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Metrics for Diagnosing Negative Mass and Stiffness when Uncoupling Experimental and Analytical Substructures

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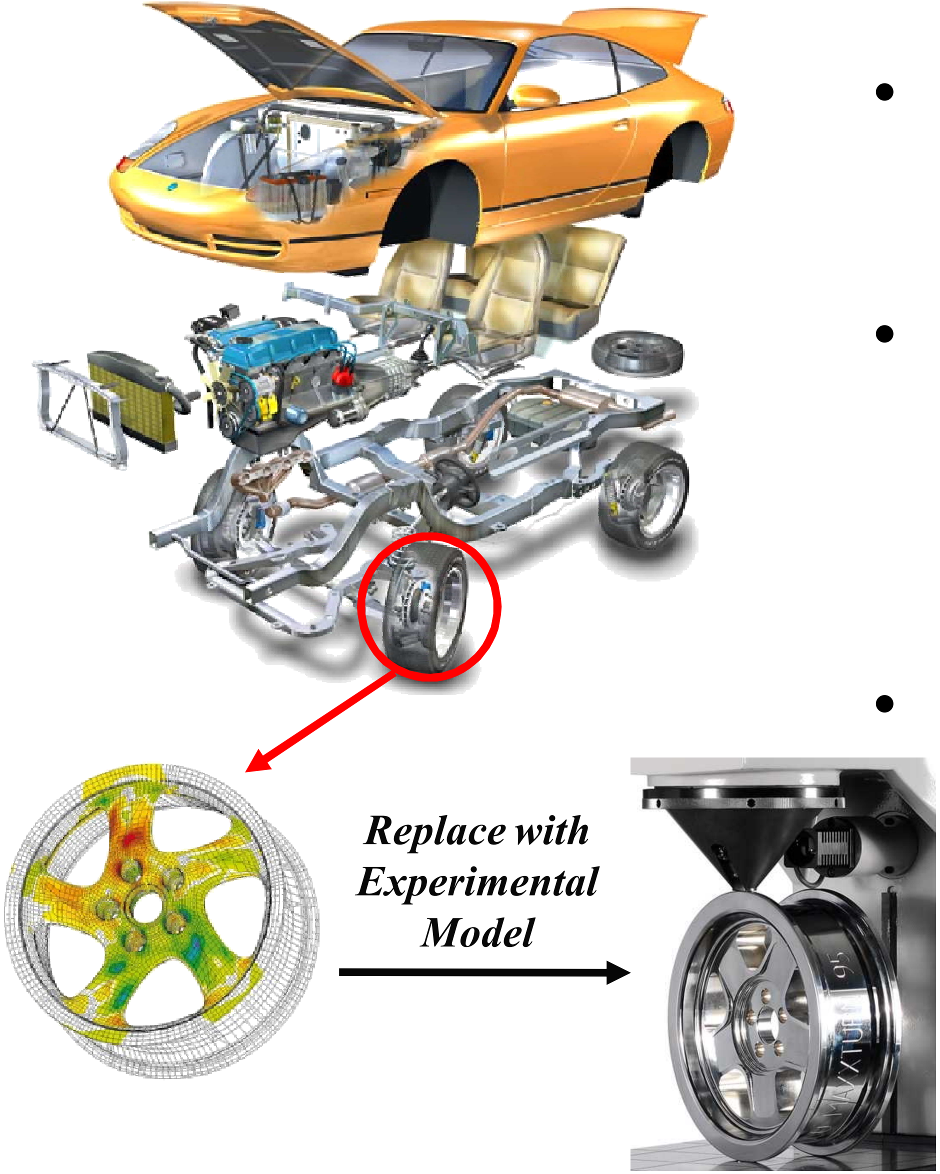
Motivation

