SAND2011-0515C

Hertzian Stress Field and Ring Crack Initiation Analysis including the Effect of Friction

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Motivation

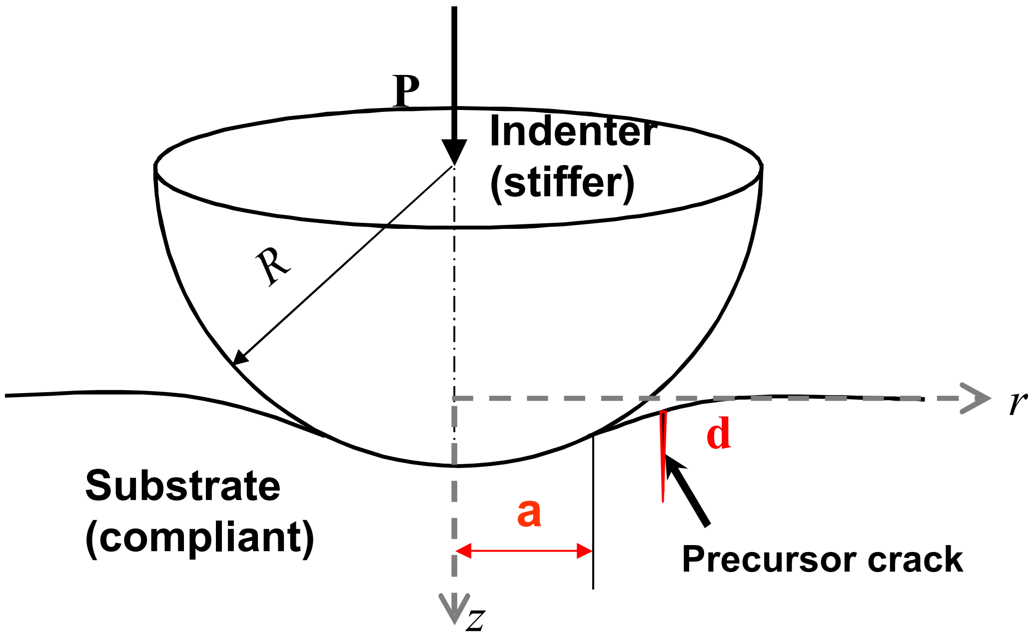
Attempts have been made to use Hertzian indentation to measure toughness and residual stress[1-6]

Conventional strength tests measure the largest flaw, but with this test it is possible to sample the smaller flaws, and maybe even the entire distribution.

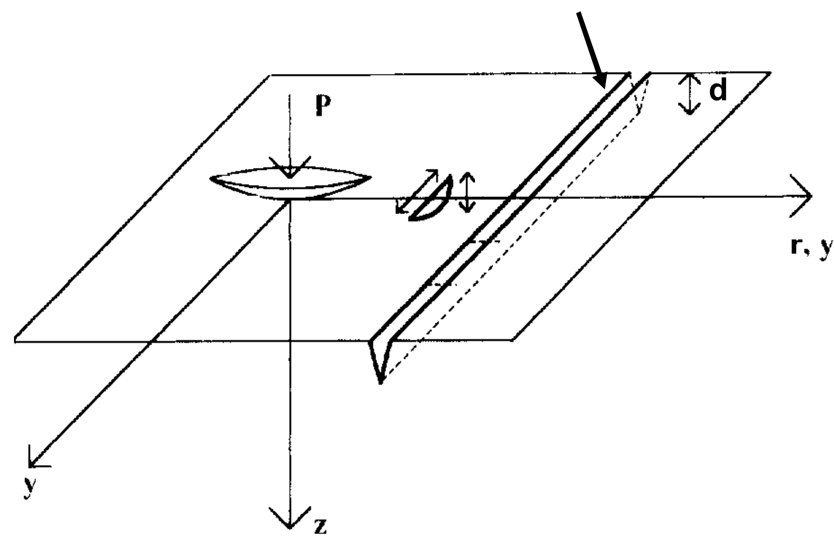
These smaller flaws are critical assessing surface quality, impact/shock behavior, crack initiation under intense local loads, and laser-damage thresholds

1. Warren PD, Hills DA. The influence of elastic mismatch between indenter and substrate on Hertzian fracture. J. Mater. Sci. 1994;29:2860.
2. Warren PD. Determining the Fracture Toughness of Brittle Materials by Hertzian Indentation. J. Eur. Cer. Soc. 1995;15:201.
3. Geandier G, Denis S, Mocellin A. Float glass fracture toughness determination by Hertzian contact: experiments and analysis. J. Non-Crystalline Solids 2003;318:284.
4. Petit F, Sartieaux AC, Gonon M, Cambier F. Fracture toughness and residual stress measurements in tempered glass by Hertzian indentation. Acta Mater. 2007;55:2765.
5. Roberts SG, Lawrence CW, Bisrat Y, Warren PD, Hills DA. Determination of Surface Residual Stresses in Brittle Materials by Hertzian Indentation: Theory and Experiment. J. Am. Ceram. Soc. 1999;82:1809.
6. Paliwal B et al., Assessing the Hertzian Indentation Approach for Measuring Fracture Toughness, J. Amer. Ceram. Soc., to be published 2011

What is the Hertzian Fracture Test ?



Assumed 2d crack geometry



From Lin et al., JECS,1998

What is stress distribution at the precursor crack site?
What is the stress intensity factor, K, there?

How can we use this K to obtain the information about the material?

<u>Toughness, residual stress, surface flaw sizes, stress to pop-in crack</u>

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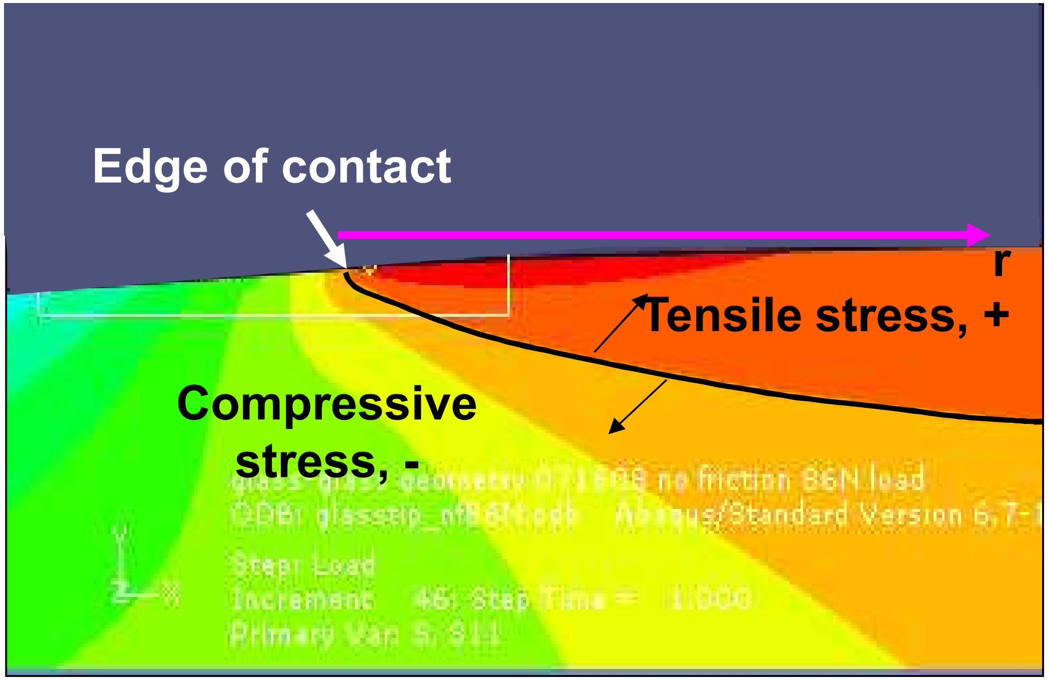
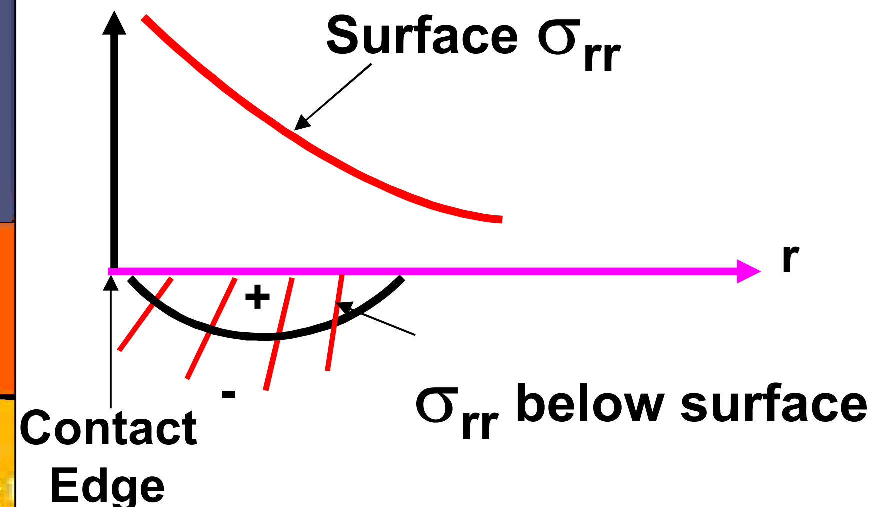
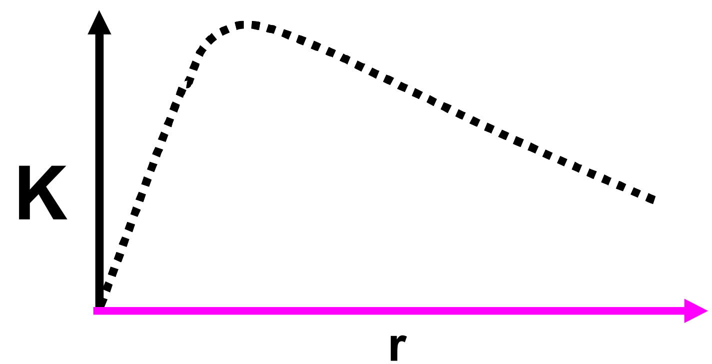
Ideal Hertzian Stress State and the Resultant Stress Intensity Factor







