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The Impact of Network Structure on the Perturbation Dynamics of a Multi-agent Economic Model

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Abstract. Complex adaptive systems have become a common tool to study the behavioral dynamics of agents in broad range of disciplines from ecology to economics. Many models concentrate on the dynamics of a CAS approaching a steady state, while a subset also studies the effect of perturbations on a CAS in equilibrium. However, there is a notable absence of work on the effect of the structure of agent interaction pathways on perturbation dynamics. We present an agent-based CAS that models a competitive economic environment. We use this framework to study the perturbation dynamics of simple and complex structures by introducing a series of catastrophic events of various intensities and observing key system metrics. First, we characterize the dynamics of simple structures. Then we combine these simple component structures to make complex networks and analyze the resulting dynamics. This yields new insights into understanding perturbations in systems and the effect of structure on component interactions.

Keywords: Structure, Perturbation, Network, Complex System

1 Introduction

The increase in computing power seen in the past 10 years has made agent based models a viable option for studying complex systems. Exciting new work has begun to show the importance of network structure in a CAS, including how a system can be driven using driver nodes [Liu et al. 2011]. Other research has focused on how structure can affect diffusion through a system [Ghoshal et al. 2011]. Some research has also explored the implications of community structures in a network [Karrer et al.]. However, there has been a notable lack of research that studies the effect of network structure on the dynamics of a system. Social networks are typically well connected, while industrial networks follow a hierarchical structure. A virus will affect these two structures differently due to their arrangements, but the inherent complexity of these systems makes it unfeasible to analytically develop a perturbation theory.

We present a new approach to characterizing a network structure, by compartmentalizing a complex network into simple component structures whose dynamics are simply defined. We then combine and test these compartment networks to gain insight on how perturbations travel across networks. This numerical approach can result in valuable generalizations about complex networks.

1.1 Model Formulation

The Nation State model builds on the framework of the Exchange2 model [Beyeler et al. 2011]. Exchange2 is a complex adaptive system consisting of coupled nonlinear first order differential equations that describe the behavior of autonomous agents. The agents must store, consume, and produce resources to maintain viability and competitiveness in their environment. The agents maintain their stability through a series of discrete interactions with markets, which create exchange pathways between agents.

The Nation State model consists of a set of agents arranged in a hierarchy. The top level agent is a Nation State. Agents in the Nation State are referred to as entities. Entities can be grouped into sectors, each of which is a collection of agents that produce and consume the same resources. Markets mediate transactions between sectors.

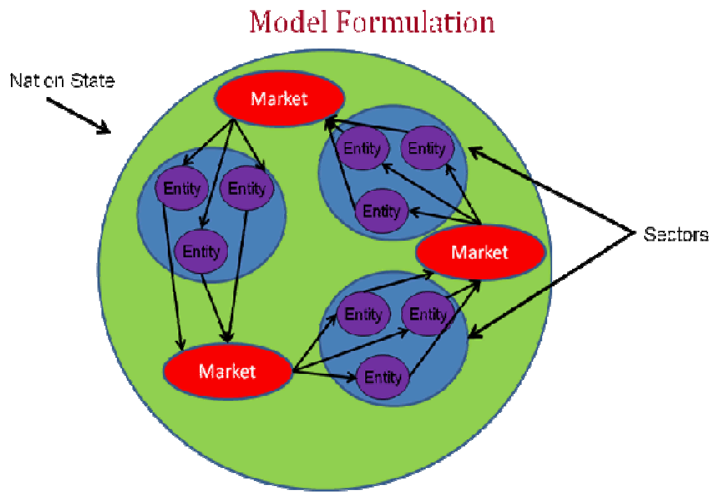

Fig 1. A diagram illustrating the structure of a nation state. Multiple entities make up a sector. Every entity in a sector goes to a market to buy and sell resources. A collection of these sectors and markets make up a nation state.

Entities interact by joining a market and bidding to buy or sell resources. Consumers and producers are matched via a double auction. Entities make decisions about market transactions based on the entity health, resource reserves, and money levels. Health is precisely defined as a scalar function that follows an agent's consumption with respect to a nominal consumption rate. Health abstractly represents a measure of an entity's success in a dynamic and competitive marketplace.

In order to study the dynamics of the model, we introduce shocks and observe the system response. We can simulate catastrophic events by removing a certain percentage of an entity's produced resource in random events that happen with a certain frequency for a set duration. The resource is removed from the entity's production tank, which prevents it from being sold to accrue a profit. This can represent a range of perturbation types. A single perturbation is analogous to a pipe bursting. We can also introduce smaller but more frequent perturbations, such as a 1% loss every time step, which simulates a leak in a pipe. This gives us considerable control over the perturbations we introduce into the model.