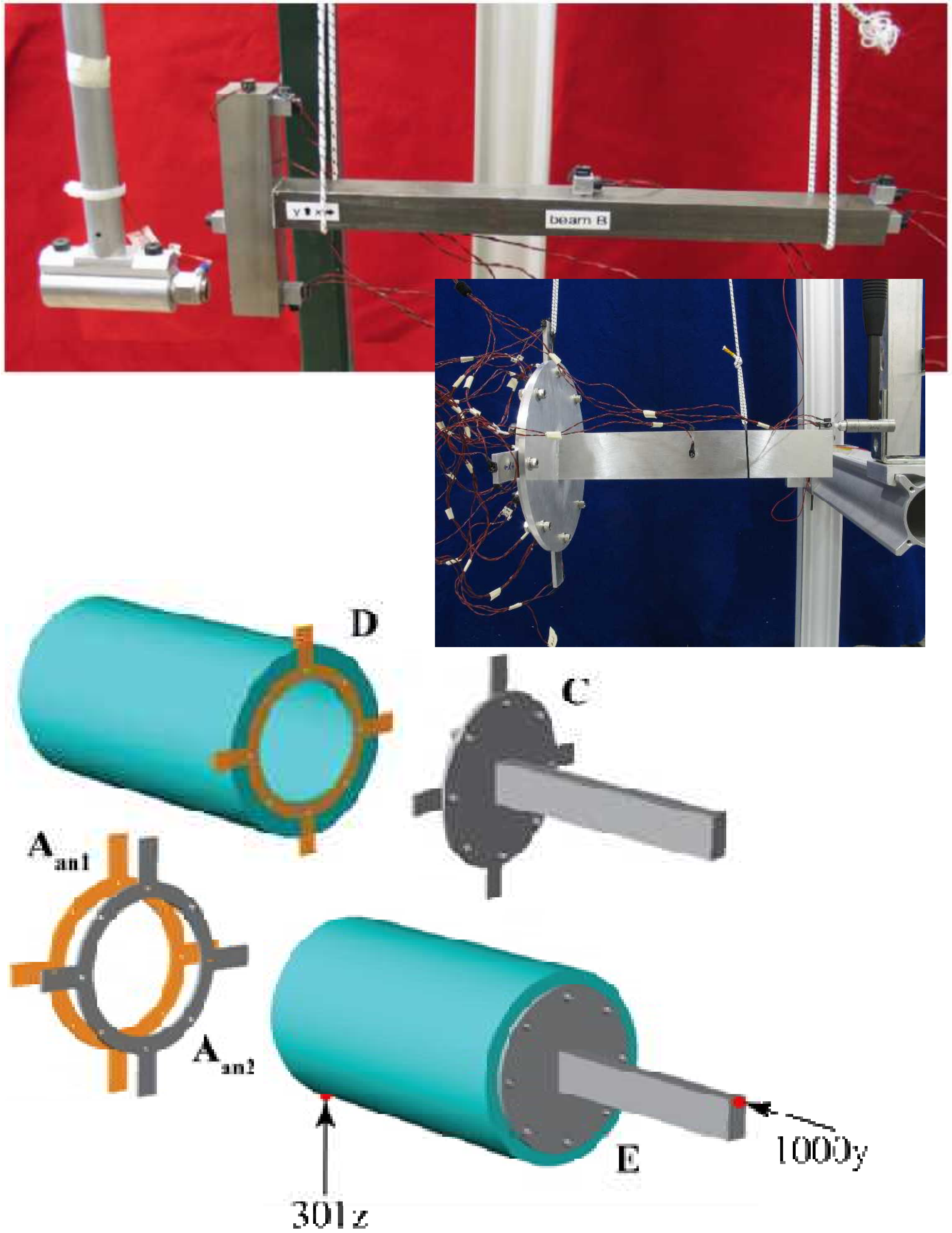


Replace with Experimental Model

- Most systems are made up of several subcomponents.
- FEA method requires detailed information about each subcomponent (geometry, material properties, etc…) that may be difficult or impractical to obtain.
- **Alternative:** Experimental-Analytical Substructuring:
  - Perform dynamic tests on subcomponent of interest to create experimental model.
  - Couple the experimental model to the analytical model for the rest of the system and predict system response.
- **Challenges:**
  - Experimental substructuring can be sensitive to small errors in the measurements.
  - Physically unrealistic results, such as negative mass, may be obtained when one component must be removed from another [Kanda et al].

H. Kanda, M. L. Wei, R. J. Allemang, and D. L. Brown, "Structural Dynamic Modification Using Mass Additive Technique," in IMAC IV, Los Angeles, California, 1986.

Why does negative mass occur?



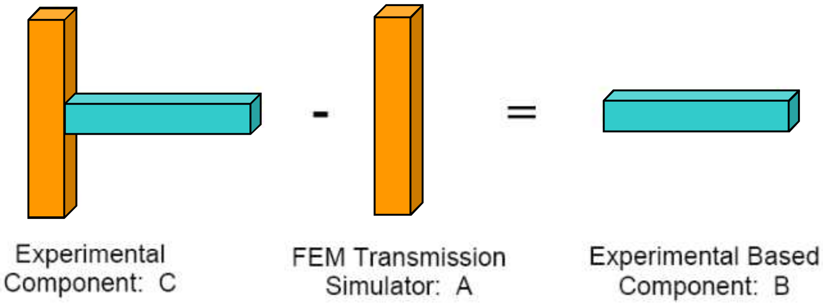
- The free-free modes of a substructure generally form a poor basis for its motion after assembly.
- Several alternatives have been proposed:
  - Fixed interface modes + constraint modes (Craig Bampton)
  - Free modes + Residual Flexibilities
  - Mass-loaded interface modes
- The last approach is very convenient, but it requires one to analytically remove a substructure from the measured model.
  - The resulting model may have negative mass!

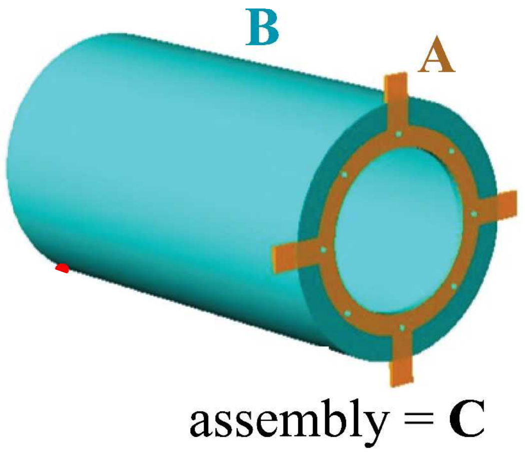
Literature Review

- Nonphysical Measurements in Substructuring
  - Modal Substructuring
    - Kanda *et al* observed negative mass when using substructuring to analytically remove rigid blocks from a structure.
      - Addressed the problem by reducing the scale factors of the rigid body modes that they were removing.
      - H. Kanda, M. L. Wei, R. J. Allemang, and D. L. Brown, "Structural Dynamic Modification Using Mass Additive Technique," in IMAC IV, Los Angeles, California, 1986.
    - Allen, Mayes & Bergman observed similar issues when removing a flexible fixture from an assembly.
      - M. S. Allen, R. L. Mayes, and E. J. Bergman, "Experimental Modal Substructuring to Couple and Uncouple Substructures with Flexible Fixtures and Multi-point Connections," JSV, vol. 329, pp. 4891–4906, 2010.
  - Frequency Based Substructuring (Admittance)
    - Imaginary part of accelerance drive-point FRF must be positive.
    - Carne & Dorhmann created DCD filtering to remove negative imaginary parts from FRFs. This was found to eliminate several spurious peaks from substructuring predictions.
      - T. G. Carne and C. R. Dohrmann, "Improving Experimental Frequency Response Function Matrices for Admittance Modeling," in IMAC XXIV, St. Louis, Missouri, 2006.

Outline

- Review of modal substructuring approach
  - Removing analytical model of a transmission simulator from the subcomponent of interest.
- Theoretical Development:
  - Metrics for the contribution of each mode to negative eigenvalues of the mass and stiffness matrices.
    - Derivation follows that of the Effective Independence (EfI) sensor placement strategy.
  - Consider the modes of each substructure:
    - Transmission Simulator (A) (or rigid mass)
    - Experimental Assembly (C)
- Examples





- Conclusions

Negative Mass after Substructure Uncoupling

- Subtracting transmission simulator, System A, from System C. The concatenated equations of motion are:

$$x_{Am} = \phi_{Am} q_A \qquad \begin{bmatrix} I_C & 0 \\ 0 & -I_A \end{bmatrix} \begin{Bmatrix} \ddot{q}_C \\ \ddot{q}_A \end{Bmatrix} + \begin{bmatrix} \omega_C^2 & 0 \\ 0 & -\omega_A^2 \end{bmatrix} \begin{Bmatrix} q_C \\ q_A \end{Bmatrix} = \begin{Bmatrix} \phi_C{}^{\mathrm{T}} f_C \\ \phi_A{}^{\mathrm{T}} f_A \end{Bmatrix}$$

- Connection point constraints:

$$x_{Cm} - x_{Am} = 0$$

- Modal constraints:

$$x_{Am} = \phi_{Am} \phi_{Am}^{\dagger} x_{Cm} = P_{Am} x_{Cm} = \hat{x}_{Cm}$$

  - where P_{Am} projects the measurements x_{Cm} onto the space spanned by the modes of A.
  - Best synthesis obtained if $R(\phi_{Cm}) \subset R(\phi_{Am})$
  - Constraints enforced and generalized coordinates reduced with:

$$\begin{Bmatrix} q_C \\ q_A \end{Bmatrix} = \begin{bmatrix} I_C \\ \tau \end{bmatrix} q_C = T q_C$$

