CALCULATING PATH-DEPENDENT TRAVEL TIME PREDICTION VARIANCE AND COVARIANCE FOR A GLOBAL TOMOGRAPHIC P-VELOCITY MODEL

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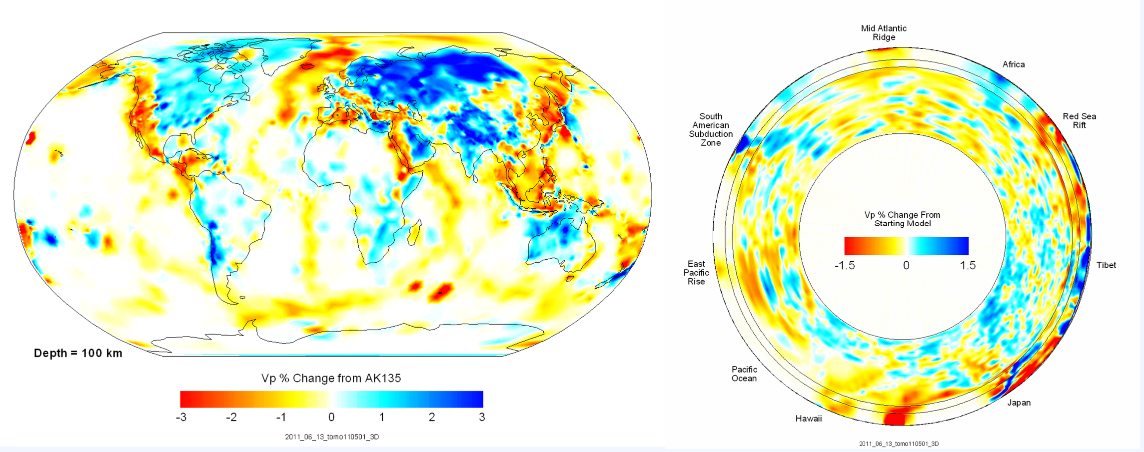
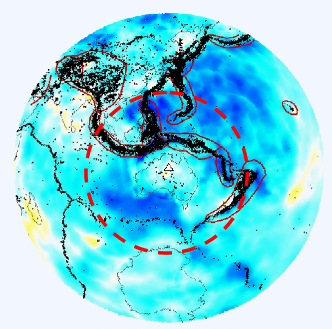
ABSTRACT

Several studies have shown that global 3D models of the compression wave speed in the Earth's mantle can provide superior first P travel time predictions at both regional and teleseismic distances. However, given the variable data quality and uneven data sampling associated with this type of model, it is essential that there be a means to calculate high-quality estimates of the path-dependent variance and covariance associated with the predicted travel times of ray paths through the model. In this paper, we show a methodology for accomplishing this by exploiting the full model covariance matrix.

Typical global 3D models have on the order of 1/2 million nodes, so the challenge in calculating the covariance matrix is formidable: 0.9 TB storage for 1/2 of a symmetric matrix, necessitating an Out-Of-Core (OOC) blocked matrix solution technique. With our approach the tomography matrix (G which includes Tikhonov regularization terms) is multiplied by its transpose (G^TG) and written in a blocked sub-matrix fashion. We employ a distributed parallel solution paradigm that solves for $(G^TG)^{-1}$ by assigning blocks to individual processing nodes for matrix decomposition update and scaling operations. We first find the Cholesky decomposition of G^TG which is subsequently inverted. Next, we employ OOC matrix multiplication methods to calculate the model covariance matrix from $(G^TG)^{-1}$ and an assumed data covariance matrix. Given the model covariance matrix, we solve for the travel-time covariance associated with arbitrary ray-paths by integrating the model covariance along both ray paths. Setting the paths equal yields the variance for that path.