2 Structural Dynamics of Simple Networks

We would like to be able to understand the dynamics of an economy. Unfortunately, networks of business relationships are often very complex. The complex feedback patterns make it very difficult to resolve any causation. In order to make sense of the dynamics, we will start by characterizing basic structures. These structures are idealized endpoints along axes of topological features commonly used to describe networks, such as path length and degree of connection. We use six sectors in each structure, and one entity in each sector. A connection from one sector to another means that a resource produced in the first sector is consumed in the second. Then we consider a structure that superimposes several of these component structures and observe the resulting dynamics.

2.1 Fully Connected Networks

A fully connected network is defined as a network where every node is connected in two directions to every other node in the system. This network is symmetrical and robust. Any perturbation will quickly reach every node, but because of the high connectivity, the impact is shared among several nodes and the system can cope with larger shocks.

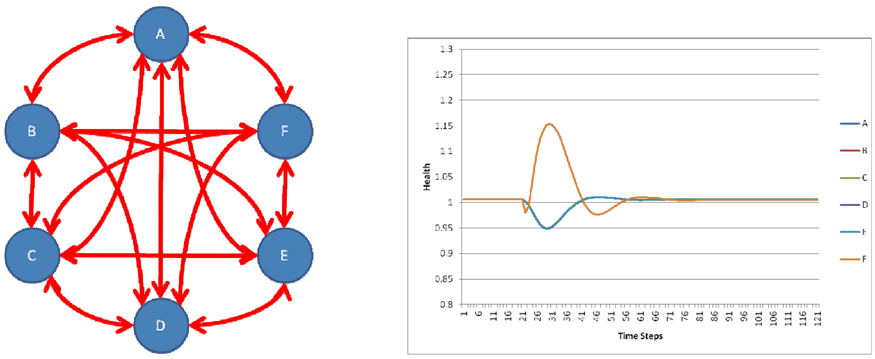

Fig 2. A fully connected network and the perturbation response when node F is shocked.

In Figure 2, we see the perturbation response when node F is perturbed by removing 100% of the produced resource stores it has in its tank. By analyzing the responses at each node, we can characterize the nodal interactions very well.

When node F sees the perturbation, it immediately stops making money, since there is no product to sell. F lowers its consumption to preserve its money reserves, and the decrease in consumption directly lowers the health level. All other sectors begin to have their health fall as well, because F's product is no longer available, and so their consumption falls, leading to a decrease in health.

Once F begins to produce again, it has a product that is very scarce with large demand. This causes the price to of F's product to spike, and F begins to make lots of money, which spurs consumption leading to a large rise in health. Meanwhile, the other sectors are competing for a scarce resource and paying a premium, which causes them to consume less and continue to decrease in health. This behavior continues to an extreme point, where there is an overabundance of F's product available. This causes a trend reversal as the sectors readjust their consumption as prices change. The health of the sectors then oscillates with a certain damping ratio until the system reaches a steady state again.

There are several notable features about this response. First, the oscillatory recovery response is a nontrivial trait of the fully connected network. Also, the perturbed node sees a health gain that is triple in magnitude the other sectors lose. This results in a net loss for the system, since there are five other sectors. Also, the perturbed node sees a large health gain compared to the other sectors, which see a net health loss. This is similar to the competitive exclusion principle shown by Beyeler. The perturbed sector is able to exploit the scarcity because our model has a fixed demand. In other systems, the market may have substitute goods and price would not spike.

2.2 Hub Networks

A hub network has a central node which consumes resources from every other node, and also produces a resource that every other node needs to consume. There are no other connections in this structure. A hub network is asymmetrical, creating a single node which is vital to the whole structure. The periphery nodes can also have a large affect on the structure, since any perturbation quickly travels to the central node and then disperses through the rest of the network. It represents an extreme case of heterogeneity in connections.

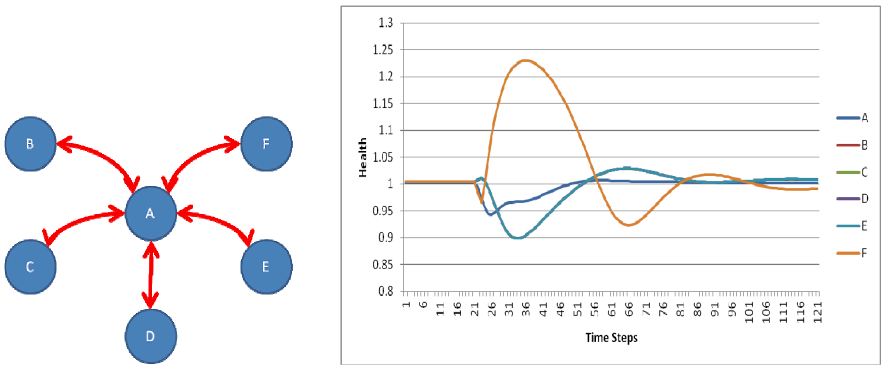

Fig 3. A hub network and a perturbation response for a shock on node F.