Negative Mass after Substructure Uncoupling (2)

- The equations of motion become: (with modal constraints)

$$m_r \ddot{q}_C + k_r q_C = 0$$

$$k_r = T^T \begin{bmatrix} \omega_C^2 & 0 \\ 0 & -\omega_A^2 \end{bmatrix} T = \omega_C^2 - \tau^T \omega_A^2 \tau \qquad m_r = T^T \begin{bmatrix} I_C & 0 \\ 0 & -I_A \end{bmatrix} T = I_C - \tau^T \tau$$

- These matrices are an approximation for the mass and stiffness (and damping) of B using the modes of C as a basis, so $m_r \approx m_B$

- Hence, $\tau = \phi_{\mathrm{A}m}{}^{\dagger} \phi_{\mathrm{C}m}$ controls the eigenvalues of m_B.
  - Since $P_{\mathrm{A}m} = \phi_{\mathrm{A}m} \phi_{\mathrm{A}m}{}^{\dagger}$ is an orthogonal projector onto the column space of $\phi_{\mathrm{A}m}$, τ is related to the projection of C's modes onto $\phi_{\mathrm{A}m}$.
  - $\tau^{\mathrm{T}} \tau$ is identical to a SEREP reduction of A onto C's modes
  - Each mode in A and C affects the estimate of B's mass.

- The mass matrix of B will be positive definite if the eigenvalues of $(\tau^{\mathrm{T}} \tau)$ and hence the singular values of τ are all less than 1.0.

Summary of Requirements

1. Number of measurements $\geq$ Number of fixture modes

   $$n_m \geq n_A$$

2. ϕ_{Am} - full column rank

3. $\phi_{Cm} \in range(\phi_{Am})$ experimental modes at measurement locations spanned by FEM fixture modes

Question:

What modes should be included in fixture and experimental structure mode sets ϕ_{Am} and ϕ_{Cm}?

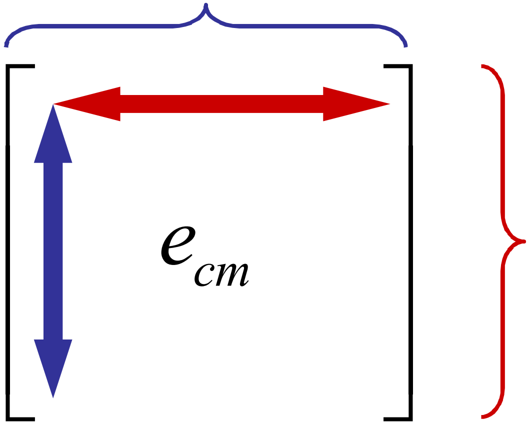
Negative Mass after Substructure Uncoupling (3)

- Case 1: Rank the contributions of each of the experimental modes (C) to the offending eigenvalues of $(\tau^{\mathrm{T}}\tau)$. Following the derivation of EfI:
  - Define $Q_{CM} = \tau\tau^T$
  - Let L_{CM} denote a sorted diagonal matrix of its eigenvalues and Ψ_{CM} the corresponding eigenvectors.
  - Then, form the matrix e_{CM}

One column for each eigenvalue of $(\tau^{\mathrm{T}}\tau)$.

Values in each column sum to each eigenvalue.

$$e_{CM} = \left[\tau^T \Psi_{CM}\right]^{\wedge 2} = \left[\; e_{cm} \;\right]$$

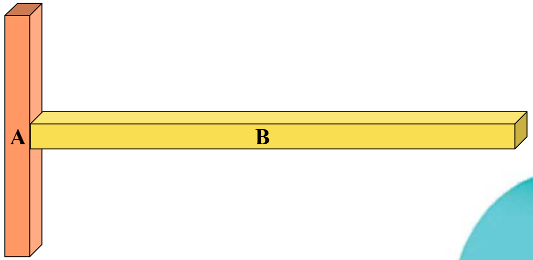


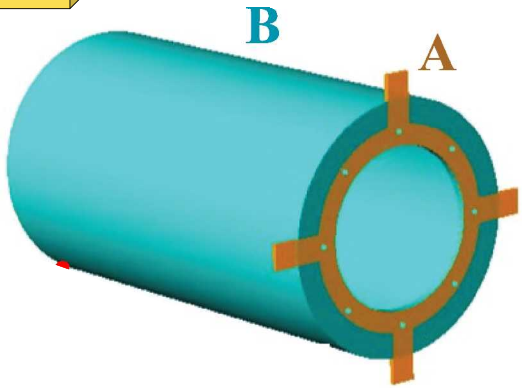
One row for each mode of ϕ_{Cm}

Values give contribution of that mode to each eigenvalue of $(\tau^{\mathrm{T}}\tau)$.

Summary of Metrics Presented

	Stiffness Matrix	Mass Matrix
Modes of A (Transmission Simulator)	e_{AK}	e_{AM}
Modes of C (Experimental Substructure)	e_{CK}	e_{CM}



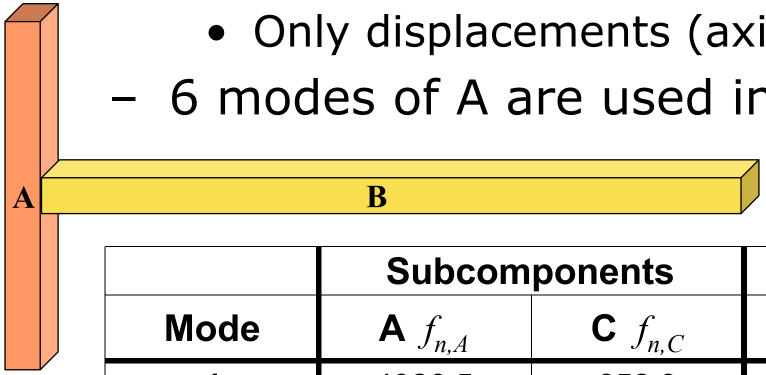


assembly = **C**

- Paper presents metrics for quantifying the effect of modes of both A and C to negative eigenvalues of both the mass and stiffness matrices
- In the authors' experience, negative mass is encountered more frequently than negative stiffness.
- The metrics relative to A and C both give information that can be valuable in understanding nonphysical results.

