Power and Fuel Temperature Based Adaptive Time Step Control for Prompt-Critical Reactivity Transients

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INTRODUCTION

Safety analysis for reactors such as the Annular Core Research Reactor (ACRR) at Sandia National Laboratories requires evaluation of rapid reactivity transients (e.g., pulses reaching 30 GW with a pulse width on the order of 10 ms). Adaptive time step control is a desirable feature in a numerical analysis code, and can help avoid the trial-and-error of properly specifying time steps for different time spans prior to code execution.

For rapid reactivity transients such as a prompt-critical pulse operation, fission energy deposition rates are high, and rapid fuel temperature rises occur before conduction can begin to significantly offset the energy deposition rate. This work considers an adaptive time step approach which is intended to address the accuracy of the fuel temperature as well as the reactor power for prompt-critical reactivity transients. The approach is tested for a large, rapid reactivity change.

DESCRIPTION OF THE ACTUAL WORK

The second derivative term of a Taylor series expansion of a function *f* may be used to estimate the relative truncation error (ε) associated with a numerical solution. This assumes the contributions of higher order derivatives to be negligible.

$$\varepsilon = \frac{1}{2}\frac{1}{f}\left|\frac{d^2 f}{dt^2}\right|(\Delta t)^2 \qquad (1)$$

The relative truncation error may then be controlled via time step size selection. In order to implement this approach, a means of evaluating the second derivative. For power (P), this is accomplished by use of the point reactor kinetics equations for *N* delayed neutron groups.

$$\frac{dP}{dt} = \frac{\rho - \beta}{\Lambda}P + \sum_{i=1}^{N}\lambda_i C_i \qquad (2)$$

$$\frac{dC_i}{dt} = \lambda_i C_i + \frac{\beta_i}{\Lambda}P \qquad (3)$$

The second derivative of power is found by differentiating Eq. 2.

$$\frac{d^2 P}{dt^2} = \frac{\rho - \beta}{\Lambda}\frac{dP}{dt} + \frac{1}{\Lambda}\frac{d\rho}{dt} + \sum_{i=1}^{N}\lambda_i \frac{dC_i}{dt} \qquad (4)$$

Thus, at a given time in the numerical analysis, the second derivative of power can then be evaluated from the current values for power, reactivity (ρ), and the delayed neutron precursors (C_i). The first derivative of the reactivity is estimated from the change in the reactivity over the previous time step.

The second derivative for the fuel temperature is obtained from the energy equation (Eq. 5) for the fuel, neglecting the conduction term.

$$\rho_F c_p \frac{dT}{dt} = \frac{P}{V_F}F_q \qquad (5)$$

The first derivative of the fuel temperature is thus related to the reactor power via the fuel density (ρ_F) and specific heat capacity (c_p), the fuel volume (V_F), and an appropriate peaking factor (F_q) for the location associated with the temperature. By taking the derivative of Eq. 5, a relation between the second derivative of the fuel temperature and the first derivative of the reactor power is obtained.

$$\frac{d^2 T}{dt^2} = \frac{F_q}{\rho c_p V_F}\frac{dP}{dt} \qquad (6)$$

Equation 4 and Equation 6 are then used with Eq. 1 to obtain a time step specification based on the first derivative of the reactor power (Δt_{1st}), and on the second derivative of the reactor power (Δt_{2nd}).

$$\Delta t_{1st} = \sqrt{\frac{2\varepsilon_t \rho c_p V_F T}{F_q\left|\frac{dP}{dt}\right|}} \qquad (7)$$

$$\Delta t_{2nd} = \sqrt{\frac{2\varepsilon_p P}{\left|\frac{d^2 P}{dt^2}\right|}} \qquad (8)$$

Equations 7 and 8 are evaluated prior to each time advancement, and the time step used is the minimum of the two values.

RESULTS

Figure 1 shows fuel temperature history for a large reactivity addition (implemented over ~100 ms beginning at t = 0.2 s) in the ACRR, resulting in a power pulse (~26 GW peak power, 7 ms pulse width). Only Eq. 8 was used for time step control. Note that little further refinement was obtained for $\varepsilon = 10^{-8}$, which was then designated as the "best estimate."

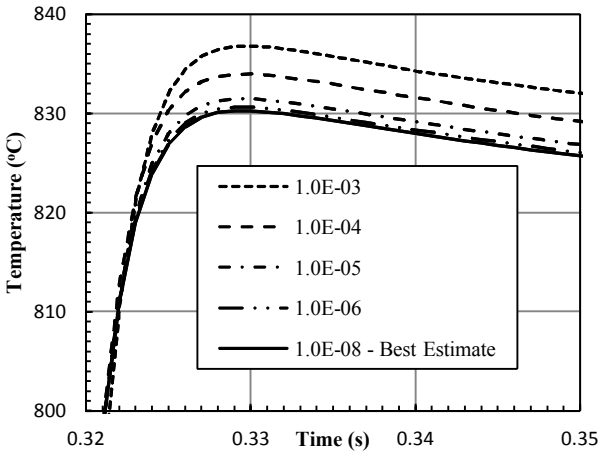

Fig. 1. Fuel temperature using 2nd derivative of power for time step control.

Figure 2 shows fuel temperature history for the same reactivity addition, using the new approach (i.e., the minimum of the time step obtained from Eq. 7 and Eq. 8). Note that the "best estimate" result was essentially obtained using $\varepsilon = 10^{-4}$ (vs. $\varepsilon = 10^{-8}$ for a 2nd derivative only approach). Code execution time decreased (~10 minutes for $\varepsilon = 10^{-4}$, vs. ~45 minutes for $\varepsilon = 10^{-8}$ using the 2nd derivative only). The specified time step does increase once the pulse is over. However, neglecting the conduction term in Eq. 5 does result in a post-pulse time step which is more restrictive than necessary.

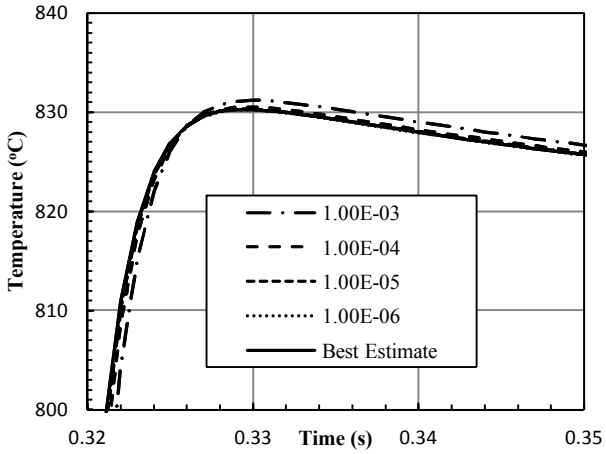

Fig. 2. Fuel temperature using 1st and 2nd derivative of power for time step control.

CONCLUSION

Adaptive time step control which considers both the second derivative of power and the first derivative of power (which is proportional to the second derivative of the fuel temperature) can be used to achieve better accuracy in the fuel temperature for rapid reactivity transients. Future work is appropriate to incorporate conduction in Eq. 5 to allow for less restrictive time steps after the initial power pulse.