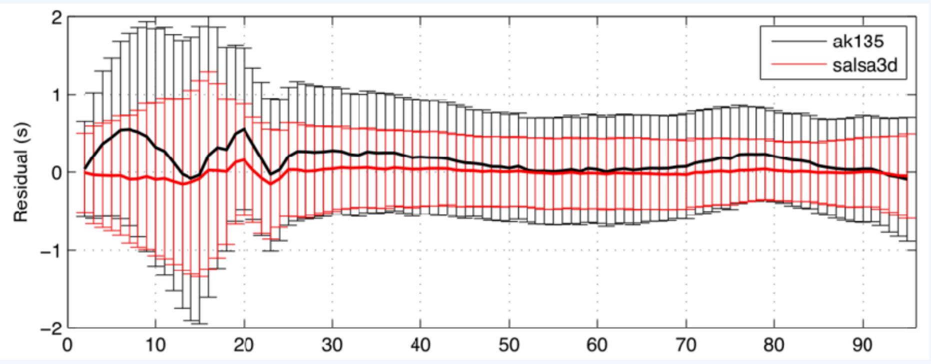
SALSA3D





SALSA3D (top) has strong heterogeneity relative to the AK135 starting model, and hence results in significant travel time differences relative to AK135 predictions (right). Clearly, the uncertainty associated with these predictions is more complex than a simple distance-dependence (below); it must reflect the source region sampling .

Station WRA



PROBLEM DEFINITION

Standard solution given an $m \times n$ set of travel time path length weights, A, associated residuals, Δd, and a set of constraints at each node, αL yields

$$G = \begin{bmatrix} A \\ \alpha L \end{bmatrix} \qquad G\Delta s = \begin{bmatrix} \Delta d + \varepsilon_{\Delta d} \\ 0 + \varepsilon_{\Delta s} \end{bmatrix} \qquad s_j^{k+1} = \Delta s_j^{k+1} + s_j^k$$

Resolution

$$R = \left(G^TG\right)^{-1} A^T A$$

Covariance

$$C_{\Delta s} = \left(G^TG\right)^{-1} G^T \begin{bmatrix} C_{\Delta d} & 0 \\ 0 & C'_{\Delta s} \end{bmatrix} G\left(G^TG\right)^{-1}$$

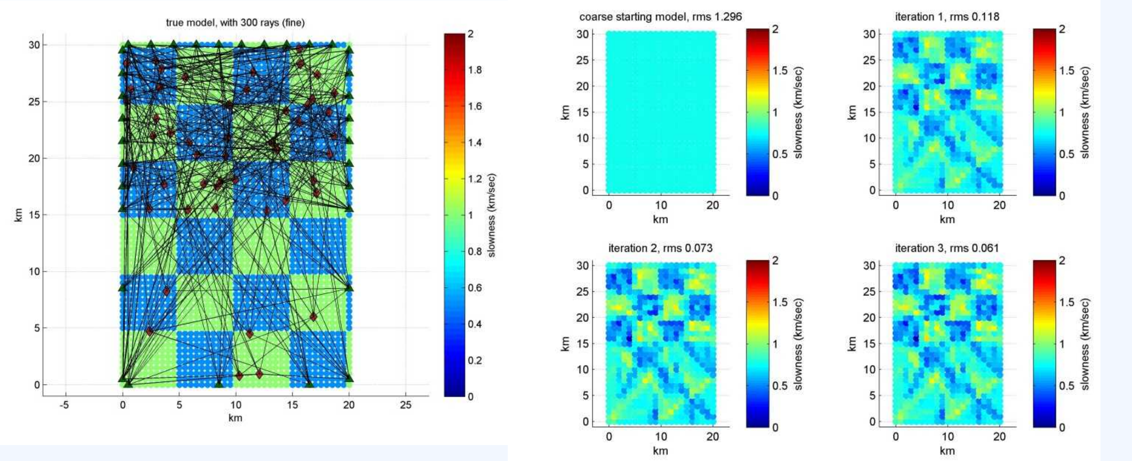
$$C'_{\Delta s} = 0 \;\rightarrow\; C_{\Delta s} = \left(G^TG\right)^{-1} A^T C_{\Delta d}\, A\left(G^TG\right)^{-1}$$