The stress intensity factor has a maximum away from the point of contact for Hertzian loading

Approach and Outline

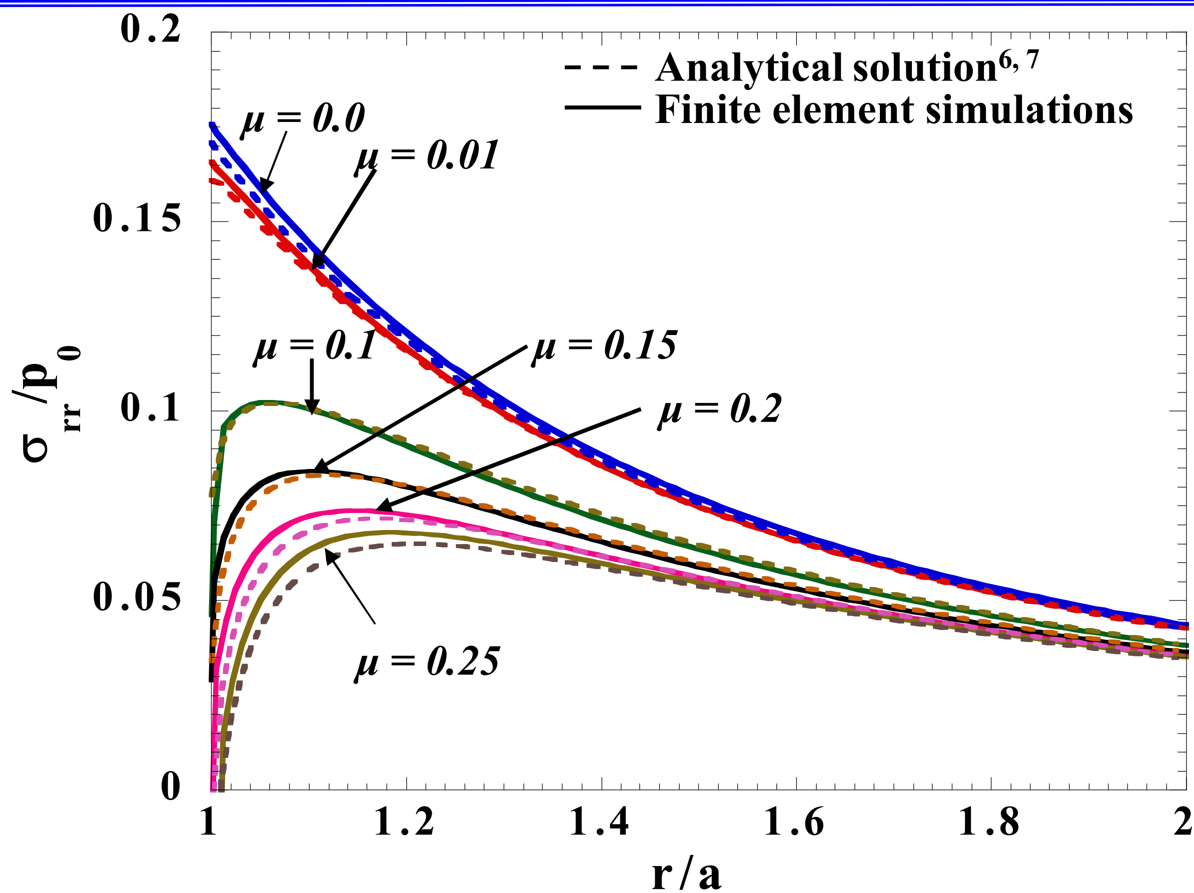
Step 1: Verify that Hertzian approach is good way for measuring toughness for a well-known material

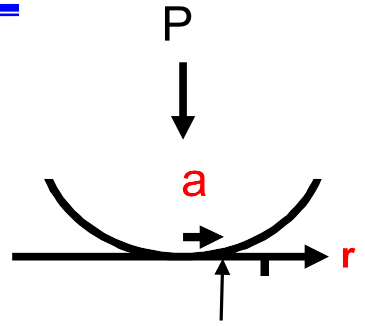
In dissimilar Hertzian contact, friction fundamentally alters the stress distribution

Outline

- Analytical & FE solution for stress and K w & w/o friction
- Experimental determination of P_{min} (per Warren et al.)
- Calculation of toughness using analytical K
- Possible reasons for failure of technique to measure K_{Ic}
- Possible practical uses of technique
- Conclusions

Comparison of Analytical and FE Stress Results for Stress with and w/o friction-Hard indenter, soft substrate





r/a=1 is contact edge

For dissimilar contacts, the surface traction on the softer material of contact creates a localized compressive stress.
This leads to reduction in absolute value of stress, and shifts the maximum in the stress farther from contact edge

6. Spence DA. J. Elasticity 1975
7. Hills DA, Sackfield A. J. Appl. Mech. 1987

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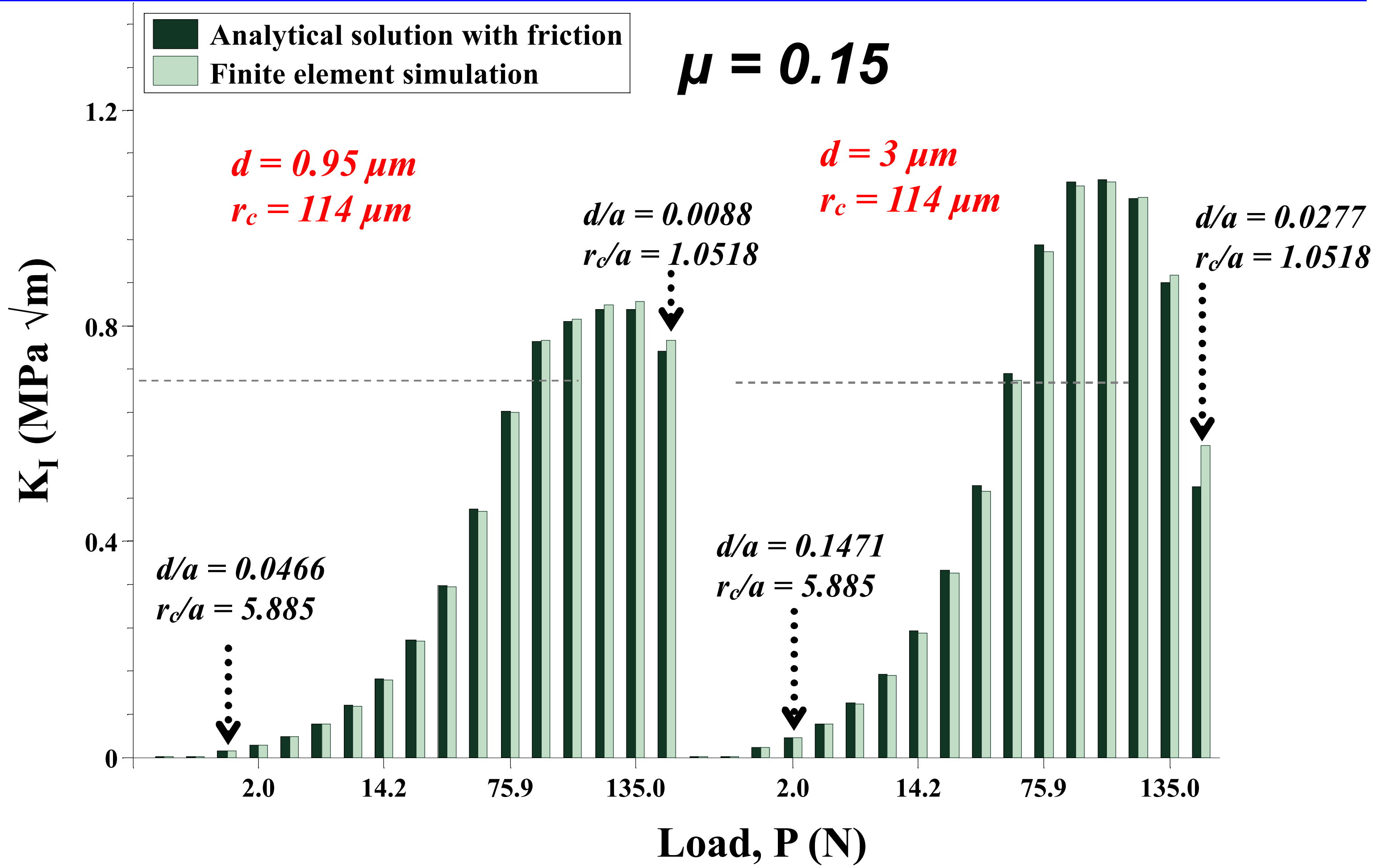
Very good agreement between analytical and FE Stress Intensity values



