  - *The sum of above infinite series depends on the Fourier content of the beam and may converge slowly.*
  - *With eigen mode expansion, fewer modes may be sufficient for accurate result.*

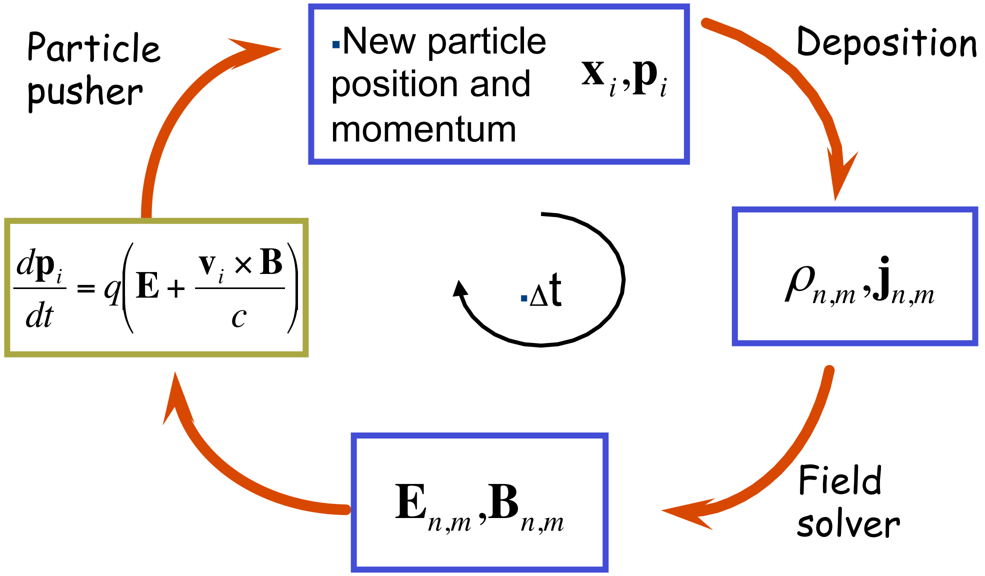
$$\begin{pmatrix}E_z(r,t)\\ H_z(r,t)\end{pmatrix} = -2q_0\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{e_{z,n}(r_0)}{C_n}\begin{pmatrix}e_{z,n}(r)\cos(k_n(z-vt))\Theta(z-vt)\cos(m\theta)\\ h_{z,n}(r)\cos(k_n(z-vt))\Theta(z-vt)\sin(m\theta)\end{pmatrix}$$

Park & Hirshfield 2000

First principle Particle-In-Cell simulation can be applied to self-consistent DWA modeling

Computational cycle

(at each step in time)



- New particle position and momentum $\mathbf{x}_i, \mathbf{p}_i$

Deposition

$\rho_{n,m}, \mathbf{j}_{n,m}$

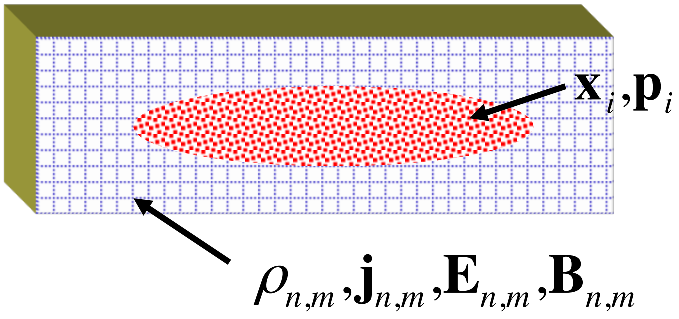
Field solver

$\mathbf{E}_{n,m}, \mathbf{B}_{n,m}$

$\frac{d\mathbf{p}_i}{dt} = q\left(\mathbf{E} + \frac{\mathbf{v}_i \times \mathbf{B}}{c}\right)$

Particle pusher

Δt



- Maxwell's equations for field solver
- Lorentz force updates particle's position and momentum

The explicit PIC is computation intensive:

$$\Delta x < \lambda, \quad c\Delta t < \Delta x \ \ (Courant \ \ Condition)$$

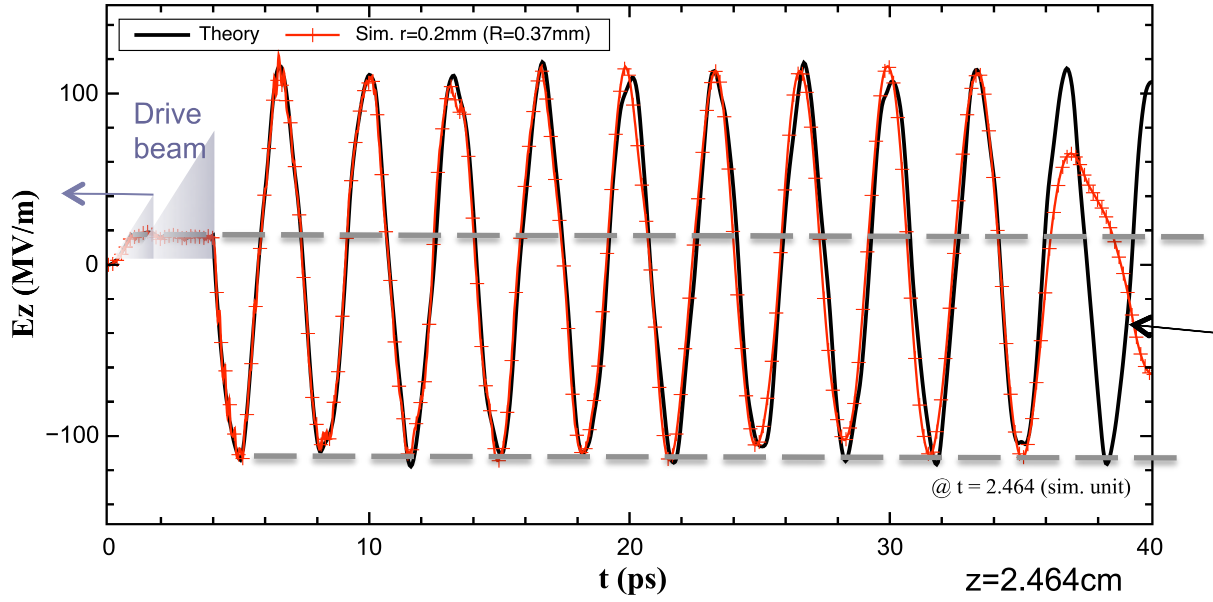
Major challenges for DWA problem:

- Numerical dispersion of EM wave
- Accuracy and computation cost

Verification of electromagnetic particle-in-cell simulations for DWA modeling with theory

- We employ relativistic electromagnetic particle-in-cell Merlin and LSP in 2D cylindrical and 3D geometry for the modeling of DWA

- Benchmark with analytic theory shows good agreement of the longitudinal wakefield and transformer ratio