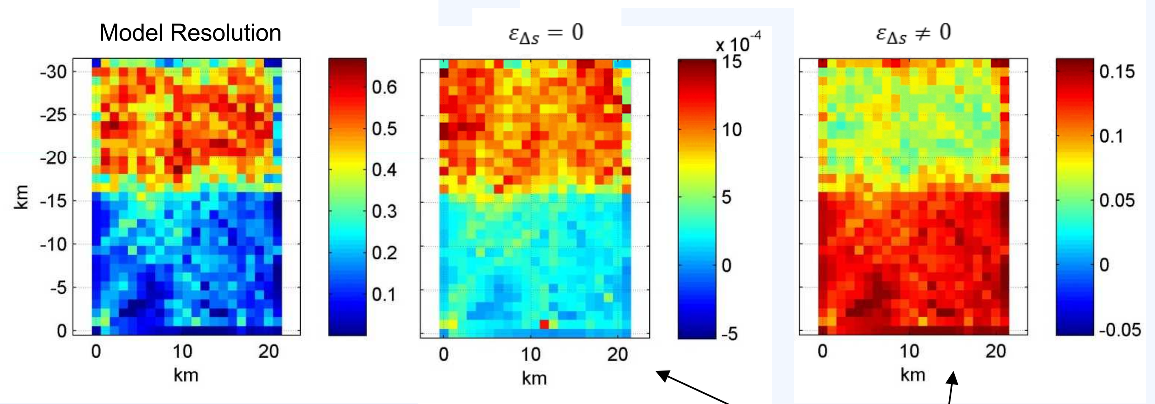
Travel Time **Uncertainty**

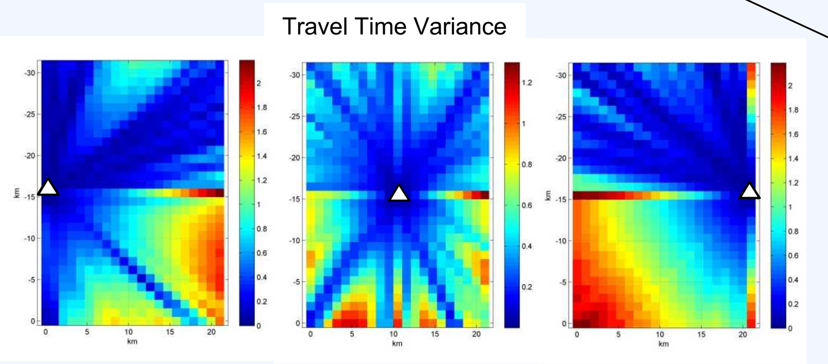
$$tt_i = \sum_{j=0} w_{ij} s_j \pm \sigma_{tt_i} \qquad \sigma_{tt}^2 = \begin{bmatrix} \frac{\partial tt_0}{\partial \Delta s_0} & \cdots & \frac{\partial tt_0}{\partial \Delta s_{n-1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial tt_{R-1}}{\partial \Delta s_0} & \cdots & \frac{\partial tt_{R-1}}{\partial \Delta s_{n-1}} \end{bmatrix} C_{\Delta s} \begin{bmatrix} \frac{\partial tt_0}{\partial \Delta s_0} & \cdots & \frac{\partial tt_{R-1}}{\partial \Delta s_0} \\ \vdots & \ddots & \vdots \\ \frac{\partial tt_0}{\partial \Delta s_{n-1}} & \cdots & \frac{\partial tt_{R-1}}{\partial \Delta s_{n-1}} \end{bmatrix}$$

$$\frac{\partial tt_i}{\partial \Delta s_j} \cong w_{ij} \;\rightarrow\; \sigma_{tt}^2 = WC_{\Delta s}W^T$$