This agreement allows us to use analytical formulation, with friction, to determine material properties

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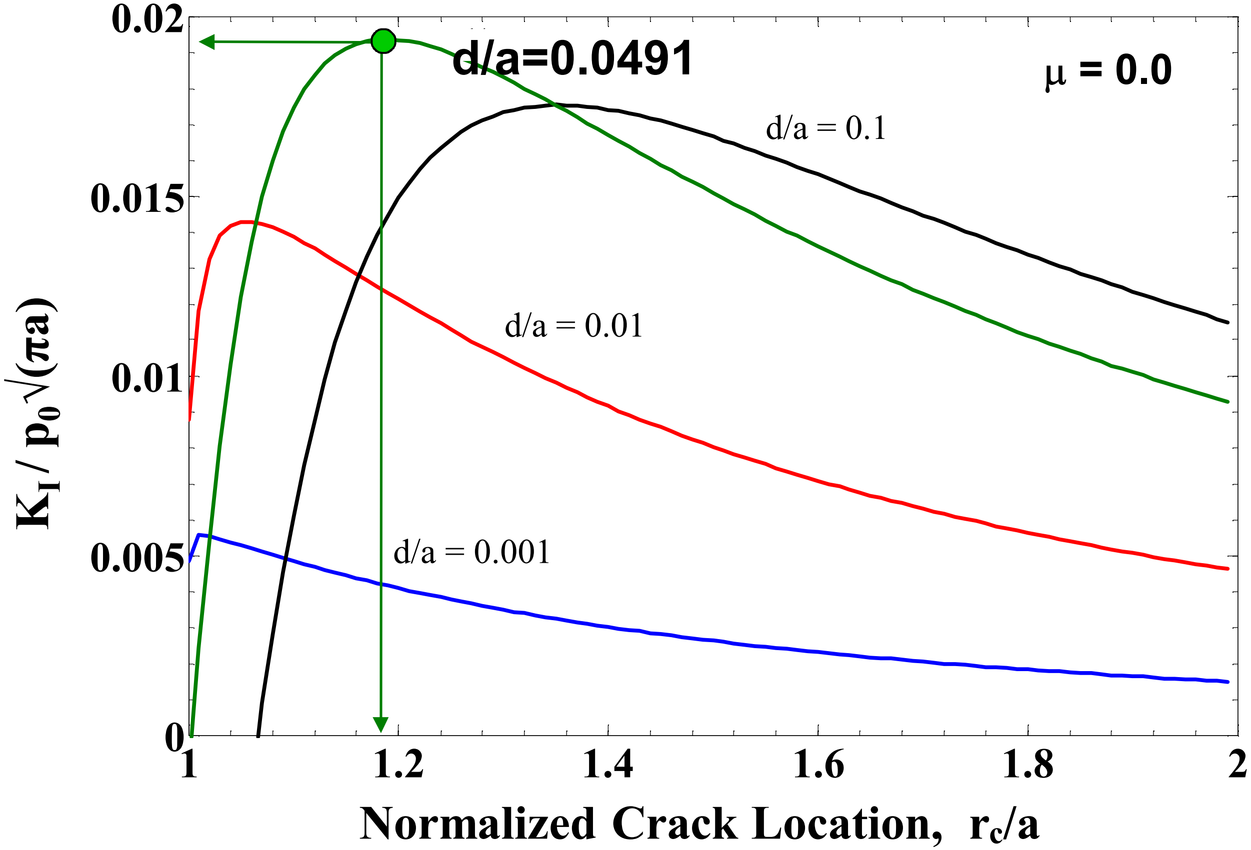
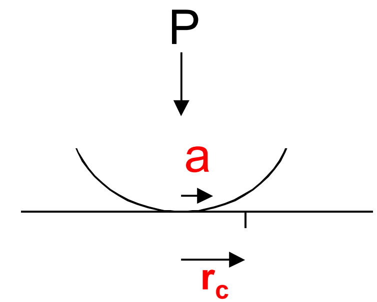
Absolute maximum in K for d/a=0.0491





Relationships

$$a = \left(\frac{3RP}{4E_{eff}}\right)^{1/3}$$

$$P_0 = \frac{3P}{2\pi a^2}$$

- Abs. maximum $K_I = 0.019 P_0 \sqrt{\pi a}$ for $r_c/a = 1.19$
- At fracture, this value=K_{Ic}=material toughness

∴ there must exist a minimum load, P_{min}, below which fracture will not occur

If we find P_{min} experimentally, K_{Ic} can be calculated

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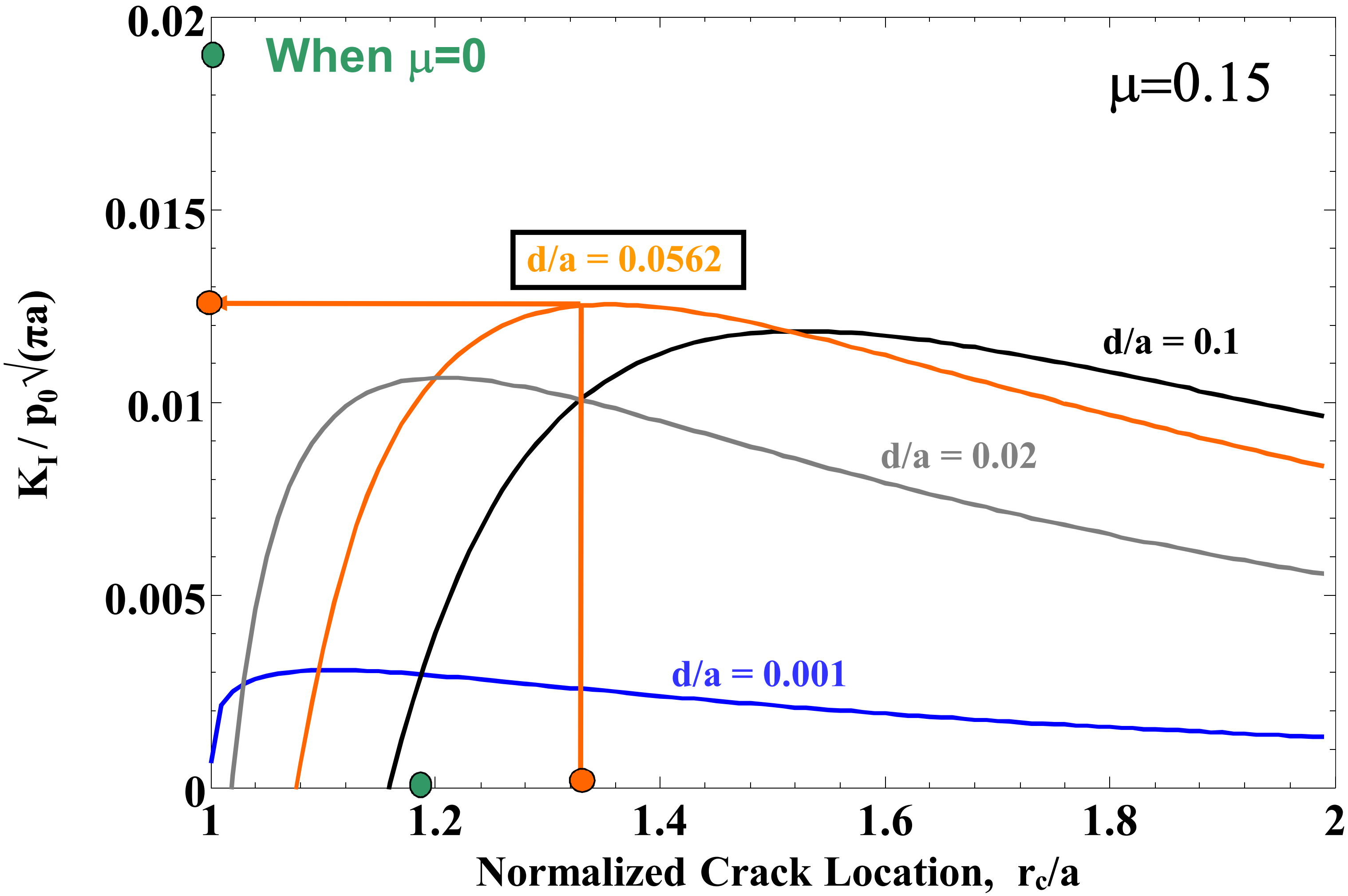
K values with friction , μ=0.15