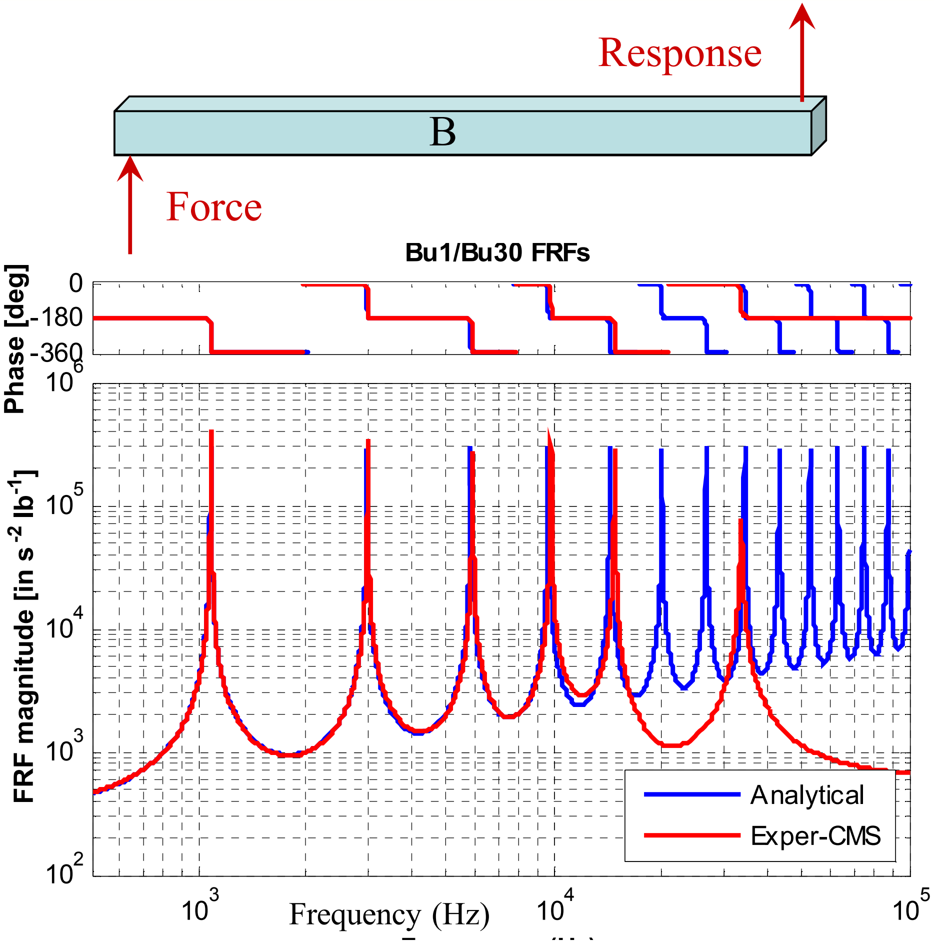
Simulation Example: T-beam system

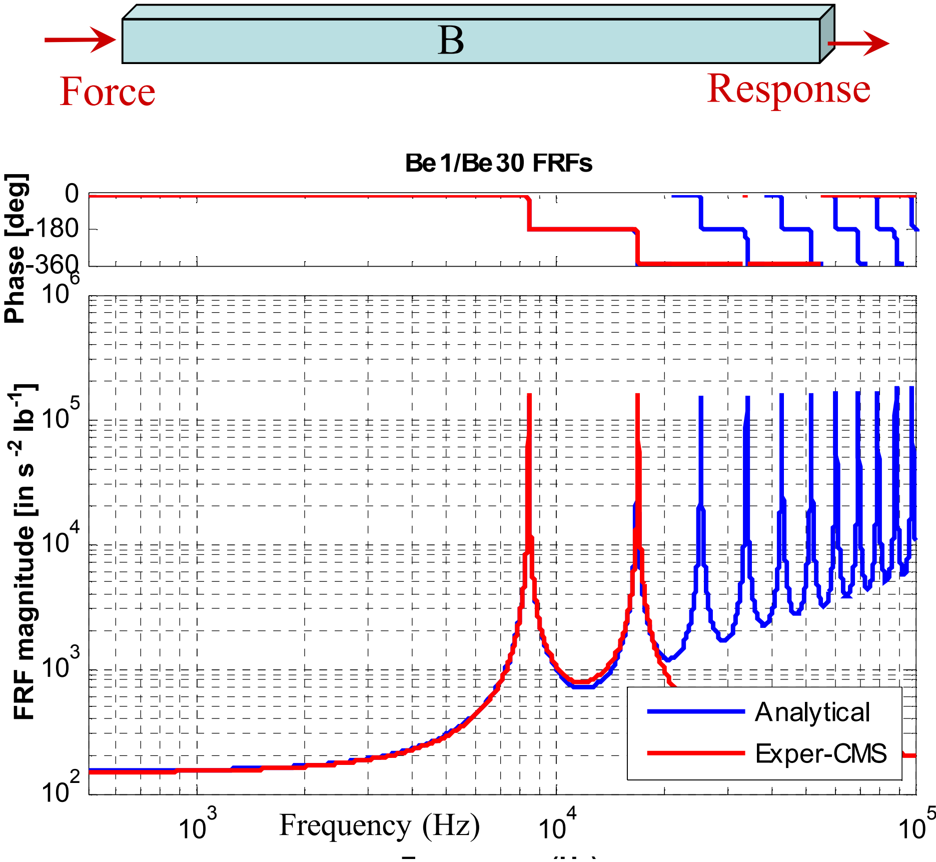
- Simulating case of interest: experimental hardware for C=(A+B), analytical model of A ! estimate a model for B.
  - Simulated using modes computed by FEA (zero noise case).
  - Assume 15 modes of C were measured (out to 20kHz)
    - Only displacements (axial and bending) are measured.
  - 6 modes of A are used in the uncoupling (3 rigid body, 3 elastic)



	Subcomponents		System B		
Mode	**A** $f_{n,A}$	**C** $f_{n,C}$	**MS est.** $f_{n,B}$	**Actual** $f_{n,B}$	**% Error**
4	4326.5	652.3	1083.3	1081.6	0.2%
5	11926.4	1453.6	0+i*1334.2	-	-
6	16853.0	2924.7	2996.4	2981.6	0.5%
7	-	3090.5	5903.3	5845.1	1.0%
8	-	5751.6	8421.5	8422.1	0.0%
9	-	7285.6	0+i*9157.5	-	-
10	-	9251.0	9824.0	9662.5	1.7%
11	-	12615.3	14832.6	14434.8	2.8%
12	-	13919.7	16965.1	16868.9	0.6%
13	-	14950.0	18066.0	20162.6	-10.4%
14	-	16853.0	0+i*20213	25365.3	-
15	-	19507.0	33706.2	26847.1	25.5%