2D Example Solution





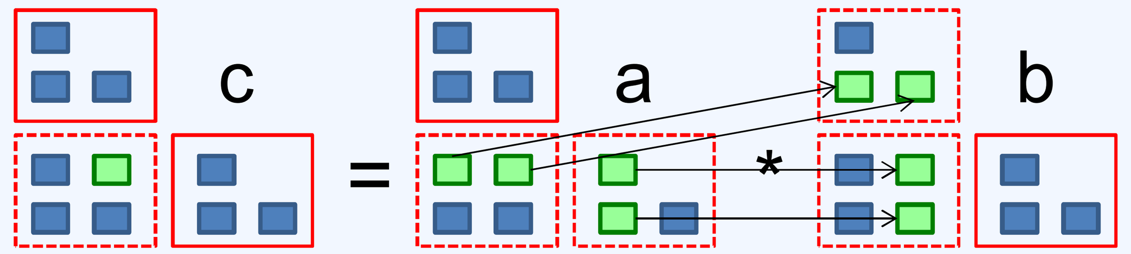


If we assume $\varepsilon_{\Delta s} = 0$, the calculated model variance is the opposite of what we would expect

OUT-OF-CORE Matrix Calculations

We solve for resolution and covariance using direct methods (Cholesky decomposition, forward and backward substitution) to obtain the inverse $(G^TG)^{-1}$ with OOC methods. For example, consider matrix multiplication in an OOC fashion. Assume a 4x4 matrix blocked into 2x2 elements for each block. Each element of c is given by the expression to the left. All elements in a block can be solved for by simply multiplying appropriate blocks as illustrated below.

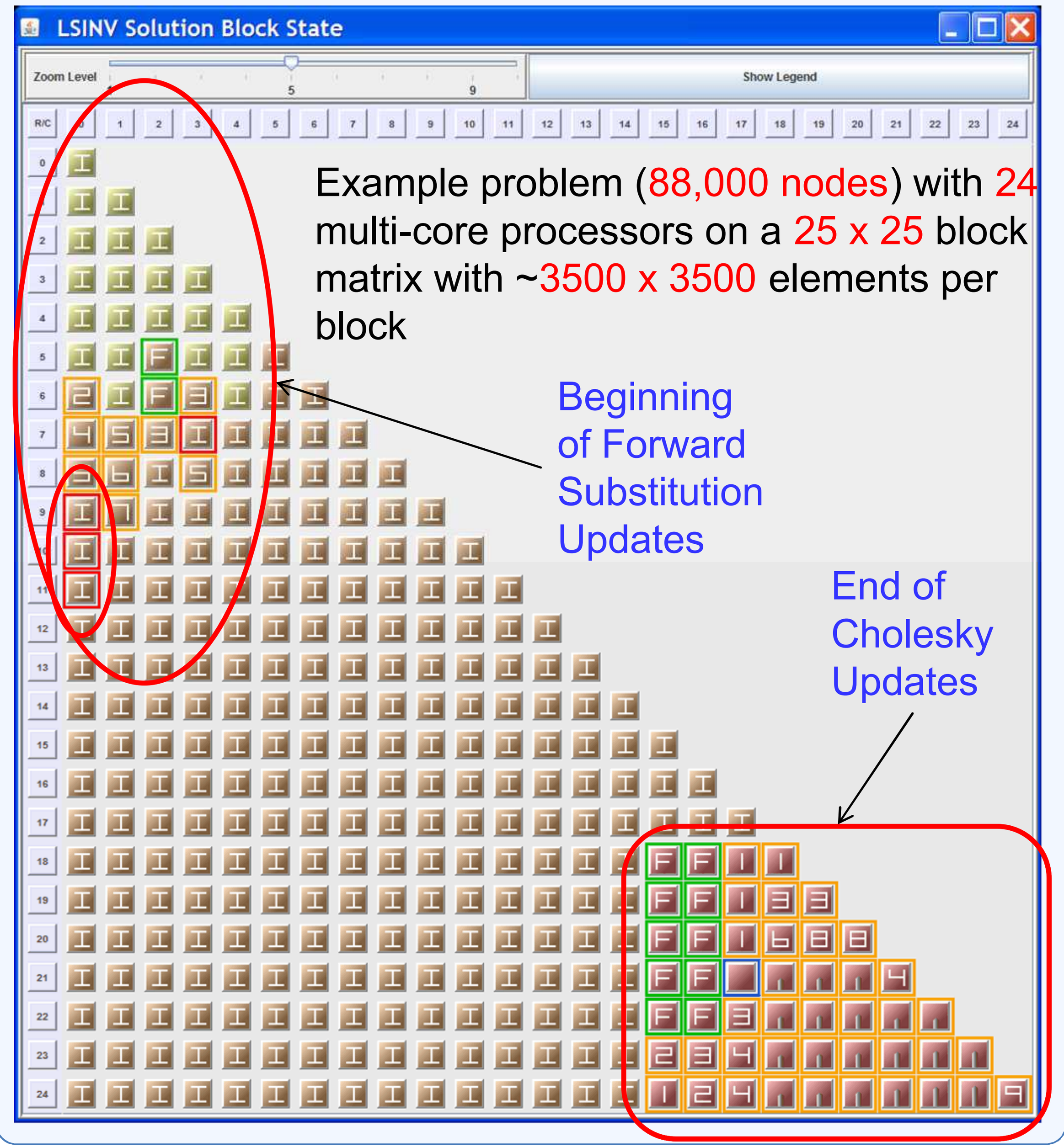
$$C_{ij} = \sum_{k=0}^{k=n-1} a_{ik} b_{kj}$$