Maximum value of K_I= 0.0125$p_0\sqrt{\pi a}$ for r_c/a=1.36
Friction lowers the K's and moves the maxima further away from contact

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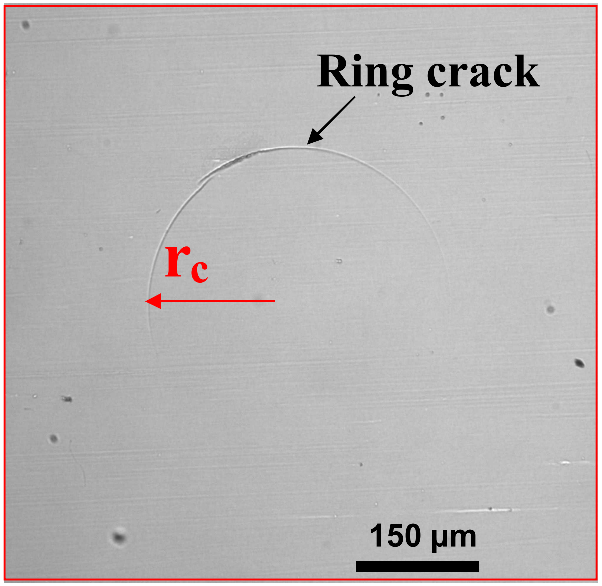
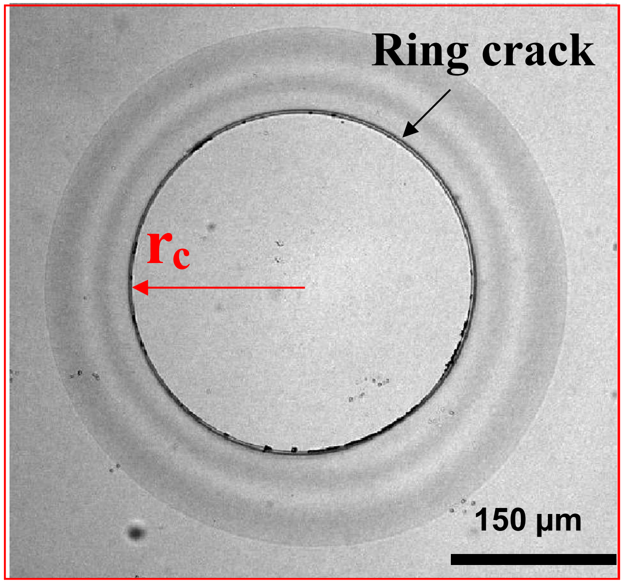
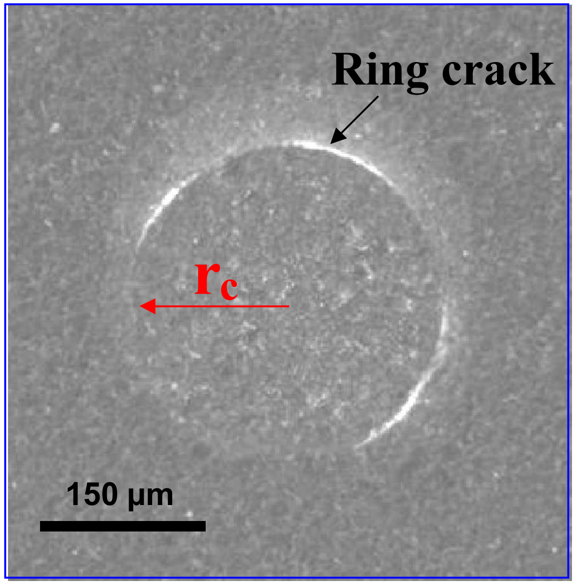
Experimental Details

- >500 experiments with WC sphere on glass (dissimilar elastic contact)
- >100 experiments with glass sphere on glass (similar elastic contact)
- Some experiments were conducted at controlled RH
- Acoustic emission to measure load at cracking
- Friction coefficient between glass and WC was measured (μ=0.15)
- Elastic properties of substrate were measured







As-received Surfaces

Abraded Surface

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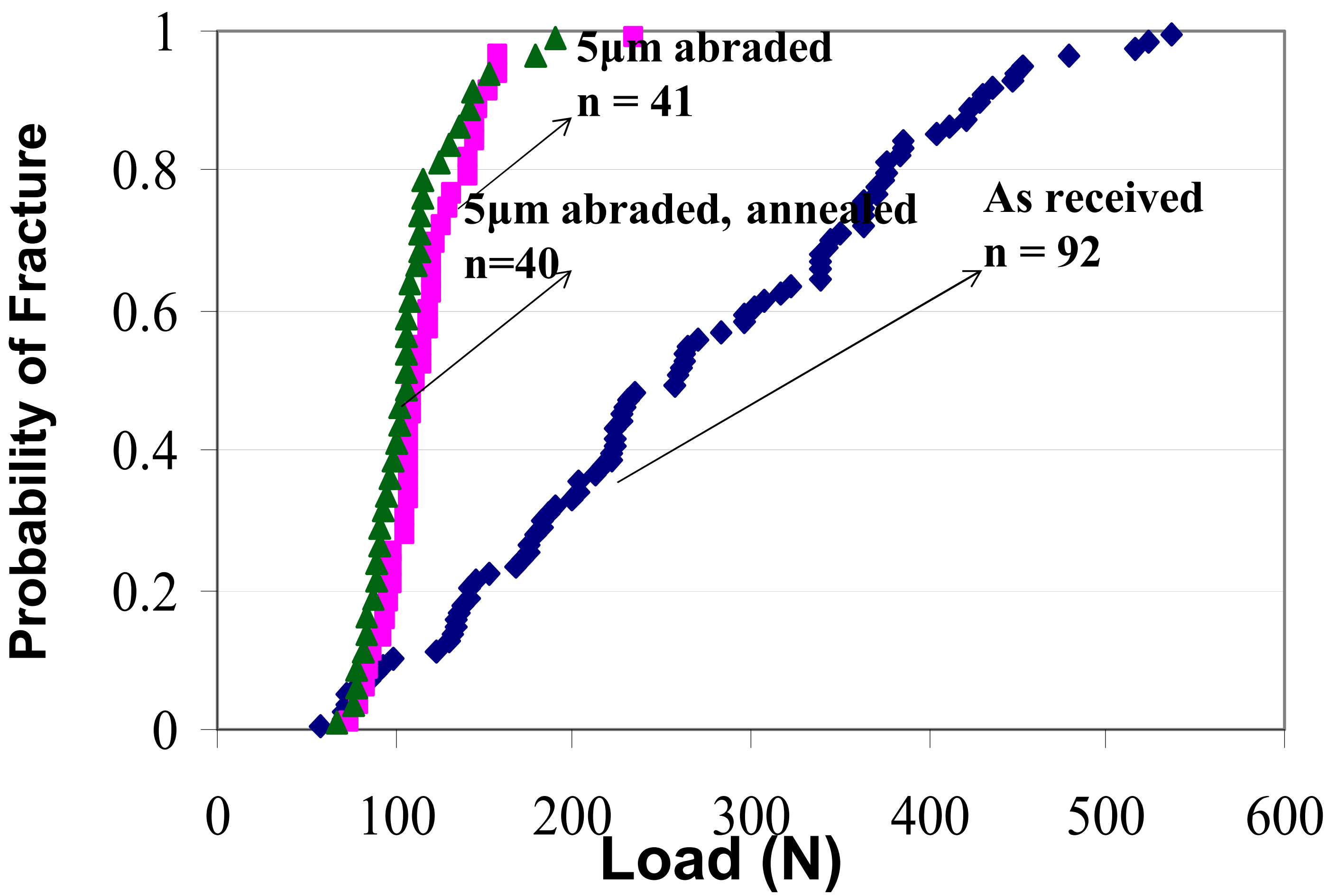
Distribution of Fracture Loads: WC-Glass