Frequency Response Functions



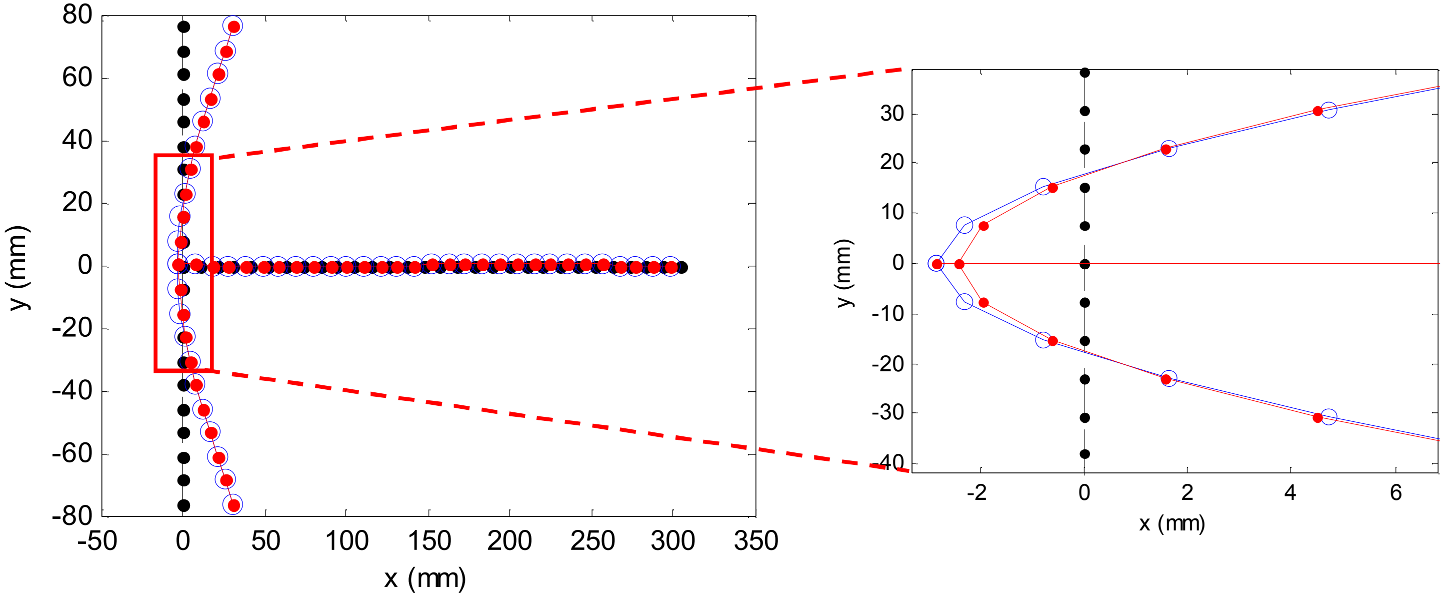


- Spurious modes, caused by negative mass, do not contribute noticeably to the FRFs for this system.
- However, they may be problematic if the model is imported into a finite element package.

Simulation Example: T-beam system (2)

- First five eigenvalues of m_B:
  - L_{BM}=[-0.012, -0.00038, 3.9e-016, 0.0077, 0.074].
  - $m_B = 1 - m_A$.
- Matrix e_{CM} shows how each mode of C contributes to the eigenvalues of m_A.
  - Recall that τ is related to a projection of A's modes onto C.
  - Max difference between C and projection onto A for Mode 6 = 1.4%

Mode of C	**Contribution to Eig. of m_A**		
	λ_A = 1.012	**λ_A = 1.00038**	**λ_A = 1**
14	0	0	1
6	0.77	0	0
5	0	0.38	0
4	0	0.37	0
3	0.08	0.01	0
7	0	0.08	0
9	0.07	0	0
11	0	0.06	0
2	0.02	0.03	0
12	0.04	0	0



Blue: Mode of C

$$\Phi_{Cm}$$

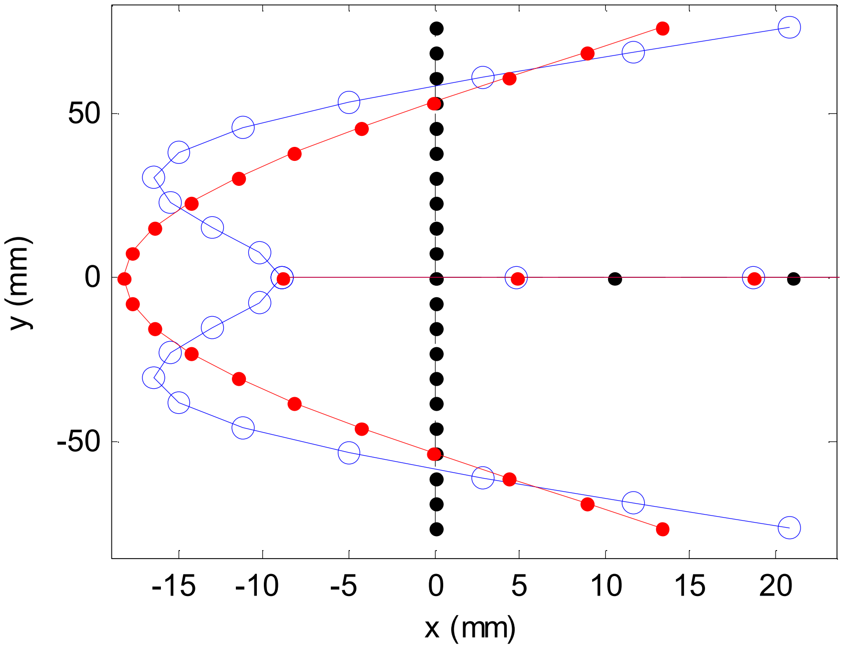
Red: Projection

$$x_{Cm,P} = P_{Am}\Phi_{Cm}$$

Simulation Example: T-beam system (3)

- Modes 9 and 12 also involve axial motion of C
  - The maximum errors in projecting Modes 9 and 12 onto the fixture's motion were 9.4% and 44.6% respectively.
  - Mode 12 shown below.
  - **Remedy:** reduce number of modes of C, or increase number of modes in A.
    - Found to be effective for this system.