E_{dec}~17.5MV/m

Transient field

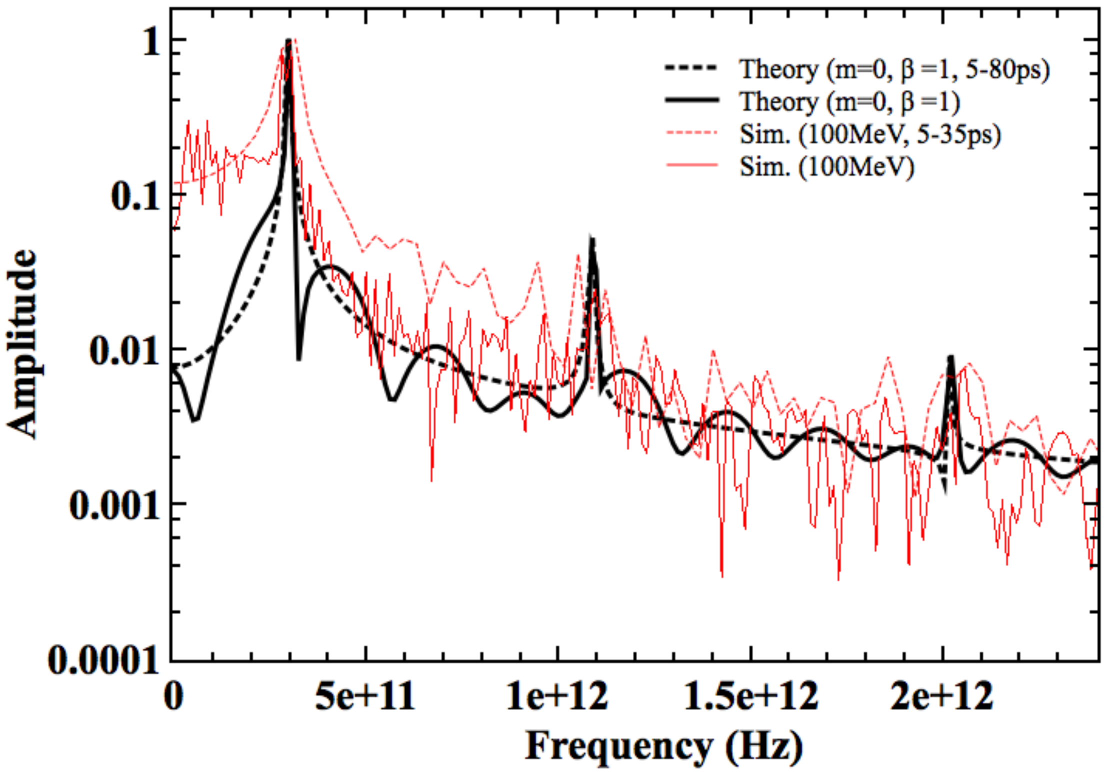
E_{acc}~117MV/m

Transformer ratio $R=E_{acc}/E_{dec}$~6.7, very close to theoretical estimate

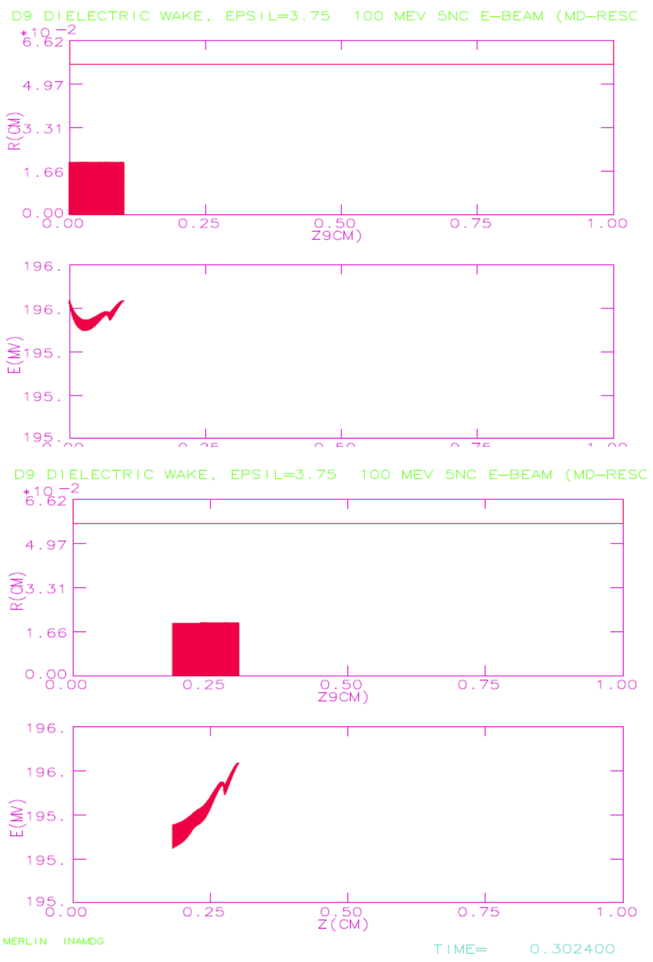
Comparison of the longitudinal wakefield mode spectrums between simulation and numerical calculation

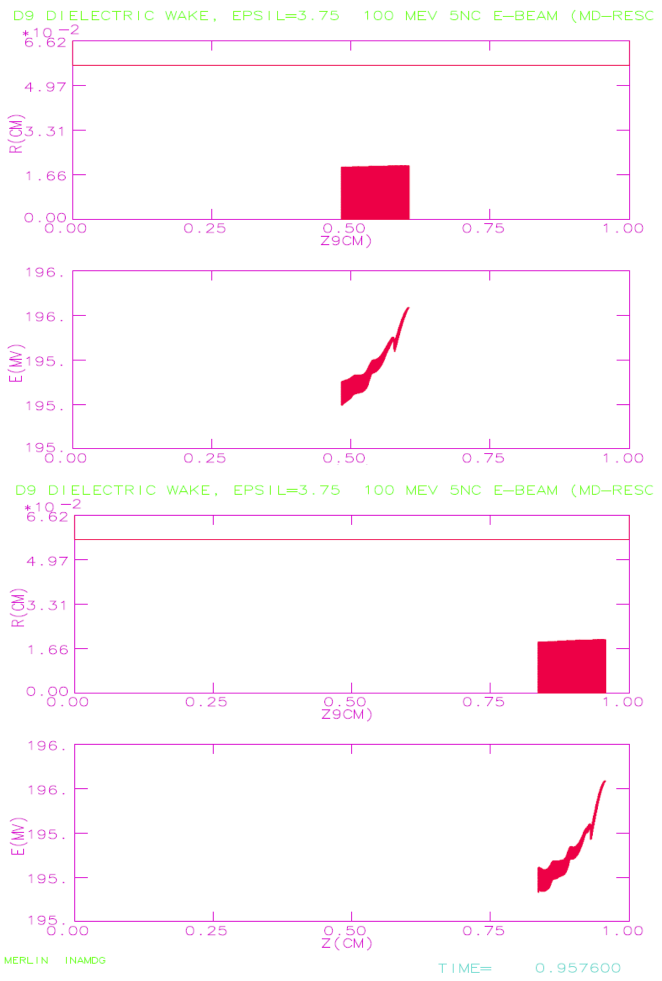
- **PIC simulation result has pronounced peaks for discrete TM modes on top of a wideband excitation from the source with sharp rise/fall. The bandwidth of the excitation is limited by simulation duration.**
- **Double-triangle beam excites a nearly uniform wakefield inside the beam which in turn modulates the discrete TM mode spectrum**

First three TM_{0n} modes excited in a 300Ghz structure by a 4ps double-triangle beam