Different surface conditions lead to ~ similar P_{min}

P_{min} for each surface condition was used in formulation to determine K_{1c}

Estimates of toughness

	P_{min} (N)	K_{Ic} ($MPam^{1/2}$)		r_c (μm)	K_{Ic} ($MPam^{1/2}$)
		μ=0	μ=0.15		
As-received	57	**1.41**	**0.91**	91.5	**0.798**
5 μm abraded	75	**1.61**	**1.04**	95.2	**0.775**
Abraded-annealed	68	**1.54**	**0.99**	105.1	**0.95**

As proposed by Warren, the technique does not require r_c measurement
In one case, the toughness is still significantly overestimated (>35%), despite measurements of location and additional assumptions

Reasons for Discrepancies and Errors

1. Unknown crack geometry- 2d vs. 3d could lead to factor of two difference in K
2. High sensitivity to elastic properties
3. High sensitivity to friction coefficient
4. How to measure the friction coefficient to be used in analysis
5. Extreme sensitivity to humidity of cracking loads

It is, therefore, difficult to recommend this technique as a method to obtain toughness of an unknown material

Mathematical errors in literature in C_f values (see Paliwal et al., JACERS, 2011)

$$C_f = \frac{3}{\pi \left(K_I^{\max} / p_0 \sqrt{\pi a} \right)^2}$$

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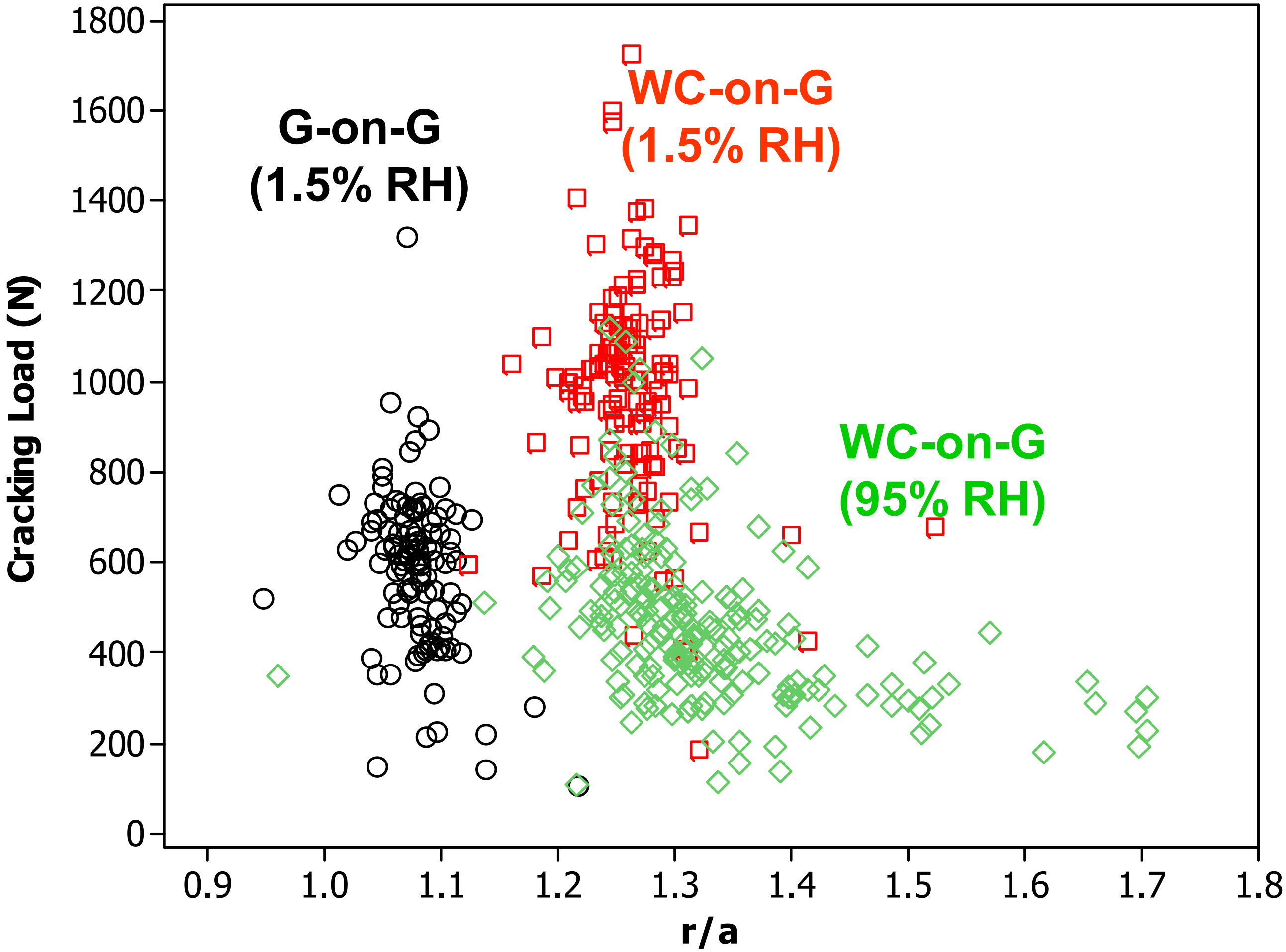
Distribution of Cracking Locations

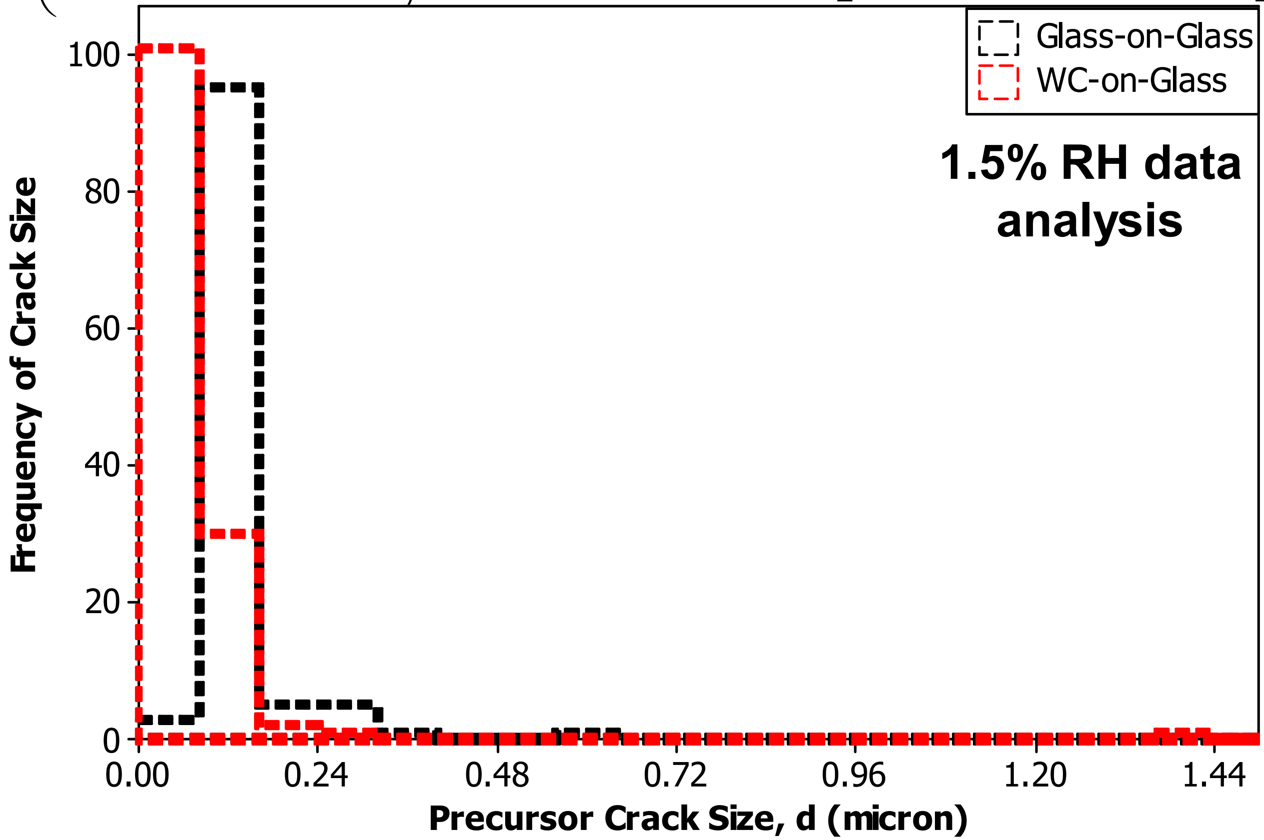


- Cracking locations are shifted away from r/a=1 for WC-on-Glass.
- Consistent with the value of peak stress being shifted away from contact edge for dissimilar contact (friction effect)
- Effect of humidity is to shift these further, possibly due to sub-critical crack growth during the test