Mode of C	Contribution to Eig. of M_A		
	$\lambda_A = 1.012$	$\lambda_A = 1.00038$	$\lambda_A = 1$
14	0	0	1
6	0.77	0	0
5	0	0.38	0
4	0	0.37	0
3	0.08	0.01	0
7	0	0.08	0
9	0.07	0	0
11	0	0.06	0
2	0.02	0.03	0
12	0.04	0	0



Blue: Mode of C

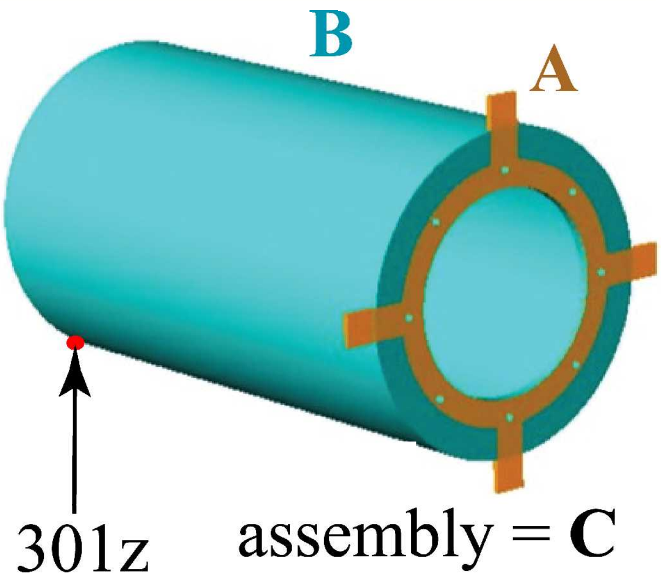
$$\Phi_{Cm}$$

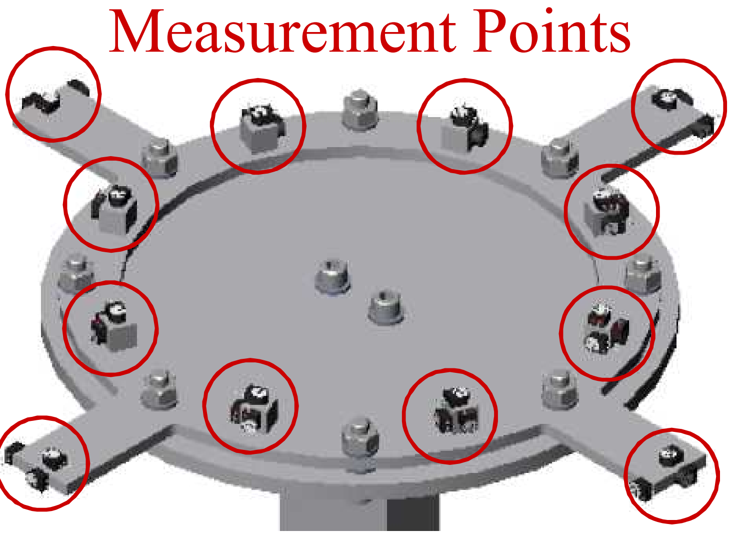
Red: Projection

$$x_{Cm,P} = P_{Am}\Phi_{Cm}$$

e_{CM} metric reveals how these modes have combined to cause the negative eigenvalue of m_B!

Simulation Example: Cylindrical System

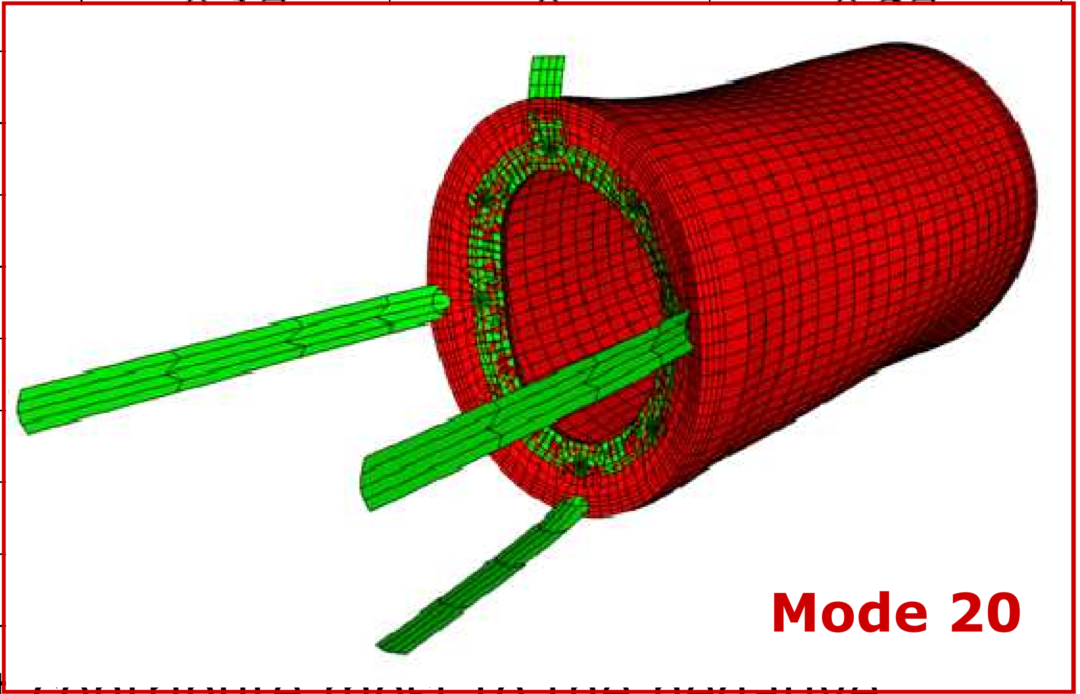




- Goal is to estimate the modes of the cylinder, B, from simulated meas. of C.
- Transmission Simulator instrumented with 12 tri-axial sensors (36 meas. points)
- Baseline case:
  - 100 modes of C combined with 18 modes of A.
  - The smallest eigenvalues of the mass matrix estimated for B are: [-0.197, -0.0764, -0.134, -0.118, 0.398, 0.679]
  - Four are negative indicating a physically impossible result.

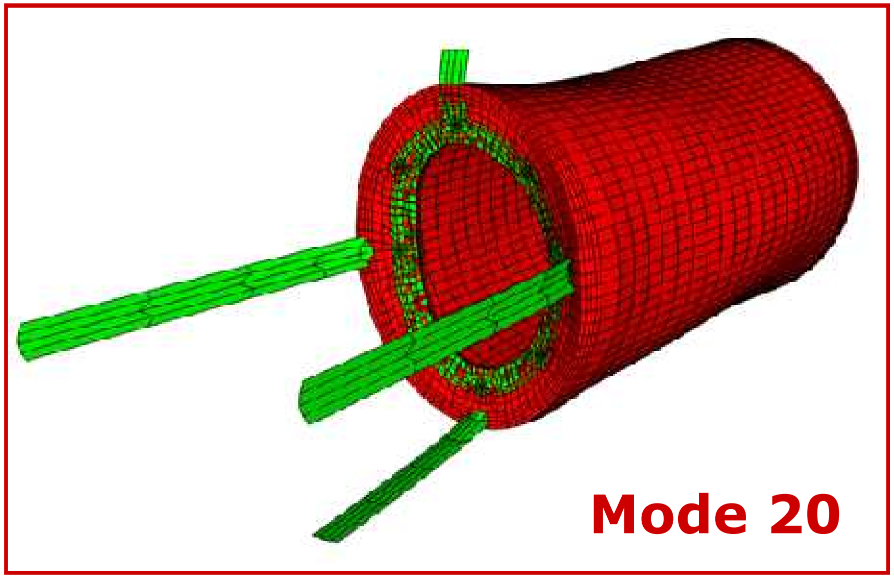
Cylindrical System: Ranking modes of C

Mode of C	Eigenvalues of Q_{CM}				
	Sum	e_{CM} for $\lambda_1 = 2.140$	e_{CM} for $\lambda_2 = 1.845$	e_{CM} for $\lambda_3 = 1.838$	e_{CM} for $\lambda_4 = 1.734$
21	1.60	0	0.01	1.59	0
20	1.48	1.04	0.17	0	0.27
19	1.34	0.21			
22	0.98	0.37			
26	0.95	0.10			
29	0.32	0.32			
25	0.17	0.02			
23	0.11	0			
34	0.08	0			
35	0.08	0			