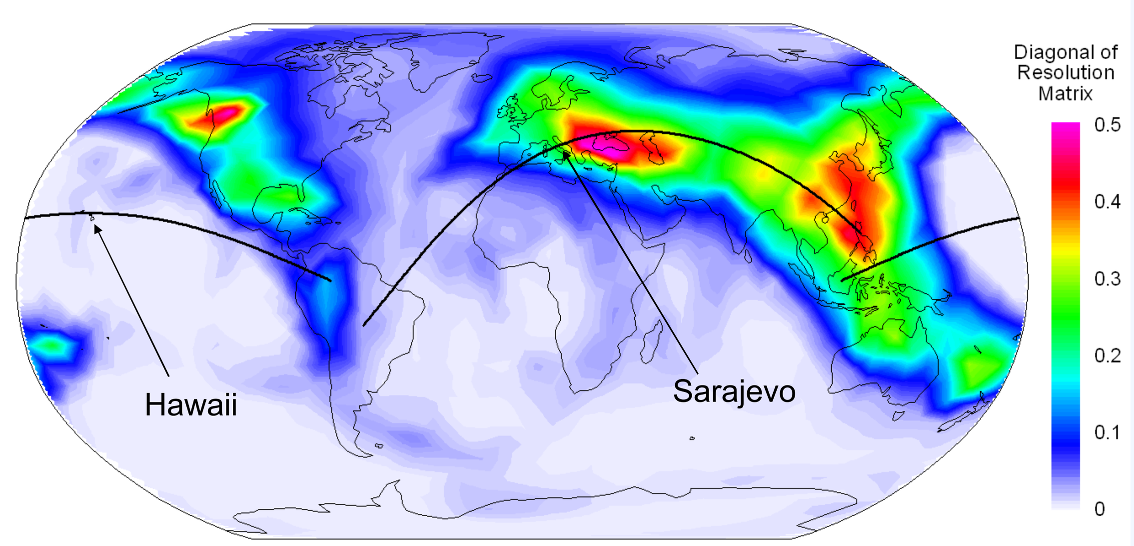
$$\begin{bmatrix} \bar{c}_{00} & 0 \\ \bar{c}_{10} & \bar{c}_{11} \end{bmatrix} = \begin{bmatrix} \bar{a}_{00} & 0 \\ \bar{a}_{10} & \bar{a}_{11} \end{bmatrix} * \begin{bmatrix} \bar{b}_{00} & 0 \\ \bar{b}_{10} & \bar{b}_{11} \end{bmatrix}$$