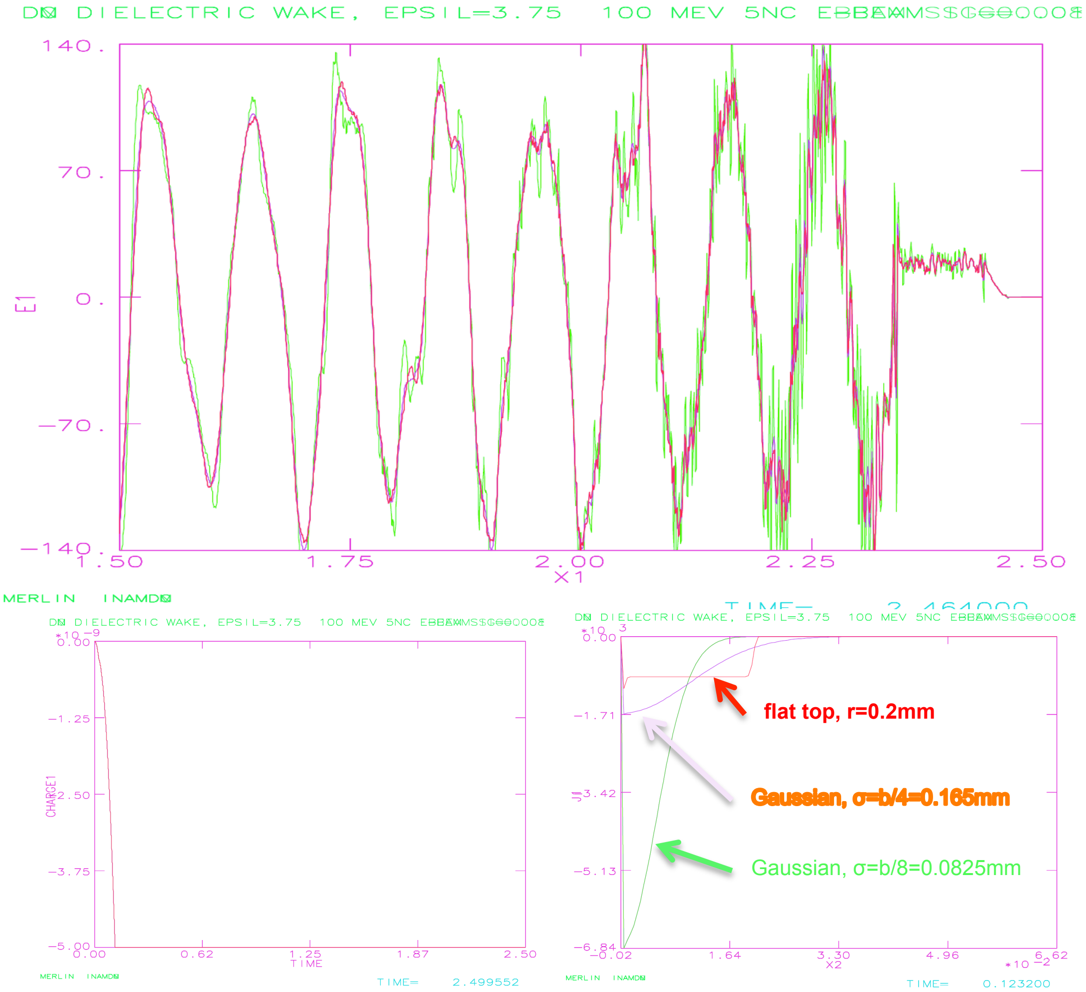


Self-consistent beam injection in 2-D simulations show small space charge effects and negligible beam divergence during its propagation in the dielectric waveguide





Two-dimension (R-Z) simulations show weak dependence of the wakefield on the radial profile of the drive electron beam