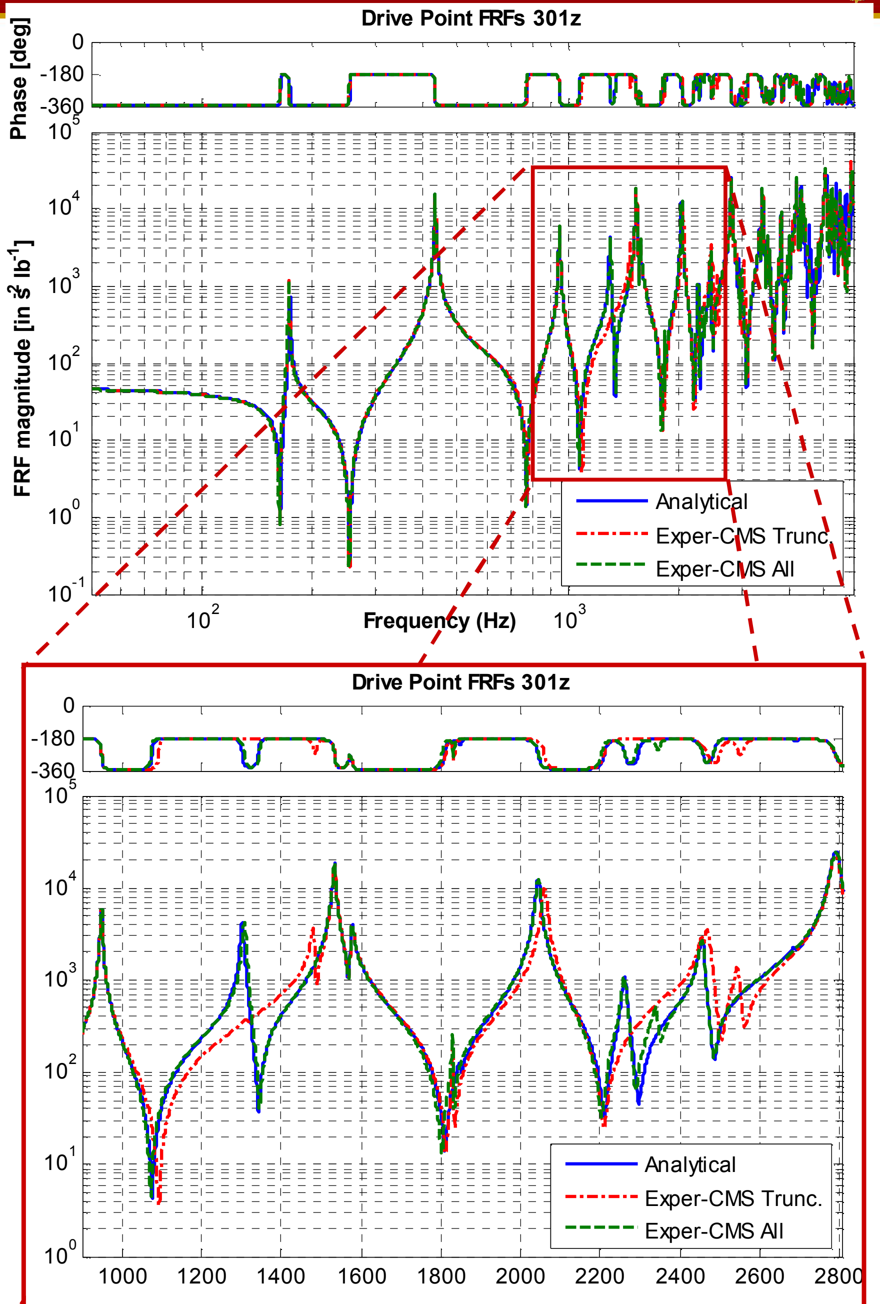


- Four modes identified that contribute most to the negative mass, all of which are dominated by motion of the transmission simulator.

Cylindrical System: Remedies



- Modes 19-22 are primarily responsible for the negative mass, but mode 26 must also be removed to obtain a positive definite mass matrix.
- When these modes are removed, the substructuring predictions degrade somewhat.



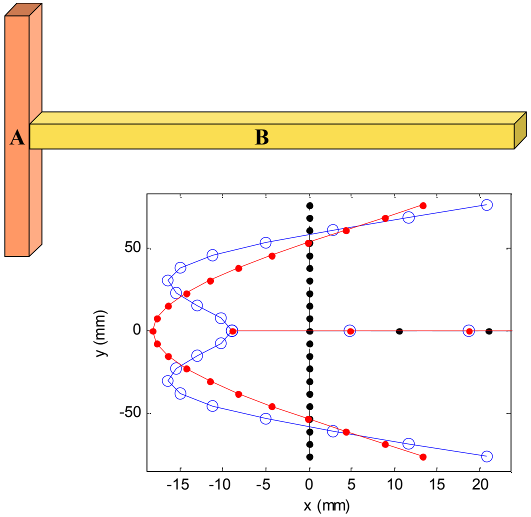
Cylindrical System: Ranking Modes of A

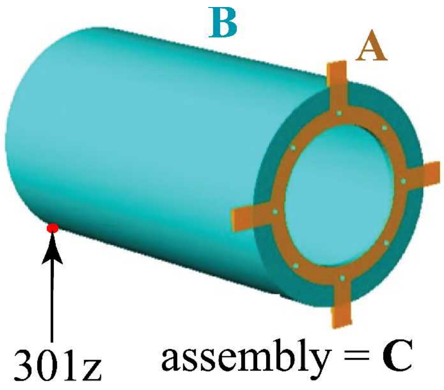
Mode of A	**Eigenvalues of Q_{AM}**				
	Sum	e_{AM} for $\lambda_1=2.140$	e_{AM} for $\lambda_2=1.845$	e_{AM} for $\lambda_3=1.838$	e_{AM} for $\lambda_4=1.734$
9	1.42	1.41	0	0	0
7	1.35	0	0.01	0	1.34
6	0.90	0.15	0.47	0.29	0
1	0.88	0.16	0.48	0.23	0
10	0.80	0	0.48	0.32	0
11	0.79	0	0.30	0.49	0
2	0.79	0.37	0	0.42	0
12	0.37	0	0.01	0	0.35
13	0.11	0	0.05	0.03	0.02
14	0.09	0	0.03	0.06	0