$$\bar{c}_{10} = \bar{a}_{10} * \bar{b}_{00} + \bar{a}_{11} * \bar{b}_{10}$$

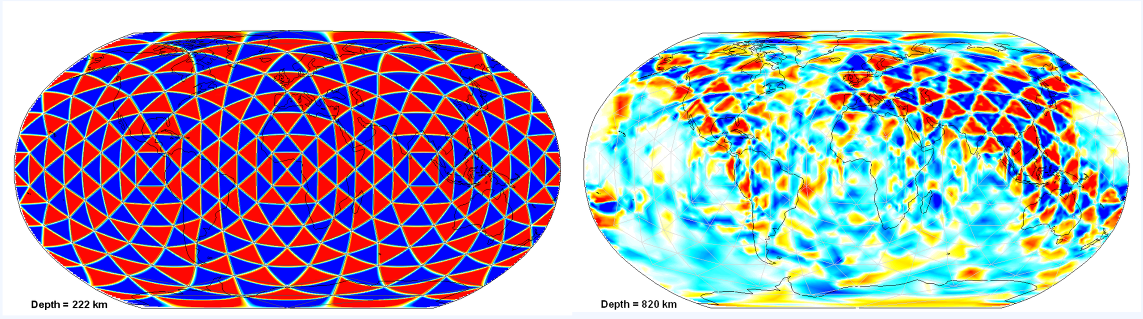
All OOC block processes are divided amongst many cores in a distributed parallel manner. A GUI was developed to enable progress tracking of the entire OOC solution.