Calculation of Crack Size Distribution

$$\sigma_{rr} = \sigma_{surf}\left(1-\alpha\left(\frac{z}{a}\right)\frac{3\left(r/a\right)^2}{\left(1-2\nu\right)}\right)$$

$$\frac{K_1}{\sqrt{\pi a}} = \sigma_{surf}\left[1.12-\frac{2\alpha}{\pi}\left(\frac{c}{a}\right)\frac{3\left(r/a\right)^2}{\left(1-2\nu\right)}\right]\left(\frac{c}{a}\right)^{1/2}$$



These crack sizes are significantly smaller than those sampled in routine strength tests (5-20 micron)

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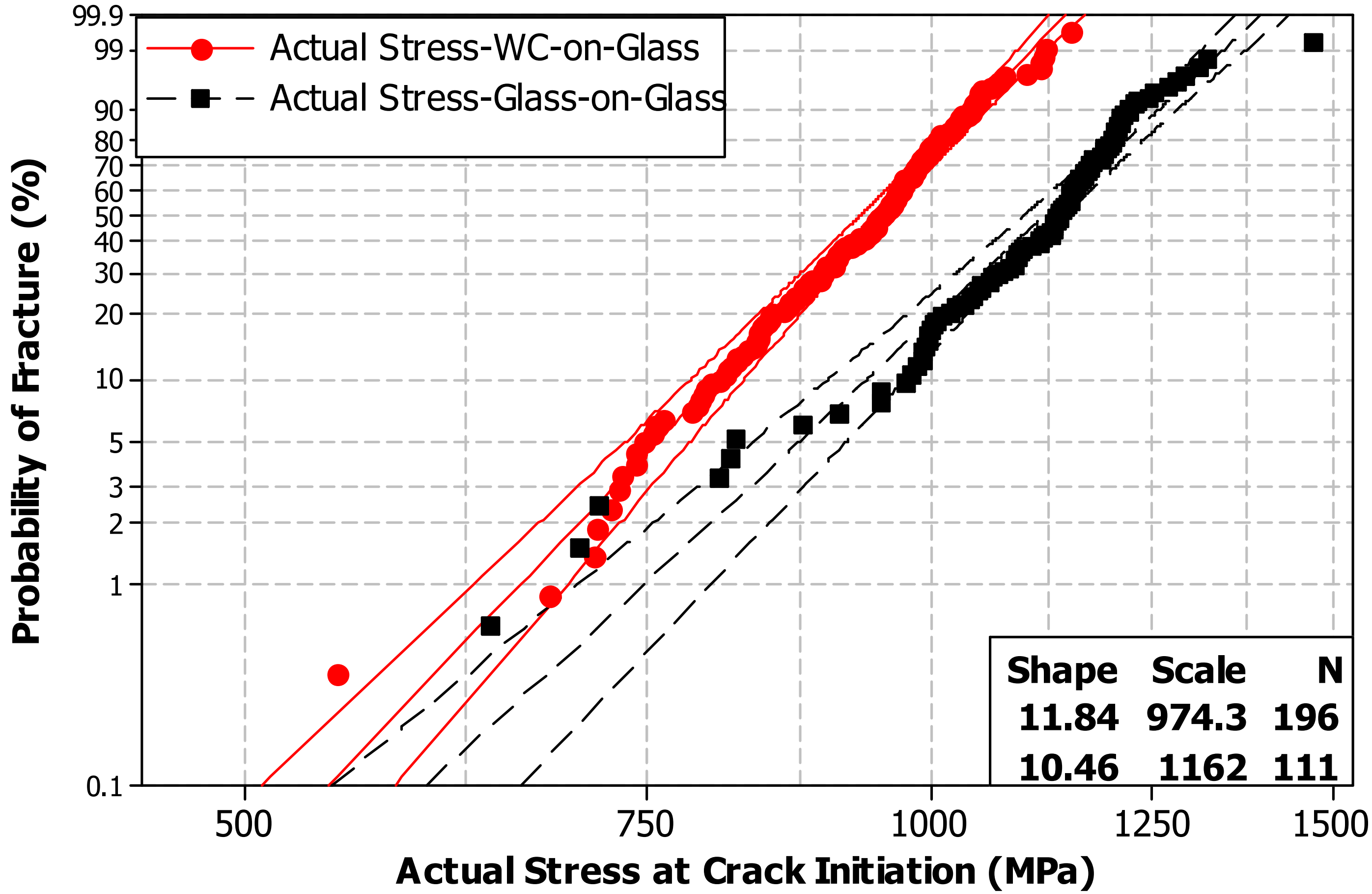
Surface Stress Distributions at Crack Initiations



1. Shape factors (m) are higher than obtained in routine strength tests (4-6) This implies that this flaw population is different from that sampled in other strength tests
2. Stresses needed for crack initiation range from ~500 MPa to ~1500 MPa

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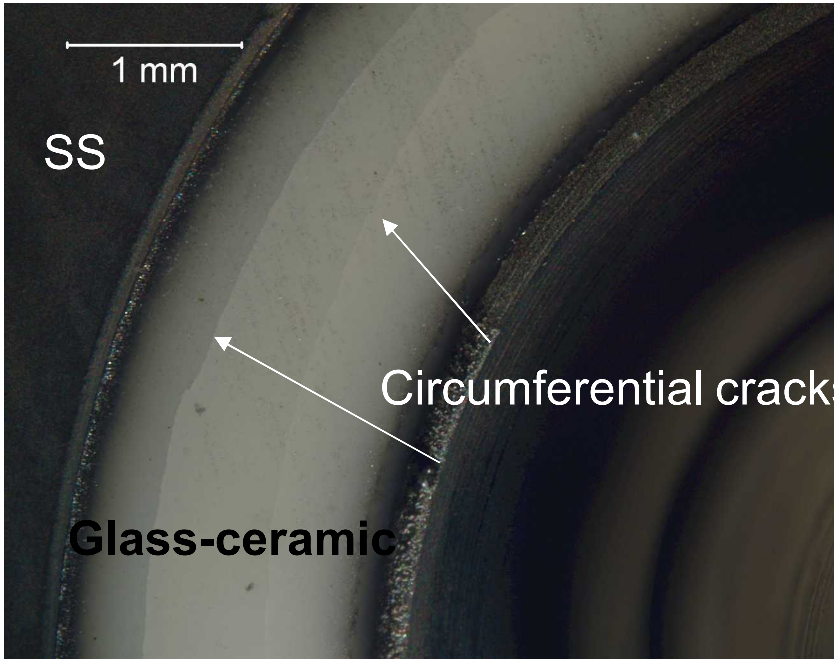
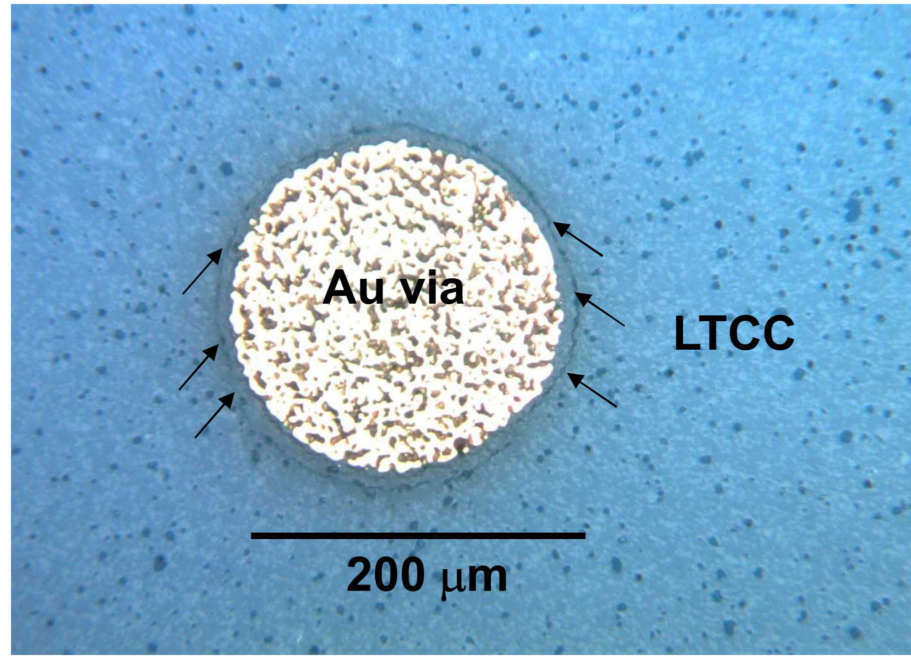
Technique may provide estimate of near surface stresses in components

Glass-to-metal seal



Electronic Substrate



Hertzian cracking surface stress on the ceramic material above may provide estimates of stresses that occur in actual components where ring cracks are seen

Estimated stress in via vicinity
~270 MPa (plane stress, elastic)
~600 MPa (plane strain, elastic)
~ 100 MPa with full plastic deformation of Au

Summary and Conclusions

Summary

- Friction in dissimilar Hertzian contact changes both the magnitude and distribution of the stresses and stress intensity factors (K)
- Accurate analytical framework for calculating K, and values of "correction factors" for K_{Ic} calculations w & w/o friction are provided
- With additional measurements and assumptions, it is possible to obtain numbers close to the true K_{Ic} value in some cases

Conclusions

- Hertzian technique is not recommended for use to estimate toughness
- Possible to estimate flaw size distribution where estimates that might be off by ~ factor of 2 would still be useful
- Useful to estimate stresses to pop-in cracks in components

Is the use of a different "m" appropriate for cracking at features with high local stresses?