The perturbation response for a peripheral node is remarkably similar to the response of a fully connected network. The hub network exhibits two responses between the symmetric nodes (B to E) verse the asymmetric node (A). The magnitude of the perturbation response is also greater than the fully connect network at the perturbed node and at the non-perturbed peripheral nodes.

The behavior becomes more interesting when we perturb the center node in the network.

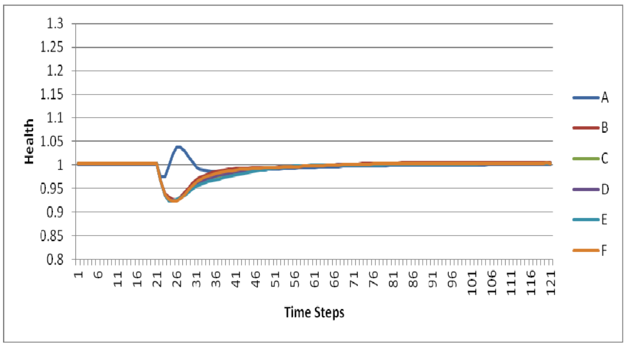

Fig 4. A perturbation response for a hub network with the center node shocked.

When a perturbation is introduced to the center node, the magnitude at the perturbed node and the recovery time are significantly reduced compared to a shock in the periphery nodes. This may seem counterintuitive, since simply changing the node that is perturbed allows the system to recover more quickly than even the fully connected network. However unlike the fully connected network, the hub network is set up so that price spikes are controlled through a subtle feedback. The perturbed node begins to see a health rise due to the price spike. The periphery nodes see a health decline that mirrors the perturbed node's increase. However, the hub arrangement means the center node is critically dependent on the periphery nodes, so as they decrease in health, they decrease production and the perturbed node is unable to consume as much. This limit on consumption effectively caps the perturbation response.

2.3 Circular Networks

A circular network is made up of several nodes that linearly connect in a circular pattern. This network is symmetrical and offers significant buffers to perturbations. Any shock must travel linearly and it will take time to reach every node in the system. This structure has the longest path length, and the minimum connection degree, of any symmetrical network. The drawback to this buffer is that the magnitude of each perturbation is passed through each node, which can tip fragile nodes to death.

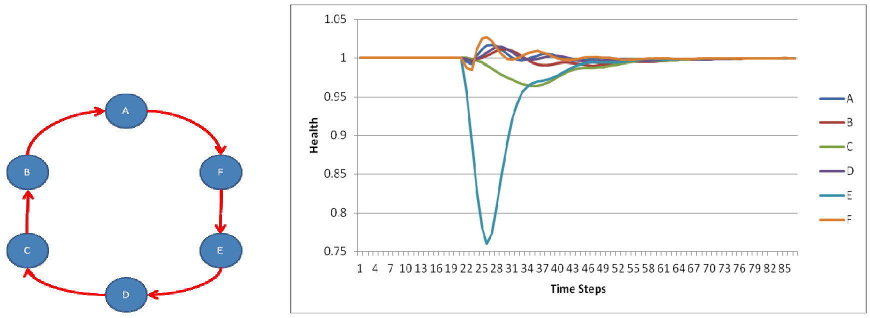

Fig 5. A circular network and a perturbation response for a shock on node F.

A circular network has a very distinct perturbation response. Figure 5 shows the response to perturbing node F by removing its product resource. Unlike the other structures, we do not see a significant health gain in the perturbed node. Instead, we have a very notable loser. The node that consumes the perturbed node's resource sees a large health loss. Nodes upstream generally see a health gain, with a phase lagging behind the perturbed node. This generalization breaks down with node C, which strangely sees a health loss. C is directly opposite from the perturbed node and has a positive and negative response rippling towards it from two directions. The negative ripple dominates and the node sees a net health loss.

3 Combined Networks

In order to understand the dynamics of complex networks, we combine several simple component structures in a new structure which is shown in Fig 6.

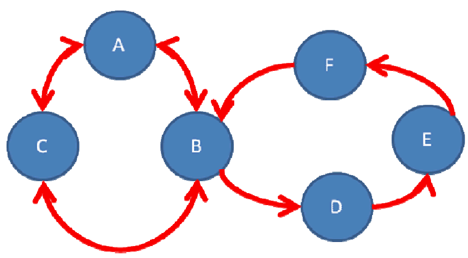

Fig 6. A fully connected network combined with a circular network.