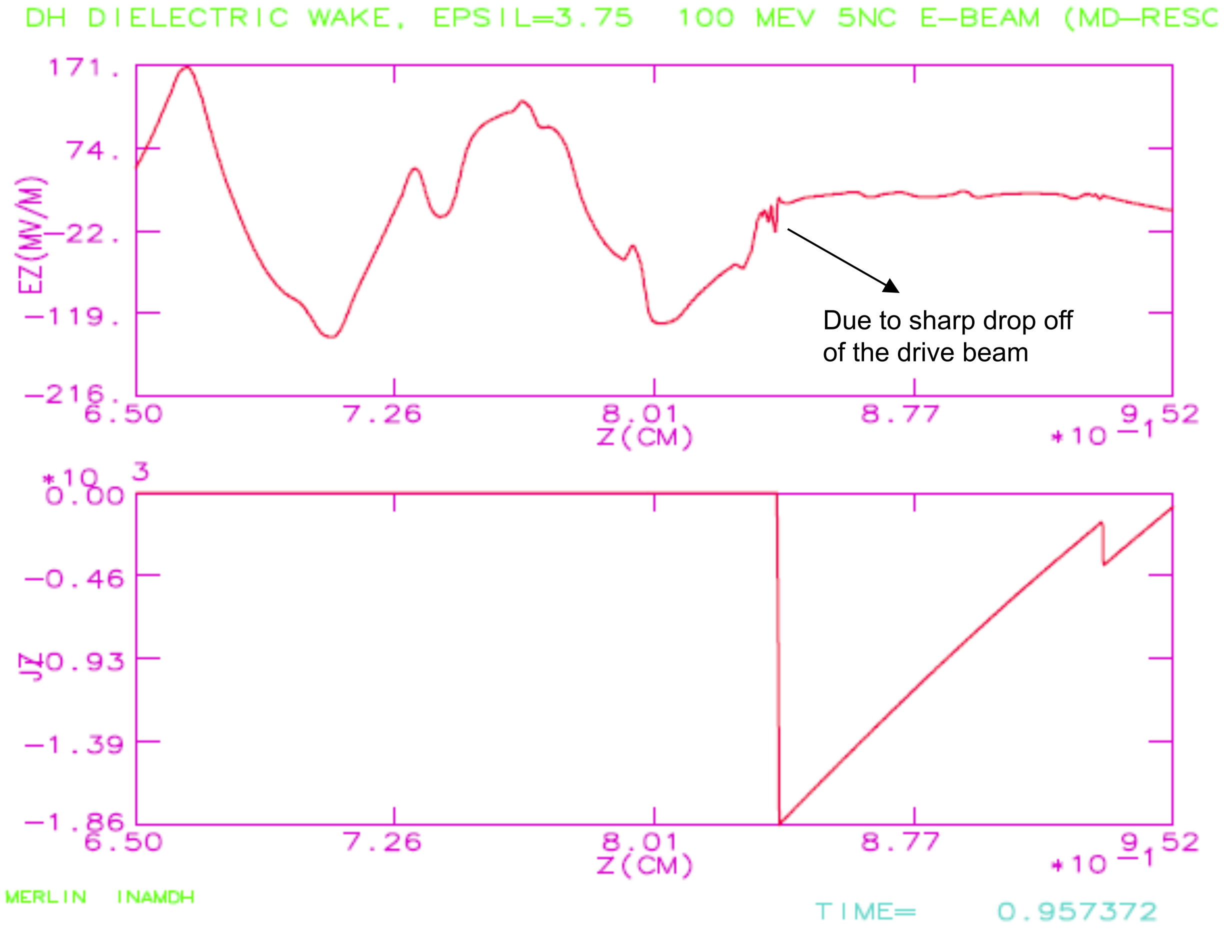


flat top, r=0.2mm

Gaussian, σ=b/4=0.165mm

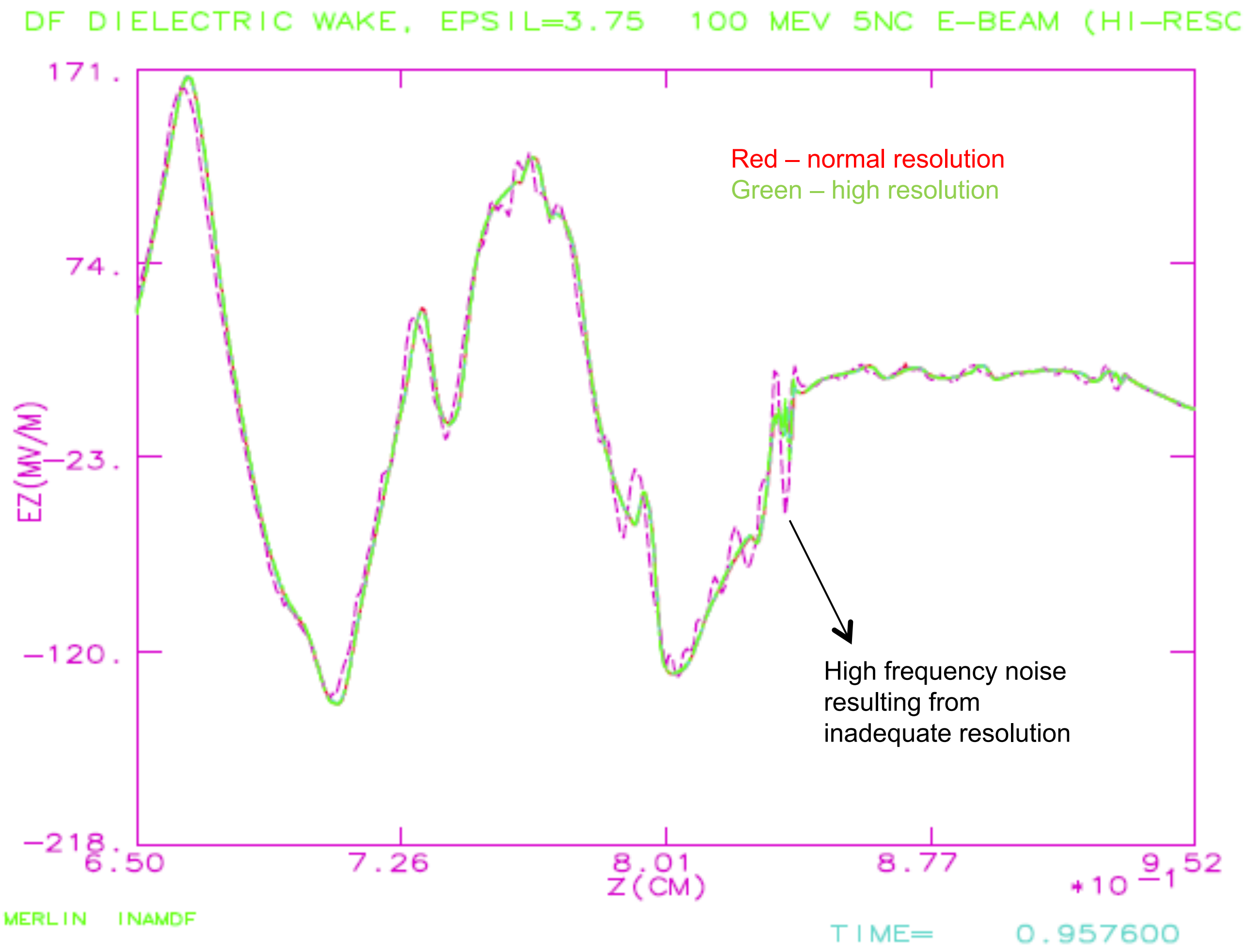
Gaussian, σ=b/8=0.0825mm

Merlin 2-D simulation shows the excitation of wakefield behind the double triangular beam pulse in the dielectric wall waveguide



Due to sharp drop off
of the drive beam

Longitudinal component of the wakefield from simulations shows the importance of numerical resolution

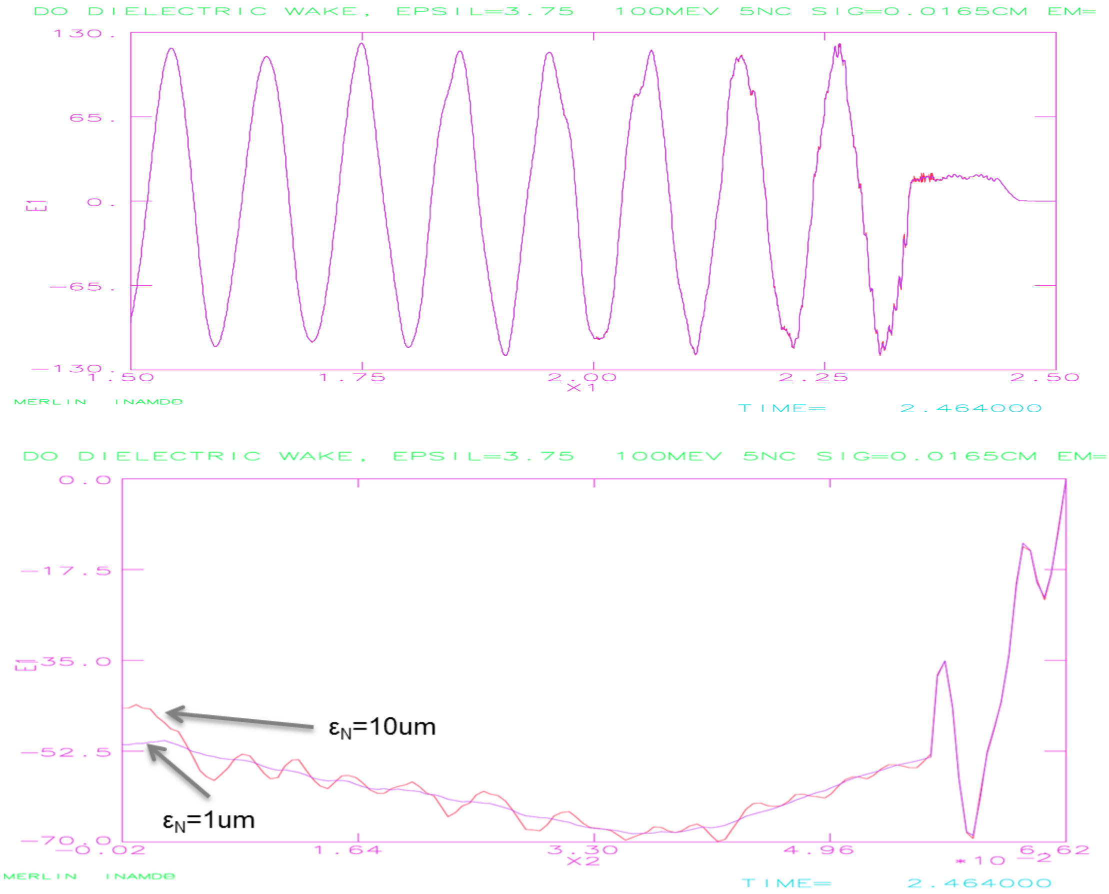


Red – normal resolution
Green – high resolution

High frequency noise resulting from inadequate resolution

The structure of the wakefield is insensitive to the emittance (ε_N=10μm and ε_N=1μm) of the drive electron beam

