SAND2011-0768C

A Statistical Method for Verifying Grid and Mesh Convergence in Direct Monte Carlo Simulations

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Many models of macroscopic phenomena are based on continuum theories represented as partial differential equations. For these models one necessary condition for verification is that numerical approximations of the solution converge with discretization refinement. Without convergence, numerical results are unreliable, rendering subsequent validation, uncertainty quantification efforts, and general use in engineering design unreliable as well. There are instances in engineering practice when a numerical method is used or enhanced to simulate physical behavior that is outside the original well-posed mathematical formulation. For example, in structural mechanics it is well known that ductile fracture is preceded by material softening which causes the governing system of equations to become ill-posed; the effect is a mesh dependency in the associated discretized problem. In addition, "element erosion" or "element death" is often used in Lagrangian finite-element simulations to model arbitrary fracture. Many physical systems that involve material softening and fracture are stochastic in nature, such as pervasive fracture processes, and fragmentation. Monte Carlo simulations of these stochastic systems will have finite-sampling errors in the empirical cumulative distribution function that are on the order of $1/\sqrt{N}$ where N is the sample size. An accurate assessment of mesh convergence must quantify these finite sampling effects, for example, with confidence intervals.

A statistical method is presented for assessing the convergence of a sequence of statistical distributions generated by direct Monte Carlo sampling. The convergence assessment is based on demonstrating empirically that a sequence of cumulative distribution functions converges in the L_∞ norm. The effect of finite sample sizes is quantified using confidence levels from the Kolmogorov-Smirnov statistic. The statistical method is independent of the underlying distributions. The statistical method is demonstrated using a fragmenting ductile ring modeled with an explicit-dynamics finite-element code. One of the main challenges in verifying convergence in distribution using direct Monte Carlo sampling is the large sample sizes required. To partially address this issue an optimization problem is formulated that minimizes total ensemble simulation cost over the space of sample sizes, given constraints on sampling accuracy. The optimization problem is solved using the method of Lagrange multipliers.

References

J.E. Bishop, O.E. Strack, A Statistical Method for Verifying Mesh Convergence in Monte Carlo Simulations with Application to Fragmentation, *International Journal for Numerical Methods in Engineering*, (to appear), 2011

*Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energys National Nuclear Security Administration under Contract DE-AC04-94AL85000.