This structure consists of two distinct parts. Nodes A, B, and C make up a fully connected network. Nodes B, D, E and F make up a circular network. Node B is a critical node that connects these two structures.

Perturbing each node will produce a unique perturbation response since the structure is asymmetric. We would expect that nodes A and C would behave similarly to a fully connected structure and nodes D, E, and F would obey circular structure dynamics. This is true, although it is not necessarily to a high degree of accuracy, since the extra structural components add new feedbacks that cause the system dynamics to change. We can say generally that the component circular structure retains many qualitative and quantitative features of its dynamics, while the fully connected network sees a few distinct changes. Therefore, when examining the components separately, we see that the circular structure is more robust to structural change.

We can be more precise by observing that the system dynamics change depending on what node is perturbed. When nodes far upstream are shocked (node D), the entire structure behaves similarly to a circular network. Conversely, when we perturb nodes close to supplying the transition node B, the entire system begins to behave with fully connected network dynamics.

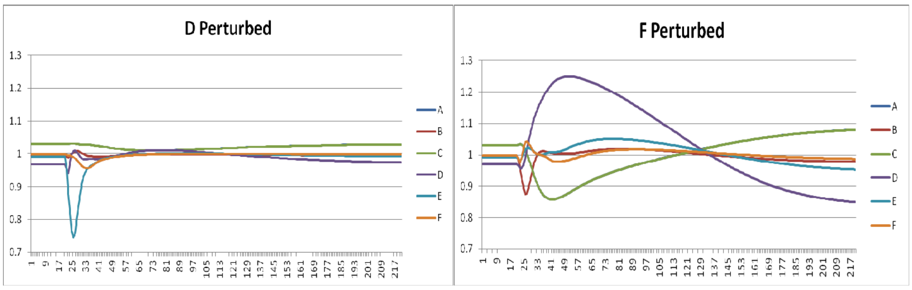

Fig 7. Note that perturbations to node D have a response similar to a characteristic circular structure, while perturbations introduced at node F behave will features of a fully connected network.

This suggests that structural components immediately downstream of a perturbation are most critical to the system's response. This makes sense, since health is tied to consumption. Perturbations disrupt downstream consumption rates, which controls the perturbation response of the system.

To explore the differences between the component structures more rigorously, we can analyze individual node responses. The most interesting node is the transition node (B). It is unclear how the component dynamics of the full and circular structures will interfere at this point. When we perturb the transition node, the response models the circular structure. The shock hits the downstream node (D) the most severely, and the perturbed node does not see drastic net gain in health observed in a fully connected system. Nodes A and C have identical responses which is expected from their symmetry. The perturbation gain that ripples through the circular structure with some phase lag is clearly seen, and the characteristic fully connected network response of a significant health gain is only seen by node D, although it is muted and a secondary response.

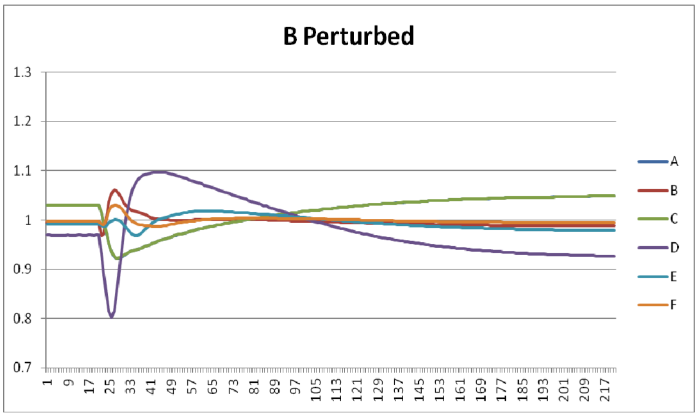

Fig 8. A combined fully connected network and circular network, where the transition node is perturbed.

This combined network provides interesting insight into structure's affect on perturbation responses. We see that downstream structure is a primary factor in determining a response. Also, we see circular structures retain their dynamics more effectively than a fully connected structure. Most importantly, we see that structural features are not simply additive and complex network structure dynamics cannot easily be analytically predicted.

4 Conclusion

Studying structure's affect on system dynamics presents many new challenges. By characterizing simple component structures and observing competing dynamics in combined structures, we can gain of lot of insight to how these networks interact. The initial results presented here already suggest that structural features alone are inadequate and perhaps misleading indicators of the system's response to perturbations. Perturbations to peripheral nodes produce much larger responses than perturbations to central nodes in the hub structure, and the node seeing maximum perturbation in the circular structure was far from the perturbed node.

The dynamics become more complex when complex structures are introduced. While analytical predictions based on the component structures of a network are possible, they are also difficult. New comprehensive and rigorous tools should be developed to characterize network structure's affect on system dynamics.

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