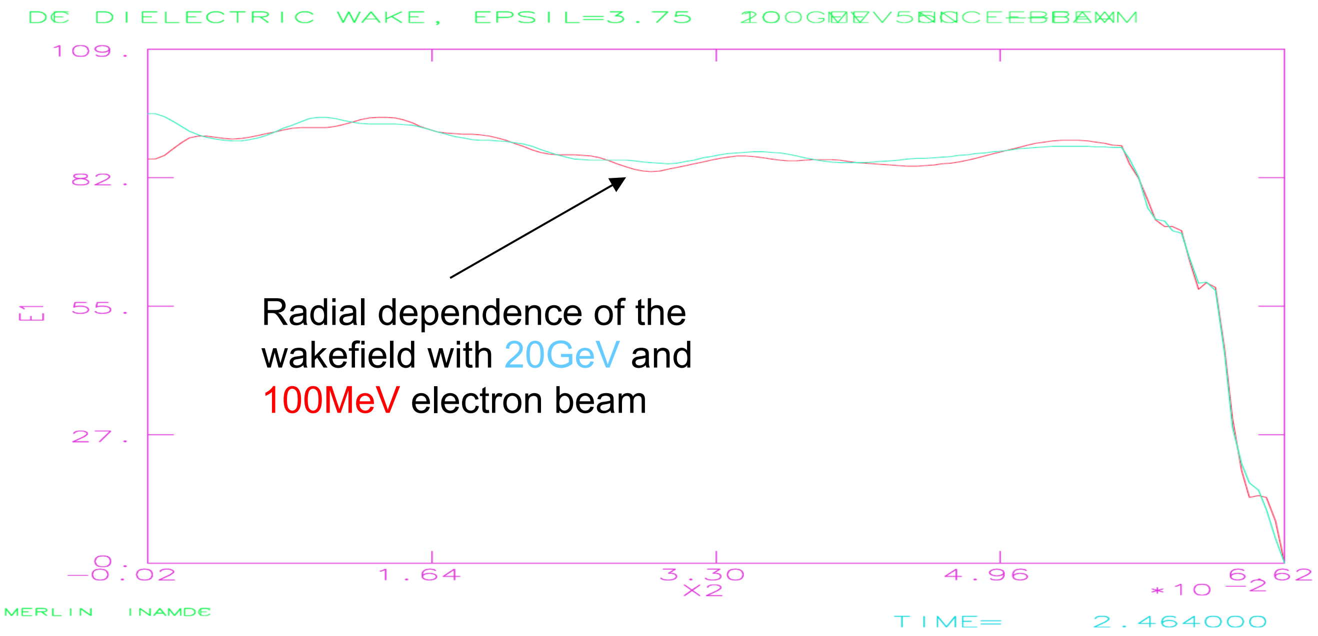
Ez @ R=0.0368cm



ε_N=10um

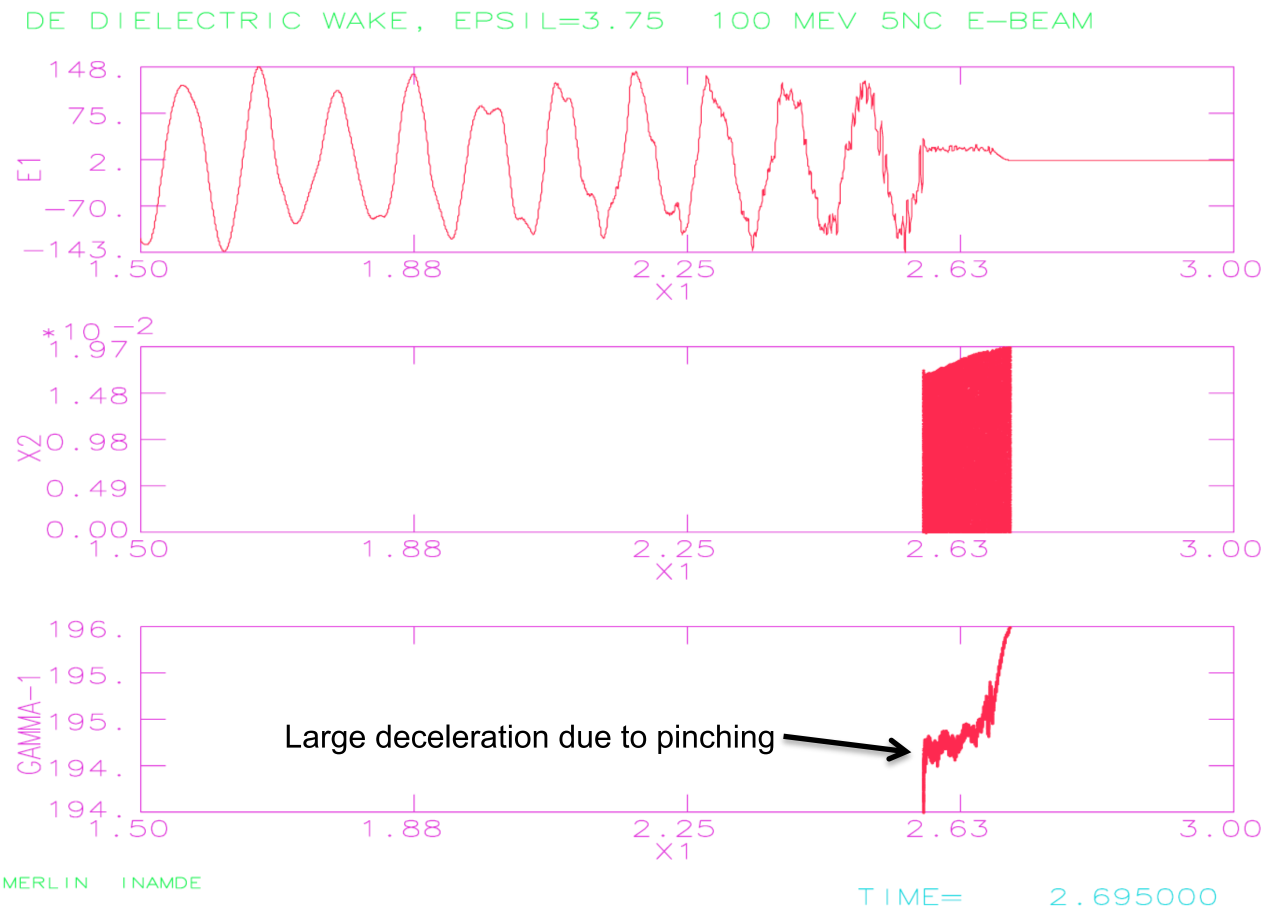
ε_N=1um

Radial dependence and structure of the wakefield is insensitive to the energy of the drive electron beam



Radial dependence of the wakefield with 20GeV and 100MeV electron beam

Merlin wakefield simulation with typical numerical resolution



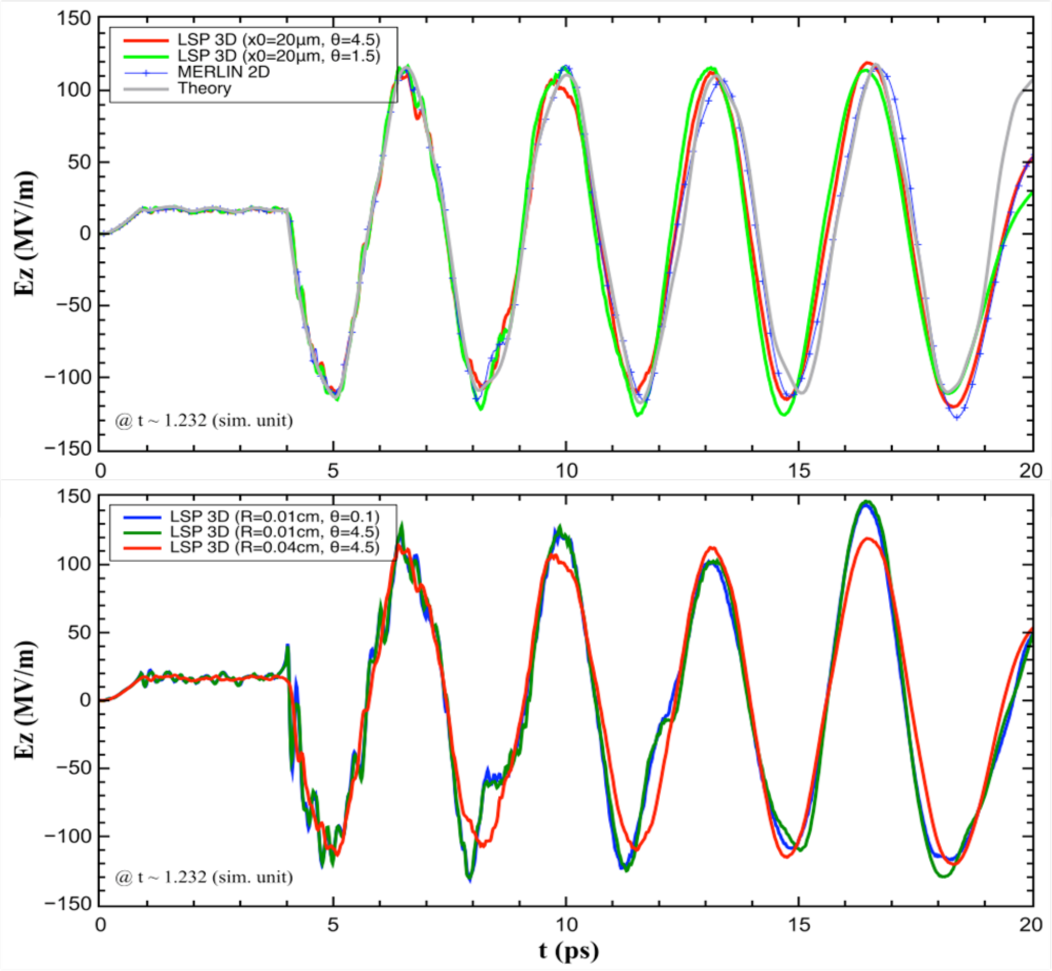
Large deceleration due to pinching

Effects of beam misalignment were studied with a 3D PIC LSP code for a flat top beam with 20 μm radial offset.