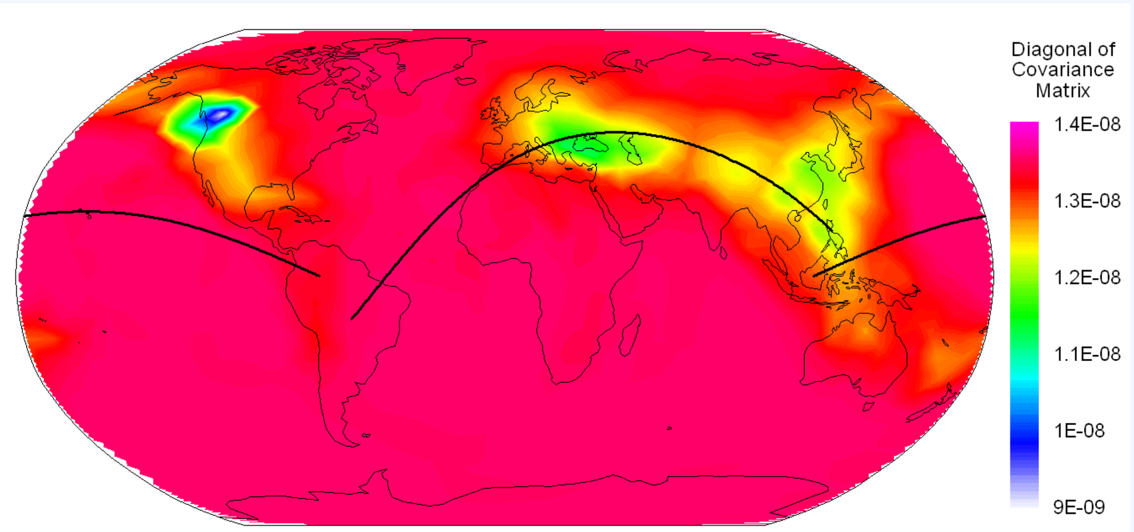


RESOLUTION and COVARIANCE

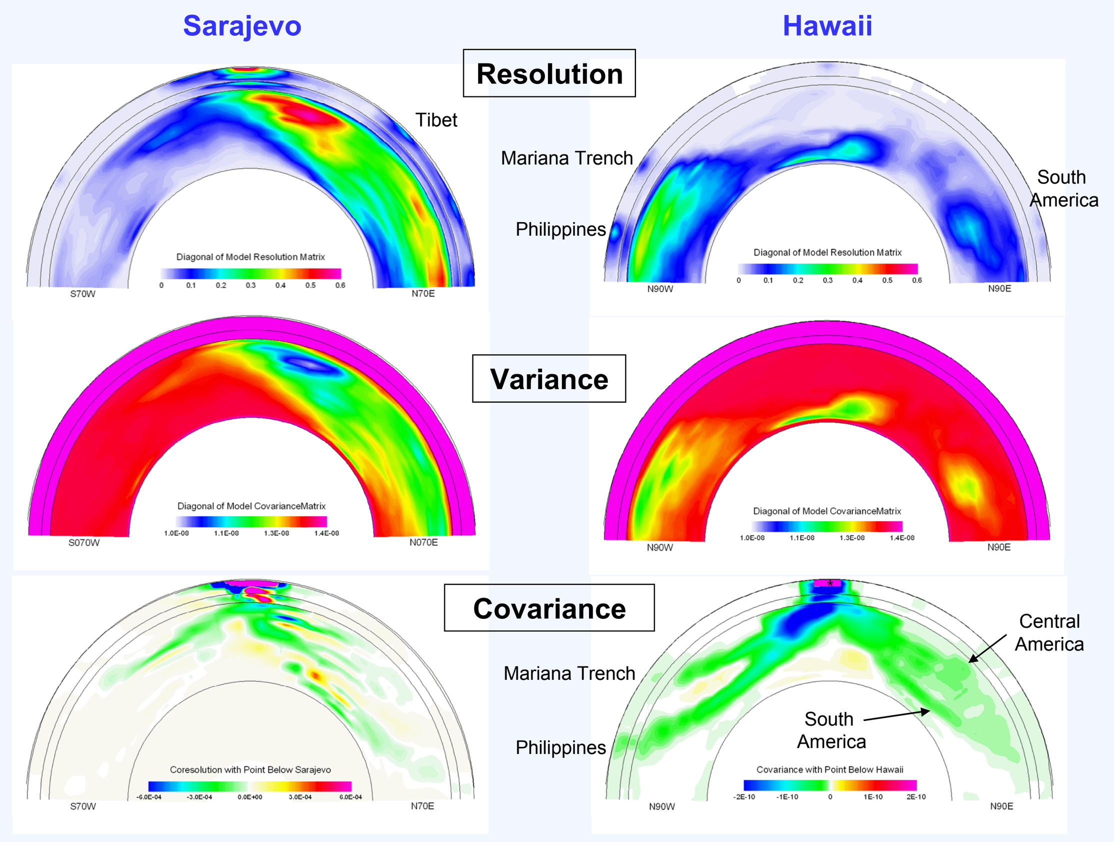


Plotting the diagonal of the model resolution matrix R gives us a much more direct assessment of how well and where our model is resolved compared to the traditional checkerboard approach.