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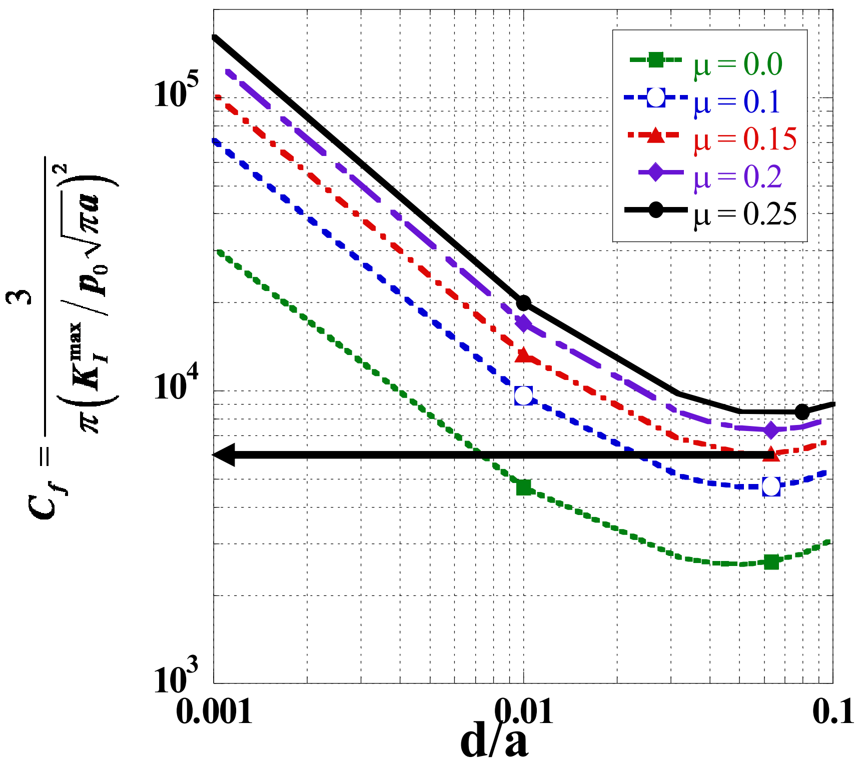
Reasons for Discrepancies and Errors

1. Crack geometry- 2d vs. 3d
2. Extreme sensitivity to elastic properties
3. Extreme sensitivity to friction coefficient
4. Extreme sensitivity to humidity
5. Mathematical errors in literature in C_f values



μ = 0.0	*Present work*		*Warren et al. (Ref. 2)*	
ν	C_f	$(d/a)_{min}$	C_f	$(d/a)_{min}$
0.1	685	0.0755	789	0.0679
0.12	797	0.072	917	0.0648
0.14	934	0.068	1074	0.0616
0.16	1105	0.0642	1270	0.0584
0.18	1321	0.0597	1517	0.0553
0.2	1598	0.0575	1883	0.0521
0.22	1957	0.054	2247	0.0488
0.24	2434	0.0501	2790	0.0456
0.244	2547	0.0491	-	-
0.26	3082	0.0449	3530	0.0423
0.28	3981	0.0425	4560	0.0391
0.3	5272	0.0388	6037	0.0357

Minimum value of this curve at any friction value used to calculate K_{Ic}

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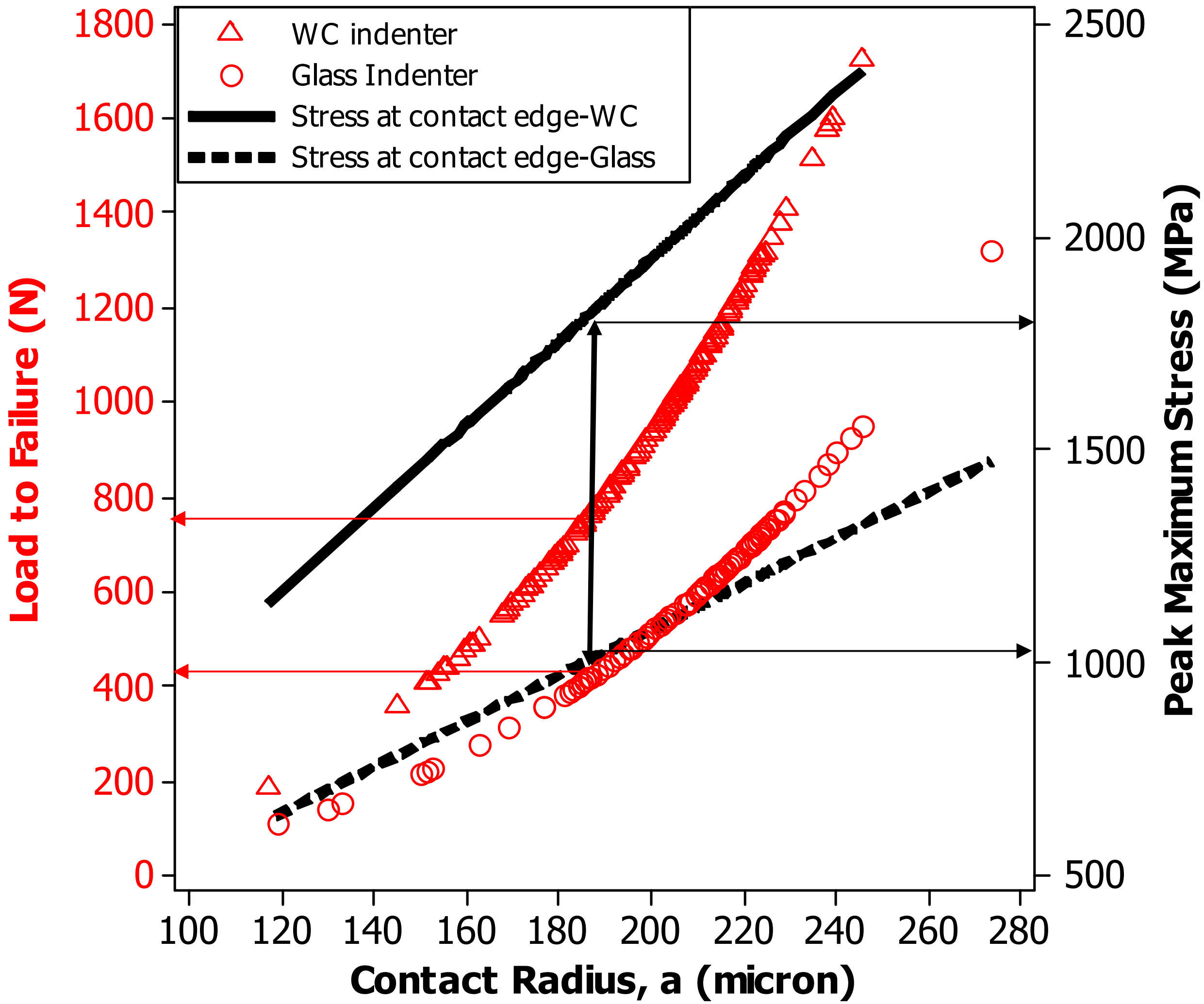
Experimental Verification of the Importance Of Friction for Dissimilar Contact



Therefore, stress in WC-Glass contact are not described by the Hertzian distribution, and are lowered due to friction