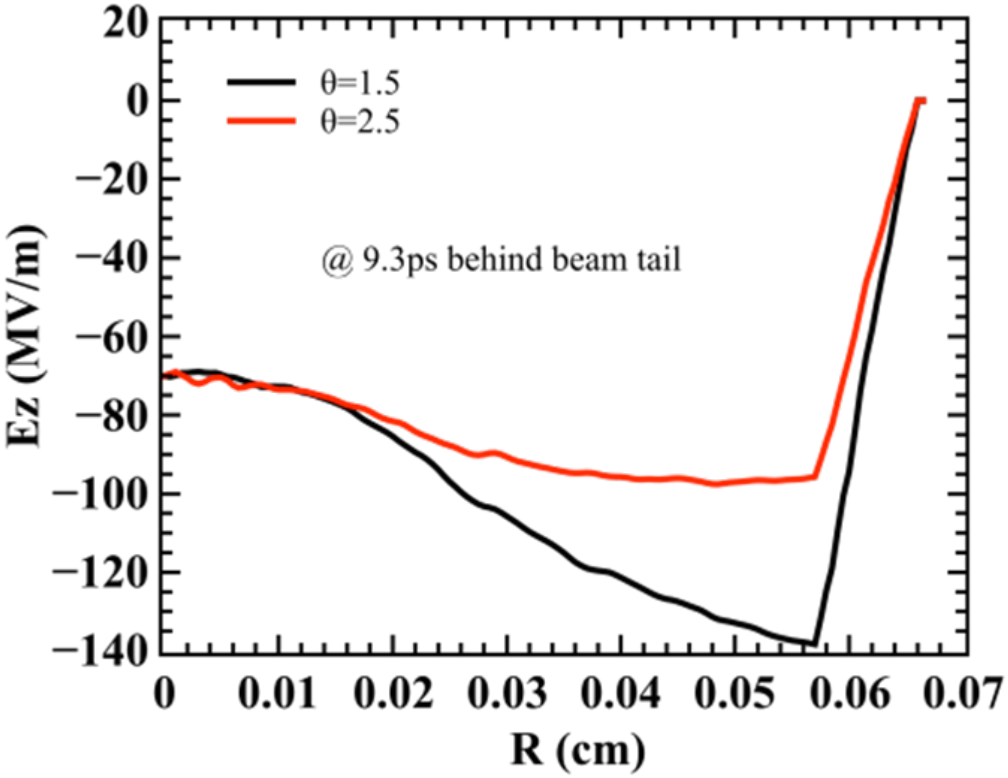
We employ relativistic electromagnetic particle-in-cell Merlin and LSP in 2D cylindrical and 3D geometry codes for modeling DWAs.

Upper panel shows LSP simulations with beam axis offset of 0.02mm. The lineouts are at different angular positions.

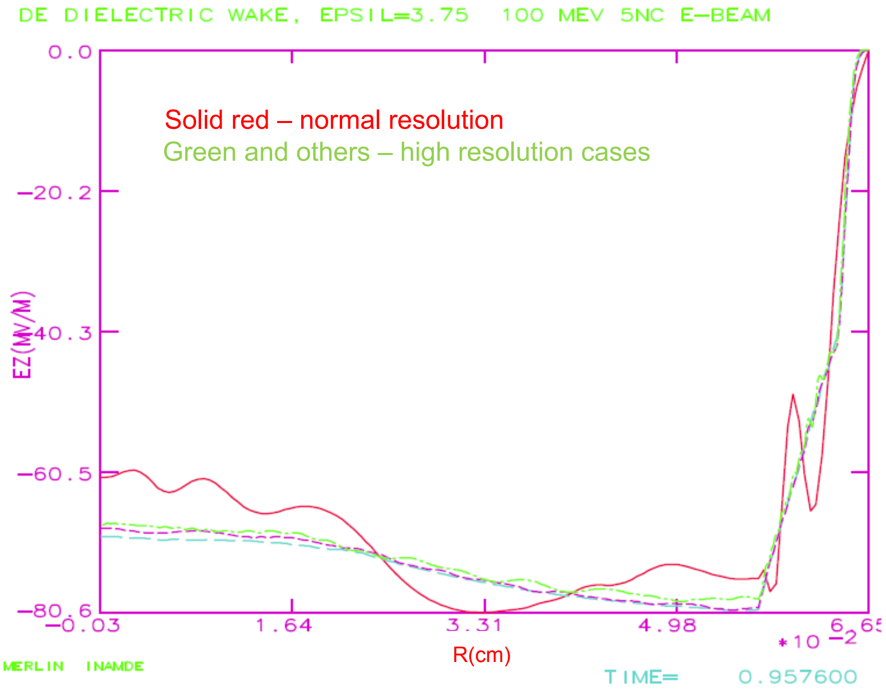
Lower panel shows LSP simulations that the wakefield is insensitive to radial and angular positions.



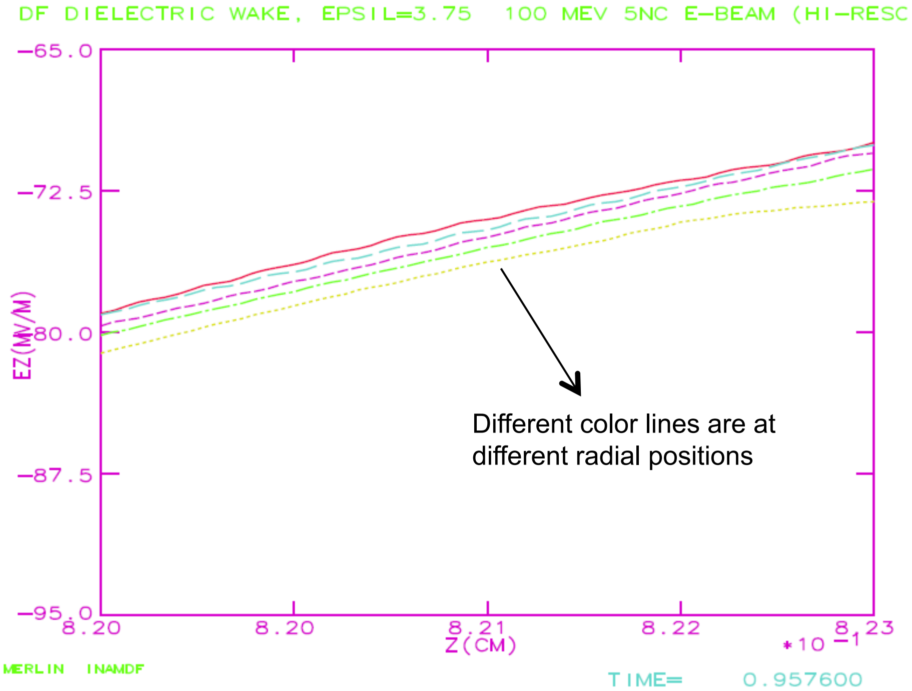
The biggest effect of the misalignment is the angular dependence of the excited accelerating field in a given plane.



Radial and axial dependence of the accelerating field is critical to achieve an accelerated beam bunch with small energy spread



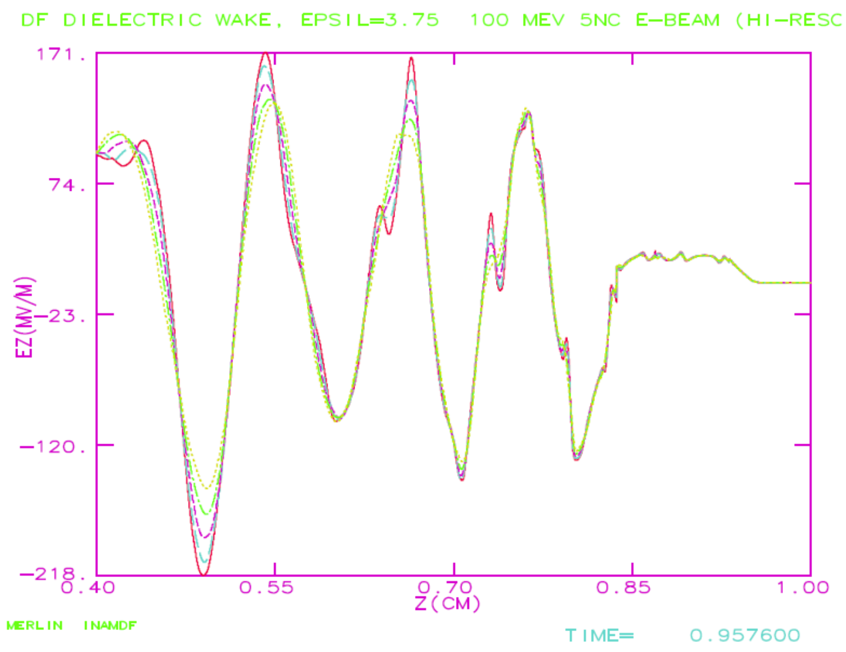
Solid red – normal resolution
Green and others – high resolution cases