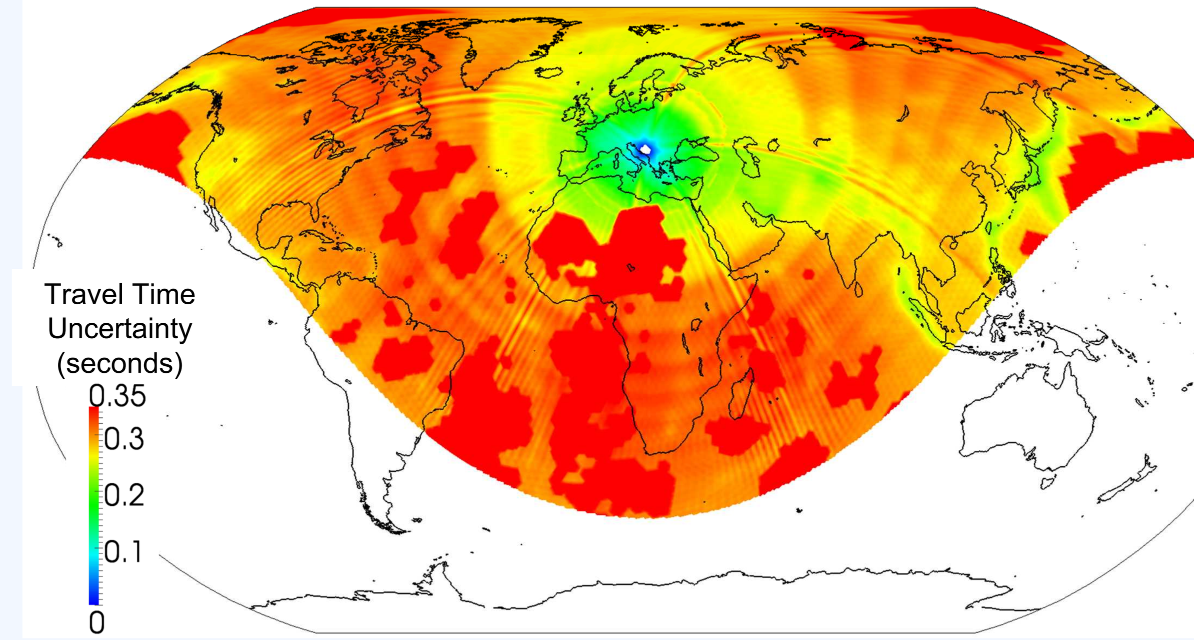
We can also plot the diagonal of the model covariance matrix, i.e. the model variance. By assuming $\varepsilon_{\Delta s} \neq 0$, we get a result that is consistent with the model resolution result. The value chosen for $\varepsilon_{\Delta s}$ determines the scaling of the variances.



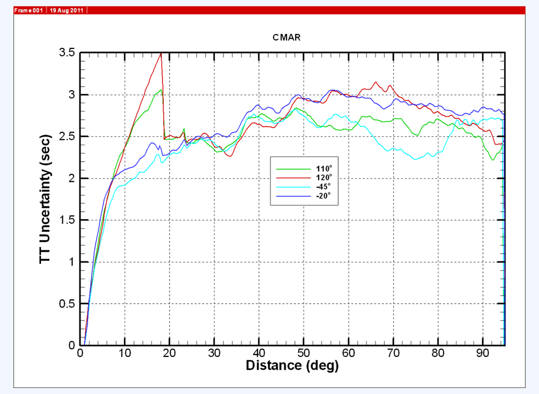
Analyzing the off-diagonal terms of the model covariance matirx, i.e. the covariances, is more difficult. To plot the information on a map, we have to chose one particular model node for which all the other nodes show covariances. Here we choose two nodes: one in the mantle beneath Sarajevo (left) and one in the mantle beneath Hawaii (right).

TRAVEL TIME UNCERTAINTY

With the model covariance matrix available, we can calculate uncertainty (variance) for any path through the model. The calculation is simple in principle but difficult in practice due to the size of the covariance matrix, hence it must be done OOC. The image below shows the results of a 28+ peta flop calculation of travel time uncertainties for ray paths through SALSA3D from a fictitious station at Sarajevo to a surface grid of source positions out to distance of 100 degrees.



Our surface has some distracting features related to paths that pass entirely though uncalibrated parts of our models (the red patches) and to the model gridding (ring-like features and spoke-like features centered on the station point from which all rays emanate). Ignoring these, we can see two important trends:



1. Travel time variance increases with path length (see figure to the right), though the relationship is not linear. This is because the longer paths sample deeper in the Earth, and our a priori slowness variance constraint decreases with depth. This relationship is very much like the standard distance-dependent relationship that is used for event location.
2. Travel time variance decreases substantially in calibrated areas (i.e. areas with sources used in the tomography). This is the effect we were expecting to see. Predictions through a tomographic model should be much more certain along paths that were represented in the tomography.

Conclusions

By using OOC methods to leverage our 400+ node distributed computing resource, we have succeeded in using the full model covariance matrix to calculate the actual path-dependent travel time variances for the SALSA3D model. Our results clearly show the expected relationship to represented source regions. Future work will focus on minimizing nuisance artifacts related to gridding.

References

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