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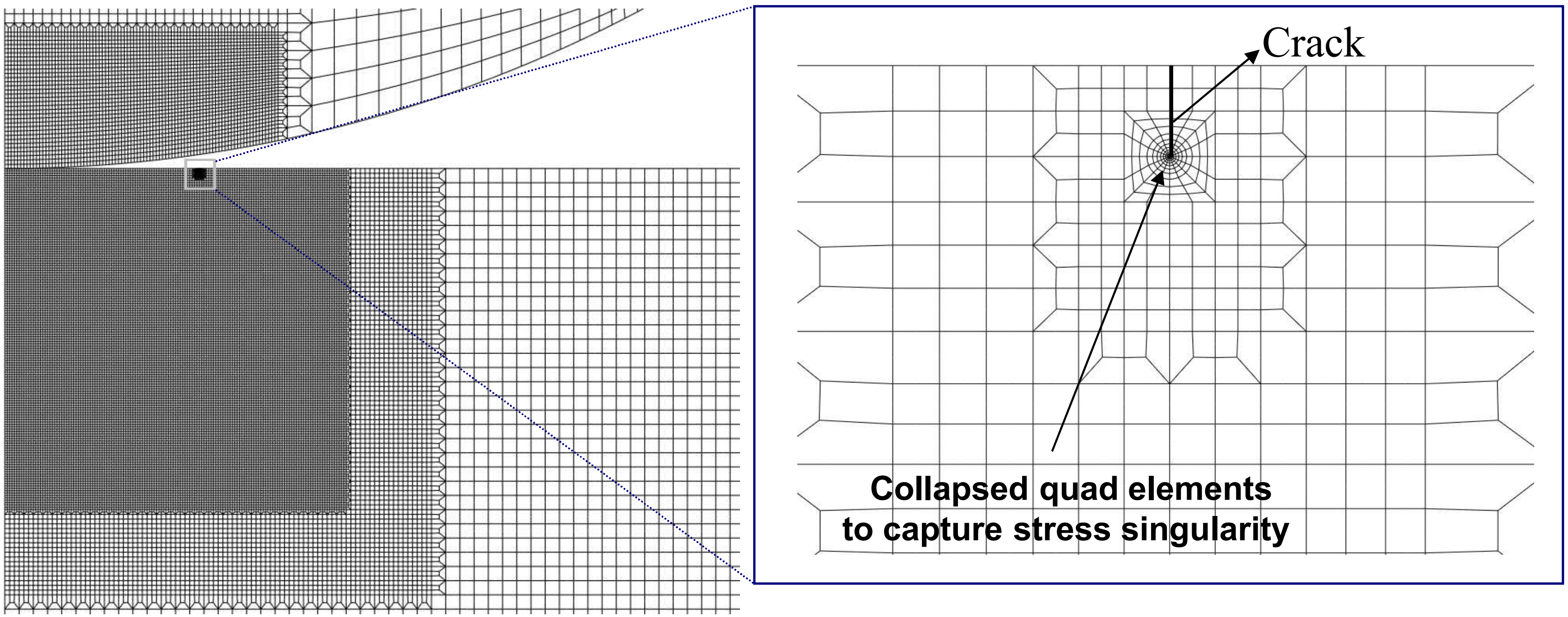
Analytical and FE Approach to K

Knowing the radial stress distribution in the presence of friction, mode I stress intensity factor can be written as

$$\frac{K_I}{p_0\sqrt{\pi a}} = \frac{2}{\pi}\sqrt{\frac{d}{a}}\int_0^{d/a}\frac{\sigma_{rr}(r_c, z)/p_0}{\sqrt{(d/a)^2-(z/a)^2}}\,d(z/a)$$



Stress decays rapidly below the surface.
Hence K verified using two approached

Analytical Approach for Radial Stress

Radial stress in non-friction Hertzian solution is:

$$\sigma_{rr}^{p}(r_a, z_a) = \left[\frac{(1-2\nu)}{3r_a^2}\left\{1-\left(\frac{z_a}{\sqrt{u}}\right)^3\right\} + \frac{z_a}{\sqrt{u}}\left\{(1+\nu)\sqrt{u}\tan^{-1}\frac{1}{\sqrt{u}} + (1-\nu)\frac{u}{1+u} + \frac{z_a^2}{u^2+z_a^2} - 2\right\}\right] p_0$$

It is modified due to friction. Modification is calculated by[6, 7]

(a) Calculating stick-slip boundary in terms of elastic parameters & friction
(b) Calculating the surface shear tractions via the function

$$q(r_a) = \mu p_0 \operatorname{sgn}(\beta)\left[\sqrt{1-r_a^2} - r\int_{r_a}^{c_a} \frac{\chi(x, c_a)}{x^2\sqrt{1-x^2}}\,dx\right]$$

(c) The modifying stress in terms of the potential function, ψ, is

$$\sigma_{rr}^{q} = 2\psi_{rr} + 2\nu\frac{\psi_r}{\bar{r}} + z_a\psi_{rrz}$$

and can be calculated using the boundary values for the problem.

6. Spence DA. The Hertz contact problem with finite friction. J. Elasticity 1975;5:297.
7. Hills DA, Sackfield A. The stress field induced by normal contact between dissimilar spheres. J. Appl. Mech. 1987;54:8.

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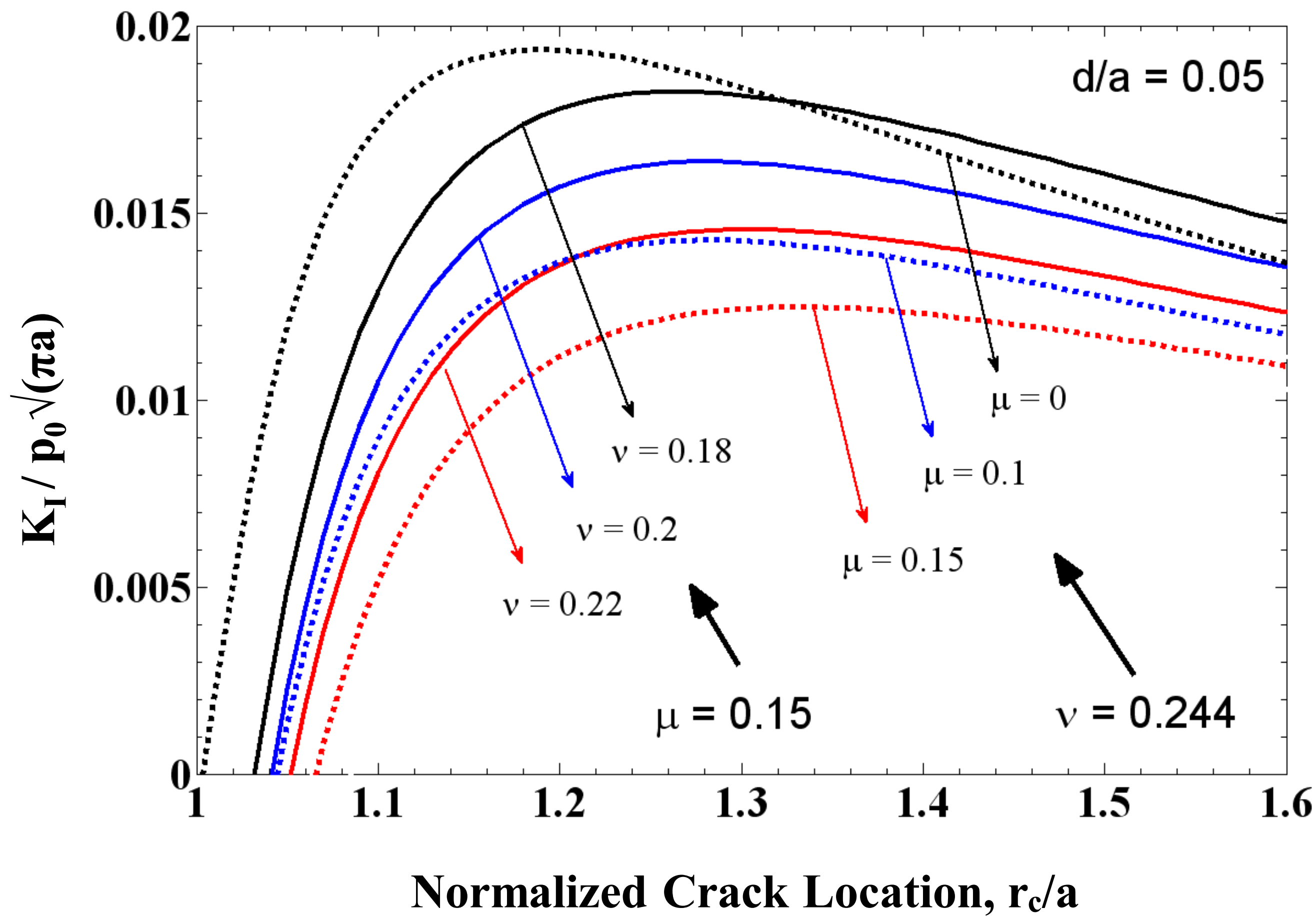
K_1 for a particular crack size, d/a=0.05



Maximum in K is shifted well away from edge of contact
Values of K are very sensitive to ν and μ

Using crack location and additional assumptions

Knowing r_c, and assuming that cracking occurred at that location at the maximum K possible there, we can calculate (better) K_{Ic}