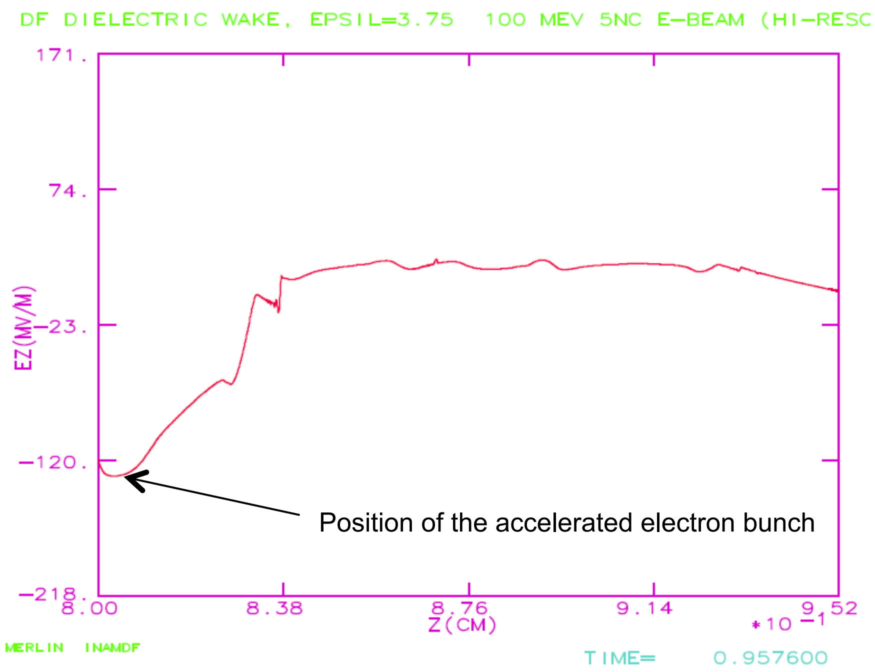
Different color lines are at different radial positions

Note that the lack of resolution can produce inaccurate results of the wakefield (the red curve).

The longitudinal component of the excited wakefield in the dielectric waveguide has only a weak radial dependence



Lineouts of the longitudinal component of the wakefield at several radial positions

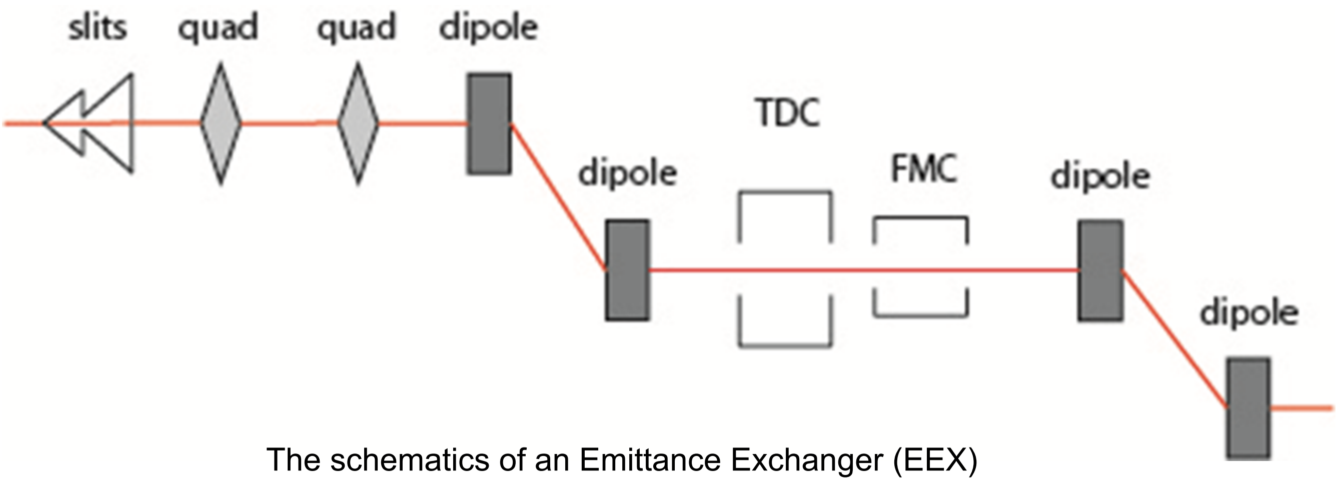


Position of the accelerated electron bunch

Longitudinal component of the wakefield where acceleration of the witness beam occurs

The use of emittance exchanger in the production of the shaped drive-beam in the dielectric wakefield accelerator

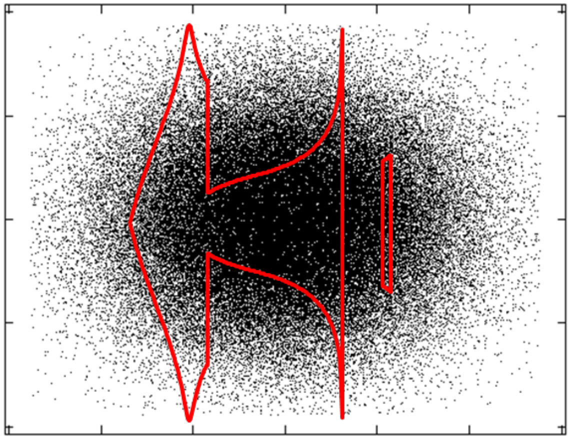
- The proposed beam pulse shaping system consists of a transverse beam shaping mask, several focusing quadrupoles, two doglegs, and a deflecting cavity in between. A fundamental mode cavity is used to compensate for uneven energy gain of the particles in the deflecting cavity.
- We model the EEX with the Elegant matrix code.
- We designed a special transverse mask which accounts for the Gaussian distribution in the initial beam and simultaneously cuts out the double-triangular drive bunch and the trapezoidal witness bunch.

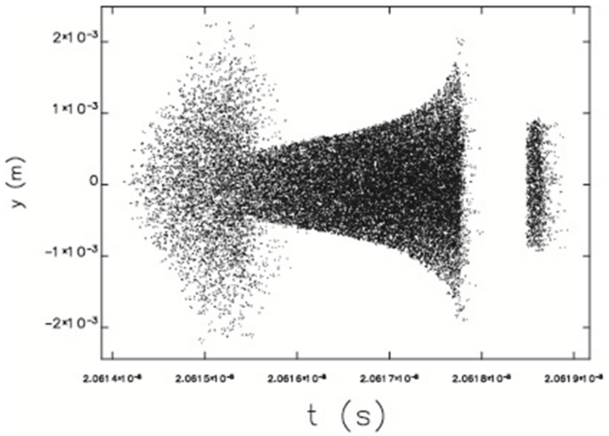


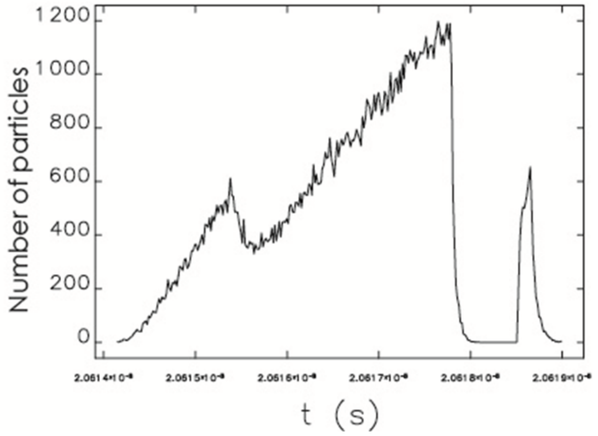
The schematics of an Emittance Exchanger (EEX)

Distribution of electrons after the mask and EEX as computed by Elegant in the second order

The shape of the mask that cuts out the correct bunch shapes out of the Gaussian distribution