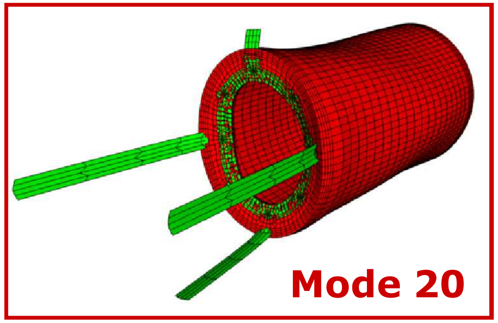
Using these metrics, they key modes causing negative mass can be targeted.

- Reducing the modal mass of these eight modes by 60% eliminates all negative mass from the model for B.
- The resulting model is still highly accurate!

Conclusions







- Method was presented for diagnosing the occurrence of negative mass after substructure uncoupling.
  - EfI metrics found to be helpful in identifying which modes of each substructure were responsible for the negative mass.
- Two causes of negative mass were identified:
  1. Modes are completely removed leaving mass near zero
  2. Transmission simulator model is inadequate to describe the motion in the C system, leading to inaccuracies and too much mass removed.
- Remedies:
  - (2) Increase the number of modes in the TS model or decrease the bandwidth of the test.
  - (1) Eliminate modes from experimental database for C (sometimes problematic)
  - (1) Reduce the mass of certain modes of the TS to avoid subtracting too much mass.

Extra Slides

Remedies – T-Beam Example

- Now that the cause of the negative eigenvalues has been determined, some remedies can be explored:
  - Add an additional mode to model A (transmission simulator)
    - Maximum error in projection of Mode 12 reduces to 6.2%
    - Corresponding negative eigenvalue of m_B dissapeared!
  - Reduce the bandwidth of the experiment (substructure C)
    - Use modes 1-6 for A and 1-11 for C (bandwidth = 14 kHz)
    - Positive definite mass matrix obtained
- Similarly, for the second negative eigenvalue:
  - Metric reveals that 7th and 8th modes are not reproduced accurately enough near the connection point (3.3 and 7.2% maximum errors).

SUBTRACT FIXTURE

Uncoupled System:

$$\begin{bmatrix} I_C & 0 \\ 0 & -I_A \end{bmatrix} \begin{Bmatrix} \ddot{q}_C \\ \ddot{q}_A \end{Bmatrix} + \begin{bmatrix} \omega_C^2 & 0 \\ 0 & -\omega_A^2 \end{bmatrix} \begin{Bmatrix} q_C \\ q_A \end{Bmatrix} = F_q$$

or

$$M_q \ddot{q} + K_q q = F_q$$

n_C - number of experimental modes.

n_A - number of fixture modes.

n_m - number of measured dof.

ALTERNATE FORM OF COUPLING TRANSFORMATION

Apply transformation to couple C and -A

$$\begin{Bmatrix} q_C \\ q_A \end{Bmatrix} = Tq_C = \begin{bmatrix} I_C \\ \left(\phi_{Am}^T \phi_{Am}\right)^{-1} \phi_{Am}^T \phi_{Cm} \end{bmatrix} q_C = \begin{bmatrix} I_C \\ \tau \end{bmatrix} q_C$$

T = null(*Z*):

Experimental representation of substructure B

$$\hat{m}_B = T^T \begin{bmatrix} I_C & 0 \\ 0 & -I_A \end{bmatrix} T \qquad \hat{k}_B = T^T \begin{bmatrix} \omega_C^2 & 0 \\ 0 & -\omega_A^2 \end{bmatrix} T$$

or

$$\boxed{\hat{m}_B = I_C - \tau^T \tau \qquad \hat{k}_B = \omega_C^2 - \tau^T \omega_A^2 \tau = \omega_C \left[I - \tau_K^T \tau_K \right] \omega_C}$$

Requirements:

$\hat{m}_B > 0$ Mass - singular values of τ < 1

$\hat{k}_B \geq 0$ Stiffness - singular values of τ_K < 1

MCFS REQUIREMENTS

1. Number of measurements $\geq$ Number of fixture modes

   $$n_m \geq n_A$$

2. ϕ_{Am} - full column rank

3. $\phi_{Cm} \in range\left(\phi_{Am}\right)$ experimental modes at measurement locations spanned by FEM fixture modes

Question:

What modes should be included in fixture and experimental structure mode sets ϕ_{Am} and ϕ_{Cm}?

FOR GIVEN FIXTURE MODE SET
RANK EXPERIMENTAL MODES

Rank *C* modes wrt Mass

Form matrix: $Q_{CM} = \tau\tau^T \quad n_A \times n_A$

Eigenvalues of Q_{CM} must be ≤ 1.0

$$e_{CM} = \left[\tau^T \Psi_{CM}\right]^{\wedge 2}$$

Rows correspond to *C* modes

Columns add to eigenvalues of Q_{CM}

Rank *C* modes wrt Stiffness

Form matrix: $Q_{CK} = \tau_K \tau_K^T \quad n_A \times n_A \qquad \tau_K = \omega_A \tau \omega_C^{\dagger}$

Eigenvalues of Q_{CK} must be ≤ 1.0

$$e_{CK} = \left[\tau_K^T \Psi_{CK}\right]^{\wedge 2}$$

Rows correspond to *C* modes

Columns add to eigenvalues of Q_{CK}

FOR GIVEN EXPERIMENTAL MODE SET
RANK FIXTURE MODES

Rank Fixture modes wrt Mass

Form matrix: $Q_{AM} = \tau^T \tau = \hat{m}_A$ $n_C \times n_C$

Eigenvalues of Q_{AM} must be ≤ 1.0

$$e_{AM} = \left[\tau \Psi_{AM}\right]^{\wedge 2}$$

Rows correspond to Fixture modes

Columns add to eigenvalues of Q_{AM}

Rank Fixture modes wrt Stiffness

Form matrix: $Q_{AK} = \tau_K^T \tau_K = \hat{k}_A$ $n_C \times n_C$ $\tau_K = \omega_A \tau \omega_C^{\dagger}$

Eigenvalues of Q_{AK} must be ≤ 1.0

$$e_{AK} = \left[\tau_K \Psi_{AK}\right]^{\wedge 2}$$

Rows correspond to C modes

Columns add to eigenvalues of Q_{AK}