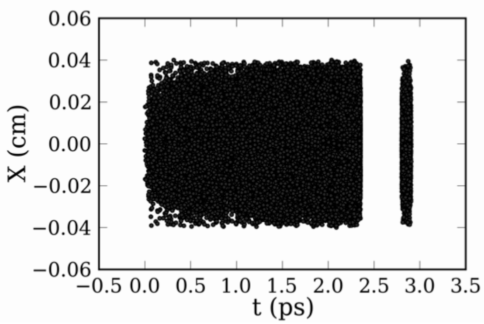


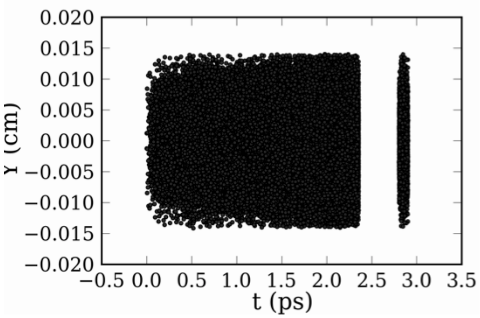


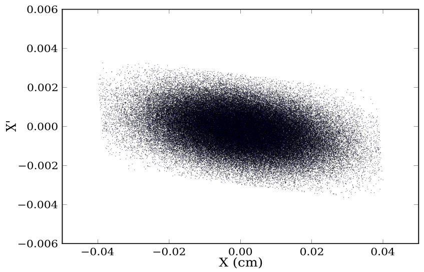


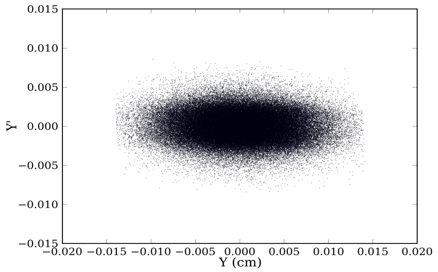
Linking Elegant and 2D PIC simulations with symmetrized particle beam distribution

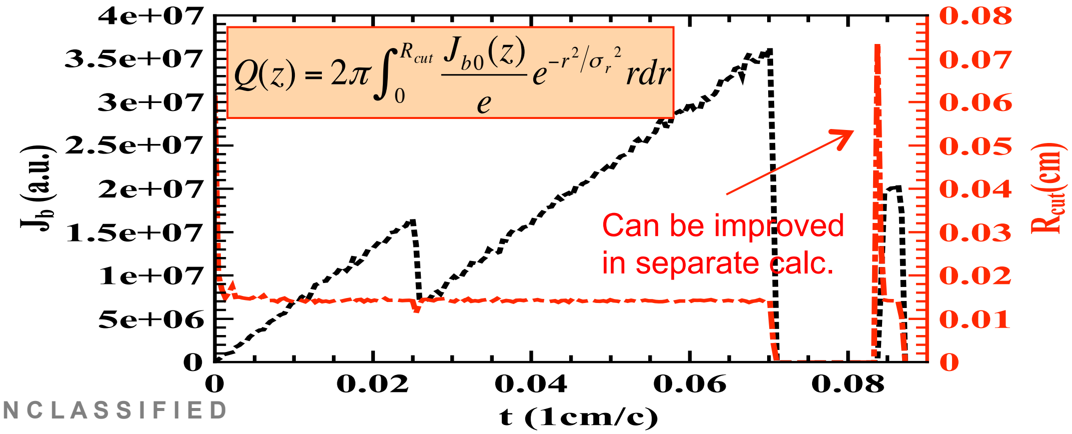
- **Particles tracked through beam mask, EEX and focusing optics in Elegant**
  - *5M particles for drive and trailing beam tracked with linear optics elements. Nonlinear effects will be included later.*
  - *Asymmetric profiles in X and Y at DWA entrance*
- **Particle distribution is sliced in time and symmetrized transversely to be a cut-Gaussian**
  - *Retain beam current profile and current density on axis.*
  - *Radial velocity is Gaussian*
  - *Energy is slice-averaged*
  - *X', Y' correlation effects deferred to 3D simulations.*









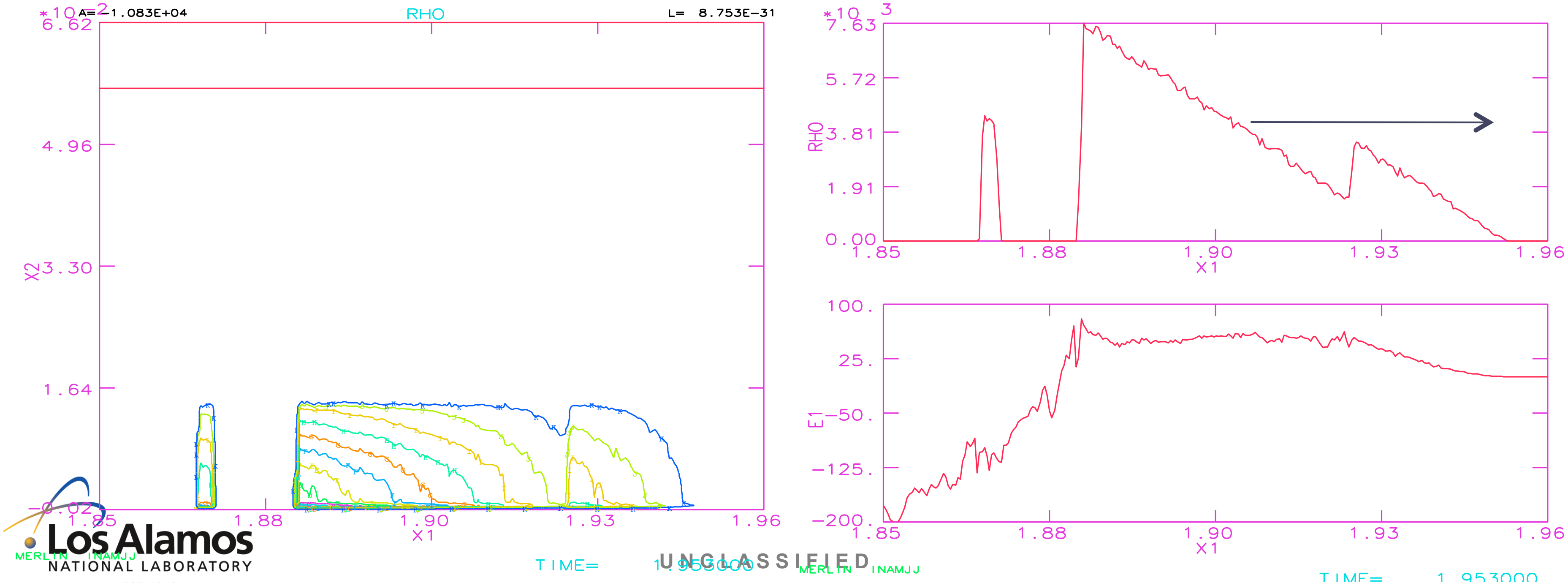


$$Q(z) = 2\pi \int_0^{R_{cut}} \frac{J_{b0}(z)}{e} e^{-r^2/\sigma_r^2} r dr$$

Can be improved in separate calc.

2D PIC simulation result with symmetrized beam and associated beam field

- **The injected beam current density follows that of the Elegant result. The preliminary result shows a wakefield with transformer ratio of about 3-4, which can be further improved by fine tuning the bunch separation**
- **Smoothness of the field may be improved with more beam particles, finer sampling and even higher simulation resolution**



Conclusions

- Particle-in-cell simulations show good agreement with theory in benchmark comparison of the wakefield in its frequency, amplitudes, and transformer ratio.
- It has been shown that adequate resolution in simulations must be ensure for accurate physics results.
- For a given waveguide configuration, wakefield excited by the drive beam is largely insensitive to the beam emittance, radial profiles, and energy.
- LSP 3-D simulations show that small radial off-set of the drive beam axis has only insignificant effects on the wakefield.
- Our simulations has showed that the design of the dielectric wakefield accelerator is shown to have the potential of achieving a high acceleration gradient and a reasonable energy spread.